III. A COUNTEREXAMPLE

Since the case in [1] is also presented and discussed for continuous systems in [2], we give a continuous system case to illustrate that the trajectories are different for finite-horizon and infinite-horizon optimization problem.

Consider the linear time-invariant linear system defined as follows:

$$\dot{x}(t) = -x(t) + u(t).$$

The cost index is defined as

$$J = 0.5x(T) + 0.5 \int_0^T [x^2(t) + u^2(t)]\, dt.$$

The optimal control law is

$$u(t) = -p(t)x(t)$$

where $p(t)$ is the solution of the following *Riccati* equation:

$$\begin{cases} \dot{p}(t) = 2p(t) + p^2(t) - 1 \\ p(T) = 1 \end{cases}.$$

Then

$$p(t) = \frac{(\sqrt{2}-1)(2+\sqrt{2}) + (\sqrt{2}+1)(2-\sqrt{2})e^{2\sqrt{2}(t-T)}}{(2+\sqrt{2}) - (2-\sqrt{2})e^{2\sqrt{2}(t-T)}}.$$

When $T \to \infty$

$$\lim_{T\to\infty} p(t) = 0.414.$$

The optimal trajectory is

$$x(t) = x(0) \exp \int_0^t [-1 - p(\tau)]\, d\tau.$$

It is obvious that the two trajectories, finite-horizon optimal trajectory in time interval $[0, \mathrm{T}]$ and infinite-horizon optimal trajectory in time interval $[0, \mathrm{T}]$, are different.

IV. CONCLUSION

References [1] and [2] present some helpful researches in the optimal controller design for the fuzzy system. However, there are some issues, such as how to simplify the computation for the optimization problem of fuzzy system and how to ensure some characteristics of the closed-loop system, that need to be resolved.

REFERENCES

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Authors' Reply

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The papers mentioned are based on the idea that the optimal decision is, in fact, a step-by-step on-going decision process. In other words, at any time state, saying $X^i(k)$, the following two decisions are to be made.

1) Minimize

$$J_\infty(R(\cdot)) = \sum_{k=k_0}^{\infty} [X^t(k)L(k)X(k) + R^t(k)W^t(Y(k))W(Y(k))R(k)]$$

regarding nonlinear system

$$\begin{aligned} X(k+1) =& H(X(k))A(k)X(k) + H(X(k))B(k)W(Y(k))R(k) \\ Y(k) =& C(k)X(k). \end{aligned}$$

2) Minimize

$$J_\infty^i(R(\cdot)) = \sum_{k=k_0^i}^{\infty} [X^t(k)LX(k) + R^t(k)W_i^t W_i R(k)]$$

regarding linear system

$$\begin{aligned} X(k+1) =& H_i AX(k) + H_i BW_i R(k) \\ Y(k) =& CX(k). \end{aligned}$$

With the aid of the dynamic decomposition algorithm (DDA), the nonlinear system behavior can be captured by the linear system for all $k \in [k_0^i, k_1^i - 1]$ and for all $i = 1, \ldots, N$. We then know these two decisions are the same for all $k \in [k_0^i, k_1^i - 1]$. Hence, we have $X_\infty^*(k) = X_\infty^{i*}(k)$. We can now imagine the original optimal trajectory as just being composed of linked segments, which are the starting sections of segmental infinite trajectories $X_\infty^{i*}(k)$ for $i = 1, \ldots, N$. If each segmental infinite trajectory is uniformly exponential stable, i.e.,

$$\begin{aligned} &\exists\, q < \infty \text{ s.t. } \|X_\infty^{i^*}(k)\| < q \qquad \forall\, k \geq k_0^i \\ &\forall\, \epsilon > 0,\ \exists \text{ an integer } T(\epsilon) > 0 \text{ s.t. } \|X_\infty^{i^*}(k)\| \leq \epsilon \\ &\forall\, k \geq T(\epsilon) \end{aligned}$$

then the original optimal trajectory is guaranteed to be exponentially stable.

This work is inferred from backward reasoning but written in a forward sense. Lemma 3 is used to connect $X_\infty^{i^*}(k)$ to $X^{i^*}(k)$. However, *we lost one condition in Lemma 3*: $X(k_1^i)$ is free by equal in these two optimal issues, i.e., $\bar{X}^*(k_1^i) = X^*(k_1^i)$. This condition can compensate the defect in Lemma 3 and, hence, cancel out the misleading switch phenomena. For readability considerations, this paper is written not only in a forward-inference way, but also not to emphasize that $X^{i^*}(k)$ is in fact from the starting section of $X_\infty^{i^*}(k)$ for the infinite-horizon issue or $X_{[k_0^i, k_1-1]}^{i^*}(k)$, finite-horizon issue.

This work is correct in stability analysis and the infinite-horizon solution is optimal positively. However, the finite-horizon solution can be called suboptimal or near-optimal in large sense to denote the less optimality of the last few segments.

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