Intersection point of magnetization curves in layered superconductors

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It was claimed recently that the magnetization of underdoped LaSCO is not consistent with the theory based on the Lawrence-Doniach model. In particular, the intersection point of the magnetization curves "moves" in the opposite directions to that predicted theoretically. We define the intersection point and study it in detail. It is shown that the intersection point always occurs below T_c. The theory is shown to be in agreement with other recent experiments on layered superconductors on HgBCCO and LaSCO.

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I. INTRODUCTION

A striking feature of magnetization curves intersecting at the same point (T^*, H^*) for a wide range of magnetic fields was observed a long time ago both in extremely anisotropic quasi-two-dimensional (quasi-2D) layered materials, such as BSCCO (Ref. 1) and Tl-based high-T_c superconductors,[2] and in more isotropic quasi-three-dimensional (quasi-3D) ones, such as the optimally doped $YBa_2Cu_3O_{7-\delta}$ (Ref. 3) and $YBa_2Cu_4O_8$ (Ref. 4). Recently, the magnetization curves of several classes of layered high-T_c superconductors, including $HgBa_2Ca_2Cu_3O_{8+\delta}$ (Ref. 5), strongly underdoped $YBa_2Cu_3O_{7-\delta}$ (Ref. 6), and LaSCO (Ref. 7), which are neither 2D nor 3D, become available. It was found that the intersection point is no longer the same for all the magnetic fields, but moves a bit from its "3D" position at low fields to its "2D" position at high fields. Normally, for superconductor materials such as YBCO, Hg, and optimally doped LaSCO, the intersection point moves from a high temperature at low fields to a lower temperature as the magnetic field increases. The intersection point is always below T_c. The only exception is the strongly underdoped $La_{2-x}Sr_xCuO_4$ with $x \leqslant 0.08$ (Ref. 7) in which at low fields the intersection point is below T_c. However, as the magnetic field is increased the intersection point moves in the opposite direction, eventually exceeding T_c.

Theoretically, the phenomenon of the intersection points in 2D (Ref. 8) and 3D materials (Ref. 9) was first described in the framework of Ginzburg-Landau theory based on the lateral fluctuations dominance scenario. Later, by using the systematic expansion, it was shown that although the intersection point is only an approximate value, it can move a negligible distance on the phase diagram in both two and three dimensions.[10] Furthermore, by using the Lawrence-Doniach (LD) model in layered materials,[6] the crossing point is still well defined, and can "move" from the 3D to 2D "position" as the magnetic field increases. In Ref. 7 the theoretical formulas of Ref. 6 in the limits of quasi-2D and quasi-3D were used to quantify the data on LaSCO. While it was possible to fit the data in the optimally doped case, it was impossible to fit the data in the far underdoped cases.

In this paper the LD model was applied to describe the magnetization curves in HgBCO and LaSCO without resorting to either 3D or 2D limiting expressions. As in YBCO, one can describe well both in HgBCO and the optimally doped, and slightly underdoped, LaSCO. The field-dependent curve of the "intersection point" is defined mathematically and, within this model, the intersection point that generally cannot move beyond T_c is proved. Moreover, upon the increasing of magnetic field, the intersection points always move to lower temperatures. Therefore, the results of strongly underdoped LaSCO are very puzzling and irreconcilable with general LD theory.

II. THE MAGNETIZATION CURVES AND THEIR INTERSECTION POINTS IN THE LAWRENCE-DONIACH MODEL

Layered superconductor with weak Josephson interlayer coupling can be effectively described by the Lawrence-Doniach free energy

$$\mathcal{G}_{LD} = \sum_n \int d^2\mathbf{r}\left[\xi_{ab}^2\left|\left(\nabla - \frac{2e}{c\hbar}\mathbf{A}\right)\psi_n\right|^2 + \frac{\gamma_t}{2}|\psi_n - \psi_{n+1}|^2 + (t-1)|\psi_n|^2 + \frac{\beta}{2}|\psi_n|^4 + \frac{1}{8\pi}(\nabla\times\mathbf{A})^2\right]. \quad (1)$$

In Eq. (1) $\psi_n(x,y)$ is the order parameter in the nth layer, ξ_{ab} is the in-plane coherence length, $t = T/T_c$, and $\gamma_t = 2(\xi_{ab}/d\gamma)^2$ is a dimensionless parameter describing the interlayer tunneling. Here, d is the interlayer spacing, and $\gamma \equiv (m_c/m_{ab})^{1/2}$ is the anisotropy. Due to the overlap of the magnetic fields of vortices near $H_{c2}(T)$ the magnetic field is constant and oriented perpendicularly to the layers (xy). We use the Landau gauge $\mathbf{A} = (0, Hx, 0)$ and the magnetic-field fluctuations are neglected.[11] The order-parameter fluctuations are consequently described by the partition function

$$Z = \int D\psi D\psi^* \exp(-\mathcal{G}_{LD}[\psi,\psi^*]/k_bT). \quad (2)$$

The standard mean-field approximation [within the lowest Landau level (LLL)] for magnetization results in (see Refs. 6 and 12)

$$M = -\frac{e\xi_{ab}^2}{\pi\beta\hbar cd}\Delta, \quad (3)$$

with dimensionless Δ defined by

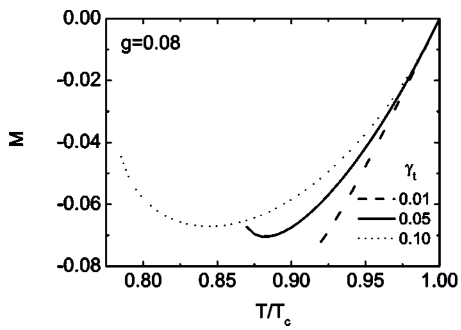

FIG. 1. The evolution of the crossing points for $g=0.08$ and $\gamma_t=0.01$, 0.05, and 0.1.

$$\Delta = g\frac{bt}{\sqrt{(b+t+\Delta+\gamma_t-1)^2-\gamma_t^2}}, \tag{4}$$

where $b \equiv H/H_{c2}$, $H_{c2}= dH_{c2}(T)/dT|_{T_c}T_c$. This magnetic field is typically significantly higher than second critical field at zero temperature $H_{c1}(T=0)$. The dimensionless coupling constant $g=2T_c\beta eH_{c2}/\pi c\hbar$ characterizing the strength of thermal fluctuations is proportional to the 2D Ginzburg number. Although the mean-field result might deviate by up to 10% from the exact result (as we know from better calculations in both the 2D and the 3D limits), it cannot be wrong beyond that level of precision.

Experimentalists often define the crossing point $T^*(H)$ as the temperature at which two "successive" magnetization curves $M(T,H)$ and $M(T,H+\Delta H)$ cross.[7] Therefore, the curves satisfy the equation

$$\left.\frac{\partial M}{\partial H}\right|_{T=T^*} = 0, \tag{5}$$

i.e., $\partial\Delta/\partial b=0$ (or equivalently $\partial\Delta/\partial t=0$). Two equations, Eqs. (4) and (5), can be solved with respect to b,

$$b = -\frac{\Delta^2(t^*+\Delta+\gamma_t-1)}{\Delta^2-g^2t^{*2}}, \tag{6}$$

where $t^*=T^*/T_c$ is the reduced temperature of T^*. Substituting Eq. (6) back into Eq. (4) one obtains the relation between the crossing temperature and magnetization Δ^*. This can be solved for Δ^*,

$$\Delta^* = gt^*\frac{gt^*(1-\gamma_t-t^*)+\gamma_t\sqrt{g^2t^{*2}+\gamma_t^2-(1-t^*-\gamma_t)^2}}{g^2t^{*2}+\gamma_t^2}. \tag{7}$$

The "motion" of the crossing points for $g=0.08$ and $\gamma_t=0.01$, 0.05, and 0.1 is depicted in Fig. 1.

Moreover, one easily sees from Eq. (7) that only for $t^*<1$ physical condition $\Delta>0$ is obeyed. Thus, in this model, the intersection point generally cannot move beyond T_c.

III. APPLICATION TO HgBCO AND LaSCO

Previously the data were analyzed by using either 2D or 3D Ginzburg-Landau models. However, unlike the optimally doped YBCO or BSCCO, the materials belong to a class that is neither 2D nor 3D. This class contains the third most studied superconductor LaSCO among others. The LaSCO system was the first to be discovered and is very extensively studied recently (see, for example, the most recent experiments on vortex matter in LaSCO as seen using muon spin rotation and neutron scattering,[13]). The importance of LaSCO was a motivation to apply the theory to a generally much less studied layered superconductor HgBCCO for which excellent magnetization measurements exist. In this section the Lawrence-Doniach model was applied to describe the magnetization curves in HgBCO and LaSCO samples.

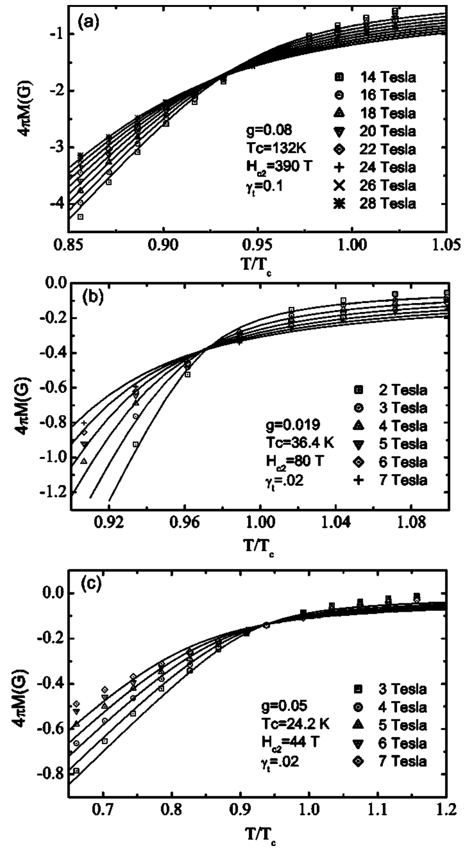

FIG. 2. Magnetic moment vs temperature in the high-field region. The solid lines are fits to Eq. (3) with the parameter discussed in the text. (a) $HgBa_2Ca_2Cu_3O_{8+\delta}$, (b) the optimally doped $La_{1.857}Sr_{0.143}CuO_4$, and (c) the strongly underdoped $La_{1.92}Sr_{0.08}CuO_4$.

A. HgBCCO

The layered superconductor $HgBa_2Ca_2Cu_3O_{8+\delta}$ clearly belongs to a wide class of high-T_c materials which lies between the extremely anisotropic, essentially 2D, superconductors, such as BSCCO and Tl, and the weakly anisotropic superconductors, such as optimally doped YBCO, in which fluctuations can be treated by using the 3D Ginzburg-Landau model. Fitting the magnetization of Ref. 5 by using the Lawrence–Doniach formulas is given above in Fig. 2(a). The transition temperature T_c derived directly from the magnetization data is 132 K. The values of $H_{c2}=390$ T, the dimensionless coupling constant $g=0.08$, and the interlayer-coupling parameter $\gamma_t=0.1$ give the best fit to experimental data. According to the result of the underdoped YBCO (Ref. 6) at temperatures exceeding the mean-field transition temperature, the LLL approximation overestimated magnetization. This can be realized by including higher Landau levels. The evolution of the crossing point is also shown in Fig. 2(a).

B. LaSCO

Data for a sample close to the optimally doped $La_{1.857}Sr_{0.143}CuO_4$ of Ref. 7 are presented in Fig. 2(b). For magnetic fields from 2 to 7 T, the crossing points move in-the direction consistent with theory and are always below T_c. The transition temperature is $T_c=36.4$ K.[7] The best fitting parameters are $g=0.019$, $H_{c2}=80$ T, and $\gamma_t=0.02$. In the strongly underdoped samples, such as $La_{1.92}Sr_{0.08}CuO_4$ and $La_{1.93}Sr_{0.07}CuO_4$, the experimental data was unusual. Two different, well-defined, crossing points were observed. As magnetic field increased from 0.3 to 7 T the crossing point unexpectedly "jumped" from a temperature below T_c to another one well above T_c. The phenomenon obviously conflicts with our previous qualitative conclusion based on the Lawrence-Doniach model. Even though, the high magnetic-field data can be fitted if a slightly higher transition temperature $T_c=24.2$ K is assumed. Figure 2(c) shows the fit for $La_{1.92}Sr_{0.08}CuO_4$ with $g=0.05$, $H_{c2}=44$ T, and $\gamma_t=0.02$.

IV. SUMMARY

The mean-field lowest Landau-level theory of thermal fluctuation is able to describe magnetization curves near T_c, including the intersection point, in both the quasi-2D superconductors, such as BSCCO, and the 3D superconductors, such as YBCO. It is therefore expected that the natural generalization of the model to include the coupling between layers, the Lawrence-Doniach model, should describe sufficiently well the thermal fluctuations in a wide range of layered materials, which exhibit neither the 2D nor the 3D behavior. A characteristic general feature is that the magnetization curves intersect always below T_c. While we show that the theory is consistent with the recent, very detailed, studies on HgBCCO and earlier studies on LaSCO, the results on the strongly underdoped LaSCO, which show the intersection point above T_c, are incompatible with the theory. Despite the fact that the theory has a number of assumptions like effects of disorder and contributions of higher Landau levels, the discrepancy is real. Since the description of the layered superconductors by the Lawrence-Doniach model is a very important part of the physics of the high-T_c superconductors, this question should be addressed experimentally.

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