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Chaos synchronization and parameter identification of three time scales brushless DC motor system

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Abstract

Chaotic anticontrol and chaos synchronization of brushless DC motor system are studied in this paper. Nondimensional dynamic equations of three time scale brushless DC motor system are presented. Using numerical results, such as phase diagram, bifurcation diagram, and Lyapunov exponent, periodic and chaotic motions can be observed. Then, chaos synchronization of two identical systems via additional inputs and Lyapunov stability theory are studied. And further, the parameter of the system is traced via adaptive control and random optimization method.

1. Introduction

Chaotic phenomena are observed in many physical systems widely. It is interesting of its apparent randomness and unpredictable behavior due to sensitive initial conditions.

Chaos synchronization has been studied extensively during the past two decades [1–3]. Traditionally, synchronization has been limited only for periodic signals. Now, chaotic signals can also be used for synchronization of either identical or different chaotic systems. Chaos synchronization has potential applications in such as construction of observer, information processing, and secure communication.

In this paper, we will present the brushless DC motor system (BLDCM), which is transformed to a nondimensionalized form at the beginning. Then, by applying the numerical results such as phase portrait, and bifurcation diagram, a variety of the phenomena of the chaotic motion can be presented. Furthermore, we discuss chaos synchronization in three aspects: the synchronization for systems with "unknown" parameters [6], the backstepping design [7], and the Gerschgorin's theorem method [8]. Finally, parameter identification via adaptive control [9] and random optimization [10] are investigated.

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2. Description of the three time scales brushless DC motor differential equations of motion

2.1. Description of brushless DC motor system and its differential equations of motion

The system considered here is shown in Fig. 1. The brushless DC motor (BLDCM) is an electromechanical system. Its equations of electrical dynamics can be described by [4,5]

$$\frac{d}{dt}i_q = \frac{1}{L_q}[-Ri_q - n\omega(L_d i_d - k_t) + v_q] \tag{2.1.1}$$

$$\frac{d}{dt}i_d = \frac{1}{L_d}[-Ri_d + nL_q\omega i_q + v_d] \tag{2.1.2}$$

and the equation of mechanical dynamics part is

$$\frac{d}{dt}\omega = \frac{1}{J}[T(I,\theta) - T_\ell(t)] \tag{2.1.3}$$

where

L_d, L_q: the fictitious inductance on the direct-axis and quadrature-axis,
v_d, v_q: the direct-axis and quadrature-axis voltage,
i_d, i_q: the direct-axis and quadrature-axis current,
n: number of permanent pole pairs,
ω: the rotor angular speed,
R: winding resistance,
J: the inertia momentum,
$k_t = \sqrt{\frac{3}{2}}k_e$: k_e is the permanent-magnet flux constant,
θ: the displacement variable,
$I = [i_q\ i_d]^T$.

$T_\ell(t)$ is the external torque caused by cogging and friction imposed on the shaft of the motor. If viscous damping is considered, then the external torque is

$$T_\ell = b\omega + T_L$$

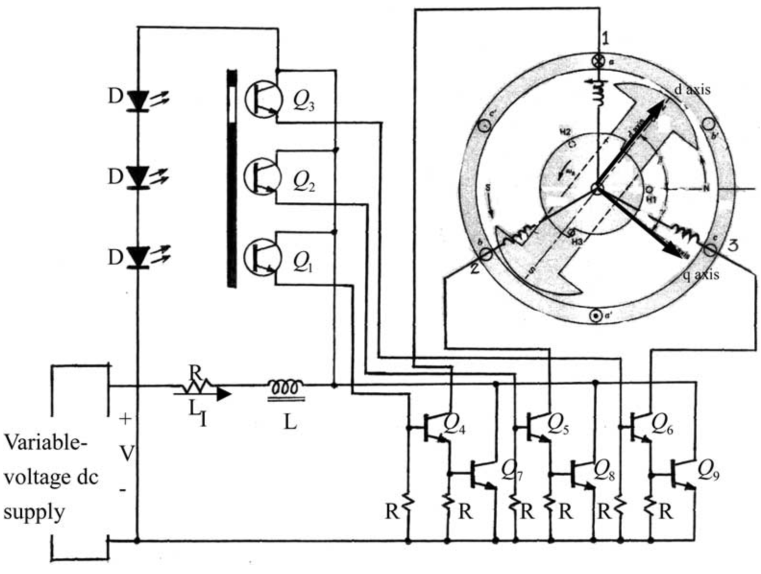

Fig. 1. Typical brushless DC motor and its commutation.

where

b: the viscous damping coefficient,
T_L: the additional terms such as external load, cogging effects, Coulomb friction, etc.

and θ is eliminated by transforming the motor dynamics to the rotating frame, the electromagnetic torque $T(I,\theta)$ is given by

$$T(i_q,i_d) = n[k_t i_q + (L_d - L_q)i_q i_d]$$

So we can get

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega = \frac{1}{J}[n[k_t i_q + (L_d - L_q)i_q i_d] + \frac{1}{J}(b\omega + T_{\mathrm{L}}) \tag{2.1.4}$$

2.2. Three time scales representation of equations of motion and the computational analysis

In this section, Eqs. (2.1.1), (2.1.2) and (2.1.4) will be transformed to another statespace model and it can reduce the number of system parameters [4]. The multiple time scales are τ_1, τ_2, and τ_3, where

$\tau_1 = \frac{L_q}{R}$: the first electrical time constant
$\tau_2 = \frac{L_d}{R}$: the second electrical time constant
$\tau_3 = \frac{JR}{k_t^2}$: the mechanical time constant

After transforming, the equations of motion are

$$\begin{aligned}
\tau_1\frac{\mathrm{d}}{\mathrm{d}t}x_1 &= V_q - x_1 - x_2x_3 - x_3 \\
\tau_2\frac{\mathrm{d}}{\mathrm{d}t}x_2 &= V_d + x_1x_3 - x_2 \\
\tau_3\frac{\mathrm{d}}{\mathrm{d}t}x_3 &= \sigma x_1 + \rho x_1x_2 - \eta x_3 - \widetilde{T}_{\mathrm{L}}
\end{aligned} \tag{2.2.1}$$

where the nondimensional variables are

$$x_1 = \frac{L_q}{k_t\sqrt{\delta}}i_q, \quad x_2 = \frac{L_q}{k_t\delta}i_d, \quad x_3 = \frac{nL_q}{R\sqrt{\delta}}\omega, \quad V_q = \frac{L_q}{k_tR\sqrt{\delta}}v_q, \quad V_d = \frac{L_q}{k_tR\delta}v_d,$$

$$\sigma = n^2, \quad \rho = (1-\delta)n^2, \quad \eta = \frac{Rb}{k_t^2}, \quad \tilde{t} = \frac{t}{\tau_3}, \quad \widetilde{T}_{\mathrm{L}} = \frac{nL_q}{k_t^2\sqrt{\delta}}T_{\mathrm{L}}, \quad \delta = \frac{L_q}{L_d}.$$

The period of autonomous system is hardly found, so we will modify the choice of Poincaré section for different inputs in the next section. Almost the same bifurcation diagrams are obtained, because we only adjust a few position of x_1 and x_3 axes from the original system. In addition, three time scales BLDCM is nondimensionalized, that means all inputs we added are dimensionless. If we change them to the original system, all inputs have their physical meanings.

We will show the computational results such as phase portrait, bifurcation diagram and Lyapunov exponents. Fig. 2 shows the phase portrait of various η. The motion is periodic for $\eta = 2.5$, 2.36, and for $\eta = 2.1$, 1.6 the motion is chaotic, and $\eta = 2.34$ is a critical value. Fig. 3 shows the bifurcation diagram and Lyapunov exponent. We observed that Lyapunov exponents $\lambda > 0$ which represent chaos in the figure.

3. Chaos synchronization of identical systems

In this section, we will show chaos synchronization by three different ways: the synchronization for systems with "unknown" parameters [6], the backstepping design [7], and the Gerschgorin's theorem method [8].

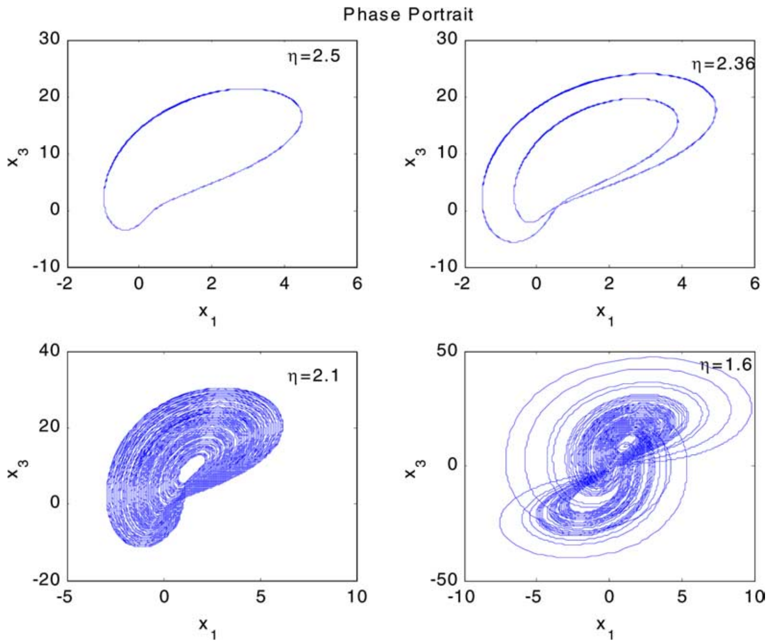

Fig. 2. Phase portrait with different η.

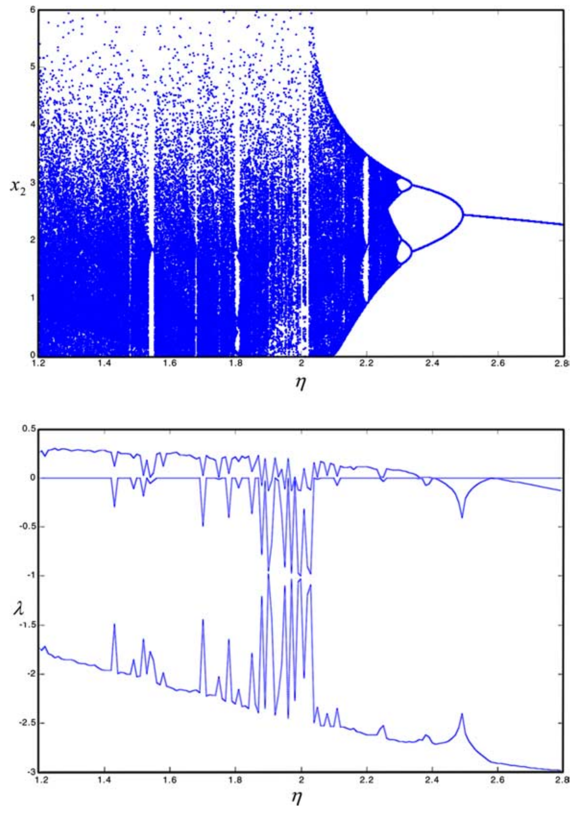

Fig. 3. Bifurcation diagram and Lyapunov exponents for three time scales system.

3.1. Synchronization of two identical systems with "unknown" parameters

We consider two identical three time scale BLDCM systems in this section. Both the parameters of the systems are unknown. The drive system is described as

$$\begin{aligned}\tau_1\frac{\mathrm{d}}{\mathrm{d}t}x_1 &= V_q - x_1 - y_1z_1 - z_1\\ \tau_2\frac{\mathrm{d}}{\mathrm{d}t}y_1 &= V_d + x_1z_1 - y_1\\ \tau_3\frac{\mathrm{d}}{\mathrm{d}t}z_1 &= \sigma x_1 + \rho x_1y_1 - \eta z_1 - \widetilde{T}_{\mathrm{L}}\end{aligned} \tag{3.1.1}$$

and the response system

$$\begin{aligned}\tau_1\frac{\mathrm{d}}{\mathrm{d}t}x_2 &= V_q - x_2 - y_2z_2 - z_2 + u_1\\ \tau_2\frac{\mathrm{d}}{\mathrm{d}t}y_2 &= V_d + x_2z_2 - y_2 + u_2\\ \tau_3\frac{\mathrm{d}}{\mathrm{d}t}z_2 &= \sigma x_2 + \rho x_2y_2 - \eta z_2 - \widetilde{T}_{\mathrm{L}} + u_3\end{aligned} \tag{3.1.2}$$

where u_1, u_2, u_3 are the controllers of the system. Subtracting Eq. (3.1.1) from Eq. (3.1.2), we can obtain the error dynamic equations

$$\begin{aligned}\tau_1\frac{\mathrm{d}}{\mathrm{d}t}e_1 &= e_2e_3 + e_2z_1 + e_3y_1 - e_1 - e_3 + u_1\\ \tau_2\frac{\mathrm{d}}{\mathrm{d}t}e_2 &= e_1e_3 + e_1z_1 + e_3x_1 - e_2 + u_2\\ \tau_3\frac{\mathrm{d}}{\mathrm{d}t}e_3 &= \sigma e_1 - \eta e_3 + \rho(e_1e_2 + e_1y_1 + e_2x_1) + u_3\end{aligned} \tag{3.1.3}$$

where $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_1$.

Then we choose the Lyapunov function of the form

$$V(e_1,e_2,e_3,\tilde{\sigma},\tilde{\rho}) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + \frac{1}{\tau_3}\tilde{\sigma}^2 + \frac{1}{\tau_3}\tilde{\rho}^2\right) > 0 \tag{3.1.4}$$

where $\tilde{\sigma} = \sigma - \hat{\sigma}$, $\tilde{\rho} = \rho - \hat{\rho}$. $\hat{\sigma}$, $\hat{\rho}$ are estimate values of the unknown parameters σ, ρ, and $\tilde{\sigma}$, $\tilde{\rho}$ are the errors. So we have the time derivative of $V(e_1,e_2,e_3,\tilde{\sigma},\tilde{\rho})$

$$\begin{aligned}\dot{V}(e_1,e_2,e_3,\tilde{\sigma},\tilde{\rho}) = {}&\frac{e_1}{\tau_1}(e_2e_3 + e_2z_1 + e_3y_1 - e_1 - e_3 + u_1) + \frac{e_2}{\tau_2}(e_1e_3 + e_1z_1 + e_3x_1 - e_2 + u_2)\\ &+ \frac{e_3}{\tau_3}(\sigma e_1 - \eta e_3 + \rho(e_1e_2 + e_1y_1 + e_2x_1) + u_3) + \frac{\tilde{\sigma}}{\tau_3}\dot{\tilde{\sigma}} + \frac{\tilde{\rho}}{\tau_3}\dot{\tilde{\rho}}\end{aligned} \tag{3.1.5}$$

We choose

$$u_1 = -(e_2e_3 + e_2z_1 + e_3y_1 - e_3)$$

$$u_2 = -(e_1e_3 + e_1z_1 + e_3x_1)$$

$$u_3 = -(\hat{\sigma}e_1 + \hat{\rho}(e_1e_2 + e_1y_1 + e_2x_1))$$

and $\dot{\tilde{\sigma}} = -\dot{\hat{\sigma}} = -e_1e_3$, $\dot{\tilde{\rho}} = -\dot{\hat{\rho}} = -e_3(e_1e_2 + e_1y_1 + e_2x_1)$, then Eq. (3.1.5) can be written as

$$\dot{V}(e_1,e_2,e_3) = -\frac{1}{\tau_1}e_1^2 - \frac{1}{\tau_2}e_2^2 - \frac{\eta}{\tau_3}e_3^2 < 0, \quad \text{for } \tau_1,\tau_2,\tau_3,\eta > 0$$

In numerical simulation, we choose the parameters as $V_q = 4.017$, $V_d = -15.305$, $\tau_1 = 6.45$, $\tau_2 = 7.125$, $\tau_3 = 1$, $\widetilde{T}_{\mathrm{L}} = 2.678$, $\eta = 2.1$, and the true values of "unknown" parameters are $\sigma = 16$, $\rho = 1.516$ for chaos condition. The initial conditions of the drive and response systems are $x_1(0) = y_1(0) = z_1(0) = 1$, $x_2(0) = y_2(0) = z_2(0) = 7$, and estimate value of "unknown" parameters $\hat{\sigma}(0) = 5$, $\hat{\rho}(0) = 0.1$, respectively. The results of synchronization are shown in Figs. 4–6, that means two identical three scale BLDCM systems can be synchronized via adaptive control with some unknown parameters.

3.2. Synchronization by backstepping design

We also use two identical three time scale BLDCM systems in this section. To synchronize two systems, the drive system is described as Eq. (3.1.1), and the response system is

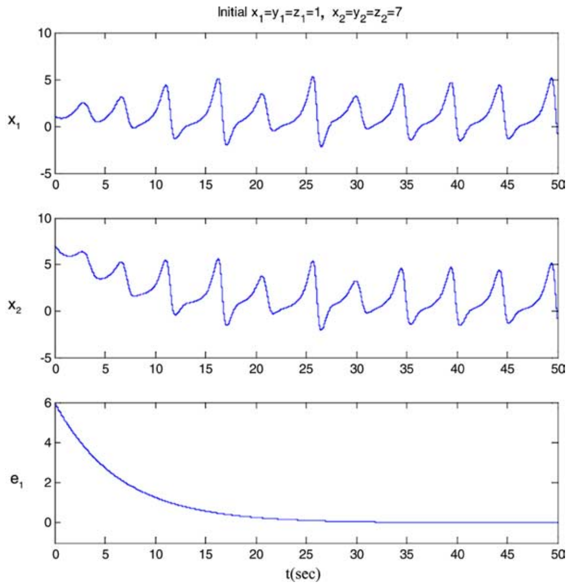

Fig. 4. Time histories of x_1, x_2, and their error.

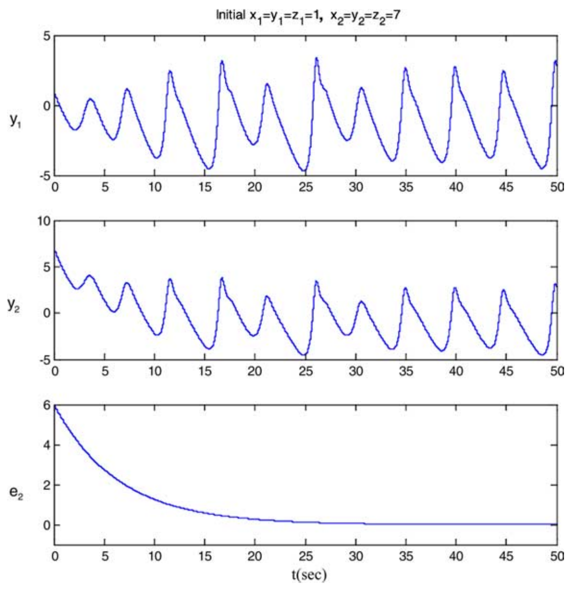

Fig. 5. Time histories of y_1, y_2, and their error.

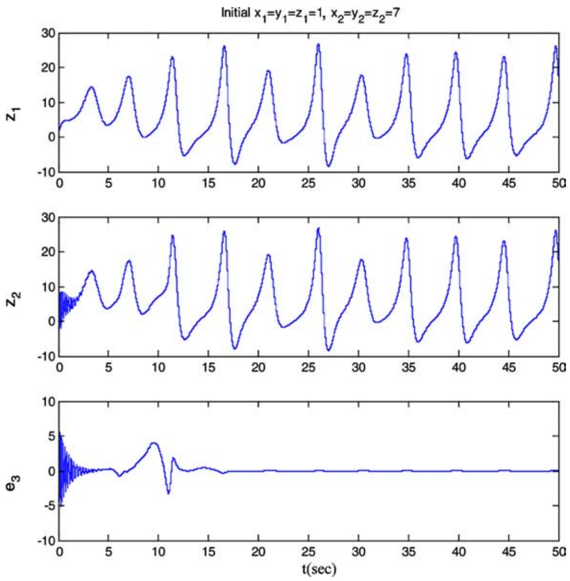

Fig. 6. Time histories of z_1, z_2, and their error.

$$\begin{aligned}\tau_1\frac{\mathrm{d}}{\mathrm{d}t}x_2 &= V_q - x_2 - y_2z_2 - z_2\\ \tau_2\frac{\mathrm{d}}{\mathrm{d}t}y_2 &= V_d + x_2z_2 - y_2 + u_2\\ \tau_3\frac{\mathrm{d}}{\mathrm{d}t}z_2 &= \sigma x_2 + \rho x_2y_2 - \eta z_2 - \widetilde{T}_{\mathrm{L}} + u_3\end{aligned} \tag{3.2.1}$$

where u_2, u_3 are the controllers of the system. Subtracting Eq. (3.1.1) from Eq. (3.2.1), we can obtain the error dynamic equations

$$\begin{aligned}\tau_1\frac{\mathrm{d}}{\mathrm{d}t}e_1 &= e_2e_3 + e_2z_1 + e_3y_1 - e_1 - e_3\\ \tau_2\frac{\mathrm{d}}{\mathrm{d}t}e_2 &= e_1e_3 + e_1z_1 + e_3x_1 - e_2 + u_2\\ \tau_3\frac{\mathrm{d}}{\mathrm{d}t}e_3 &= \sigma e_1 - \eta e_3 + \rho(e_1e_2 + e_1y_1 + e_2x_1) + u_3\end{aligned} \tag{3.2.2}$$

where $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_1$, namely, $x_2 = e_1 + x_1$, $y_2 = e_2 + y_1$, $z_2 = e_3 + z_1$.

Now, variables x_1, y_1, z_1 in the error dynamic equations (3.2.2) can be considered as input signals from the master system. Without u_2 and u_3, the error dynamic equations (3.2.2) has an equilibrium point $(0,0,0)$, which means some u_2 and u_3 would not change the equilibrium point if we select u_2 and u_3 carefully. So, the synchronization problem will change to stabilization of the error dynamics.

First, we consider the stability of the first equation of Eq. (3.2.2)

$$\tau_1\frac{\mathrm{d}}{\mathrm{d}t}e_1 = e_2e_3 + e_2z_1 + e_3y_1 - e_1 - e_3 \tag{3.2.3}$$

where e_2 and e_3 are controllers.

Then, by choosing a Lyapunov function

$$V_1(e_1) = \frac{1}{2}e_1^2 > 0 \tag{3.2.4}$$

Its time derivative is

$$\dot{V}_1(e_1) = -\frac{1}{\tau_1}e_1^2 + \frac{1}{\tau_1}e_1[e_2z_1 + (e_2 + y_1 - 1)e_3] \tag{3.2.5}$$

Assuming controllers $e_2 = \alpha_1(e_1)$, $e_3 = \alpha_2(e_1)$, Eq. (3.2.5) can be written as

$$\dot{V}_1(e_1) = -\frac{1}{\tau_1}e_1^2 + \frac{1}{\tau_1}e_1[\alpha_1 z_1 + (\alpha_1 + y_1 - 1)\alpha_2] \tag{3.2.6}$$

If $\alpha_1(e_1) = \alpha_2(e_2) = 0$, Eq. (3.3.6) can be written as

$$\dot{V}_1(e_1) = -\frac{1}{\tau_1}e_1^2 < 0$$

This means that the zero solution of Eq. (3.2.3) is asymptotically stable.

When e_2 and e_3 are considered as controllers, $\alpha_1(e_1)$ and $\alpha_2(e_1)$ are estimative functions.

Defining

$$\begin{aligned} w_2 &= e_2 - \alpha_1(e_1) \\ w_3 &= e_3 - \alpha_2(e_1) \end{aligned} \tag{3.2.7}$$

then we study the (e_1, w_2, w_3) system

$$\begin{aligned} \tau_1 \frac{\mathrm{d}}{\mathrm{d}t}e_1 &= w_2 w_3 + w_2 z_1 + w_3 y_1 - e_1 - w_3 \\ \tau_2 \frac{\mathrm{d}}{\mathrm{d}t}w_2 &= e_1 w_3 + e_1 z_1 + w_3 x_1 - w_2 + u_2 \\ \tau_3 \frac{\mathrm{d}}{\mathrm{d}t}w_3 &= \sigma e_1 - \eta w_3 + \rho(e_1 w_2 + e_1 y_1 + w_2 x_1) + u_3 \end{aligned} \tag{3.2.8}$$

We choose a Lyapunov function

$$V_2(e_1,w_2,w_3) = V_1(e_1) + \frac{1}{2}w_2^2 + \frac{1}{2}w_3^2$$

Its time derivative is

$$\dot{V}_2(e_1,w_2,w_3) = \frac{-1}{\tau_1}e_1^2 + \frac{w_2}{\tau_2}(e_1 w_3 + e_1 z_1 + w_3 x_1 - w_2 + u_2) + \frac{w_3}{\tau_3}(\sigma e_1 - \eta w_3 + \rho(e_1 w_2 + e_1 y_1 + w_2 x_1) + u_3) \tag{3.2.9}$$

From here, we design the controllers

$$\begin{aligned} u_2 &= -(e_1 w_3 + e_1 z_1 + w_3 x_1) \\ u_3 &= -(\sigma e_1 + \rho(e_1 w_2 + e_1 y_1 + w_2 x_1)) \end{aligned}$$

Then Eq. (3.2.9) can be written as

$$\dot{V}(e_1,w_2,w_3) = \frac{-1}{\tau_1}e_1^2 - \frac{1}{\tau_2}w_2^2 - \frac{\eta}{\tau_3}w_3^2 < 0 \quad \text{for } \tau_1,\tau_2,\tau_3,\eta > 0$$

That means the zero solution of Eq. (3.2.8) is asymptotically stable. By addition of u_2 and u_3, the equilibrium point $(0,0,0)$ of the error dynamics Eq. (3.2.8) is unchanged.

For numerical simulation, we choose the same parameters as in Section 3.1, and $\sigma = 16$, $\rho = 1.516$ for chaos condition. The initial conditions of the drive and response systems are $x_1(0) = y_1(0) = z_1(0) = 1$, $x_2(0) = y_2(0) = z_2(0) = 7$, respectively.

The results of synchronization are shown in Figs. 7–9, that means two identical three scales BLDCM systems can be synchronized via backstepping design.

3.3. Synchronization by Gerschgorin's theorem

We consider two identical three time scale BLDCM systems in this section. The drive system is described as Eq. (3.1.1), and the slave system is described as Eq. (3.1.2).

To synchronizing two identical systems, we add three coupling terms for controllers, $u_1 = k_1(x_1 - x_2)$, $u_2 = k_2(y_1 - y_2)$, $u_3 = k_3(z_1 - z_2)$, and Eq. (3.1.2) can be described as

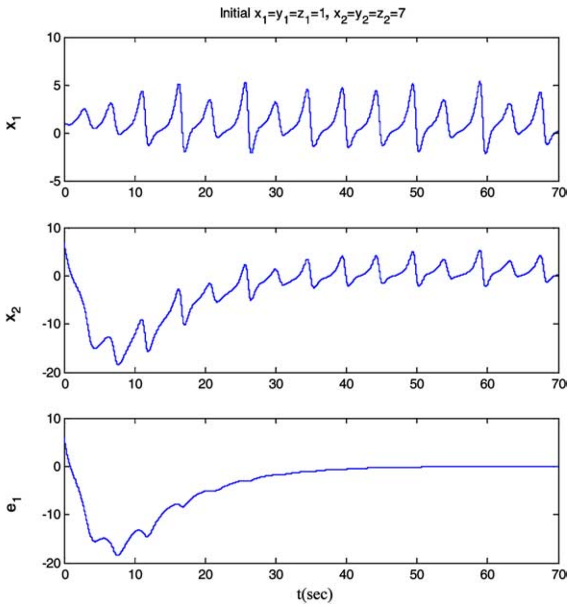

Fig. 7. Time histories of x_1, x_2, and their error.

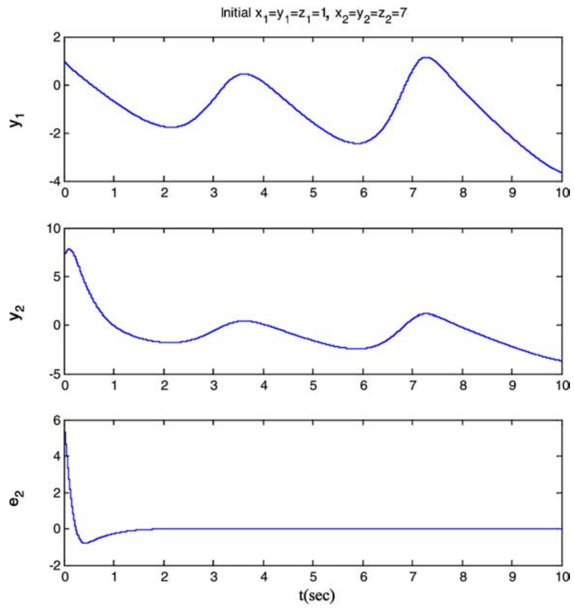

Fig. 8. Time histories of y_1, y_2, and their error.

$$
\begin{aligned}
\tau_1 \frac{d}{dt} x_2 &= V_q - x_2 - y_2 z_2 - z_2 + k_1(x_1 - x_2) \\
\tau_2 \frac{d}{dt} y_2 &= V_d + x_2 z_2 - y_2 + k_2(y_1 - y_2) \\
\tau_3 \frac{d}{dt} z_2 &= \sigma x_2 + \rho x_2 y_2 - \eta z_2 - \widetilde{T}_L + k_3(z_1 - z_2)
\end{aligned}
\tag{3.3.1}
$$

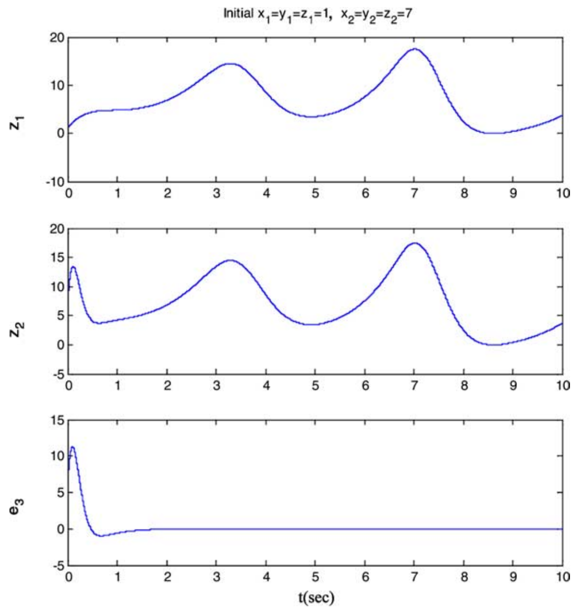

Fig. 9. Time histories of z_1, z_2, and their error.

The drive and response systems can be written as

$$\begin{aligned}\dot{X}_1 &= \mathbf{A}X_1 + g(X_1)\\ \dot{X}_2 &= \mathbf{A}X_2 + g(X_2) + \mathbf{K}(X_1 - X_2)\end{aligned} \tag{3.3.2}$$

where $\mathbf{A} \in R^{n\times n}$ is a constant matrix, $\mathbf{g(X)}$ is a nonlinear function, and $\mathbf{u} \in R^n$ is the external input vector.

Assuming that

$$\mathbf{g}(\mathbf{X}_1) - \mathbf{g}(\mathbf{X}_2) = \mathbf{M}_{\mathbf{X}_1,\mathbf{X}_2}(\mathbf{X}_1 - \mathbf{X}_2) \tag{3.3.3}$$

where the elements in $\mathbf{M}_{\mathbf{X}_1,\mathbf{X}_2}$ are dependent on $\mathbf{X}_1$ and $\mathbf{X}_2$.

From Eq. (3.3.2), we can obtain the error dynamic equation

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{X}_1,\mathbf{X}_2})\mathbf{e}, \tag{3.3.4}$$

where $\mathbf{e} = \mathbf{X}_1 - \mathbf{X}_2$.

We choose $\mathbf{P} = \mathbf{diag}(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n)$, a positive definite diagonal matrix and the Lyapunov function

$$\mathbf{V} = \mathbf{e}^\mathbf{T}\mathbf{P}\mathbf{e} > 0.$$

Its derivative is

$$\begin{aligned}\dot{\mathbf{V}} &= \dot{\mathbf{e}}^\mathbf{T}\mathbf{P}\mathbf{e} + \mathbf{e}^\mathbf{T}\mathbf{P}\dot{\mathbf{e}} = [(\mathbf{A} - \mathbf{K})\mathbf{e} + \mathbf{g}(\mathbf{X}_1) - \mathbf{g}(\mathbf{X}_2)]^\mathbf{T}\mathbf{P}\mathbf{e} + \mathbf{e}^\mathbf{T}\mathbf{P}[(\mathbf{A} - \mathbf{K})\mathbf{e} + \mathbf{g}(\mathbf{X}_1) - \mathbf{g}(\mathbf{X}_2)]\\ &= \mathbf{e}^\mathbf{T}[(\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{X}_1,\mathbf{X}_2})^\mathbf{T}\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{X}_1,\mathbf{X}_2})]\mathbf{e} = \mathbf{e}^\mathbf{T}\mathbf{Q}\mathbf{e}\end{aligned} \tag{3.3.5}$$

where $\mathbf{Q} = (\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{X}_1,\mathbf{X}_2})^\mathbf{T}\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{X}_1,\mathbf{X}_2})$.

Rewrite $\mathbf{Q}$ as

$$\begin{aligned}\mathbf{Q} &= (\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{x},\mathbf{y}})^\mathbf{T}\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{x},\mathbf{y}}) = [\mathbf{P}(\mathbf{A} + \mathbf{M}_{\mathbf{x},\mathbf{y}}) + (\mathbf{A} + \mathbf{M}_{\mathbf{x},\mathbf{y}})^\mathbf{T}\mathbf{P}] - [\mathbf{P}\mathbf{K} + \mathbf{K}^\mathbf{T}\mathbf{P}]\\ &= [\bar{a}_{ij}] - [b_{ij}]\end{aligned} \tag{3.3.6}$$

where $[b_{ij}] = \mathrm{diag}(2k_1p_1, 2k_2p_2, \ldots, 2k_n\ \mathrm{p}n)$.

Gerschgorin theorem guarantees that each eigenvalue of $\mathbf{Q}$, when plotted in the complex plane, must lie on or within Gerschgorin's circle. The center of circle is $\bar{a}_{ii} - 2k_ip_i$, the radii are r_i, where $r_i = \sum_{j=1,j\neq i}^{n}|\bar{a}_{ij}|$.

Since $\mathbf{Q} = \mathbf{Q}^\mathrm{T}$, if all eigenvalues of $\mathbf{Q}$ are negative, $\dot{V}$ would be negative definite. This means that the error dynamics Eq. (3.3.4) would be asymptotically stable about $(0,0,0)$. In the other word, two identical BLDCM systems would be synchronized.

To achieve synchronization, we assume all eigenvalues of $\mathbf{Q}$ are negative

$$\lambda_i \leqslant \mu < 0, \quad i = 1,2,\ldots,n \tag{3.3.7}$$

where μ is a negative constant.

From Gerschgorin theorem and Eq. (3.3.7), we can get that $\bar{a}_{ii} - 2k_i p_i + r_i \leqslant \mu$, and the range of k_i can be obtained.

$$k_i \geqslant \frac{1}{2p_i}(\bar{a}_{ii} + r_i - \mu), \quad i = 1,2,\ldots,n \tag{3.3.8}$$

Choosing $\mathbf{P} = \mathbf{I}$, Eq. (3.3.8) can be rewritten as

$$k_i \geqslant \frac{1}{2}(\bar{a}_{ii} + r_i - \mu), \quad i = 1,2,\ldots,n \tag{3.3.9}$$

Considering two identical three time scale BLDCM systems, we can obtain

$$\mathbf{A} = \begin{bmatrix} \frac{-1}{\tau_1} & 0 & \frac{-1}{\tau_1} \\ 0 & \frac{-1}{\tau_2} & 0 \\ \frac{\sigma}{\tau_3} & 0 & \frac{-\eta}{\tau_3} \end{bmatrix}, \quad \mathbf{M}_{X_1,X_2} = \begin{bmatrix} 0 & \frac{-1}{\tau_1} z_2 & \frac{-1}{\tau_1} y_1 \\ \frac{1}{\tau_2} z_2 & 0 & \frac{-1}{\tau_2} x_1 \\ 0 & \frac{-\rho}{\tau_3} x_1 & \frac{-\rho}{\tau_3} y_2 \end{bmatrix} \tag{3.3.10}$$

$$[a_{ij}] = \mathbf{P}(\mathbf{A} + \mathbf{M}) + (\mathbf{A} + \mathbf{M})^{\mathrm{T}}\mathbf{P} = \begin{bmatrix} \frac{-2}{\tau_1} & \left(\frac{-1}{\tau_1} + \frac{1}{\tau_2}\right) z_2 & \frac{-1}{\tau_1}(y_1 + 1) + \frac{1}{\tau_3}\sigma \\ \left(\frac{-1}{\tau_1} + \frac{1}{\tau_2}\right) z_2 & \frac{-2}{\tau_2} & \left(\frac{1}{\tau_2} + \frac{1}{\tau_3}\right) x_1 \\ \frac{-1}{\tau_1}(y_1 + 1) + \frac{1}{\tau_3}\sigma & \left(\frac{1}{\tau_2} + \frac{1}{\tau_3}\right) x_1 & \frac{2}{\tau_2}(y_2 - \eta) \end{bmatrix} \tag{3.3.11}$$

By choosing $\mu = -1$, we obtain the coupling strength as $k_1 = 24$, $k_2 = 12.5$, $k_3 = 6$. The results are shown in Figs. 10–12, and the synchronization is achieved.

4. Parameter identification

We consider the parameters identification in this section. Two methods are presented: the adaptive control [9], and the random optimization method [10].

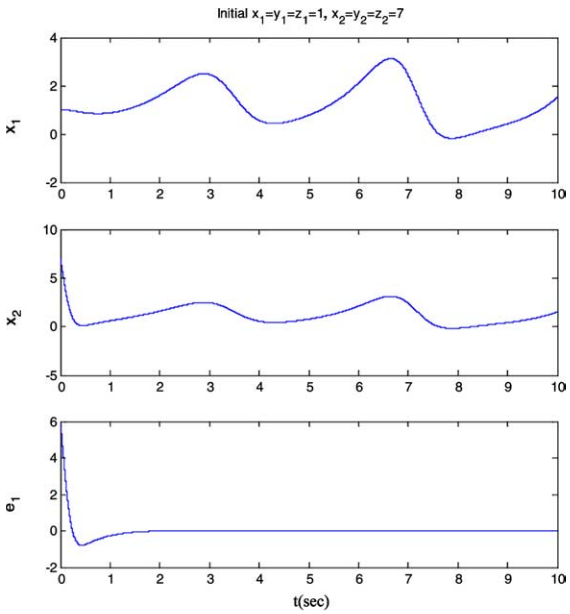

Fig. 10. Time histories of x_1, x_2, and their error.

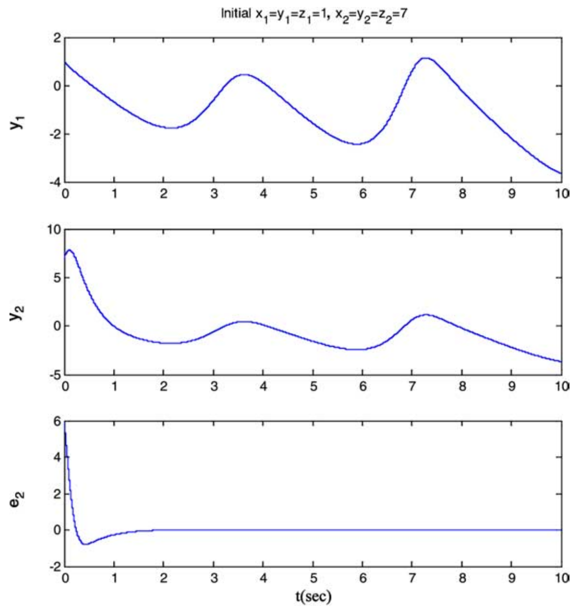

Fig. 11. Time histories of y_1, y_2, and their error.

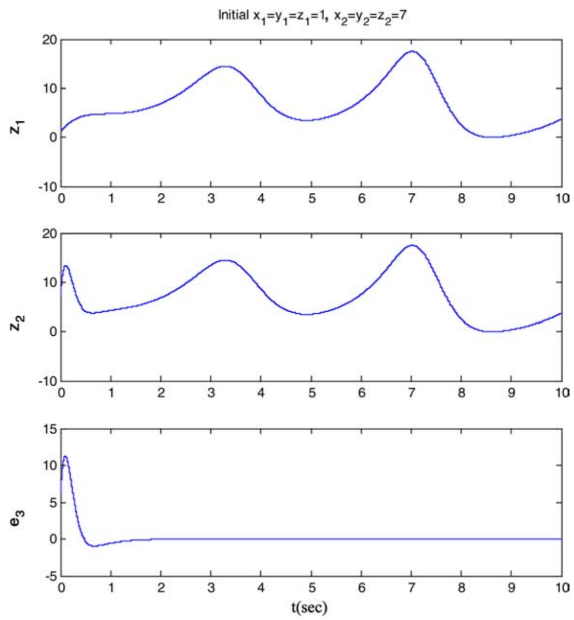

Fig. 12. Time histories of z_1, z_2, and their error.

4.1. Parameter identification by adaptive control

We discuss two identical BLDCM systems in this section. Both systems have the same unknown parameters, and one parameter of the slave system is uncertain. Our object is to identify the uncertain parameter. The drive system is described as

$$\begin{aligned}
\tau_1 \frac{\mathrm{d}}{\mathrm{d}t} x_1 &= V_q - x_1 - y_1 z_1 - z_1 \\
\tau_2 \frac{\mathrm{d}}{\mathrm{d}t} y_1 &= V_d + x_1 z_1 - y_1 \\
\tau_3 \frac{\mathrm{d}}{\mathrm{d}t} z_1 &= \sigma x_1 + \rho x_1 y_1 - \eta z_1 - \widetilde{T}_{\mathrm{L}}
\end{aligned} \tag{4.1.1}$$

and the response system

$$\begin{aligned}
\tau_1 \frac{\mathrm{d}}{\mathrm{d}t} x_2 &= V_q - x_2 - y_2 z_2 - z_2 + u_1 \\
\tau_2 \frac{\mathrm{d}}{\mathrm{d}t} y_2 &= V_d + x_2 z_2 - y_2 + u_2 \\
\tau_3 \frac{\mathrm{d}}{\mathrm{d}t} z_2 &= \sigma x_2 + \rho x_2 y_2 - \eta_h z_2 - \widetilde{T}_{\mathrm{L}} + u_3
\end{aligned} \tag{4.1.2}$$

where u_1, u_2, u_3 are the controllers of the system, and η_h is the only unknown parameter.

Before solving this problem, we consider a special case first. If the drive and response systems' parameters are the same, that is $\eta_h = \eta$, and the error equations are

$$\begin{aligned}
\tau_1 \frac{\mathrm{d}}{\mathrm{d}t} e_1 &= e_2 e_3 + e_2 z_1 + e_3 y_1 - e_1 - e_3 + u_1 \\
\tau_2 \frac{\mathrm{d}}{\mathrm{d}t} e_2 &= e_1 e_3 + e_1 z_1 + e_3 x_1 - e_2 + u_2 \\
\tau_3 \frac{\mathrm{d}}{\mathrm{d}t} e_3 &= \sigma e_1 - \eta e_3 + \rho(e_1 e_2 + e_1 y_1 + e_2 x_1) + u_3
\end{aligned} \tag{4.1.3}$$

where $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_1$.

Then we choose the Lyapunov function of the form

$$V_1(e_1,e_2,e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) > 0 \tag{4.1.4}$$

and we choose

$$\begin{aligned}
u_1 &= -(e_2 e_3 + e_2 z_1 + e_3 y_1 - e_3) \\
u_2 &= -(e_1 e_3 + e_1 z_1 + e_3 x_1) \\
u_3 &= -[\sigma e_1 + \rho(e_1 e_2 + e_1 y_1 + e_2 x_1) + (1 - \eta) e_3]
\end{aligned} \tag{4.1.5}$$

so we can get

$$\dot{V}_1(e_1,e_2,e_3) = \frac{-1}{\tau_1} e_1^2 - \frac{1}{\tau_2} e_2^2 - \frac{\eta}{\tau_3} e_3^2 < 0$$

Now if one of the response system parameters, η_h will be $\eta_h = \eta_h(t)$, the third equation of Eq. (4.1.5) will change to

$$u_3 = -[\sigma e_1 + \rho(e_1 e_2 + e_1 y_1 + e_2 x_1) + (1 - \eta_h(t)) e_3]$$

We choose the Lyapunov equation

$$V_2(e_1,e_2,e_3,\tilde{\eta}) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + \tilde{\eta}^2) > 0$$

where $\tilde{\eta} = \eta_h - \eta$, and its derivative is

$$\begin{aligned}
\dot{V}_2(e_1,e_2,e_3,\tilde{\eta}) &= \frac{e_1}{\tau_1}(e_2 e_3 + e_2 z_1 + e_3 y_1 - e_1 - e_3 + u_1) + \frac{e_2}{\tau_2}(e_1 e_3 + e_1 z_1 + e_3 x_1 - e_2 + u_2) \\
&\quad + \frac{e_3}{\tau_3}(\sigma e_1 - \eta_h e_3 + \rho(e_1 e_2 + e_1 y_1 + e_2 x_1) + u_3) + \tilde{\eta}\dot{\eta}_h \\
&= \frac{-1}{\tau_1} e_1^2 - \frac{1}{\tau_2} e_2^2 + \frac{e_3}{\tau_3}(-e_3 - \tilde{\eta} z) + \tilde{\eta}\dot{\eta}_h
\end{aligned} \tag{4.1.6}$$

We take $\dot{\eta}_h = \frac{1}{\tau_3} z e_3 + (\eta_h - \eta)$, and Eq. (4.1.6) can be described as

$$\dot{V}(e_1,e_2,e_3,\tilde{\eta}) = \frac{-1}{\tau_1} e_1^2 - \frac{1}{\tau_2} e_2^2 - \frac{1}{\tau_3} e_3^2 - \tilde{\eta}^2 < 0 \quad \text{for } \tau_1,\tau_2,\tau_3 > 0$$

In numerical simulation, the initial conditions of the drive and response systems are $x_1(0) = y_1(0) = z_1(0) = 1$, $x_2(0) = y_2(0) = z_2(0) = 7$, and $\eta_h(0) = 0$, respectively. The result of parameter identification is shown in Fig. 13, and chaos synchronization are shown in Figs. 14–16. With the specific controller and parameters estimate update law, a parameter can be identified and two systems can be synchronized.

4.2. Parameter identification by random optimization

We consider two identical three time scale BLDCM systems in this section. Both systems have the same parameters, but one of three parameters of the slave system are unknown. Our object is to identify the unknown parameters. The drive system is described as

$$\begin{aligned}
\tau_1 \frac{\mathrm{d}}{\mathrm{d}t} x_1 &= V_q - x_1 - y_1 z_1 - z_1 \\
\tau_2 \frac{\mathrm{d}}{\mathrm{d}t} y_1 &= V_d + x_1 z_1 - y_1 \\
\tau_3 \frac{\mathrm{d}}{\mathrm{d}t} z_1 &= \sigma x_1 + \rho x_1 y_1 - \eta z_1 - \tilde{T}_{\mathrm{L}}
\end{aligned} \tag{4.2.1}$$

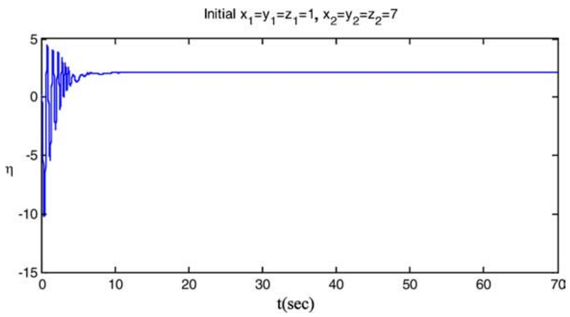

Fig. 13. Time histories of η_h.

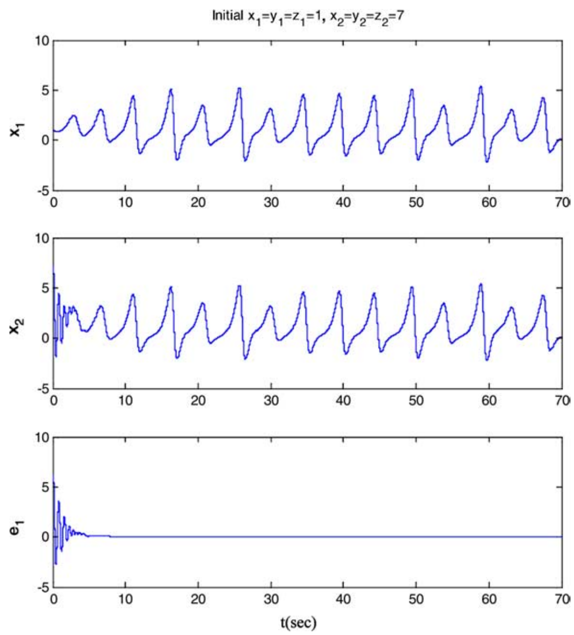

Fig. 14. Time histories of x_1, x_2, and their error.

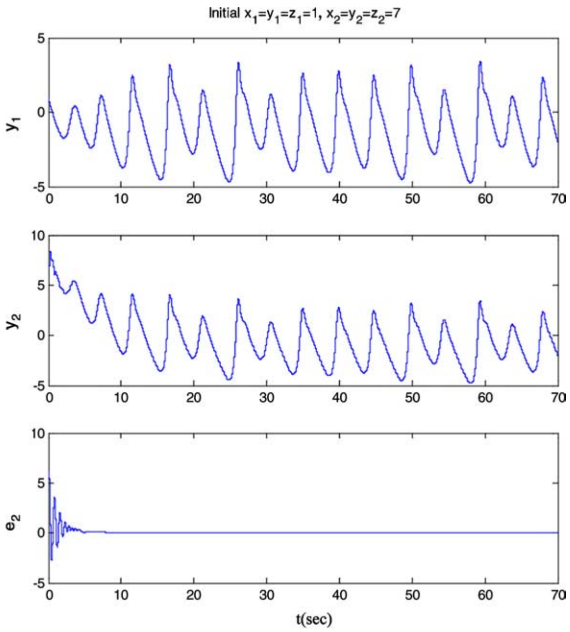

Fig. 15. Time histories of y_1, y_2, and their error.

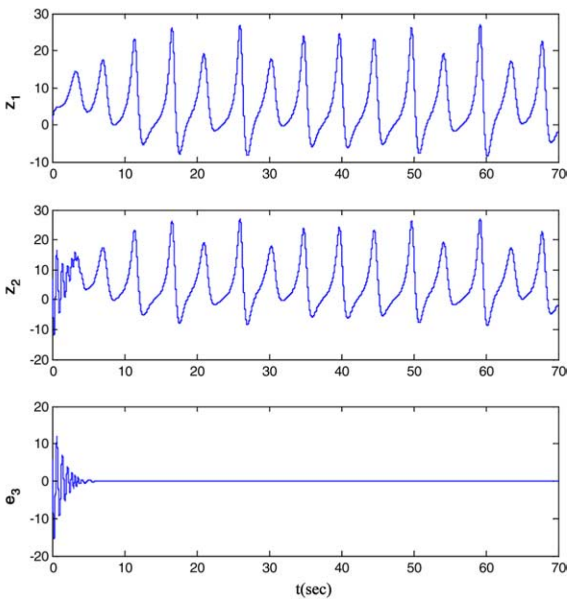

Fig. 16. Time histories of z_1, z_2, and their error.

and the response system is

$$
\begin{aligned}
&\tau_1 \frac{\mathrm{d}}{\mathrm{d}t} x_2 = V_q - x_2 - y_2 z_2 - z_2 + u_1 = V_q - x_2 - y_2 z_2 - z_2 + k(x_1 - x_2) \\
&\tau_2 \frac{\mathrm{d}}{\mathrm{d}t} y_2 = V_d + x_2 z_2 - y_2 \\
&\tau_3 \frac{\mathrm{d}}{\mathrm{d}t} z_2 = \sigma' x_2 + \rho' x_2 y_2 - \eta' z_2 - \tilde{T}_{\mathrm{L}}
\end{aligned}
\tag{4.2.2}
$$

Eqs. (4.2.1) and (4.2.2) can be written as

$$\begin{aligned}\dot{\mathbf{x}}_1 &= \mathbf{f}(\mathbf{x},\{p\})\\ \dot{\mathbf{x}}_2 &= \mathbf{f}(\mathbf{y},\{p'\}) + \mathbf{K}(\mathbf{x}_1 - \mathbf{x}_2)\end{aligned} \tag{4.2.3}$$

where $\{p\} = \{\sigma, \rho, \eta\}$, $\{p'\} = \{\sigma', \rho', \eta'\}$ are parameter sets, and $\mathbf{K} = [k \quad 0 \quad 0]$.

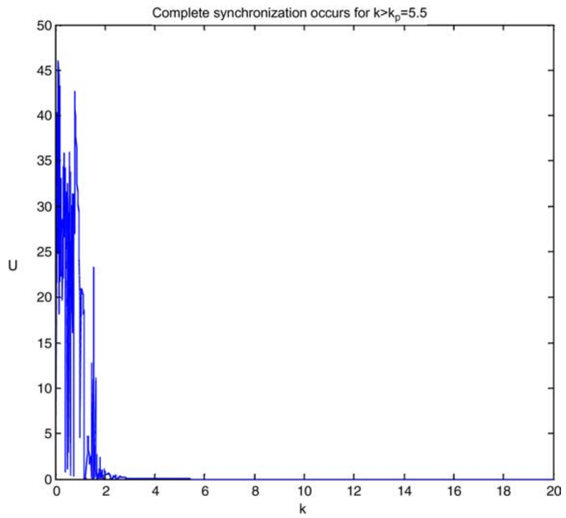

Fig. 17. Difference with respect to the coupling strength k.

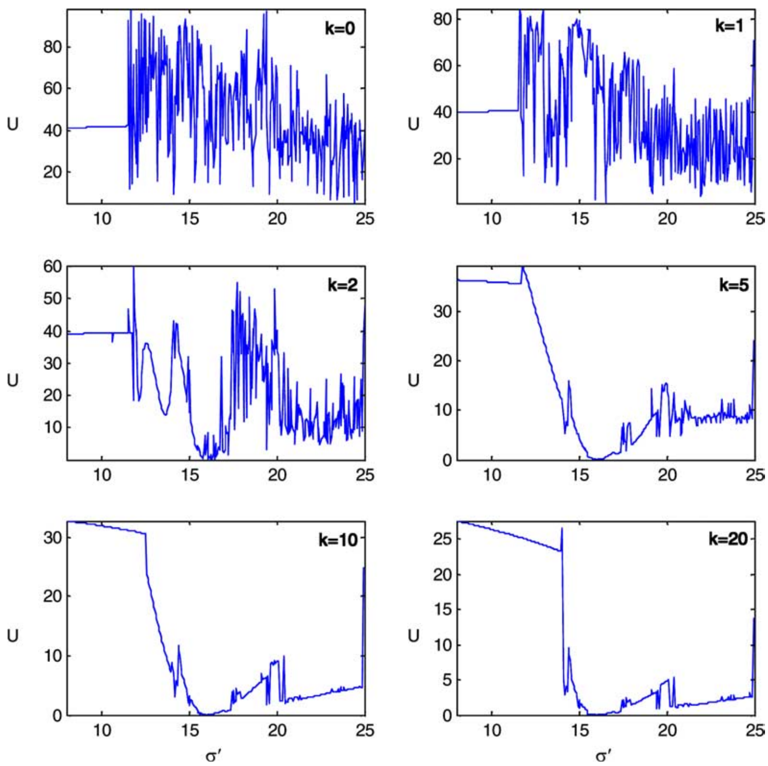

Fig. 18. Difference with respect to the parameter σ' for different k.

Define the difference by

$$U = \int_{0.95T}^{T} |x_1 - x_2|^2 \mathrm{d}t \tag{4.2.4}$$

where T is the simulation time.

With the same parameter sets, $\{p'\} = \{p\}$, the synchronization can be achieved for $k > k_p$. In numerical simulation, we obtain that $k_p = 5.50$. The numerical result is shown in Fig. 17.

The difference U can be considered as a function of one parameter of $\{p'\}$ and k. If k is sufficiently large and one of $\{p'\}$ is close to that of $\{p\}$, the difference U would tend to zero. In the other words, with sufficiently large value of k, if U is small, one of $\{p'\}$ would be close to that of $\{p\}$. In numerical simulation, we assume that only one parameter of $\{p'\}$ is unknown. The result is shown in Figs. 18–20.

To identify the unknown parameters of the slave system, we use the random optimization method. The algorithm is as follows.

First, choose a sufficiently large value of k. In our case, we choose $k = 20$. By estimating each initial value of $\{p'\}$, we can calculate the difference U.

Each parameter p' (one of σ', ρ', η') in the parameter set $\{p'\}$ is randomly modified as

$$p'_m = p' + r \tag{4.2.5}$$

where r is a random number which obeys the Gaussian distribution with variance $\sigma = 0.01$.

Substituting the modified parameter p'_m into Eq. (4.2.3), we can obtain x'_2. The difference between two systems is

$$U' = \int_{0.95T}^{T} |x_1 - x'_2|^2 \mathrm{d}t \tag{4.2.6}$$

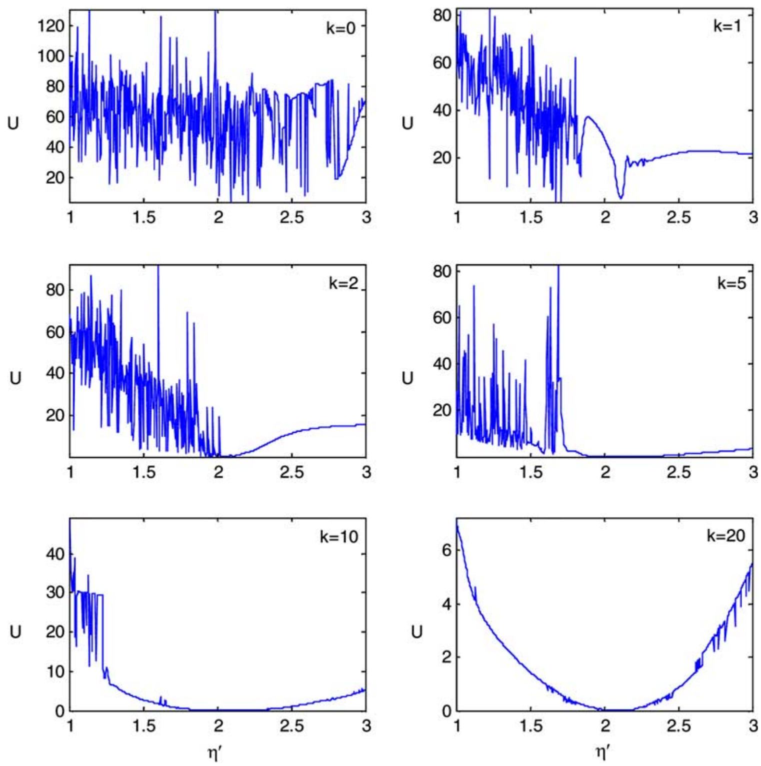

Fig. 19. Difference with respect to the parameter η' for different k.

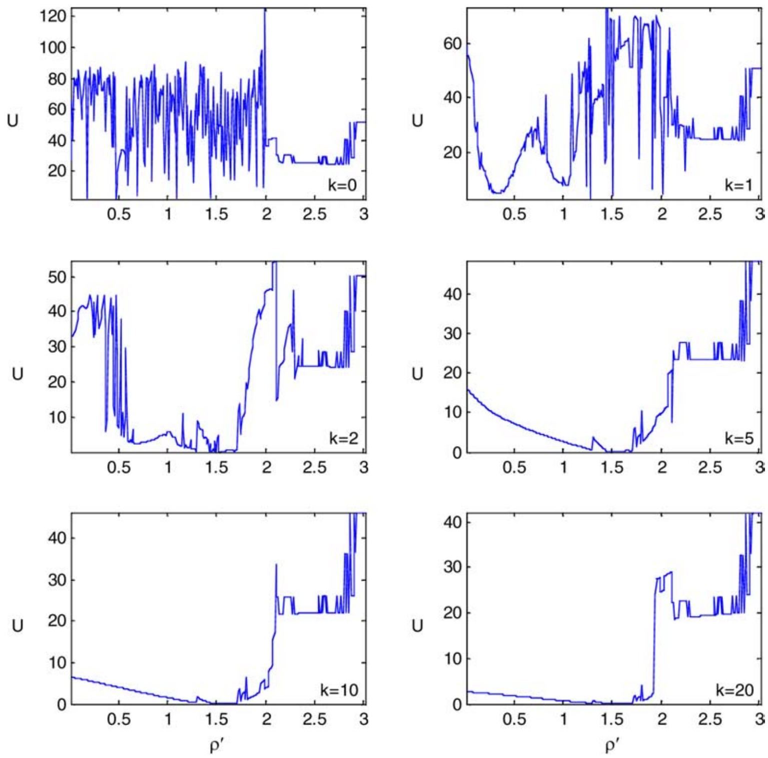

Fig. 20. Difference with respect to the parameter ρ' for different k.

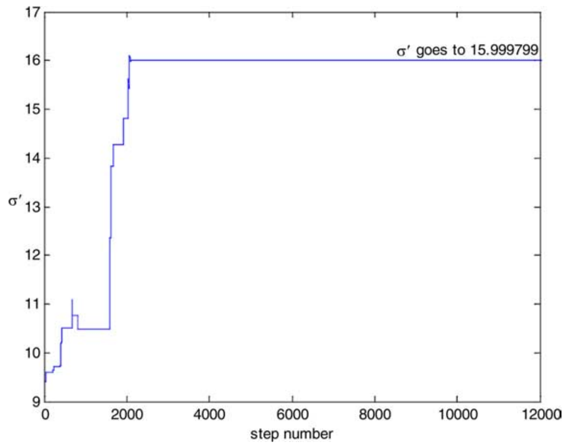

Fig. 21. Time evolution of σ' by random optimization process.

If the difference U' is smaller than U, the parameter is changed from p' to p'_m. On the other hand, if the difference U' is larger than U, the parameter is unchanged and kept to be p'. The processes are repeated until the difference U tends to zero.

In numerical simulation, we assume that only one parameter of $\{p'\}$ is unknown. Parameters identification can be achieved. The result is shown in Figs. 21–23.

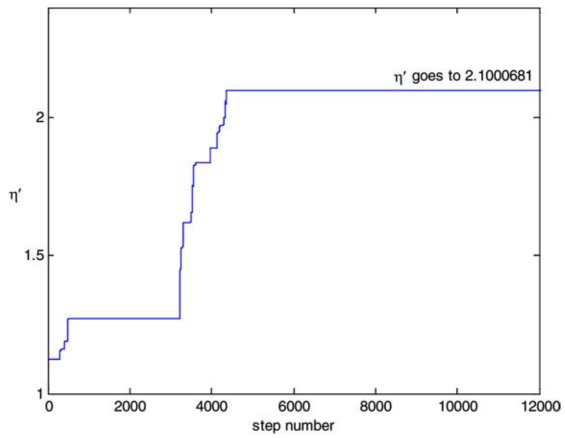

Fig. 22. Time evolution of η' by random optimization process.

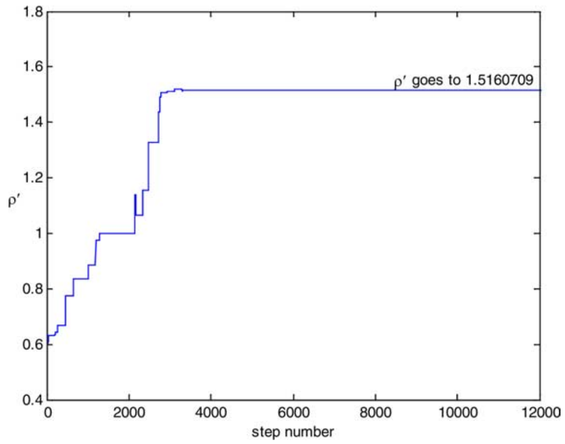

Fig. 23. Time evolution of ρ' by random optimization process.

5. Conclusions

First, we present the brushless DC motor (BLDCM) system that is transformed to a nondimensionalized form, and study the behavior of BLDCM via numerical simulation. After applying the numerical results such as phase portrait, and bifurcation diagram, a variety of the phenomena of the chaotic motion can be presented. Then, we discuss chaos synchronization in three aspects: the synchronization for systems with ''unknown'' parameters, the backstepping design, and the Gerschgorin's theorem method, and that make two identical systems synchronized successfully. Finally, parameter identification can be achieved via adaptive control and random optimization.

Acknowledgment

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