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考量公司違約下的經理人股票選擇權的評價及激勵效果

The Valuation and Incentive Effects of Executive Stock Options under the Consideration of Firm Default

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摘要：經理人股票選擇權契約的評價不僅考量公司股票價格是否超越履約價格，也涉及公司價值是否足夠支付應許的債務及薪資支付。考慮公司潛在的違約風險，本文發展一個經理人股票選擇權的定價模型，著重於分析契約評價、經理人激勵效果、公司違約效果及兩者的策略行為。我們的結果指出經理人有動機去努力工作及承擔風險因而提升契約價值，但這些動機水準與經理人股票選擇權契約的設計及公司的策略行為息息相關。另外，若公司應付債務及薪資支付是較大的，則公司較可能違約。

關鍵詞：激勵效果、違約效果、經理人股票選擇權、承擔風險

Abstract: The valuation of executive stock options (ESOs) depends not only on whether the stock price exceeds a strike price, but also depends on whether the firm value is sufficient to cover debt

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repayment and salary compensation owed to employees. We propose an option pricing model for ESO contracts with a risk generated from the option writer's potential for defaulting. This study focuses on the analysis of valuations, executive incentives, firm defaults, and the two parties' respective strategic behaviors. We support that executives have incentives to enhance contract value by increasing their effort or risk-taking, but these incentives are correlated to the design of the contract and a firm's strategies. If the total of debt repayments and salary compensation is greater, the firm has a greater likelihood of defaulting on the ESO contract.

Keywords: Incentive Effect, Default Effect, Executive Stock Options, Risk-taking

1. Introduction

Executive stock options (ESOs) are issued by a board of directors for the purpose of motivating the executives to strive harder for the ultimate benefits of the corporate shareholders. To increase the opportunity for profits under an ESO contract, executives have incentives to create firm value and advance shareholder interests. In many countries, ESOs are widely regarded as an effective tool in terms of incentive compensation plans (See, Alon and Sisli-Ciamarra, 2013, Bryan *et al*., 2002; Burns *et al*., 2015). Under some conditions, however, the board of directors does not always make sufficient payments to the executives, even though the ESOs are exercisable upon maturity. Specifically, if the firm's value of assets does not cover its promised liabilities, the firm may default on the payments of ESO contracts due to considerations of payment priority. In response to the option writer's default possibility, the executives may wish to adjust their efforts and management decisions. On the other hand, the board of directors may adjust corporate policies and design contracts specifically with the aim of motivating the executives. Thus, ESO valuations involve the default risk of the option writers and the incentive effects on the part of the executives.

Given some circumstances, ESO holders may suffer losses as a result of the default risk they bear. First, before making the option payoffs to the executives and paying the dividends to the shareholders, the firm's value must be distributed to outside debt holders in order to satisfy promised liabilities, and to inside employees for paying out salary compensation. Thus, whether or not an ESO contract is defaulted upon depends on the relative levels of firm's value, salary policy, and promised debt. During an economic depression, some firms have been known to default on their ESO obligations. A second reason why ESOs may be defaulted upon involves a trading mechanism.

An ESO contract issued by the board of directors of a firm is a derivative of OTC markets, and is not traded in an exchange market, in which any OTC contract may have a possibility of default. As a result, the pricing of ESOs must take into account the firm's default probability.

This study analyzes ESOs under the consideration of risk generated from the option writer's potential for default. If a firm's value is not sufficient to pay outside liabilities and cover salary compensation, the holders of ESOs receive only a portion of the full payoffs they are owed, even if the ESO contract is in-the-money. Under other conditions, the holders of ESOs can receive payoffs in full, so long as the firm's value is high enough. Consequently, the valuations of ESO contracts depend on how likely the firm is to default on making the promised payoffs.

A lot of the financial literature has discussed default effects on option contracts, specifically focusing on vulnerable options, in terms of their default mechanisms (see, Basak and Shapiro, 2005; Fard 2015; Hull and White, 1995; Jarrow and Turnbull, 1995; Johnson and Stulz, 1987; Klein, 1996; Klein and Inglis, 2001; Klein and Yang, 2010; Lu *et al*., 2013; Merton, 1974; Moraux, 2004; Uhrig-Homburg, 2002). Klein (1996) considers option writers' outside liabilities to be a default threshold under occurrences of financial distress. Considering a default threshold which includes the potential liability represented by the options and the other debts of option writers, Klein and Inglis (2001) value European options subject to the risk of financial distress generated by the option writers. Thus, the holders of option contracts bear default risk, even though the contracts stay in-the-money upon maturity.

Another major issue in the area of ESOs concerns the incentive effects on the executives (see, Armstrong and Vashishtha, 2012; Baixauli-Soler *et al*., 2015; Burns and Kedia, 2008; Brockman *et al*., 2010; Chen *et al*., 2006; Cronqvist and Fahlenbrach, 2013; Dong *et al*., 2010; Johnson and Tian, 2000; Tang, 2012). Comparing the incentive effects of numerous ESO plans, Johnson and Tian (2000) suggest that changing specifications of option parameters brings about significant changes in executive incentives. Chen *et al*. (2006) stress the importance of the relation between option-based executive compensation and risk-taking incentives, finding that stock with an option-based value can induce risk-taking. Tang (2012) constructs a multi-period framework to analyze the incentive effects of ESOs, suggesting that restricted stock generates greater incentives to change stock prices.

Our article is related to the default risk issues related to executive stock options, but our article differs from the previous studies in three ways. First, the executives have incentives to alter their risk-taking and expenditure of effort as they suffer from the default possibility. Second, the boards of directors also have the ability to control their policies so as to encourage the executives to implement appropriate management strategies for the advancement of stockholders' interests. In particular, the boards of directors may adjust outside liabilities and salary payments, with the end result of changing the default probabilities. Third, our study also focuses on the valuation of ESO contracts under a consideration of the writer's default risk. While previous studies have analyzed ESO valuation (Carpenter, 1998; Colwell *et al*., 2015; Johnson and Tian, 2000; Tang, 2012); they have not considered the impact of default risk. In short, our study contributes to the growing literature on ESO valuations by considering both default risks and strategic inter-dependences.

We derive a closed-form solution of the ESO contracts under default risks originating from a firm's value being insufficient to pay outside liabilities and to make salary payments. Next, this study analyzes executive's incentives to increase stock price and return volatility, or to raise the firm's dividend payout policy threshold, in the context of the ESO contracts. We also discuss how the salary policy of boards of directors and contract designs reflect the executive incentives. In addition, the study derives a default probability of ESOs and predicts the boards of directors' default behaviors in terms of salary policy. Finally, we discuss how the executives implement their strategies in terms of dividend policy, risk management policy, and their overall efforts in response to the firm's default behavior.

The rest of the paper is organized as follows. The following section develops an option pricing model for deriving option premiums, incentives, and default probabilities. The third section implements numerical examples to characterize the ESO premiums, incentives, and default probabilities in terms of specified factors. The final section concludes with a brief discussion.

2. Valuation Model, Incentives, and Default Probability

Using a martingale approach, we derive a closed-form solution for ESOs with default risk. Executive incentives and default probability in the context of the ESOs are also analyzed.

2.1 ESOs with Default Risk

To solve a closed-form result for ESOs, this article sets out numerous underlying assumptions and constructs a preliminary model of option pricing. Our underlying assumptions are consistent with the work of Black and Scholes (1973), Klein (1996), and Klein and Inglis (2001). A representative firm has movements of the firm's value (V) and stock price (S) following a geometric *Brownian* motion (GBM) process framed in a complete probability space (Ω,Ψ,P) with filtration $\{\Psi_t\}$, as follows:

$$dV = rVdt + \theta VdZ^{Q}(t) \tag{1}$$

and

$$dS = (r-q)Sdt + \sigma SdW^{Q}(t) \tag{2}$$

where Z and W denote *Wiener processes* in a risk-neutral probability Q-measure, in which they correlate with coefficient ρ. r represents a risk-free rate of interest. θ and σ are the instantaneous volatility rates of the firm value and stock price, respectively. q represents a continuous dividend rate that the firm pays to the shareholders.

For simplicity's sake and without the loss of generality, the ESO is in the form of a plain vanilla European call option, and is free of attached restrictions on it[1]. For rewarding the executives, the board of directors issues the option contracts on the firm's stock price (S) with a strike price (K), whereby the executives have an incentive to enhance the ESOs' value by redoubling their efforts in terms of firm management. On maturity, the ESO holder receives payoffs of S-K, depending on whether the stock price is over the strike price. However, whether or not the executives obtain the payoffs also involves whether the firm's value exceeds a default threshold. Specifically, the executives cannot receive full payoffs upon maturity (T) if the firm value is not sufficient to make payments to outside bondholders and the firm's employees. Before paying out the option payoffs, the firm should pay off outside debts to the bondholders and the salaries it owes to its employees. For this reason, the valuations of ESOs involve a condition of whether or not the option writer defaults. In total, the ESO value at time t is expressed as follows:

$$C_t = e^{-r(T-t)} E \left\{ \begin{array}{ll} S_T - K, & if \quad S_T \geq K, V_T \geq B + mV_T \\ (S_T - K)\left[\dfrac{(1-\alpha)V_T}{B + mV_T}\right], & if \quad S_T \geq K, V_T < B + mV_T \\ 0, & otherwise \end{array} \right\} \tag{3}$$

where B represents a fixed default threshold, which can be regarded as the face value of the firm's debt (see, Klein 1996). α is a rate of dead-weight costs generated from the occurrence of financial defaults. mV denotes a salary payment that is expressed as a fixed portion m of the firm value, because a firm with a greater scale of firm value generally has more employees to whom it must pay

1 We analyze European-type ESOs rather than American ESOs, even though many firms issue American-type ESOs in practice. Following a lot of previous studies (Cheung and Corrado 2009, Korn *et al*. 2012, Johnson and Tian 2000), European ESOs are analyzed because we can derive analytical solutions. We appreciate the reviewer's suggestions about differentiating between the analysis of American and European options.

salaries. Thus, a higher value of *m* means that a firm is liable to pay more compensation to its employees. Under these settings, this study explores how the dividend policy (*q*), debt policy (*B*), and salary policy (*m*) affect the ESO valuations, incentive effects, and default effects.

ESO value varies depending on which of two situations it falls under, as shown in equation (3). The first line of this equation states a standard form of the payoffs for the call options on the stock prices, in the case where the firm's board of directors definitely makes the full payoffs because the firm value covers the total of the promised debt payments and the salaries of its employees. In the other situation, where the firm's value is lower to make the promised debt payouts and pay the salaries of its employees, the executive receives only a portion of the full payoffs, as indicated in the second line of the equation. As the option writer defaults, the ESO holder receives a percentage of the available amount of the net firm value (i.e., $(1\text{-}\alpha)V$) with respect to the full terminal payoffs. In particular, the dividend payouts and option payoffs are not taken into account in the total promised necessary payments (i.e., B+mV) before the firm pays debts and makes salary payments, so they do not appear in the denominator portion of the second line of the equation.

The ESO price (C_t) at time t can be derived by discounting the expected closing price (C_T) by a martingale approach. The closed-form solution for the value of the ESOs is given as follows:

$$\begin{aligned} C_t &= S_t e^{-q(T-t)} N(b_1, b_2; \rho) - K e^{-r(T-t)} N(a_1, a_2; \rho) \\ &\quad + \frac{(1-\alpha)V_t}{B + mV_t} \left(S_t e^{(r-q-y+\rho\sigma\theta)(T-t)} N(c_1, c_2; -\rho) - K e^{-y(T-t)} N(d_1, d_2; -\rho) \right) \end{aligned} \tag{4}$$

where, $a_1 = \dfrac{\ln(S_t / K) + (r - q - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$, $a_2 = \dfrac{\ln((1-m)V_t / B) + (r - \theta^2/2)(T-t)}{\theta\sqrt{T-t}}$,

$b_1 = \dfrac{\ln(S_t / K) + (r - q + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$, $b_2 = \dfrac{\ln((1-m)V_t / B) + (r - \theta^2/2 + \theta\rho\sigma)(T-t)}{\theta\sqrt{T-t}}$,

$c_1 = \dfrac{\ln(S_t / K) + (r - q + \sigma^2/2 + \rho\sigma\theta)(T-t)}{\sigma\sqrt{T-t}}$, $c_2 = -\dfrac{\ln((1-m)V_t / B) + (r + \theta^2/2 + \rho\sigma\theta)(T-t)}{\theta\sqrt{T-t}}$,

$d_1 = \dfrac{\ln(S_t / K) + (r - q - \sigma^2/2 + \rho\sigma\theta)(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = -\dfrac{\ln((1-m)V_t / B) + (r + \theta^2/2)(T-t)}{\theta\sqrt{T-t}}$,

$y = r - \dfrac{B}{B + mV}\left(r - \dfrac{m\theta^2 V}{B + mV}\right)$, $\eta = \dfrac{\theta B}{B + mV}$,

and where $N(x_1, x_2; \rho)$ denotes the density function of a standard bivariate normal distribution. The ESO solution is similar to the vulnerable option price developed by Klein (1996), but our model considers salary policy as it applies to the area of executive stock options with default risks, and focuses on the incentive effects. The Appendix contains the derivation of this formula.

The closed-form solution for ESOs with default risks becomes comparatively more complicated than a closed-form solution for ESOs without default risks. First, the payoff structure of the option can be separated into two standard types, represented, respectively, by holding a plain vanilla call option on the stock price with a strike price of K, and by longing another call option given certain conditions, but the executives receive only a portion of the full payoffs in the case of the second option. Second, there is one point that must be mentioned with respect to the normal distributions: a univariate normal distribution is used in the plain vanilla option contract, but a bivariate normal distribution is employed for the ESO contract since the option value depends on two processes: stock prices and firm value. Finally, the default risk, which originates from the condition where the firm value is less than the total of the promised outside debts and the salaries owed to its employees, directly impacts the values of ESOs through the interactions between numerous factors, such as the return volatility (θ) of firm value, salary policy (m), debt boundary (B), and so on.

2.2 Incentive Effects for Executives

After the issuing of an ESO contract, the executives have a motive to act so as to affect the underlying stock prices, return volatility, and dividend policy[2] in order to increase the value of the ESO contract. That is, an increase in the stock price, an increase in the return volatility, or a decrease in the dividend payout, may increase the potential payoffs upon maturity, so the executives have an incentive to expend greater efforts on their business management and decision planning. This study therefore explores the executive incentives in terms of the strategic behavior of the two parties.

To analyze the incentives of the executives, the partial derivatives of the contract premium to the underlying stock price (*Delta*), the return volatility (*Vega*) of the stock price, and the dividend rate (*Psi*) are used to measure the level of executive incentive to change stock price, risk-taking and dividend payout. High values for the *Greeks* indicate that the executives have stronger incentives to affect terminal payoffs by redoubling their efforts when it comes to risk management or business operations. The formulas of these *Greeks* for the ESOs are listed as follows:

---

[2] Dividend policy is determined by the executives, in this study, as it is the work of Johnson and Tian (2000).

$$
\begin{aligned}
Delta \equiv \frac{\partial C_t}{\partial S_t} &= e^{-q(T-t)}N(b_1,b_2;\rho) + S_t e^{-q(T-t)}n(b_1)N(\frac{b_2-b_1\rho}{\sqrt{1-\rho^2}})\frac{1}{\sigma S_t\sqrt{T-t}} \\
&- Ke^{-r(T-t)}n(a_1)N(\frac{a_2-a_1\rho}{\sqrt{1-\rho^2}})\frac{1}{\sigma S_t\sqrt{T-t}} \\
&+ \frac{(1-\alpha)V_t}{B+mV_t}e^{(r-q-y+\rho\sigma\theta)(T-t)}[N(c_1,c_2;-\rho)+S_t n(c_1)N(\frac{c_2+c_1\rho}{\sqrt{1-\rho^2}})\frac{1}{\sigma S_t\sqrt{T-t}}] \\
&- \frac{(1-\alpha)V_t K}{B+mV_t}e^{-y(T-t)}n(d_1)N(\frac{d_2+d_1\rho}{\sqrt{1-\rho^2}})\frac{1}{\sigma S_t\sqrt{T-t}}
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
Vega \equiv \frac{\partial C_t}{\partial \sigma} &= S_t e^{-q(T-t)}n(b_1)N(\frac{b_2-b_1\rho}{\sqrt{1-\rho^2}})(\sqrt{T-t}-\frac{b_1}{\sigma}) \\
&+ S_t e^{-q(T-t)}n(b_2)N(\frac{b_1-b_2\rho}{\sqrt{1-\rho^2}})(\rho\sqrt{T-t}) \\
&+ Ke^{-r(T-t)}n(a_1)N(\frac{a_2-a_1\rho}{\sqrt{1-\rho^2}})(\frac{b_1}{\sigma}) + \frac{(1-\alpha)V_t K}{B+mV_t}e^{-y(T-t)}n(d_1)N(\frac{d_2+d_1\rho}{\sqrt{1-\rho^2}})(\frac{b_1}{\sigma}) \\
&+ \frac{(1-\alpha)S_t V_t}{B+mV_t}e^{(r-q-y+\rho\sigma\theta)(T-t)}[N(c_1,c_2;-\rho)\rho\eta(T-t) \\
&+ n(c_1)N(\frac{c_2+c_1\rho}{\sqrt{1-\rho^2}})(\sqrt{T-t}-\frac{b_1}{\sigma}) - n(c_2)N(\frac{c_1+c_2\rho}{\sqrt{1-\rho^2}})\rho\sqrt{T-t}]
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
Psi \equiv \frac{-\partial C_t}{\partial q} &= S_t e^{-q(T-t)}n(b_1)N(\frac{b_2-b_1\rho}{\sqrt{1-\rho^2}})\frac{\sqrt{T-t}}{\sigma} \\
&+ (T-t)S_t e^{-q(T-t)}N(b_1,b_2;\rho) - Ke^{-r(T-t)}n(a_1)N(\frac{a_2-a_1\rho}{\sqrt{1-\rho^2}})\frac{\sqrt{T-t}}{\sigma} \\
&+ \frac{(1-\alpha)S_t V_t}{B+mV_t}e^{(r-q-y+\rho\sigma\theta)(T-t)}[n(c_1)N(\frac{c_2+c_1\rho}{\sqrt{1-\rho^2}})\frac{\sqrt{T-t}}{\sigma} + (T-t)N(c_1,c_2;-\rho)] \\
&- \frac{(1-\alpha)V_t K}{B+mV_t}e^{-y(T-t)}n(d_1)N(\frac{d_2+d_1\rho}{\sqrt{1-\rho^2}})\frac{\sqrt{T-t}}{\sigma}
\end{aligned} \tag{7}
$$

where *Psi* is defined as a negative partial derivative with respect to the dividend yield for expressing an incentive to reduce dividend payout for the executives (see Johnson and Tian, 2000). $n(\cdot)$ denotes a probability density function of normal distribution.

The *Delta* of ESO allows us to measure the degree of how executives expend greater effort in business management. If the *Delta* is positive, firm executives have an incentive to create firm value by redoubling their efforts. The reason is that executive putting greater effort into business management tends to increase a firm's value, which, in turn, increases the ESO's value. The *Vega* measures the incentive for risk-taking. A positive *Vega* for an ESO means that executives have the incentive to undertake high-risk projects in order to enhance the firm's value, and, therefore, the contract value. The *Psi* measures executives' willingness to reduce the dividend rate in order to increase firm value. A higher value for contract *Psi* indicates that the executives have a stronger incentive to reduce the dividend payout rate.

When compared with the results of ESOs *without* default risks, these values of the *Greek* letters for ESOs with default risks display complicated forms. As shown in equations (5)-(7), it is difficult to find specific trends for these *Greeks* in terms of determinants. Thus, for the purpose of clearly recognizing the essential characteristics of contract *Greeks* and executive incentives, it is necessary to implement numerical analyses.

2.3 Default Probability and Strategic Behaviors

The probability of whether or not the firm value is sufficient to satisfy the promised payments is critical for the valuation of ESOs with default risks. We further derive a closed-form result of full-payable probability (P_f) and default probability (P_d) for the ESO contracts, as follows:

$$P_f = P(S_T \geq K, V_T \geq B + mV) = N(a_1, a_2; \rho) \tag{8}$$

$$P_d = P(S_T \geq K, V_T < B + mV) = N(a_1, -a_2; -\rho) \tag{9}$$

$$P_o = 1 - N(a_1, a_2; \rho) - N(a_1, -a_2; -\rho) \tag{10}$$

where P_o represents a probability of other cases; for example, where the option is not exercised (in a situation of $S_T < K$).

These probabilities follow a bivariate normal distribution and are conditional upon whether or not the stock price and firm value exceed the strike price and the total of the default threshold and salary payments, respectively. In particular, the dividend policy (q), salary policy (m), debt policy (B), time to maturity (T-t), return volatility (σ) of stock prices, return volatility (θ) of firm values, and dead-weight cost (α) can directly determine these probabilities.

Next, the default effects of boards of directors' activities are measured using the partial derivative (*Zeta*) of the default probability with respect to the salary policy. That is, the *Zeta* refers to a possibility of the firm defaulting on the ESOs as the firm changes its salary policy. An increase in the salary payout rate (m) can increase the probability of a firm defaulting. This study uses *Zeta* as a measure of default incentives in terms of salary policy (m). It is derived as follows:

$$\begin{aligned} Zeta &\equiv \frac{\partial P_d}{\partial m} \\ &= \frac{n(-a_2)}{\theta(1-m)\sqrt{T-t}} N\left(\frac{a_1 - a_2\rho}{\sqrt{1-\rho^2}}\right) \end{aligned} \tag{11}$$

Finally, the study further discusses the relationship between the strategic behavior of the board of directors and the executives. Specifically, taking into account the default behavior of the board of directors, the executives may adjust those factors which are within their controls in response to the firm's likelihood of default.

3. Numerical Calibration

Using the numerical analysis, the study examines, analyzes, and explains the premium characteristics, *Greeks*, incentive effects, and default probabilities of ESOs.

3.1 Premium Characteristics

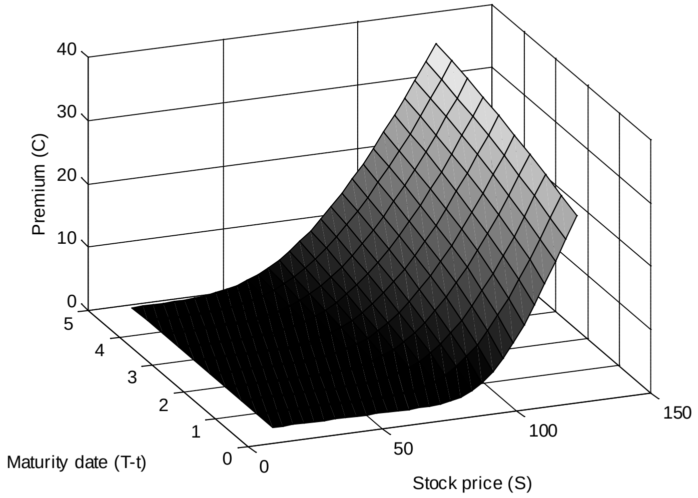
Premium characteristics of ESO contracts vary with numerous factors, such as the stock price, return volatility, debt threshold, and so on. Table 1 lists the premium changes for these factors, given certain specifications of parameters. The initial stock price (S) is assumed to be \$120. The strike price (K) of the ESO is assumed to be \$100. Given values of other parameters are listed in the note to Table 1.

We summarize the main results as follows. First, as shown in Panel A of Table 1, the contract premium (C) is negatively correlated with strike price and is positively correlated with time-to-maturity. The result indicates that a higher strike price decreases the potential for payoff upon maturity. Second, the ESO premium gradually increases with the return volatility (σ) of the stock price (Panel B). In addition, Figure 1 graphically displays the numerical results for the contract premium with respect to the stock price and its return volatility. The premium is higher for those stocks with a higher initial stock price and a greater return volatility. Third, a high salary rate (m) and higher payments (B) of outside liabilities depreciate the contract premiums (C) because the firm is obliged to pay the promised debts and salary compensation first, due to their higher priority (Panels C and D in Table 1). Fourth, in accordance with our theoretical expectation, the dividend payout rate (q) has a negative impact on the contract premium (Panel E). In short, the premium of

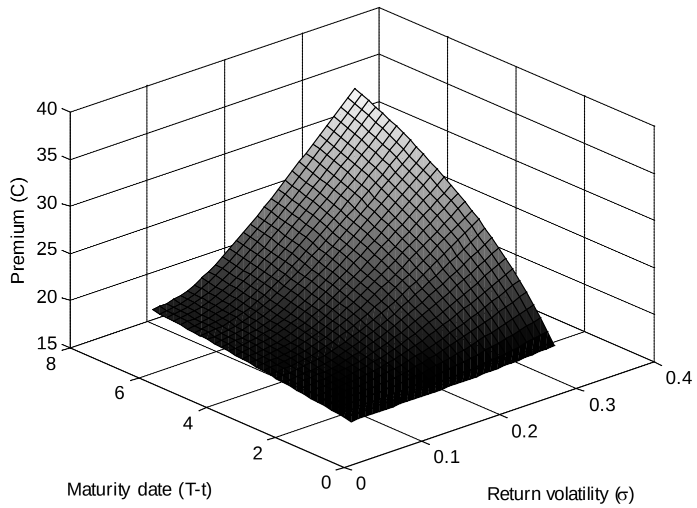
Table 1 Option Premium

Panel A: Exercise price (K)						
Time (τ = T-t)	$95	$100	$105	$110	$115	$120
Short-life (1 year)	25.5504	21.7019	18.1537	14.9519	12.1260	9.6861
3 years	29.7131	26.6714	23.8493	21.2491	18.8689	16.7035
Long-life (5 years)	32.6954	30.0523	27.5800	25.2757	23.1350	21.1523
Panel B: Return volatility (σ)						
	5%	10%	15%	20%	25%	30%
Short-life (1 year)	19.4401	19.7427	20.4992	21.7019	23.1861	24.8399
3 years	20.5869	21.7396	23.9464	26.6714	29.6372	32.7188
Long-life (5 years)	21.6203	23.4456	26.4998	30.0523	33.8061	37.6337
Panel C: Salary policy (m)						
	0%	5%	10%	15%	20%	25%
Short-life (1 year)	22.3932	22.1813	21.7019	20.8903	19.8653	18.8611
3 years	27.2002	27.0052	26.6714	26.1567	25.4472	24.5795
Long-life (5 years)	30.4921	30.3220	30.0523	29.6512	29.0952	28.3824
Panel D: Debt threshold (B)						
	$390	$410	$430	$450	$470	$490
Short-life (1 year)	22.4487	22.3491	22.1170	21.7019	21.1053	20.3845
3 years	27.2619	27.1399	26.9474	26.6714	26.3072	25.8581
Long-life (5 years)	30.5397	30.4277	30.2675	30.0523	29.7780	29.4441
Panel E: Dividend rate (q)						
	2%	3%	4%	5%	6%	7%
Short-life (1 year)	26.6714	24.1894	21.8619	19.6867	17.6614	15.7829
3 years	30.0523	26.1772	22.6633	19.4966	16.6613	14.1402
Long-life (5 years)	32.4930	27.3305	22.7893	18.8295	15.4091	12.4841

The table reports option premiums of ESOs with default risk varying with various factors. Parameters are given as follows: initial stock price of S = $120, strike price of K = $100, risk free rate of r = 3%, dividend yield of q = 2%, return volatility of stock price of σ = 20%, debt threshold of B = $450, firm value of V = $500, return volatility of firm value of θ = 10%, salary policy of m = 10%, coefficient of correlation of ρ = 0.5, and dead-weight cost of α = 10%.



Panel A: Varying with stock prices



Panel B: Varying with return volatilities

Figure 1 Option Values Varying with Stock Prices and Return Volatilities

The figure displays option values of ESOs with a default risk varying with stock price and volatility rate. Parameters are given as follows: initial stock price of S = \$120, strike price of K = \$100, risk free rate of r = 3%, dividend yield of q = 2%, time to maturity of τ = 1 year, return volatility of stock price of σ = 20%, debt threshold of B = \$450, firm value of V = \$500, return volatility of firm value of θ = 10%, salary policy of m = 10%, coefficient of correlation of ρ = 0.5, and dead-weight cost of α = 10%.

an ESO contract with default risk displays a standard characteristic of vanilla call options; however, the premium is also determined by salary policy and debt policy.

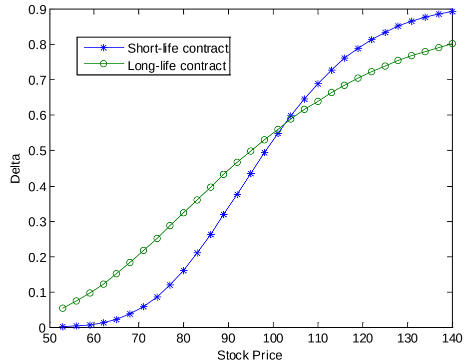
3.2 Contract Greeks and executive incentives

Figure 2 shows how the option's *Greeks* (*Delta*, *Vega*, and *Psi*) change with the stock price and time to maturity. Although the *Greeks* of ESO contracts have complicated forms, as shown in equations (5)-(7), they displays a consistent tendency.

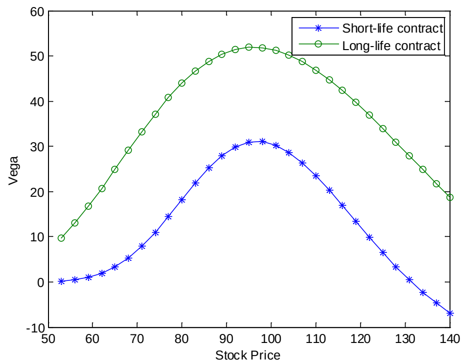
The ESO *Delta* shows a similar trend to the *Delta* of the plain vanilla call option. First, as the stock price is over the strike price (i.e., K = \$100), the *Delta* approaches a maximum. Otherwise, the *Delta* approaches zero if the stock price is less than the strike price. Second, the option *Delta* changes dramatically near the strike price. Moreover, the *Delta* of a short-life contract changes more sensitively with respect to the underlying stock price than that of a long-life contract does. Finally, positive values of the option *Delta* indicate that the ESO holder has an incentive to increase the stock price to create contract value. That is, the ESO contract provides an incentive mechanism for executives to work harder at managing the company's business.

As shown in Panel B of Figure 2, we further examine the *Vega* to analyze the executive incentive to take risks. Although the *Vega* may appear as a negative value, in most ranges of stock price the option *Vega* is positive. Specifically, the option *Vega* displays a peak as the stock price locates near the strike price, at which point the ESO holder has the strongest incentive to enhance the contract's value by taking more risks in business operations. In addition, in contrast to the theoretical results for standard call options, the ESO *Vega* presents negative values as the stock price becomes high enough. That is, the executive has a disincentive to create contract value by taking more risks because the board of directors may default as the stock price reaches a high enough level. What's more, long-life ESO contracts have larger *Vegas* than short-life ESO contracts do.

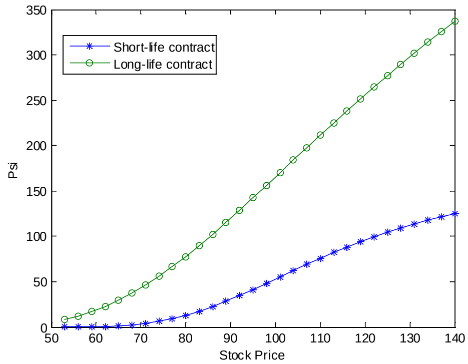
A higher value for contract *Psi* indicates that the executives have stronger incentives to reduce dividend payout rates. As shown in Panel C of Figure 2, the ESO holders have weak incentives to alter dividend policy as the stock prices are lower, and have strong incentives to do so as the stock prices are higher. Specifically, the executives have strong incentives to reduce dividend payouts in order to avoid a decrease in the ESO values as the contract stays in-the-money. In addition, the sensitivities of option *Psi* are larger for a long-life ESO contract, in the sense that a long-life contract's incentive effect in relation to reducing dividend payouts is stronger.



Panel A: ***Delta***



Panel B: *Vega*



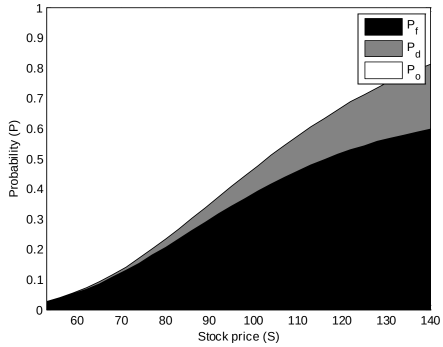
Panel C: *Psi*

Figure 2 *Greeks* of Executive Stock Options with Writer's Default

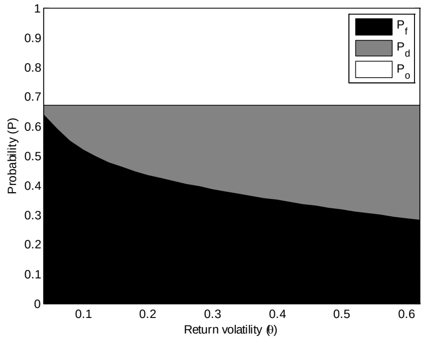
The figure displays *Greeks* of ESOs with a default risk varying with stock prices. Short-life contracts and long-life contracts have times to maturity of τ = year and 3 years, respectively. Parameters are given as follows: initial stock price of S = \$120, strike price of K = \$100, risk free rate of r = 3%, dividend yield of q = 2%, return volatility of stock price of σ = 20%, debt threshold of B = \$450, firm value of V = \$500, return volatility of firm value of θ = 10%, salary policy of m = 10%, coefficient of correlation of ρ = 0.5, and dead-weight cost of α = 10%.

3.3 Probability Analysis and Default Effects

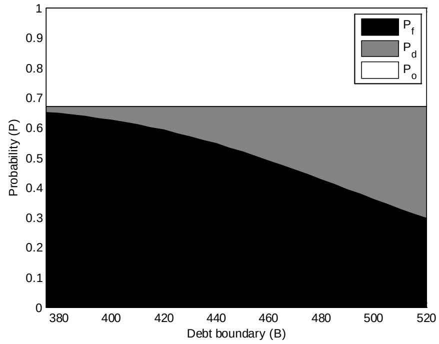
We analyze a default probability (P_d), a full-payable probability (P_f), and a non-exercisable probability (P_o), with the results shown in Figure 3. First, both the full-payable probability (P_f) and default probability (P_d) increase with the stock price (S), indicating that ESOs tend to be exercisable.



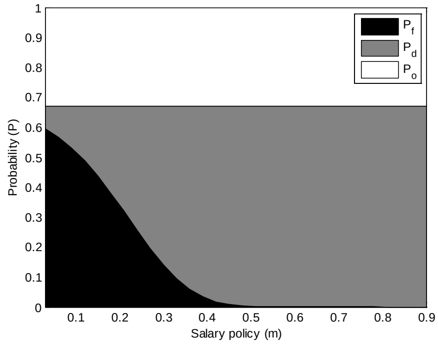
Panel A: Varying with stock prices



Panel B: Varying with return volatilities of firm value



Panel C: Varying with debt boundaries



Panel D: Varying with salary policies

Figure 3 Probabilities

The figure displays how full-payable probabilities, default probabilities, and non-exercisable probabilities of ESOs with a default risk vary with various factors. Parameters are given as follows: initial stock price of S = \$120, strike price of K = \$100, risk free rate of r = 3%, dividend yield of q = 2%, time to maturity of τ = 3 years, return volatility of stock price of σ = 20%, debt threshold of B = \$450, firm value of V = \$500, return volatility of firm value of θ = 10%, salary policy of m = 10%, coefficient of correlation of ρ = 0.5, and dead-weight cost of α = 10%.

However, it also means that the default risk tends to increase if the stock price increases. Second, the default probability (P_d) is positively related to the return volatility (θ) of the firm value, but the full-payable probability (P_f) is not (Panel B). Other conditions being equal, a greater value of return volatility for firm value increases the possibility of the firm value being less than the total of the promised payments. Third, as the debt payments (B) and salary compensation (m) gradually increase, the default probability tends to increase and the full-payable probability tends to decrease (Panels C and D). Finally, the return volatilities of the firm value, debt payments, and salary compensation do not change the non-exercisable probability (P_o) because the factors involve whether the firm defaults or not, and do not relate to whether or not the ESOs are exercised, in the model. Thus, the distributions of the three probabilities give us some insights regarding *whether* and *how* the default risk changes based on stock prices, return volatilities, capital structures, and salary policies.

Next, the current study focuses on the default probability to explore the properties of the default risks of ESO contracts, as shown in Table 2. First, based on the initial stock prices, the default probability displays a growth trend, as shown in Panel A of Table 2. That is, the ESO contract has a higher default possibility if the stock price increases. Second, the return volatility (σ) of the stock price can cause a negative effect on the default probability; conversely, the return volatility (θ) of the firm value has a positive impact on the probability. Third, the default probability increases with the debt payments (B) and salary compensation (m). Fourth, the dividend rate (q) reduces the default probability because it devalues the stock price, which then reduces the possibility of the ESOs being exercised.

Specifically, we find that the time-to-maturity has inconsistent impacts on the default probability. A long-life contract is less likely to be defaulted on than a short-life contract because of the firm's positive growth drift in terms of firm value, resulting in the firm having a better ability to service outside debt and make salary payments. Our results shown in Panels B-F of Table 2 show a consistent relation between maturity and default probability. However, in Panel A, we see that an out-of-the-money ESO's default probability is positively correlated to the time-to-maturity (i.e., $S <$ \$100). As the maturity is extended, the likelihood of the ESO being exercised by the executives upon reaching the maturity date increases because the stock price and firm value both increase gradually. However, even though the firm's value slightly increases, the firm's value is still insufficient to pay its liabilities and salary. That is, the firm's default risk increases.

Next, for analyzing the board of directors' default behavior, the study uses contract *Greeks* to measure the extent of default effects. The *Zeta* is defined as the partial derivative of the default probability with respect to the salary payouts, and it measures the sensitivity of the default risk in response to the firm's behavior.

The board of directors has a stronger default incentive as the ESO contract stays in a deep in-the-money condition. As shown in Figure 4, as the stock price gradually increases, which means the contract tends to stay deep in-the-money, the *Zeta* value tends to be greater. In addition, our results show that a positive *Zeta* value indicates that the firm's default risk increases if the firm expands salary payouts.

Table 2 Default Probability

Panel A: Initial stock price (S)					
Time (τ =T-t)	\$50	\$70	\$90	\$110	\$130
2 years	0.00%	0.10%	1.36%	5.84%	13.38%
3 years	0.01%	0.25%	1.74%	5.46%	10.97%
4 years	0.02%	0.38%	1.89%	5.02%	9.33%
5 years	0.04%	0.48%	1.93%	4.60%	8.10%
Panel B: Return volatility (σ)					
	5%	10%	15%	20%	25%
2 years	35.97%	29.80%	23.29%	19.05%	16.22%
3 years	32.56%	24.77%	18.75%	15.13%	12.74%
4 years	29.65%	21.25%	15.76%	12.58%	10.49%
5 years	27.16%	18.62%	13.60%	10.74%	8.86%
Panel C: Debt payments (B)					
	\$400	\$420	\$440	\$460	\$480
2 years	4.53%	8.91%	15.25%	23.19%	31.99%
3 years	4.53%	7.89%	12.44%	18.04%	24.35%
4 years	4.29%	6.99%	10.52%	14.81%	19.67%
5 years	4.00%	6.23%	9.08%	12.52%	16.44%
Panel D: Return volatility (θ)					
	5%	10%	15%	20%	25%
2 years	9.37%	19.05%	23.78%	26.81%	29.11%
3 years	5.76%	15.13%	20.25%	23.64%	26.25%
4 years	3.76%	12.58%	17.93%	21.60%	24.44%
5 years	2.55%	10.74%	16.24%	20.11%	23.16%
Panel E: Dividend rate (q)					
	0%	1%	2%	3%	4%
2 years	21.28%	20.17%	19.05%	17.93%	16.80%
3 years	17.66%	16.40%	15.13%	13.87%	12.64%
4 years	15.28%	13.92%	12.58%	11.27%	10.01%
5 years	13.53%	12.12%	10.74%	9.41%	8.16%
Panel F: Salary policy (m)					
	0%	5%	10%	15%	20%
2 years	5.44%	10.71%	19.05%	30.41%	43.39%
3 years	5.25%	9.20%	15.13%	23.20%	33.01%
4 years	4.89%	8.02%	12.58%	18.78%	26.55%
5 years	4.49%	7.06%	10.74%	15.72%	22.07%

The table reports default probabilities of ESOs with a default risk varying with various factors. Parameters are given as follows: initial stock price of S = \$120, strike price of K = \$100, risk free rate of r = 3%, dividend yield of q = 2%, return volatility of stock price of σ = 20%, debt threshold of B = \$450, firm value of V = \$500, return volatility of firm value of θ = 10%, salary policy of m = 10%, coefficient of correlation of ρ = 0.5, and dead-weight cost of α = 10%.

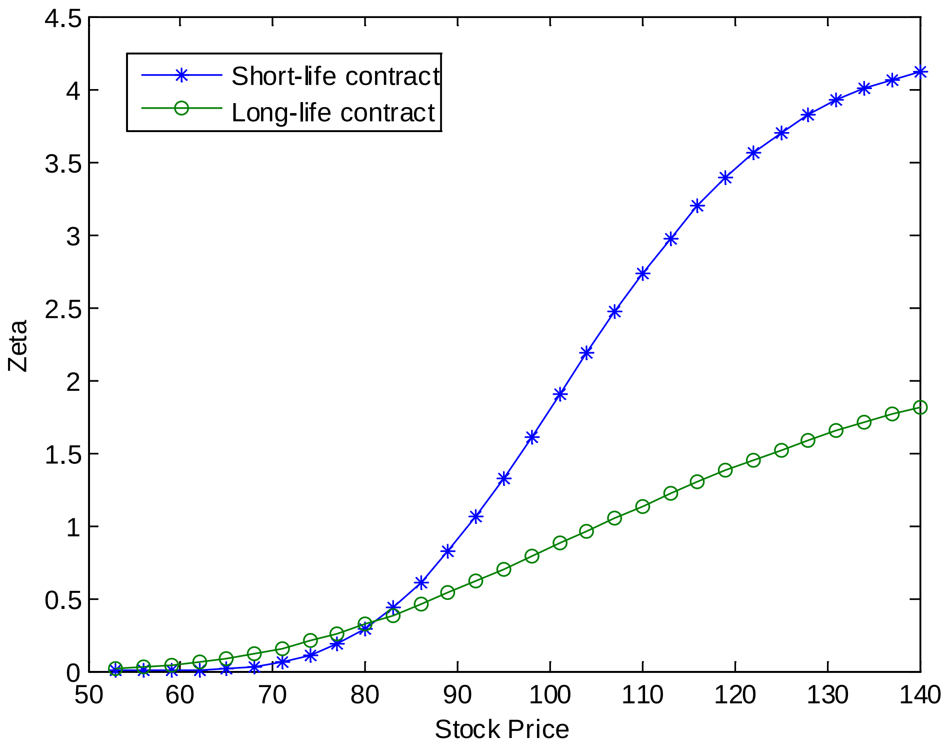


Figure 4　*Greeks* of Default Probability

The figure shows *Zeta* of default probabilities for ESOs with a default risk. Short-life contracts and long-life contracts have times to maturity of τ = 1 year and 3 years, respectively. Parameters are given as follows: initial stock price of S = \$120, strike price of K = \$100, risk free rate of r = 3%, dividend yield of q = 2%, return volatility of stock price of σ = 20%, debt threshold of B = \$450, firm value of V = \$500, return volatility of firm value of θ = 10%, salary policy of m = 10%, coefficient of correlation of ρ = 0.5, and dead-weight cost of α = 10%.

3.4 Executives' Incentives

The boards of directors issue ESOs to the executives for the ultimate purpose of motivating the executives to create shareholder value, so the boards of directors can adjust the provisions of ESO contracts or salary policy so as to affect the executive incentives. In this section, we numerically analyze how the executive incentives change in response to the adoption of specific strategies on the part of a board of directors.

We first observe the impacts of strike price on the executive incentives, as shown in Panel A of Table 3. The executive incentives (*Delta*) of increasing the stock price are a negative function of

Table 3 Executive's Incentive

Board of directors' strategies	Executive's incentive effects		
	Incentive to increase stock price (*Delta*)	Incentive to increase return volatility (*Vega*)	Incentive to reduce dividend (*Psi*)
Panel A: Using strike price			
K = \$80	0.8299	7.3003	298.7641
K = \$90	0.7781	22.9767	280.1300
K = \$100	0.7110	38.7774	255.9733
K = \$110	0.6335	52.2355	228.0585
K = \$120	0.5513	61.8017	198.4818
Panel B: Adjusting maturity date			
T-t = 1 year	0.8617	1.5239	51.7044
T-t = 2 years	0.8250	7.2134	74.2523
T-t = 3 years	0.7987	12.2584	95.8471
T-t = 4 years	0.7790	16.7247	116.8457
T-t = 5 years	0.7635	20.7262	137.4294
Panel C: Changing salary policy			
m = 0%	0.7305	48.8356	262.9722
m = 10%	0.7110	38.7774	255.9733
m = 20%	0.6715	29.6307	241.7268
m = 30%	0.6193	31.2855	222.9550
m = 40%	0.5715	40.1033	205.7232

The table displays incentive effects of ESO holders varying with various strategies of a board of directors. Parameters are given as follows: initial stock price of S = \$120, strike price of K = \$100, risk free rate of r = 3%, dividend yield of q = 2%, time to maturity of τ = 3 years, return volatility of stock price of σ = 20%, debt threshold of B = \$450, firm value of V = \$500, return volatility of firm value of θ = 10%, salary policy of m = 10%, coefficient of correlation of ρ = 0.5, and dead-weight cost of α = 10%.

the strike price, indicating that an ESO contract with a low strike price provides a strong incentive to create contract value or shareholder value by increasing stock price. Next, if the strike price increases, the executive incentive (*Vega*) representing the willingness to take more risks also increases. That is, the executives have a stronger motivation to undertake risky strategies in their management of the firm's business, for the purpose of increasing the contract value if the ESO contract has a high strike price. Besides, an ESO contract with a low strike price also creates a strong incentive to reduce dividend payouts in order to enhance the contract's value. As a result, based on this analysis, a board of directors can increase the executive's incentive to increase stock prices, decrease the executive's incentive to increase return volatility, and increase the executive's incentive to reduce dividend payouts, in order to create shareholder value resulting from an increase in firm value, by designing an ESO contract with a low strike price.

A short-life ESO contract is preferred by the boards of directors for increasing executives' incentives in terms of the amount of effort they put into their work, and for reducing executives' incentives to engage in risk-taking. However, a short-life ESO contract may eliminate the executives' incentives with respect to the dividend payouts. As shown in Panel B, the price sensitivity (i.e., the *Delta* indicating the executives' incentive to increase their effort) is negatively correlated with the time to maturity of the ESO, while the risk sensitivity (i.e., the *Vega*, indicating the incentive effect to engage in risk-taking) and dividend sensitivity (i.e., *Psi*, indicating the incentive effect to alter the firm's dividend payout) are positively correlated to the time to maturity of the ESO contract. That is, the firm can design a short-life ESO contract that encourages executives to increase the amount of effort they put into managing the firm's business and adopt low-risk plans in their operational decisions.

If a firm has more employees to pay (i.e., a greater value of m), the firm's promised payments increase before it can payout the potential payoffs to the ESO holders. Panel C of Table 3 shows how three executive incentives vary with the salary payouts (m). As the salary-payout rate gradually increases from 0% to 40%, the executives' incentive effect to increase stock price (*Delta*) decreases, and the incentive to reduce dividend payouts (*Psi*) also decreases, yet the risk-taking incentive (*Vega*) exhibits a mixed result. These results indicate that the executives have little motivation to enhance contract value when a firm's policy involves employing a large number of workers to whom it must make salary payments because there is a higher possibility that the ESO contracts in such cases will be defaulted upon.

In short, subject to the default risk, the executives' incentive to create contract value by altering stock prices, return volatility, and dividend payouts involves the both the contract's design and the firm's strategies.

3.5 Firm Default

We further analyze how the firm's default behavior changes in response to the executives' strategies. The default behavior of a board of directors is directly correlated with the contract design and the executives' efforts. First, as shown in Panel A of Table 4, an ESO contract with a high strike price (K) will decrease the default effects of the firm. One possible explanation for this is that ESOs with a high strike price have a low probability of being exercised (i.e., including full-payable probability and default probability). The firms thus have weak default incentives to change their salary policy for the purpose of raising the default possibility. Second, as shown in Panel B of Table

Table 4 Firm's Default

Executive's strategies	Incentive to change salary policy (*Zeta*)
Panel A: Using strike price (K)	
K = \$80	1.9614
K = \$90	1.6981
K = \$100	1.4021
K = \$110	1.1094
K = \$120	0.8463
Panel B: Adjusting maturity date (T-t)	
T-t = 1 year	5.6211
T-t = 2 years	4.2546
T-t = 3 years	3.4478
T-t = 4 years	2.9087
T-t = 5 years	2.5201
Panel C: Changing effort for increasing stock price (S)	
S = \$80	0.3193
S = \$100	0.8463
S = \$120	1.4021
S = \$140	1.8171
S = \$160	2.0695
Panel D: Changing effort for increasing stock volatility (σ)	
σ = 5%	2.3169
σ = 10%	1.6467
σ = 15%	1.2266
σ = 20%	0.9844
σ = 25%	0.8155
Panel E: Changing dividend policy (q)	
q = 0%	1.5762
q = 1%	1.4907
q = 2%	1.4021
q = 3%	1.3111
q = 4%	1.2187

The table presents default effects of ESO writers varying with various strategies of executives. Parameters are given as follows: initial stock price of S = \$120, strike price of K = \$100, risk free rate of r = 3%, dividend yield of q = 2%, time to maturity of τ = 3 years, return volatility of stock price of σ = 20%, debt threshold of B = \$450, firm value of V = \$500, return volatility of firm value of θ = 10%, salary policy of m = 10%, coefficient of correlation of ρ = 0.5, and dead-weight cost of α = 10%.

4, the default effect is stronger for a short-life ESO contract, or, in other words, the firms have less incentive to default on a long-life ESO contract. Third, if the executives undertake little effort to increase stock price, decrease return volatility, and decrease dividend payouts in order to create more contract values, the firms have little motivation to default. That is, given these conditions, the ESO contract has a lower possibility of being exercised, and the firm thus has less incentive to change its salary policy for the purpose of affecting the default probability.

Consequently, to mitigate the default effects of the board of directors' behavior, the executives can adopt the following strategies: increasing strike price and time to maturity in the ESO contract design, increasing stock price, decreasing the return volatility of the stock price, and decreasing dividend payouts. However, the default effects of the board of directors' actions and the incentive effects of the executives' behavior can affect each other by altering both parties' strategies.

3.6 Robustness Tests

In this section, we report on robustness tests for the ESOs without a default risk. Our objective is to contrast both premiums of ESOs with and without a default risk. Additionally, we also assess the executives' incentives when the ESO's holders do not bear a default risk.

The results of premiums of ESO contract without a default risk are listed in Table 5. We find that the premiums of ESOs without a default risk are slightly greater over compared to these of ESOs with a default risk, as shown in Tables 1 and 5. First, given a short-life contract and a strike price of \$95, the option premiums are \$25.5504 and \$26.5535 for ESOs with and without a default risk, respectively (Panel A in Tables 1 and 5). We also find consistent results over various maturities and strike prices. Thus, our results support that the option premiums of ESOs without a default risk are greater. That is, as the default risk gradually vanishes, ESO contracts tend to be more valuable. Second, the premium of an ESO without a default risk is higher than that of one with a default risk, if the return volatility of the stock price changes (Panel B of Tables 1 and 5). Finally, we find that the option premiums of ESOs without a default risk are not always higher than the premiums of ESOs with a default risk, when we change the dividend rate (Panel C of Table 5). The premium of ESOs without a default risk is greater for firms who pay a higher rate of dividends, while the premium is smaller for firms which pay a lower rate of dividends. The reason is that the dividend rate decreases the ESO's value and affects the ESO's probability of being exercised, which with the latter more obviously depreciating the value of ESOs with a default risk, in a condition of where dividends are paid by the firm at a high rate.

Table 5　Premium of European ESOs without Default Risk

Panel A: Exercise price (K)						
Time ($\tau = T\text{-}t$)	\$95	\$100	\$105	\$110	\$115	\$120
Short-life (1 year)	26.5535	22.4876	18.7544	15.4004	12.4532	9.9196
3 years	30.5660	27.3860	24.4444	21.7422	19.2756	17.0374
Long-life (5 years)	33.4427	30.6980	28.1368	25.7551	23.5473	21.5062
Panel B: Return volatility (σ)						
	5%	10%	15%	20%	25%	30%
Short-life (1 year)	20.5794	20.6903	21.3356	22.4876	23.9521	25.6023
3 years	21.6391	22.5526	24.6820	27.3860	30.3507	33.4383
Long-life (5 years)	22.5839	24.1743	27.1635	30.6980	34.4478	38.2746
Panel C: Dividend rate (q)						
	2%	3%	4%	5%	6%	7%
Short-life (1 year)	22.4876	21.4927	20.5217	19.5746	18.6520	17.7540
3 years	27.3860	24.8102	22.3981	20.1474	18.0550	16.1172
Long-life (5 years)	30.6980	26.7049	23.0904	19.8388	16.9327	14.3531

The table reports the premiums of ESOs without default risk varying with various factors. Parameters are given as follows: initial stock price of S = \$120, strike price of K = \$100, risk free rate of r = 3%, dividend yield of q = 2%, and return volatility of stock price of σ = 20%.

Table 6　Executive's Incentive for ESOs without Default Risk

Board of directors' strategies	Executive's incentive effects		
	Incentive to increase stock price (*Delta*)	Incentive to increase return volatility (*Vega*)	Incentive to reduce dividend policy (*Psi*)
Panel A: Using strike price			
K = \$80	0.8699	28.0785	41.4507
K = \$90	0.8120	43.0995	33.9786
K = \$100	0.7384	57.3323	27.3860
K = \$110	0.6548	68.5323	21.7422
K = \$120	0.5674	75.4982	17.0374
Panel B: Adjusting maturity date			
$T\text{-}t$ = 1 year	0.8388	26.7103	22.4876
$T\text{-}t$ = 2 years	0.7727	45.0660	25.1979
$T\text{-}t$ = 3 years	0.7384	57.3323	27.3860
$T\text{-}t$ = 4 years	0.7155	66.4255	29.1880
$T\text{-}t$ = 5 years	0.6979	73.4921	30.6980

The table displays incentive effects of ESO holders varying with various strategies of a board of directors. Parameters are given as follows: initial stock price of S = \$120, strike price of K = \$100, risk free rate of r = 3%, dividend yield of q = 2%, return volatility of stock price of σ = 20%, and time to maturity of τ = 3 years.

Table 6 presents the executives’ incentives. The results indicate that the executive’s incentives for ESOs without a default risk are consistent with these for ESOs with a default risk. The executives tend to expend more effort to create firm value as the ESO contract is short-term and has a lower strike price. They also have stronger incentives to take on high-risk projects as the ESO contract is long-term and has a higher strike price. Finally, they have stronger incentives to pay out more dividends if the ESO contract has a lower strike price and a longer time-to-maturity.

4. Conclusion

An ESO holder may receive some payoff depending on whether the stock price is over the strike price and whether the firm value exceeds a default threshold. In this study, we provide a closed-form solution for ESO contracts and analyze executives’ incentives to create contract value by altering the stock price, risk-taking, and dividend payout policy. In addition, we also discuss firm defaults in terms of their salary policy. The two parties' behavior also exhibits strategic inter-dependences in terms of their controllable actions and the ESO contract’s designs.

This study finds a number of interesting results with important implications. First, the formula of an ESO contract with a default risk is more complicated than that of an ESO contract without a default risk, with the formula of the former involving whether the firm’s value is sufficient to pay outside debt-holders and its employees. Second, under some conditions, executives have an incentive to create contract value by increasing their efforts or increasing the level of risk-taking in their management decisions, or by altering the firm's dividend payout policy. Third, if the total amount of promised debt payments and salary compensation is greater, the firm may be more likely to default on an ESO contract. Finally, the two parties can design an appropriate contract (in terms of strike price and time to maturity) or implement their own controllable policies (i.e., dividend policy, capital structure, or salary policy) to affect each other’s incentives and default behavior.

Overall, this article contributes to literature on the consideration of risk originating in option writers’ defaults, and highlights the specific issues of executive incentives, firm defaults, and strategic inter-dependences, for the purpose of strengthening executive’s incentives and decreasing firms’ default possibilities.

Appendix

This Appendix presents a derivation of the formula (4) of ESO contracts. According to option pricing theory, the initial price of an executive stock option with default risk can be written as follows:

$$
\begin{aligned}
C_t &= e^{-r(T-t)}E^Q(C_T) \\
&= e^{-r(T-t)}E^Q((S_T-K)I_1)+e^{-r(T-t)}E^Q(\frac{(1-\alpha)(S_T-K)V_T}{B+mV_T}I_2) \\
&= e^{-r(T-t)}E^Q(S_TI_1)-e^{-r(T-t)}E^Q(KI_1)+e^{-r(T-t)}E^Q(\frac{(1-\alpha)S_TV_T}{B+mV_T}I_2) \\
&\quad -e^{-r(T-t)}E^Q(\frac{(1-\alpha)KV_T}{B+mV_T}I_2) \\
&= A_1-A_2+A_3-A_4 \qquad \text{(A.1)}
\end{aligned}
$$

where,

$$
\begin{aligned}
A_2 &= e^{-r(T-t)}E^Q(KI_1) \\
&= Ke^{-r(T-t)}E^Q(I_1) \\
&= Ke^{-r(T-t)}P^Q[S_T \ge K, V_T \ge B+mV_T] \\
&= Ke^{-r(T-t)}P^Q[\frac{dW^Q}{\sqrt{T-t}} \ge \frac{\ln(K/S_t)-(r-q-\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}, \\
&\qquad \frac{dZ^Q}{\sqrt{T-t}} \ge \frac{\ln(B/(1-m)V)-(r-\theta^2/2)(T-t)}{\theta\sqrt{T-t}}] \\
&= Ke^{-r(T-t)}N(a_1,a_2;\rho) \qquad \text{(A.2)}
\end{aligned}
$$

$$
\begin{aligned}
A_1 &= e^{-r(T-t)}E^Q(S_TI_1) \\
&= S_te^{-q(T-t)}E^Q(\xi_S I_1) \\
&= S_te^{-q(T-t)}E^R(I_1) \\
&= S_te^{-q(T-t)}P^R[S_T \ge K, V_T \ge B+mV_T] \\
&= S_te^{-q(T-t)}N(b_1,b_2;\rho) \qquad \text{(A.3)}
\end{aligned}
$$

$$
\begin{aligned}
A_3 &= e^{-r(T-t)}E^Q(\frac{(1-\alpha)S_TV_T}{B+mV_T}I_2) \\
&= e^{-r(T-t)}E^Q((1-\alpha)S_TH_TI_2) \\
&= e^{-r(T-t)}E^Q((1-\alpha)S_tH_tI_2e^{(r-q-\sigma^2/2)(T-t)+\sigma dW^Q}e^{(r-y-\eta^2/2)(T-t)+\eta dW^Q})
\end{aligned}
$$

$$= (1-\alpha)S_t H_t e^{(r-q-y+\rho\sigma\eta)(T-t)} E^Q(\xi_{S,H} I_2)$$
$$= \frac{(1-\alpha)S_t V_t}{B+mV_t} e^{(r-q-y+\rho\sigma\eta)(T-t)} E^R(I_2) \quad \text{(A.4)}$$
$$= \frac{(1-\alpha)S_t V_t}{B+mV_t} e^{(r-q-y+\rho\sigma\eta)(T-t)} N(c_1, c_2; -\rho)$$

$$A_4 = e^{-r(T-t)} E^Q\left(\frac{(1-\alpha)KV_T}{B+mV_T} I_2\right)$$
$$= (1-\alpha)Ke^{-r(T-t)} E^Q(H_T I_2)$$
$$= (1-\alpha)Ke^{-r(T-t)} E^Q(H_t e^{(r-y-\eta^2/2)(T-t)+\eta dW^Q} I_2)$$
$$= (1-\alpha)H_t Ke^{-y(T-t)} E^Q(\xi_H I_2)$$
$$= (1-\alpha)H_t Ke^{-y(T-t)} E^R(I_2) \quad \text{(A.5)}$$
$$= \frac{(1-\alpha)V_t K}{B+mV_t} e^{-y(T-t)} N(d_1, d_2; -\rho)$$

here,

$$H_t = \frac{V_t}{B+mV_t} \quad \text{(A.6)}$$

The closed-form solution (4) is naturally derived as follows:

$$C_t = A_1 - A_2 + A_3 - A_4$$
$$= S_t e^{-q(T-t)} N(b_1, b_2; \rho) - Ke^{-r(T-t)} N(a_1, a_2; \rho)$$
$$+ \frac{(1-\alpha)V_t}{B+mV_t}\left(S_t e^{(r-q-y+\rho\sigma\theta)(T-t)} N(c_1, c_2; -\rho) - Ke^{-y(T-t)} N(d_1, d_2; -\rho)\right)$$

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