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# Fuzzy-identification-based adaptive controller design via backstepping approach

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## Abstract

This paper proposes a fuzzy-identification-based adaptive control scheme for the chaotic dynamic systems using backstepping control approach, which is referenced as adaptive fuzzy backstepping control (AFBC). The proposed AFBC offers a design approach to drive the chaotic trajectory to track a desired trajectory, and it is comprised of a fuzzy backstepping controller and a robust controller. The fuzzy backstepping controller containing a fuzzy estimation system is the principal controller, and the robust controller is designed to dispel the effect of minimum approximation error introduced by the fuzzy estimation system. Moreover, the Taylor linearization technique is employed to derive the linearized model of the fuzzy estimation system so that all the parameters in the fuzzy system could be updated according. The adaptation laws of the control system are derived in the sense of Lyapunov function and Barbalat's lemma, thus the stability of the system can be guaranteed. For comparison, the partial- and full-tuned cases for the parameters in the fuzzy system are simulated. Finally, simulation results verify that the proposed AFBC system can achieve favorable tracking performance for the chaotic system with regard to parameter variations and unknown dynamic function.

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## 1. Introduction

Fuzzy system has supplanted a conventional technology in some scientific applications and engineering systems, especially in control systems [11]. The fuzzy system consists of a set of fuzzy if-then rules.

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Though it is one of the most effective methods using expert knowledge, it has not been viewed as a rigorous approach due to the lack of formal synthesis techniques that can guarantee global stability of the fuzzy systems. To tackle this problem, some researchers have focused on the use of the Lyapunov synthesis approach to construct a stable adaptive fuzzy controller. The key element of the adaptive fuzzy control is the merger of adaptive systems with fuzzy approximation theory, where the fuzzy system can approximate the unknown plants. Based on the universal approximation theory [24], the adaptive fuzzy control design method can provide stabilizing controllers in the Lyapunov sense. An important class of the adaptive fuzzy control is constructed using a radial basis function neural network. Most of these approaches only tune the consequent part of the rule sets; however, the membership functions are fixed [4,13,17,20,24]. Recently, some algorithms are derived to tune the membership functions and the rule sets simultaneously based on the Taylor series [7,12,14,25] or the genetic algorithm [9,15].

In the past decade, research on backstepping control has increased [2,3,10,21–23,26]. The backstepping control is a systematic and recursive design methodology for nonlinear systems. The backstepping approach offers a choice to accommodate the unmodelled nonlinear effects and parameter uncertainties. The idea of backstepping design is to select recursively some appropriate functions of state variables as pseudo-control inputs for lower dimension subsystems of the overall system. Each backstepping stage results in a new pseudo-control design, expressed in terms of the pseudo-control design from preceding design stages. The procedure terminates a feedback design for the true control input which achieves the original design objective by virtue of a final Lyapunov function which is formed by summing the Lyapunov functions associated with each individual design stage [10].

The chaotic dynamic systems can be observed in many nonlinear circuits and mechanical systems. Recently, control of the chaotic dynamic system has become a significant research topic in the physics, mathematics and engineering communities [1,5,6,8,16,19]; however, some of them cannot achieve favorable control performance and some of them require overly complex design procedures. To overcome these drawbacks, this paper develops an adaptive fuzzy backstepping control (AFBC) system, which creates a bridge between backstepping control approach and adaptive fuzzy control design, to control the chaotic dynamic systems. The developed AFBC system is implemented without using any knowledge of the chaotic dynamic system, and it is comprised of a fuzzy backstepping controller and a robust controller. The fuzzy backstepping controller containing a fuzzy estimation system is designed in the sense of the backstepping control, and the robust controller is designed to dispel the effect of approximation error introduced by the fuzzy estimation system. The adaptive laws of the AFBC system are derived in the sense of Lyapunov function and Barbalat's lemma; thus the stability of the system can be guaranteed. The developed tuning algorithms of the fuzzy estimator can on-line tune all the parameters of the fuzzy system (e.g., centers and widths of membership function and consequent parts of the rules set) based on the Taylor linearization technique to reduce the approximation error and to improve the tracking performance. Finally, simulation results are provided to verify the effectiveness of the developed AFBC scheme for the chaotic dynamic system with regard to plant parameter variations and unknown dynamic functions.

## 2. Problem formulation of chaotic dynamic systems

Chaotic systems have been studied and known to exhibit complex dynamical behavior. The interest in chaotic systems lies mostly upon their complex, unpredictable behavior, and extreme sensitivity to initial conditions as well as parameter variations. Consider a second-order chaotic system such as well known

Duffing’s equation describing a special nonlinear circuit or a pendulum moving in a viscous medium under control [1,8,16]

$$\ddot{x}(t) = -p\dot{x}(t) - p_1x(t) - p_2x^3(t) + q \cos(\omega t) + u(t) = f(x, \dot{x}) + u(t), \tag{1}$$

where  $t$  is the time variable,  $\omega$  is the frequency,  $f(x, \dot{x}) = -p\dot{x}(t) - p_1x(t) - p_2x^3(t) + q \cos(\omega t)$  is the system dynamic function,  $u(t)$  is the control effort and  $p, p_1, p_2$  and  $q$  are real constants. Depending on the choice of these constants, it is known that the solutions of system (1) may exhibit periodic, almost periodic and chaotic behavior [1]. For observing the chaotic unpredictable behavior, the open-loop system behavior with  $u(t) = 0$  was simulated with  $p = 0.4, p_1 = -1.1, p_2 = 1.0$  and  $\omega = 18$ . The phase plane plots from an initial condition point (0,0) are shown in Figs. 1(a) and (b) for  $q = 2.10$  and  $7.00$ , respectively. It is shown that the uncontrolled chaotic dynamic system has different chaotic trajectories for different  $q$  values. The control objective is to find a control law so that the chaotic trajectory can track the desired periodic orbit.

### 3. Description of fuzzy systems

There are four principal parts in fuzzy systems: fuzzifier, fuzzy rule base, fuzzy inference engine and defuzzifier. Assume that there are  $N$  rules in the fuzzy rule base in the following form [11]:

$$\text{Rule } j: \text{ IF } x_1 \text{ is } F_1^j \text{ and } \dots x_n \text{ is } F_n^j \text{ THEN } y \text{ is } G^j, \tag{2}$$

where  $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_n]^T \in R^n$  and  $y$  are the input and output variables of the fuzzy system, respectively; and  $F_l^j, l = 1, 2, \dots, n$  and  $G^j$  are the linguistic terms characterized by their corresponding fuzzy membership functions of the fuzzy sets  $\mu_{F_l^j}(x_l)$  and  $\mu_{G^j}(y)$ , respectively. In this study, the membership function  $\mu_{F_l^j}(x_l)$  is chosen as a Gaussian function, and the membership function  $\mu_{G^j}(y)$  is chosen as a singleton. The fuzzy system is constructed with a singleton fuzzification, a product inference and a weighted sum defuzzification. The neural network scheme of the fuzzy system with  $n$  inputs,  $N$  rules (hidden units) and one output is shown in Fig. 2. The fuzzy system performs the mappings according to [12,24]

$$y = \sum_{j=1}^N \mathbf{w}_j \phi_j(\boldsymbol{\sigma}_j, \|\mathbf{x} - \mathbf{c}_j\|), \tag{3}$$

where  $\mathbf{c}_j$  and  $\boldsymbol{\sigma}_j$  are the center and width vectors of the Gaussian membership, respectively; and  $\mathbf{w}_j$  is the connection weight between the hidden layer and output layer. The Gaussian membership  $\phi_j$  represents as

$$\phi_j(\boldsymbol{\sigma}_j, \|\mathbf{x} - \mathbf{c}_j\|) = \prod_{i=1}^n \exp[-(x_i - c_j^i)^2 / \sigma_j^i], \tag{4}$$

where  $\mathbf{c}_j = [c_j^1 \ c_j^2 \ \dots \ c_j^n]^T \in R^n$  and  $\boldsymbol{\sigma}_j = [\sigma_j^1 \ \sigma_j^2 \ \dots \ \sigma_j^n]^T \in R^n$ . For ease of notation, define the vectors  $\mathbf{w} = [\mathbf{w}_1 \ \mathbf{w}_2 \ \dots \ \mathbf{w}_N]^T \in R^N, \mathbf{c} = [\mathbf{c}_1^T \ \mathbf{c}_2^T \ \dots \ \mathbf{c}_N^T]^T \in R^{nN}$  and  $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_1^T \ \boldsymbol{\sigma}_2^T \ \dots \ \boldsymbol{\sigma}_N^T]^T \in R^{nN}$ , then the output of the fuzzy system can be represented as [7,24,25]

$$y(\mathbf{x}, \mathbf{c}, \boldsymbol{\sigma}, \mathbf{w}) = \mathbf{w}^T \boldsymbol{\Phi}(\mathbf{x}, \mathbf{c}, \boldsymbol{\sigma}), \tag{5}$$

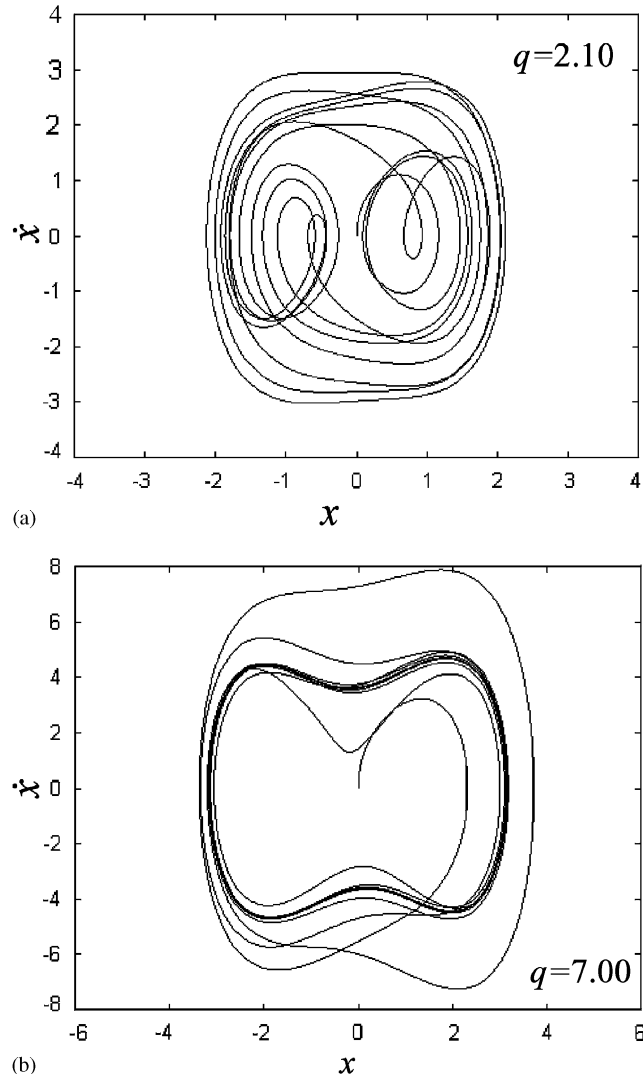


Fig. 1. Typical chaotic orbits of the chaotic dynamic system.

where  $\Phi(\mathbf{x}, \mathbf{c}, \boldsymbol{\sigma}) = [\phi_1(\boldsymbol{\sigma}_1, \|\mathbf{x} - \mathbf{c}_1\|) \phi_2(\boldsymbol{\sigma}_2, \|\mathbf{x} - \mathbf{c}_2\|) \dots \phi_N(\boldsymbol{\sigma}_N, \|\mathbf{x} - \mathbf{c}_N\|)]^T$ . It has been proven that there exists a fuzzy system of (5) such that it can uniformly approximate a nonlinear even time-varying function  $\Theta$ . By the universal approximation theorem, there exists an ideal fuzzy system  $y^*$  such that [24]

$$\Theta = y^*(\mathbf{x}, \mathbf{c}^*, \boldsymbol{\sigma}^*, \mathbf{w}^*) + \Delta = \mathbf{w}^{*T} \Phi(\mathbf{x}, \mathbf{c}^*, \boldsymbol{\sigma}^*) + \Delta, \tag{6}$$

where  $\Delta$  denotes the approximation error and is assumed to be bounded by  $|\Delta| \leq \Delta^*$ , in which  $\Delta^*$  is a positive constant; and  $\mathbf{w}^*$ ,  $\mathbf{c}^*$  and  $\boldsymbol{\sigma}^*$  are the optimal parameter vectors of  $\mathbf{w}$ ,  $\mathbf{c}$  and  $\boldsymbol{\sigma}$ , respectively. There should exist constants  $\bar{\mathbf{w}}$ ,  $\bar{\mathbf{c}}$  and  $\bar{\boldsymbol{\sigma}}$  satisfying  $\|\mathbf{w}^*\| \leq \bar{\mathbf{w}}$ ,  $\|\mathbf{c}^*\| \leq \bar{\mathbf{c}}$  and  $\|\boldsymbol{\sigma}^*\| \leq \bar{\boldsymbol{\sigma}}$ . In fact, the optimal parameter vectors that are needed to best approximate a given nonlinear function  $\Theta$  are difficult to determine.

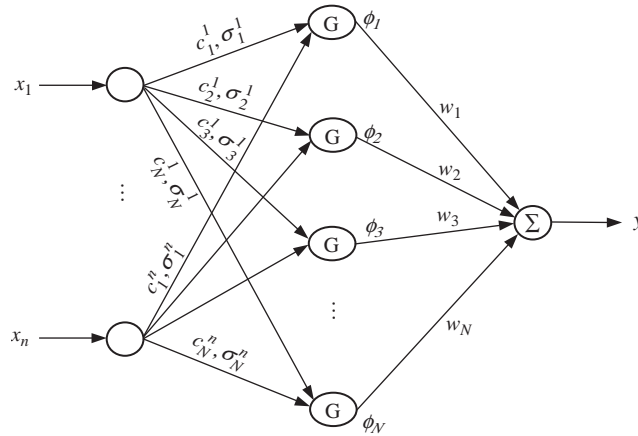


Fig. 2. The structure of fuzzy system using a radial basis function neural network.

Thus, an estimation function is defined as

$$\hat{y}(\mathbf{x}, \hat{\mathbf{c}}, \hat{\boldsymbol{\sigma}}, \hat{\mathbf{w}}) = \hat{\mathbf{w}}^T \boldsymbol{\Phi}(\mathbf{x}, \hat{\mathbf{c}}, \hat{\boldsymbol{\sigma}}), \tag{7}$$

where  $\hat{\mathbf{w}}$ ,  $\hat{\mathbf{c}}$  and  $\hat{\boldsymbol{\sigma}}$  are the estimated vectors of  $\mathbf{w}^*$ ,  $\mathbf{c}^*$  and  $\boldsymbol{\sigma}^*$ , respectively. For notational convenience, denote  $\tilde{\boldsymbol{\Phi}}^* = \boldsymbol{\Phi}(\mathbf{x}, \mathbf{c}^*, \boldsymbol{\sigma}^*)$  and  $\hat{\boldsymbol{\Phi}} = \boldsymbol{\Phi}(\mathbf{x}, \hat{\mathbf{c}}, \hat{\boldsymbol{\sigma}})$ . Define the estimated error  $\tilde{y}$  as

$$\tilde{y} = \theta - \hat{y} = y^* - \hat{y} + \Delta = \tilde{\mathbf{w}}^T \hat{\boldsymbol{\Phi}} + \hat{\mathbf{w}}^T \tilde{\boldsymbol{\Phi}} + \tilde{\mathbf{w}}^T \tilde{\boldsymbol{\Phi}} + \Delta, \tag{8}$$

where  $\tilde{\mathbf{w}} = \mathbf{w}^* - \hat{\mathbf{w}}$  and  $\tilde{\boldsymbol{\Phi}} = \boldsymbol{\Phi}^* - \hat{\boldsymbol{\Phi}}$ . In the following, some tuning laws will be derived to on-line tune the parameters of the fuzzy system to achieve favorable estimation of the nonlinear function. To achieve this goal, the Taylor expansion linearization technique is employed to transform the nonlinear function into a partially linear form [7,12], i.e.

$$\tilde{\boldsymbol{\Phi}} = \boldsymbol{\Phi}'_{\mathbf{c}}|_{\mathbf{c}=\hat{\mathbf{c}}} \tilde{\mathbf{c}} + \boldsymbol{\Phi}'_{\boldsymbol{\sigma}}|_{\boldsymbol{\sigma}=\hat{\boldsymbol{\sigma}}} \tilde{\boldsymbol{\sigma}} + \mathbf{h}(\mathbf{x}, \tilde{\mathbf{c}}, \tilde{\boldsymbol{\sigma}}), \tag{9}$$

where  $\tilde{\mathbf{c}} = \mathbf{c}^* - \hat{\mathbf{c}}$ ;  $\tilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}^* - \hat{\boldsymbol{\sigma}}$ ;  $\mathbf{h}(\mathbf{x}, \tilde{\mathbf{c}}, \tilde{\boldsymbol{\sigma}})$  denotes the sum of high-order arguments in the Taylor's serious expansion; and  $\boldsymbol{\Phi}'_{\mathbf{c}}|_{\mathbf{c}=\hat{\mathbf{c}}}$  and  $\boldsymbol{\Phi}'_{\boldsymbol{\sigma}}|_{\boldsymbol{\sigma}=\hat{\boldsymbol{\sigma}}}$  are derivatives of  $\boldsymbol{\Phi}$  with respect to  $\mathbf{c}$  and  $\boldsymbol{\sigma}$  at  $(\hat{\mathbf{c}}, \hat{\boldsymbol{\sigma}})$ , respectively, that are expressed as

$$\boldsymbol{\Phi}'_{\mathbf{c}} = \boldsymbol{\Phi}'_{\mathbf{c}}|_{\mathbf{c}=\hat{\mathbf{c}}} = [\phi'_{c_1}(\hat{\boldsymbol{\sigma}}_1, \|\mathbf{x} - \hat{\mathbf{c}}_1\|) \phi'_{c_2}(\hat{\boldsymbol{\sigma}}_2, \|\mathbf{x} - \hat{\mathbf{c}}_2\|) \dots \phi'_{c_N}(\hat{\boldsymbol{\sigma}}_N, \|\mathbf{x} - \hat{\mathbf{c}}_N\|)]^T, \tag{10}$$

$$\boldsymbol{\Phi}'_{\boldsymbol{\sigma}} = \boldsymbol{\Phi}'_{\boldsymbol{\sigma}}|_{\boldsymbol{\sigma}=\hat{\boldsymbol{\sigma}}} = [\phi'_{\sigma_1}(\hat{\boldsymbol{\sigma}}_1, \|\mathbf{x} - \hat{\mathbf{c}}_1\|) \phi'_{\sigma_2}(\hat{\boldsymbol{\sigma}}_2, \|\mathbf{x} - \hat{\mathbf{c}}_2\|) \dots \phi'_{\sigma_N}(\hat{\boldsymbol{\sigma}}_N, \|\mathbf{x} - \hat{\mathbf{c}}_N\|)]^T \tag{11}$$

with  $\phi'_{c_j} = \partial\phi_j/\partial c_j$  and  $\phi'_{\sigma_j} = \partial\phi_j/\partial\sigma_j$ . The high-order term  $\mathbf{h}(\mathbf{x}, \tilde{\mathbf{c}}, \tilde{\boldsymbol{\sigma}})$  is bounded by

$$\begin{aligned} \|\mathbf{h}(\mathbf{x}, \tilde{\mathbf{c}}, \tilde{\boldsymbol{\sigma}})\| &= \|\tilde{\boldsymbol{\Phi}} - \boldsymbol{\Phi}'_{\mathbf{c}}\tilde{\mathbf{c}} - \boldsymbol{\Phi}'_{\boldsymbol{\sigma}}\tilde{\boldsymbol{\sigma}}\| \\ &\leq \|\tilde{\boldsymbol{\Phi}}\| + \|\boldsymbol{\Phi}'_{\mathbf{c}}\|\|\tilde{\mathbf{c}}\| + \|\boldsymbol{\Phi}'_{\boldsymbol{\sigma}}\|\|\tilde{\boldsymbol{\sigma}}\| \\ &\leq \mathbf{k}_1 + \mathbf{k}_2\|\tilde{\mathbf{c}}\| + \mathbf{k}_3\|\tilde{\boldsymbol{\sigma}}\|, \end{aligned} \tag{12}$$

where  $\|\tilde{\boldsymbol{\Phi}}\| \leq \mathbf{k}_1$ ,  $\|\boldsymbol{\Phi}'_{\mathbf{c}}\| \leq \mathbf{k}_2$ ,  $\|\boldsymbol{\Phi}'_{\boldsymbol{\sigma}}\| \leq \mathbf{k}_3$  and  $\mathbf{k}_1$ ,  $\mathbf{k}_2$  and  $\mathbf{k}_3$  are some bounded positive constants due to the fact that Gaussian function and its derivative are always bounded by constants [7]. And the  $\tilde{\mathbf{w}}$ ,  $\tilde{\mathbf{c}}$  and

$\tilde{\sigma}$  should be also bounded as

$$\|\tilde{\mathbf{w}}\| = \|\mathbf{w}^* - \hat{\mathbf{w}}\| \leq \|\mathbf{w}^*\| + \|\hat{\mathbf{w}}\| \leq \bar{\mathbf{w}} + \|\hat{\mathbf{w}}\|, \quad (13)$$

$$\|\tilde{\mathbf{c}}\| = \|\mathbf{c}^* - \hat{\mathbf{c}}\| \leq \|\mathbf{c}^*\| + \|\hat{\mathbf{c}}\| \leq \bar{\mathbf{c}} + \|\hat{\mathbf{c}}\|, \quad (14)$$

$$\|\tilde{\sigma}\| = \|\sigma^* - \hat{\sigma}\| \leq \|\sigma^*\| + \|\hat{\sigma}\| \leq \bar{\sigma} + \|\hat{\sigma}\|. \quad (15)$$

Substituting (9) into (8), gives

$$\begin{aligned} \tilde{y} &= \tilde{\mathbf{w}}^T \hat{\Phi} + \hat{\mathbf{w}}^T (\Phi'_c \tilde{\mathbf{c}} + \Phi'_\sigma \tilde{\sigma} + \mathbf{h}) + \tilde{\mathbf{w}}^T \tilde{\Phi} + \Delta \\ &= \tilde{\mathbf{w}}^T \hat{\Phi} + \tilde{\mathbf{c}}^T \Phi'_c{}^T \hat{\mathbf{w}} + \tilde{\sigma}^T \Phi'_\sigma{}^T \hat{\mathbf{w}} + \hat{\mathbf{w}}^T \mathbf{h} + \tilde{\mathbf{w}}^T \tilde{\Phi} + \Delta \\ &= \tilde{\mathbf{w}}^T \hat{\Phi} + \tilde{\mathbf{c}}^T \Phi'_c{}^T \hat{\mathbf{w}} + \tilde{\sigma}^T \Phi'_\sigma{}^T \hat{\mathbf{w}} + \varepsilon, \end{aligned} \quad (16)$$

where  $\hat{\mathbf{w}}^T \Phi'_c \tilde{\mathbf{c}} = \tilde{\mathbf{c}}^T \Phi'_c{}^T \hat{\mathbf{w}}$  and  $\hat{\mathbf{w}}^T \Phi'_\sigma \tilde{\sigma} = \tilde{\sigma}^T \Phi'_\sigma{}^T \hat{\mathbf{w}}$  are used since they are scales; and the sum of matching error  $\varepsilon \equiv \hat{\mathbf{w}}^T \mathbf{h} + \tilde{\mathbf{w}}^T \tilde{\Phi} + \Delta$ . Then the term  $\varepsilon$  should be bounded as

$$\begin{aligned} |\varepsilon| &= |\mathbf{w}^{*T} \mathbf{h} + \tilde{\mathbf{w}}^T \Phi'_c \tilde{\mathbf{c}} + \tilde{\mathbf{w}}^T \Phi'_\sigma \tilde{\sigma} + \Delta| \\ &\leq (\mathbf{k}_1 + \mathbf{k}_2 \|\tilde{\mathbf{c}}\| + \mathbf{k}_3 \|\tilde{\sigma}\|) \bar{\mathbf{w}} + \mathbf{k}_2 \|\tilde{\mathbf{c}}\| (\bar{\mathbf{w}} + \|\hat{\mathbf{w}}\|) + \mathbf{k}_3 \|\tilde{\sigma}\| (\bar{\mathbf{w}} + \|\hat{\mathbf{w}}\|) + \Delta^* \\ &\leq [\mathbf{k}_1 + \mathbf{k}_2 (\bar{\mathbf{c}} + \|\hat{\mathbf{c}}\|) + \mathbf{k}_3 (\bar{\sigma} + \|\hat{\sigma}\|)] \bar{\mathbf{w}} + \mathbf{k}_2 (\bar{\mathbf{c}} + \|\hat{\mathbf{c}}\|) (\bar{\mathbf{w}} + \|\hat{\mathbf{w}}\|) \\ &\quad + \mathbf{k}_3 (\bar{\sigma} + \|\hat{\sigma}\|) (\bar{\mathbf{w}} + \|\hat{\mathbf{w}}\|) + \Delta^* \\ &= (\mathbf{k}_1 + 2\mathbf{k}_2 \bar{\mathbf{c}} + 2\mathbf{k}_3 \bar{\sigma}) \bar{\mathbf{w}} + \Delta^* + (\mathbf{k}_2 \bar{\mathbf{c}} + \mathbf{k}_3 \bar{\sigma}) \|\hat{\mathbf{w}}\| + 2\mathbf{k}_2 \bar{\mathbf{w}} \|\hat{\mathbf{c}}\| + 2\mathbf{k}_3 \bar{\mathbf{w}} \|\hat{\sigma}\| \\ &\quad + \mathbf{k}_2 \|\hat{\mathbf{w}}\| \cdot \|\hat{\mathbf{c}}\| + \mathbf{k}_3 \|\hat{\mathbf{w}}\| \cdot \|\hat{\sigma}\| \\ &= [\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_4, \bar{\varepsilon}_5, \bar{\varepsilon}_6] \cdot [1, \|\hat{\mathbf{w}}\|, \|\hat{\mathbf{c}}\|, \|\hat{\sigma}\|, \|\hat{\mathbf{w}}\| \cdot \|\hat{\mathbf{c}}\|, \|\hat{\mathbf{w}}\| \cdot \|\hat{\sigma}\|]^T \\ &= \bar{\Xi}^T \Gamma, \end{aligned} \quad (17)$$

where  $\bar{\varepsilon}_1 = (\mathbf{k}_1 + 2\mathbf{k}_2 \bar{\mathbf{c}} + 2\mathbf{k}_3 \bar{\sigma}) \bar{\mathbf{w}} + \Delta^*$ ,  $\bar{\varepsilon}_2 = \mathbf{k}_2 \bar{\mathbf{c}} + \mathbf{k}_3 \bar{\sigma}$ ,  $\bar{\varepsilon}_3 = 2\mathbf{k}_2 \bar{\mathbf{w}}$ ,  $\bar{\varepsilon}_4 = 2\mathbf{k}_3 \bar{\mathbf{w}}$ ,  $\bar{\varepsilon}_5 = \mathbf{k}_2$ ,  $\bar{\varepsilon}_6 = \mathbf{k}_3$  and  $\Gamma = [1, \|\hat{\mathbf{w}}\|, \|\hat{\mathbf{c}}\|, \|\hat{\sigma}\|, \|\hat{\mathbf{w}}\| \cdot \|\hat{\mathbf{c}}\|, \|\hat{\mathbf{w}}\| \cdot \|\hat{\sigma}\|]^T$ . Since  $\bar{\Xi}$  is a bounded vector, if  $\Gamma$  can be guaranteed to be bounded (This will be proven in the following section.) then the term  $\varepsilon$  can be bounded by  $|\varepsilon| \leq E$  where  $E$  is a positive constant; however, the error bound  $E$  is difficult to determine for practical applications.

## 4. Ideal backstepping control and adaptive fuzzy backstepping control

### 4.1. Design of ideal backstepping controller

Assume that the parameters of the system (1) are known, the design of ideal backstepping control for the chaotic dynamic system is described step-by-step as follows:

*Step 1:* Define the tracking error

$$e_1(t) = x(t) - x_c(t), \quad (18)$$

where  $x_c(t)$  is the command trajectory; and the derivative of tracking error is defined as

$$\dot{e}_1(t) = \dot{x}(t) - \dot{x}_c(t). \quad (19)$$

The  $\dot{x}(t)$  can be viewed as a virtual control in the equation. Define the following stabilizing function

$$\alpha(t) = -\tau_1 e_1(t) + \dot{x}_c(t), \quad (20)$$

where  $\tau_1$  is a positive constant.

*Step 2:* Define

$$e_2(t) = \dot{x}(t) - \alpha(t) \quad (21)$$

then the derivative of  $e_2(t)$  is expressed as

$$\dot{e}_2(t) = \ddot{x}(t) - \dot{\alpha}(t) = \ddot{x}(t) - (-\tau_1 \dot{e}_1(t) + \ddot{x}_c(t)) = \ddot{e}_1(t) + \tau_1 \dot{e}_1(t). \quad (22)$$

*Step 3:* If the dynamic system is known, an ideal backstepping controller can be obtained as

$$u_{ib}(t) = \ddot{x}_c(t) - f(x, \dot{x}) - \tau_1 \dot{e}_1(t) - \tau_2 e_2(t) - e_1(t), \quad (23)$$

where  $\tau_2$  is a positive constant. Substituting (23) into (1), it is obtained that

$$\dot{e}_2(t) = -\tau_2 e_2(t) - e_1(t). \quad (24)$$

*Step 4:* Define the Lyapunov function as

$$V_1(e_1(t), e_2(t)) = \frac{e_1^2(t)}{2} + \frac{e_2^2(t)}{2}. \quad (25)$$

Differentiating (25) with respect to time and using (19), (22) and (24), it is obtained that

$$\begin{aligned} \dot{V}_1(e_1(t), e_2(t)) &= e_1(t)\dot{e}_1(t) + e_2(t)\dot{e}_2(t) \\ &= e_1(t)(e_2(t) - \tau_1 e_1(t)) + e_2(t)(-\tau_2 e_2(t) - e_1(t)) \\ &= -\tau_1 e_1^2(t) - \tau_2 e_2^2(t) \leq 0. \end{aligned} \quad (26)$$

Since  $\dot{V}_1(e_1(t), e_2(t)) \leq 0$ , that is  $V_1(e_1(t), e_2(t)) \leq V_1(e_1(0), e_2(0))$ , it implies that  $e_1(t)$  and  $e_2(t)$  are bounded. Now define the following term:

$$\Omega(t) = \tau_1 e_1^2(t) + \tau_2 e_2^2(t) = -\dot{V}_1(e_1(t), e_2(t)), \quad (27)$$

then

$$\int_0^t \Omega(\tau) d\tau = V_1(e_1(0), e_2(0)) - V_1(e_1(t), e_2(t)). \quad (28)$$

Because  $V_1(e_1(0), e_2(0))$  is bounded and  $V_1(e_1(t), e_2(t))$  is nonincreasing and bounded, the following result can be obtained

$$\lim_{t \rightarrow \infty} \int_0^t \Omega(\tau) d\tau < \infty. \quad (29)$$

Also  $\dot{\Omega}(t)$  is bounded, so by Barbalat's Lemma [18], it can be shown that  $\lim_{t \rightarrow \infty} \Omega(t) = 0$ . This implies that  $e_1(t)$  and  $e_2(t)$  converge to zero as  $t \rightarrow \infty$ . Therefore, the ideal backstepping controller in (23) will asymptotically stabilize the system.

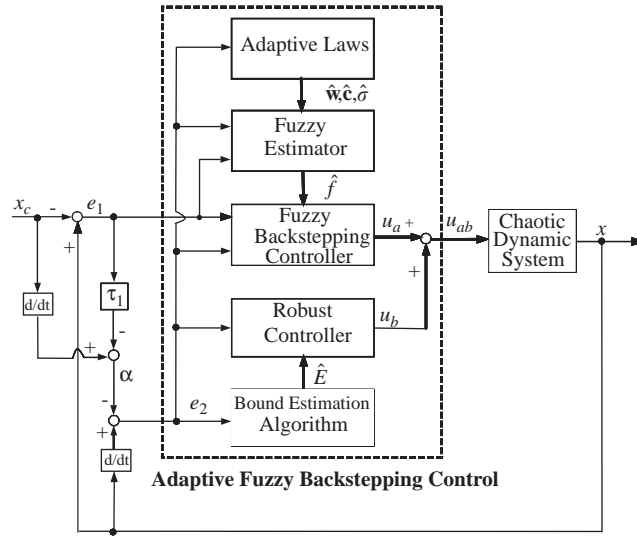


Fig. 3. The block diagram of the adaptive fuzzy backstepping control chaotic system.

#### 4.2. Design of adaptive fuzzy backstepping controller

Since the system dynamic function  $f(x, \dot{x})$  may be unknown or perturbed in practical application, the ideal backstepping controller (23) cannot be precisely obtained. Thus, an AFBC system is proposed as shown in Fig. 3. In the AFBC system, the fuzzy system is designed to estimate the system dynamic function. The design of AFBC for the chaotic dynamic system is described step-by-step as follows:

*Step 1:* Define the tracking error  $e_1(t)$  as (18), a stabilizing function  $\alpha(t)$  as (20) and  $e_2(t)$  as (21).

*Step 2:* The control law of the AFBC is developed in the following equation:

$$u_{ab}(t) = u_a(t) + u_b(t), \quad (30)$$

where

$$u_a(t) = \ddot{x}_c(t) - \hat{f} - \tau_1 \dot{e}_1(t) - \tau_2 e_2(t) - e_1(t), \quad (31)$$

$$u_b(t) = -\hat{E} \operatorname{sgn}(e_2(t)). \quad (32)$$

In the fuzzy backstepping controller  $u_a$ , the system dynamic  $f$  is estimated by a fuzzy system  $\hat{f}$  described in Section 3; and in the robust controller  $u_b$ ,  $\hat{E}$  is an estimated error bound of  $E$ . Substituting (30) into (1), it can be obtained that

$$\dot{e}_2(t) = f - \hat{f} - \tau_2 e_2(t) - e_1(t) - \hat{E} \operatorname{sgn}(e_2(t)). \quad (33)$$

By defining the approximation error as (16), Eq. (33) can be rewritten as

$$\dot{e}_2 = \tilde{\mathbf{w}}^T \hat{\Phi} + \tilde{\mathbf{c}}^T \Phi_c'^T \hat{\mathbf{w}} + \tilde{\sigma}^T \Phi_\sigma'^T \hat{\mathbf{w}} + \varepsilon - \tau_2 e_2 - e_1 - \hat{E} \operatorname{sgn}(e_2(t)). \quad (34)$$



Step 3: Define the Lyapunov function as

$$V_2(e_1(t), e_2(t), \tilde{E}(t), \tilde{\mathbf{w}}, \tilde{\mathbf{c}}, \tilde{\boldsymbol{\sigma}}) = \frac{e_1^2(t)}{2} + \frac{e_2^2(t)}{2} + \frac{\tilde{E}^2(t)}{2\eta_1} + \frac{\tilde{\mathbf{w}}^T \tilde{\mathbf{w}}}{2\eta_2} + \frac{\tilde{\mathbf{c}}^T \tilde{\mathbf{c}}}{2\eta_3} + \frac{\tilde{\boldsymbol{\sigma}}^T \tilde{\boldsymbol{\sigma}}}{2\eta_4}, \quad (35)$$

where  $\tilde{E}(t) \equiv E - \hat{E}(t)$ ; and  $\eta_1, \eta_2, \eta_3$  and  $\eta_4$  are positive constants. Differentiating (35) with respect to time and using (33) and (34), it is obtained that

$$\begin{aligned} \dot{V}_2 &= e_1(t)\dot{e}_1(t) + e_2(t)\dot{e}_2(t) + \frac{\tilde{E}(t)\dot{\tilde{E}}(t)}{\eta_1} + \frac{\tilde{\mathbf{w}}^T \dot{\tilde{\mathbf{w}}}}{\eta_2} + \frac{\tilde{\mathbf{c}}^T \dot{\tilde{\mathbf{c}}}}{\eta_3} + \frac{\tilde{\boldsymbol{\sigma}}^T \dot{\tilde{\boldsymbol{\sigma}}}}{\eta_4} \\ &= e_1(t)(e_2(t) - \tau_1 e_1(t)) + e_2(t)(\tilde{\mathbf{w}}^T \hat{\boldsymbol{\Phi}} + \tilde{\mathbf{c}}^T \boldsymbol{\Phi}'^T \hat{\mathbf{w}} + \tilde{\boldsymbol{\sigma}}^T \boldsymbol{\Phi}_\sigma'^T \hat{\mathbf{w}} + \varepsilon - \tau_2 e_2 - e_1 - \hat{E} \operatorname{sgn}(e_2(t))) \\ &\quad + \frac{\tilde{E}(t)\dot{\tilde{E}}(t)}{\eta_1} + \frac{\tilde{\mathbf{w}}^T \dot{\tilde{\mathbf{w}}}}{\eta_2} + \frac{\tilde{\mathbf{c}}^T \dot{\tilde{\mathbf{c}}}}{\eta_3} + \frac{\tilde{\boldsymbol{\sigma}}^T \dot{\tilde{\boldsymbol{\sigma}}}}{\eta_4} \\ &= -\tau_1 e_1^2(t) - \tau_2 e_2^2(t) + \tilde{\mathbf{w}}^T (e_2 \hat{\boldsymbol{\Phi}} + \dot{\tilde{\mathbf{w}}}/\eta_2) + \tilde{\mathbf{c}}^T (e_2 \boldsymbol{\Phi}'^T \hat{\mathbf{w}} + \dot{\tilde{\mathbf{c}}}/\eta_3) + \tilde{\boldsymbol{\sigma}}^T (e_2 \boldsymbol{\Phi}_\sigma'^T \hat{\mathbf{w}} + \dot{\tilde{\boldsymbol{\sigma}}}/\eta_4) \\ &\quad + \varepsilon e_2(t) - \hat{E}|e_2(t)| + \frac{\tilde{E}(t)\dot{\tilde{E}}(t)}{\eta_1}. \end{aligned} \quad (36)$$

If the adaptive laws of the error bound and the fuzzy estimator are chosen as

$$\dot{\hat{E}}(t) = -\dot{\tilde{E}}(t) = \eta_1 |e_2(t)|, \quad (37)$$

$$\dot{\hat{\mathbf{w}}} = -\dot{\tilde{\mathbf{w}}} = \eta_2 e_2(t) \hat{\boldsymbol{\Phi}}, \quad (38)$$

$$\dot{\hat{\mathbf{c}}} = -\dot{\tilde{\mathbf{c}}} = \eta_3 e_2(t) \boldsymbol{\Phi}'^T \hat{\mathbf{w}}, \quad (39)$$

$$\dot{\hat{\boldsymbol{\sigma}}} = -\dot{\tilde{\boldsymbol{\sigma}}} = \eta_4 e_2(t) \boldsymbol{\Phi}_\sigma'^T \hat{\mathbf{w}}, \quad (40)$$

then (36) can be rewritten as

$$\begin{aligned} \dot{V}_2 &= -\tau_1 e_1^2(t) - \tau_2 e_2^2(t) + \varepsilon e_2(t) - E|e_2(t)| \\ &\leq -\tau_1 e_1^2(t) - \tau_2 e_2^2(t) - (E - |\varepsilon|)|e_2(t)| \\ &\leq -\tau_1 e_1^2(t) - \tau_2 e_2^2(t) \leq 0. \end{aligned} \quad (41)$$

Similar to the discussion of (26), it can be concluded that  $\tilde{E}, \tilde{\mathbf{w}}, \tilde{\mathbf{c}}$  and  $\tilde{\boldsymbol{\sigma}}$  are bounded and  $e_1$  and  $e_2$  converge to zero as  $t \rightarrow \infty$ . This also guarantees that  $\hat{\mathbf{w}}, \hat{\mathbf{c}}$  and  $\hat{\boldsymbol{\sigma}}$  are bounded and then the vector  $\Gamma$  in (17) is bounded.

## 5. Simulation results

Since the dynamic characteristics of the chaotic system are nonlinear and the precise model is difficult to obtain, the AFBC system has been proposed for the chaotic control system to track a desired periodic orbit. It should be emphasized that the derivation of AFBC does not need to know the dynamic function of the controlled system. The block diagram of the AFBC chaotic feedback control system is shown in Fig. 3, where  $x_c$  is the trajectory command and  $x$  is the system trajectory state. A fuzzy estimator with

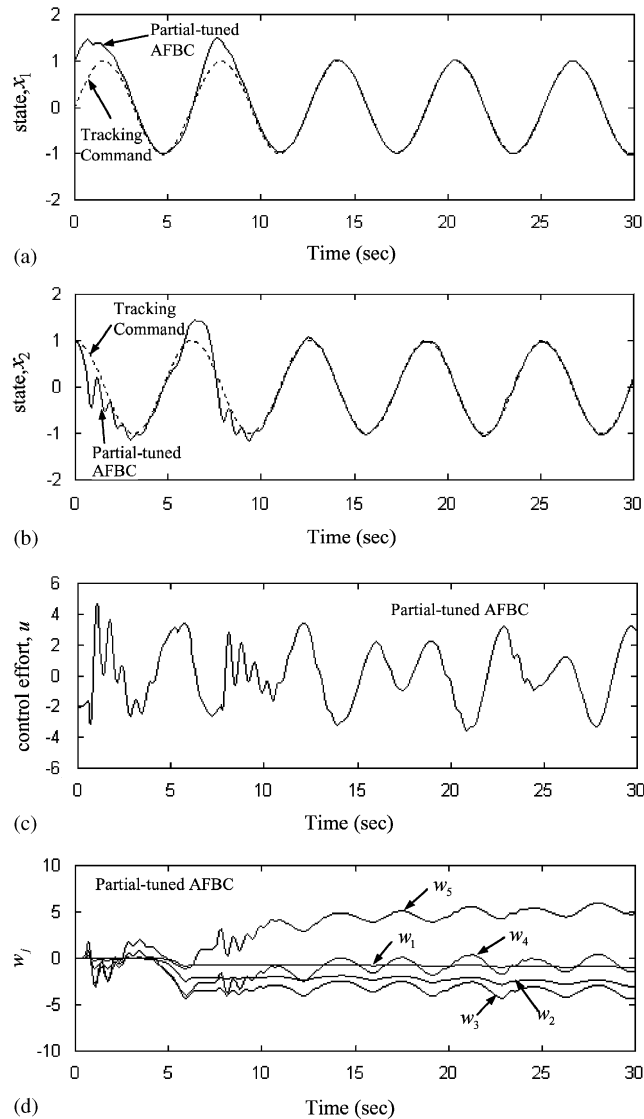


Fig. 4. The simulation results of partial-tuned AFBC for the chaotic system with  $q = 2.10$ .

5 fuzzy rules is utilized to on-line estimate the system dynamics. For comparison, two fuzzy estimation cases are revealed. One is the partial-tuned case, in which the Gaussian membership functions are fixed and only the output connections  $w_j$  are tuned. The other one is the full-tuned case, in which the Gaussian membership functions  $\phi_j$  and the output connections  $w_j$  are all tuned. For the partial-tuned case the Gaussian membership functions are given with  $\mathbf{c}_j = [-2, -1, 0, 1, 2]^T$  and  $\boldsymbol{\sigma}_j = [0.8, 0.8, 0.8, 0.8, 0.8]^T$ ; and the output connections are initiated from zeros. For full-tuned case, the parameters of the fuzzy estimator are all initiated from zeros and are learned by the adaptive laws (38)–(40) and the approximator error bound is learned by the estimation algorithm (37). For both cases, the control parameters are selected as

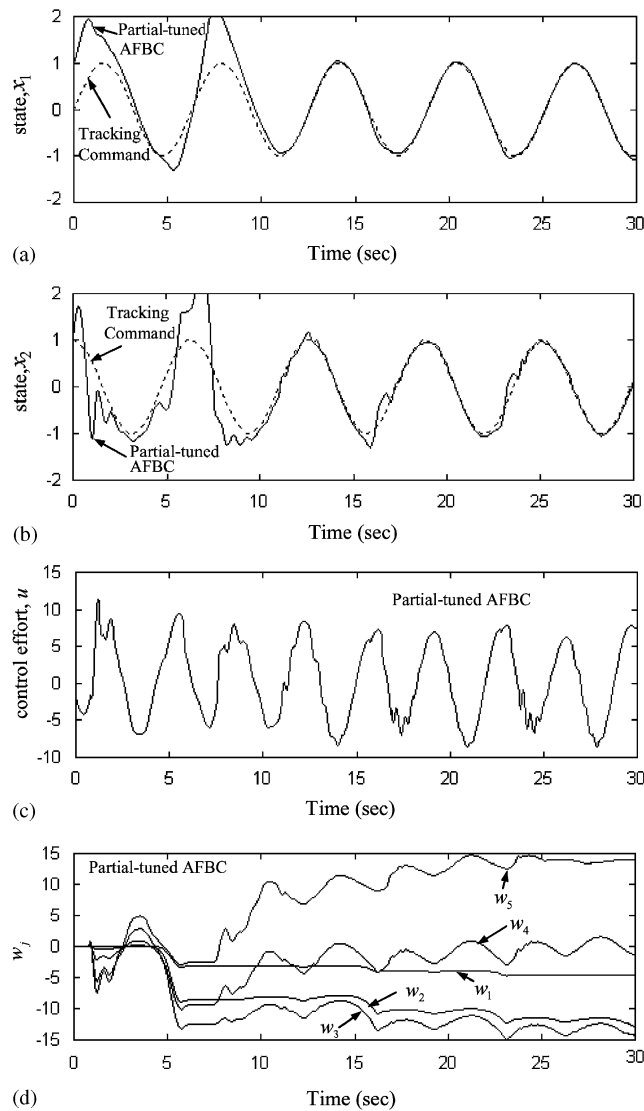


Fig. 5. The simulation results of partial-tuned AFBC for the chaotic system with  $q = 7.00$ .

$\tau_1 = \tau_2 = 1$ ,  $\eta_1 = 0.1$  and  $\eta_2 = \eta_3 = \eta_4 = 50$ . These parameters are chosen to achieve favorable transient control performance considering the requirement of asymptotic stability and the possible operating conditions. The simulation results of the AFBC chaotic systems for  $q = 2.10$  and  $7.00$  are shown. For partial-tuned case, the simulation results are shown in Figs. 4 and 5. The tracking responses of  $x_1$  are shown in Figs. 4(a) and 5(a); the tracking responses of  $x_2$  are shown in Figs. 4(b) and 5(b); the associated control efforts are shown in Figs. 4(c) and 5(c), and the learned output connections  $\hat{w}_j$  are shown in Figs. 4(d) and 5(d). For full-tuned case, the simulation results are shown in Figs. 6 and 7. The tracking responses of  $x_1$  are shown in Figs. 6(a) and 7(a); the tracking responses of  $x_2$  are shown in Figs. 6(b)

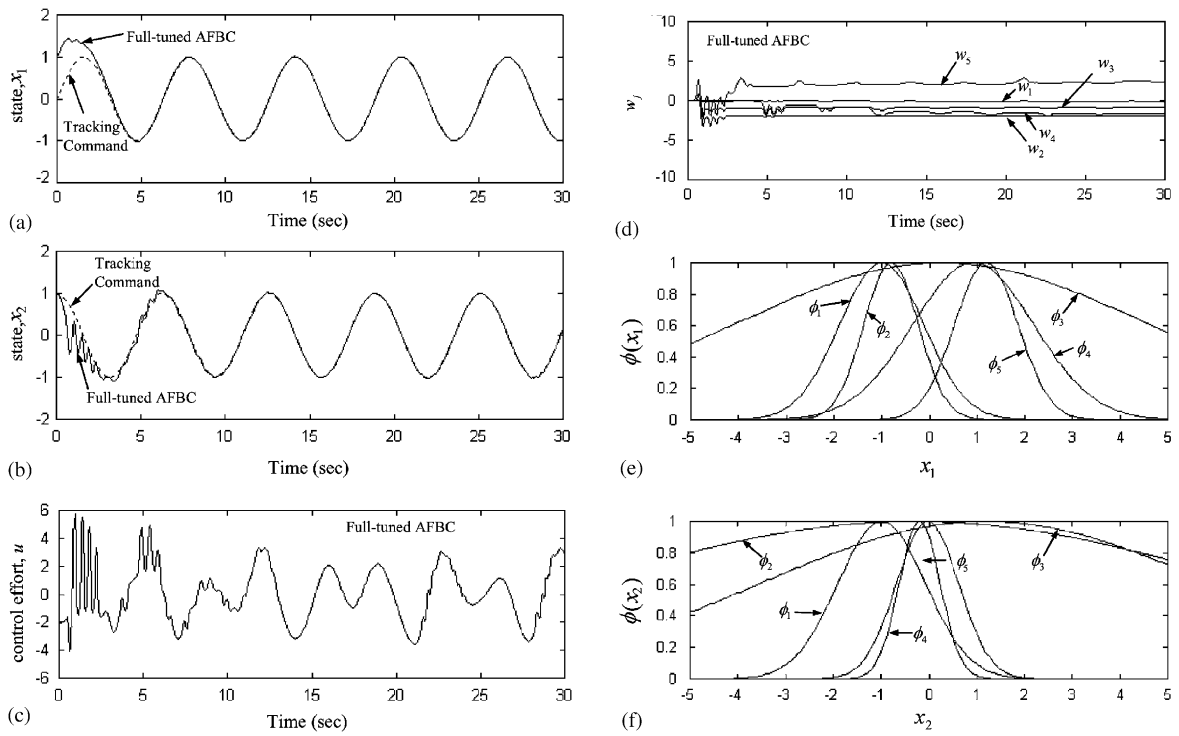


Fig. 6. The simulation results of full-tuned AFBC for the chaotic system with  $q = 2.10$ .

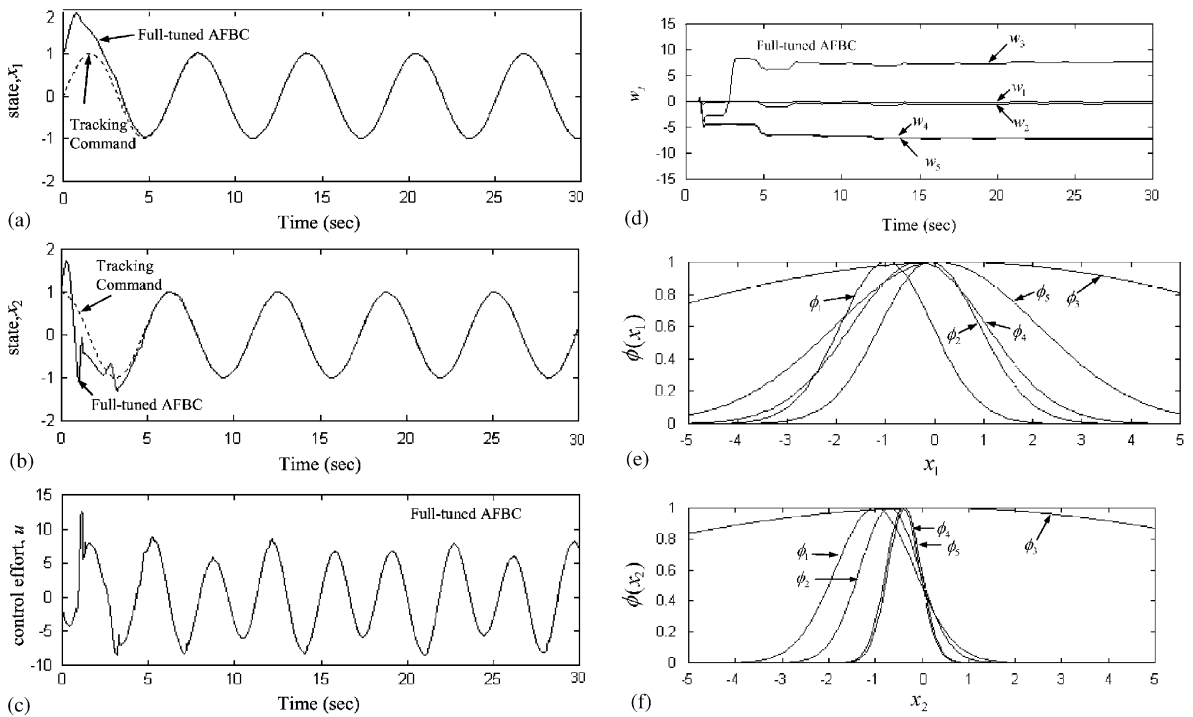


Fig. 7. The simulation results of full-tuned AFBC for the chaotic system with  $q = 7.00$ .

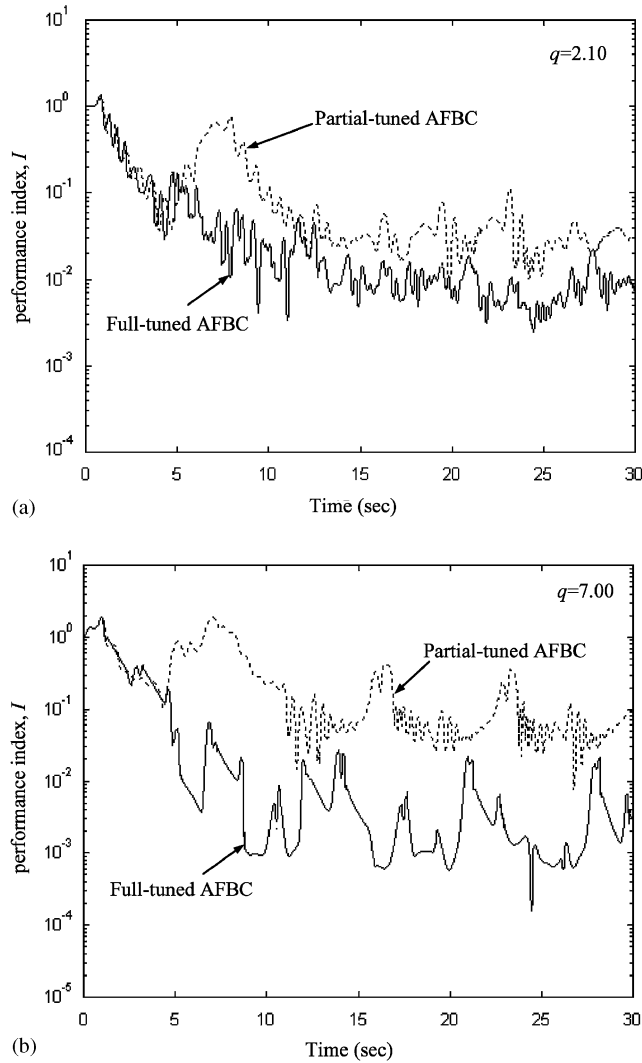


Fig. 8. Performance index of partial-tuned and full-tuned AFBC schemes.

and 7(b); the associated control efforts are shown in Figs. 6(c) and 7(c), the learned output connections  $\hat{w}_j$  are shown in Figs. 6(d) and 7(d), the learned Gaussian membership functions for  $x_1$  are shown in Figs. 6(e) and 7(e), and the learned Gaussian membership functions for  $x_2$  are shown in Figs. 6(f) and 7(f). These results show that the proposed AFBC design method can achieve favorable tracking performance; and the full-tuned case achieves better tracking performance than the partial-tuned case by paying the price of computational load for more parameter adaptive laws. A performance index  $I$  is defined as  $I = \sqrt{e_1^2(t) + \dot{e}_1^2(t)}$ . The performance index  $I$  of partial-tuned and full-tuned AFBC with  $q = 2.10$  and  $7.00$  are shown in Fig. 8. It is shown that the performance index of the proposed full-tuned algorithm is smaller than that of the partial-tuned algorithm.

## 6. Conclusions

Since the dynamic characteristics of the chaotic system are nonlinear and the precise model is difficult to obtain, an AFBC system has been proposed for the chaotic dynamic system control to track a desired periodic orbit. The developed AFBC system is comprised of a fuzzy backstepping controller with a fuzzy estimation system and a robust controller. The fuzzy estimation system is used to estimate the system dynamic function. The adaptive laws for the bound of matching error and the parameter adjustment of fuzzy system are synthesized using the Lyapunov function and Barbalat's lemma, so that the stability of the control system can be guaranteed. The developed updating laws of the fuzzy estimator can on-line tune all the parameters of the fuzzy system based on the Taylor linearization technique. Finally, simulation results verified that the developed control algorithm can achieve favorable tracking performance.

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