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Stringy symmetries and their high-energy limits

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Abstract

We derive stringy symmetries with conserved charges of arbitrarily high spins from the decoupling of two types of zero-norm states in the old covariant first quantized (OCFQ) spectrum of open bosonic string. These symmetries are valid to all energy α' and all loop orders χ in string perturbation theory. The high-energy limit $\alpha' \rightarrow \infty$ of these stringy symmetries can then be used to fix the proportionality constants between scattering amplitudes of different string states algebraically *without* referring to Gross and Mende's saddle point calculation of high-energy string-loop amplitudes. These proportionality constants are, as conjectured by Gross, independent of the scattering angle ϕ_{CM} and the order χ of string perturbation theory. However, we also discover some *new* nonzero components of high-energy amplitudes not found previously by Gross and Manes. These components are essential to preserve massive gauge invariances or decouple massive zero-norm states of string theory. A set of massive scattering amplitudes and their high energy limit are calculated explicitly to justify our results.

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In the traditional formulation of a local quantum field theory, a symmetry principle was postulated, which can be used to determine the interaction of the theory, e.g., Yang–Mills theories and general relativity. The idea of “symmetry dictates interaction” has thus become one of the fundamental philosophy to pursue new physics such as GUTs and supergravities for the last few decades. One of the most important consequences of these symmetries is the resulting softer

ultraviolet structure of field theories which, in some cases, makes them consistent or renormalizable quantum field theories when incorporating with quantum mechanics. In these cases, the Ward identities, the direct consequence of symmetry on the n -point Green functions of the theory, are intensively used to remove the unwanted loop divergences in perturbation theory. In contrast to the local quantum field theory, string theory is very different in this respect. In string theory, on the contrary, it is the interaction, prescribed by the very tight quantum consistency conditions due to the extendedness of string rather than point particle, which determines the form of the symmetry. In fact,

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once we settle on the quantum theory of a free string, the forms of the interactions and thus symmetries of all string states are fixed by the quantum consistency of the theory. For example, the massless gauge symmetries of 10D heterotic string [1] were determined to be $SO(32)$ or E_8^2 by the string one-loop consistency or modular invariance of the theory. Some stringy Einstein–Yang–Mills type symmetries with symmetry parameters containing both Einstein and Yang–Mills index were proposed in Ref. [2]. Being a consistent quantum theory with no free parameter and an infinite number of states, it is conceivable that there exists an huge symmetry group or Ward identities, which are responsible for the ultraviolet finiteness of string theory. To uncover the structure of this huge hidden symmetry group has become one of the most challenging problem ever since the discovery of string theory.

In 1988 Gross [3] made an important progress on this subject (see also [4] for the subsequent developments). With the calculation of high-energy limit of closed string scattering amplitudes for an arbitrary string-loop order G through the use of a semi-classical, saddle point technique developed by Gross and Mende [5], he was able to derive an infinite number of linear relations among high-energy scattering amplitudes of different string states with the same momenta. These relations were shown to be valid order by order and were of the *identical form* in string perturbation theory. As a result, the high-energy scattering amplitudes of all string states can be expressed in terms of, say, the dilaton scattering amplitudes. A similar result was obtained for the open string by Gross and Manes [6]. However, the physical origin of these symmetries and thus the meaning of proportionality constants between the high-energy scattering amplitudes of different string states were unknown to those authors, and their values were not calculated.

In this Letter, we propose an infinite number of stringy Ward identities derived from the decoupling of two types of zero-norm states [7] in the OCFQ string spectrum. These Ward identities are valid to *all* energy α' and to all loop orders in string perturbation theory since zero-norm states should be decoupled from the correlation functions at each order of perturbation theory by unitarity. The simplest example is the familiar massless *on-shell* Ward identity of string QED. In this sense, the stringy Ward identities we proposed in this Letter serve as a natural generalization of Ward

identity in gauge field theory. As the first test of these stringy Ward identities, the high-energy limit $\alpha' \rightarrow \infty$ of them are used to produce Gross's [3] linear relations among high-energy scattering amplitudes of different string states with the same momenta. Moreover, the proportionality constants between scattering amplitudes of different string states are calculated for the second massive level algebraically *without* referring to Gross and Mende's saddle point calculation of high-energy string-loop amplitudes. Our calculation thus serves as a consistent check of the saddle point technique of string-loop diagram developed by Gross and Mende [5]. We find that these high-energy proportionality constants are, as conjectured by Gross [3], independent of scattering angle ϕ_{CM} and the order χ of string perturbation theory. However, the proportionality coefficients do depend on the scattering angle ϕ_{CM} through the dependence of momentum at *low* energy. *More importantly, we also discover some new nonzero components of high-energy amplitudes not found previously by Gross and Manes [6]. These components are essential to preserve massive gauge invariances or decouple massive zero-norm states of string theory.* As an explicit example, we calculate the high energy limit of a set of massive scattering amplitudes of the second massive level derived in [8] to justify our results. The fact that zero-norm states imply inter-particle symmetries was demonstrated previously by two other independent approaches based on the massive worldsheet sigma-model [9] and Witten's string field theory [10]. To further uncover the group theoretical structure of these stringy symmetries, it is important to explicitly calculate the complete set of zero-norm states with arbitrarily high spins in the spectrum. Recently, a simplified method to generate zero-norm states in OCFQ bosonic string was proposed [11]. General formulas of some zero-norm tensor states at an arbitrary mass level were given. Unfortunately, general formulas for the *complete* set of zero-norm states are still lacking mostly due to the high dimensionality of spacetime $D = 26$. However, in the toy 2D string model [12], a general formula of zero-norm states with discrete Polyakov's momenta at an arbitrary mass level was given in terms of Schur polynomials [13]. These zero-norm states were shown to carry the spacetime ω_∞ charges. On the other hand, the complete spacetime symmetry group of toy 2D string was known to be the same ω_∞ , and the corresponding ω_∞ Ward iden-

titles were powerful enough to determine the tachyon scattering amplitudes *without* any integration. These observations in 2D and 26D string theories signal the importance of the existence of zero-norm states in the OCFQ string spectrum, not shared by other quantization schemes of string theory, e.g., light-cone quantization. The advantage of using the decoupling of zero-norm states to derive stringy Ward identities is that one can avoid the difficult calculation of string-loop amplitudes. Another one is that the resulting Ward identities are valid to *all* energy α' , in contrast to the high-energy $\alpha' \rightarrow \infty$ result of Gross.

Let us begin with a brief review of QED Ward identity

$$k_{\mu_1} \mathcal{T}^{\mu_1 \mu_2 \dots \mu_n}(k_1 k_2 \dots k_n) = 0, \quad (1)$$

where \mathcal{T} is the *off-shell* n -point Green function for n external photons of polarizations μ_1, \dots, μ_n and momenta k_1, \dots, k_n . Eq. (1) means that the amplitude \mathcal{T} vanishes if the polarization of one of the external photons is taken to be longitudinal. Note that Eq. (1) holds even off-shell. This seemingly simple equation, which originated from $U(1)$ gauge symmetry, turns out to be one of the most far-reaching property of QED. In the old covariant Gupta–Bleuler quantization of QED, the polarization vector ϵ_μ of photon is constrained by the covariant gauge condition $k \cdot \epsilon = 0$. One of the three allowed physical polarizations, the longitudinal one $\epsilon = k$, is zero-norm due to the massless condition of on-shell photon. The theory thus ends up with only two physical transverse propagating modes, and the longitudinal degree of freedom turns out to serve as the $U(1)$ symmetry parameter of the theory. In the OCFQ spectrum of open bosonic string theory, there exists a natural stringy generalization of this zero-norm longitudinal degree of freedom. They are (we use the notation in Ref. [7])

Type I:

$$L_{-1}|x\rangle, \quad \text{where } L_1|x\rangle = L_2|x\rangle = 0, \quad L_0|x\rangle = 0; \quad (2)$$

Type II:

$$\left(L_{-2} + \frac{3}{2} L_{-1}^2 \right) |\tilde{x}\rangle, \quad \text{where } L_1|\tilde{x}\rangle = L_2|\tilde{x}\rangle = 0, \\ (L_0 + 1)|\tilde{x}\rangle = 0. \quad (3)$$

While type I states have zero-norm at any spacetime dimension, type II states have zero-norm *only* at $D =$

26. The existence of type II zero-norm states turns out to be crucial for the discussion in the rest of this Letter. The simplest zero-norm state $k \cdot \alpha_{-1}|0, k\rangle$, $k^2 = 0$ with polarization k is the massless solution of Eq. (2), which reproduces the longitudinal photon discussed in Eq. (1). A simple prescription to systematically solve Eqs. (2) and (3) for an infinite number of zero-norm states was given recently in Ref. [11]. A more thorough understanding of the solution of these equations and their relation to spacetime ω_∞ symmetry of toy $D = 2$ string was discussed in Ref. [13].

In the first quantized approach of string theory, the string generalization of Eq. (1), or the stringy *on-shell* Ward identities are proposed to be (for our purpose we choose four-point amplitudes in this Letter)

$$\mathcal{T}_\chi(k_i) = g_c^{2-\chi} \int \frac{Dg^{\alpha\beta}}{\mathcal{N}} DX^\mu \\ \times \exp\left(-\frac{\alpha'}{2\pi} \int d^2\xi \sqrt{g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu\right) \\ \times \prod_{i=1}^4 v_i(k_i) = 0, \quad (4)$$

where at least one of the 4 vertex operators corresponds to the zero-norm state solution of Eqs. (2) or (3). In Eq. (4) g_c is the closed string coupling constant, \mathcal{N} is the volume of the group of diffeomorphisms and Weyl rescalings of the worldsheet metric, and $v_i(k_i)$ are the on-shell vertex operators with momenta k_i . The integral is over orientable open surfaces of Euler number χ parametrized by moduli \vec{m} with punctures at ξ_i . To illustrate the power of this seemingly trivial equation, the four Ward identities of the second massive level (spin-three) were calculated to be [8]

$$k_\mu \theta_{\nu\lambda} \mathcal{T}_\chi^{(\mu\nu\lambda)} + 2\theta_{\mu\nu} \mathcal{T}_\chi^{(\mu\nu)} = 0, \quad (5)$$

$$\left(\frac{5}{2} k_\mu k_\nu \theta'_\lambda + \eta_{\mu\nu} \theta'_\lambda\right) \mathcal{T}_\chi^{(\mu\nu\lambda)} + 9k_\mu \theta'_\nu \mathcal{T}_\chi^{(\mu\nu)} \\ + 6\theta'_\mu \mathcal{T}_\chi^\mu = 0, \quad (6)$$

$$\left(\frac{1}{2} k_\mu k_\nu \theta_\lambda + 2\eta_{\mu\nu} \theta_\lambda\right) \mathcal{T}_\chi^{(\mu\nu\lambda)} + 9k_\mu \theta_\nu \mathcal{T}_\chi^{[\mu\nu]} \\ - 6\theta_\mu \mathcal{T}_\chi^\mu = 0, \quad (7)$$

$$\left(\frac{17}{4} k_\mu k_\nu k_\lambda + \frac{9}{2} \eta_{\mu\nu} k_\lambda\right) \mathcal{T}_\chi^{(\mu\nu\lambda)} \\ + (9\eta_{\mu\nu} + 21k_\mu k_\nu) \mathcal{T}_\chi^{(\mu\nu)} + 25k_\mu \mathcal{T}_\chi^\mu = 0, \quad (8)$$

where $\theta_{\mu\nu}$ is transverse and traceless, and θ'_λ and θ_λ are transverse vectors. In each equation, we have chosen, say, $v_2(k_2)$ to be the vertex operators constructed from zero-norm states and $k_\mu \equiv k_{2\mu}$. Note that Eq. (7) is the inter-particle Ward identity corresponding to D_2 vector zero-norm state obtained by antisymmetrizing those terms which contain $\alpha_{-1}^\mu \alpha_{-2}^\nu$ in the original type I and type II vector zero-norm states. We will use 1 and 2 for the incoming particles and 3 and 4 for the scattered particles. In Eqs. (5)–(8), 1, 3 and 4 can be any string states (including zero-norm states) and we have omitted their tensor indices for the cases of excited string states. For example, one can choose $v_1(k_1)$ to be the vertex operator constructed from another zero-norm state which generates an inter-particle Ward identity of the third massive level. The resulting Ward identity of Eq. (7) then relates scattering amplitudes of particles at different mass level. T'_χ s in Eqs. (5)–(8) are the second massive level, χ th order string-loop amplitudes. For the string-tree level $\chi = 1$ with three tachyons $v_{1,3,4}$, the three scattering amplitudes T'_χ s were explicitly calculated and the Ward identities Eqs. (5)–(8) were verified [8]. At this point, $\{T'_\chi^{(\mu\nu\lambda)}, T'_\chi^{(\mu\nu)}, T'_\chi^\mu\}$ is identified to be the *amplitude triplet* of the spin-three state. In fact, it can be shown that $T'_\chi^{(\mu\nu)}$ and T'_χ^μ are fixed by $T'_\chi^{(\mu\nu\lambda)}$ due to the stringy Ward identities, Eqs. (5) and (6), constructed from the type I spin-two zero-norm state and another vector zero-norm state obtained by symmetrizing those terms which contain $\alpha_{-1}^\mu \alpha_{-2}^\nu$ in the original type I and type II vector zero-norm states. $T'_\chi^{[\mu\nu]}$ is obviously identified to be the scattering amplitude of the antisymmetric spin-two state with the same momenta as $T'_\chi^{(\mu\nu\lambda)}$. Eq. (7) thus relates the scattering amplitudes of two different string states at the second massive level. Note that Eqs. (5)–(8) are valid order by order and are *automatically* of the identical form in string perturbation theory. This is consistent with Gross's argument through the calculation of high-energy scattering amplitudes. However, it is important to note that Eqs. (5)–(8) are, in contrast to the high-energy $\alpha' \rightarrow \infty$ result of Gross, valid to *all* energy α' and their coefficients do depend on the center of mass scattering angle ϕ_{CM} , which is defined to be the angle between \vec{k}_1 and \vec{k}_3 , through the dependence of momentum k . To produce Gross's high-energy result and fix the proportionality constants, which were not dwelt on

in Refs. [3,6] due to lack of the physical origin of the proposed high-energy symmetries, one needs to calculate high-energy limit of Eqs. (5)–(8).

We will calculate high energy limit of Eqs. (5)–(8) without referring to the saddle point calculation in [3,5,6]. Let us define the normalized polarization vectors, $e_P = \frac{1}{m_2}(E_2, \mathbf{k}_2, 0) = \frac{k_2}{m_2}$, $e_L = \frac{1}{m_2}(\mathbf{k}_2, E_2, 0)$ and $e_T = (0, 0, 1)$ in the CM frame contained in the plane of scattering. They satisfy the completeness relation $\eta^{\mu\nu} = \sum_{\alpha,\beta} e_\alpha^\mu e_\beta^\nu \eta^{\alpha\beta}$, where $\mu, \nu = 0, 1, 2$ and $\alpha, \beta = P, L, T$. $\text{Diag } \eta^{\mu\nu} = (-1, 1, 1)$. One can now transform all μ, ν coordinates in Eqs. (5)–(8) to coordinates α, β . For Eq. (5), we have $\theta^{\mu\nu} = e_L^\mu e_L^\nu - e_T^\mu e_T^\nu$ or $\theta^{\mu\nu} = e_L^\mu e_T^\nu + e_T^\mu e_L^\nu$. In the high energy $E \rightarrow \infty$, fixed angle ϕ_{CM} limit, one identifies $e_P = e_L$ and Eq. (5) gives (we drop loop order χ here to simplify the notation)

$$\mathcal{T}_{LLL}^{6 \rightarrow 4} - \mathcal{T}_{LTT}^4 + \mathcal{T}_{(LL)}^4 - \mathcal{T}_{(TT)}^2 = 0, \quad (9)$$

$$\mathcal{T}_{LLT}^{5 \rightarrow 3} + \mathcal{T}_{(LT)}^3 = 0. \quad (10)$$

In Eqs. (9) and (10), we have assigned a relative energy power for each amplitude. For each L component, the order is E^2 (the naive order of $e_L \cdot k$ is E^2) and for each transverse T component, the order is E (the naive order of $e_T \cdot k$ is E). This is due to the definitions of e_L and e_T above, where e_L got one energy power more than e_T . Thus, for example, the naive order of \mathcal{T}_{LLL} is E^6 . However, by Eq. (9), the E^6 term of the energy expansion for \mathcal{T}_{LLL} is forced to be zero. As a result, the leading order term of \mathcal{T}_{LLL} is at most E^4 . We have used $6 \rightarrow 4$ in Eq. (9) to represent this energy reduction. Similar rule applies to \mathcal{T}_{LLT} in Eq. (10). For Eq. (6), we have $\theta'^\mu = e_L^\mu$ or $\theta'^\mu = e_T^\mu$ and one gets, in the high-energy limit,

$$10\mathcal{T}_{LLL}^{6 \rightarrow 4} + \mathcal{T}_{LTT}^4 + 18\mathcal{T}_{(LL)}^4 + 6\mathcal{T}_L^2 = 0, \quad (11)$$

$$10\mathcal{T}_{LLT}^{5 \rightarrow 3} + \mathcal{T}_{TTT}^3 + 18\mathcal{T}_{(LT)}^3 + 6\mathcal{T}_T^1 = 0. \quad (12)$$

For the D_2 Ward identity, Eq. (7), we have $\theta^\mu = e_L^\mu$ or $\theta^\mu = e_T^\mu$ and one gets, in the high-energy limit,

$$\mathcal{T}_{LLL}^{6 \rightarrow 4} + \mathcal{T}_{LTT}^4 + 9\mathcal{T}_{[LL]}^{4 \rightarrow 2} - 3\mathcal{T}_L^2 = 0, \quad (13)$$

$$\mathcal{T}_{LLT}^{5 \rightarrow 3} + \mathcal{T}_{TTT}^3 + 9\mathcal{T}_{[LT]}^3 - 3\mathcal{T}_T^1 = 0. \quad (14)$$

Note that $\mathcal{T}_{[LL]}$ in Eq. (13) originate from the high-energy limit of $\mathcal{T}_{[PL]}$, and the antisymmetric property of the tensor forces the leading E^4 term to be zero. Finally the singlet zero norm state Ward identity, Eq. (8),

imply, in the high-energy limit,

$$34\mathcal{T}_{LLL}^{6 \rightarrow 4} + 9\mathcal{T}_{LTT}^4 + 84\mathcal{T}_{(LL)}^4 + 9\mathcal{T}_{(TT)}^2 + 50\mathcal{T}_L^2 = 0. \quad (15)$$

It is important to note that all components of high-energy amplitudes of symmetric spin-three and antisymmetric spin-two states appear at least once in Eqs. (9)–(15). It is now easy to see that the naive leading order amplitudes corresponding to E^4 appear in Eqs. (9), (11), (13) and (15). However, a simple calculation shows that $\mathcal{T}_{LLL}^4 = \mathcal{T}_{LTT}^4 = \mathcal{T}_{(LL)}^4 = 0$. So the real leading order amplitudes correspond to E^3 , which appear in Eqs. (10), (12) and (14). A simple calculation shows that

$$\mathcal{T}_{TTT}^3 : \mathcal{T}_{LLT}^3 : \mathcal{T}_{(LT)}^3 : \mathcal{T}_{[LT]}^3 = 8 : 1 : -1 : -1. \quad (16)$$

Note that these proportionality constants are, as conjectured by Gross, independent of the scattering angle ϕ_{CM} and the loop order χ of string perturbation theory. *Most importantly, we now understand that they originate from zero-norm states in the OCFQ spectrum of the string!* The subleading order amplitudes corresponding to E^2 appear in Eqs. (9), (11), (13) and (15). One has 6 unknown amplitudes and 4 equations. Presumably, they are not proportional to each other or the proportional coefficients do depend on the scattering angle ϕ_{CM} . We will justify this point later in our sample calculation. Our calculation here is, similar to the toy 2D string case, purely algebraic without any integration and is independent of saddle point calculation in [3,5,6]. It is important to note that our result in Eq. (16) is gauge invariant as it should be since we derive it from Ward identities (5)–(8). On the other hand, the result obtained in [6] with $\mathcal{T}_{TTT} \propto \mathcal{T}_{[LT]}$, and $\mathcal{T}_{LLT} = \mathcal{T}_{(LT)} = 0$ in the leading order energy at this mass level is, on the contrary, *not* gauge invariant. In fact, with only two nonzero amplitudes of \mathcal{T}_{TTT} and $\mathcal{T}_{[LT]}$, an inconsistency arises between Eqs. (6) and (7) or Eqs. (12) and (14). To further justify our result, we give a sample calculation. For the string-tree level $\chi = 1$, with one tensor and three tachyons $v_{1,3,4}$, the four scattering amplitudes $\mathcal{T}^{(\mu\nu\lambda)}$, $\mathcal{T}^{(\mu\nu)}$, $\mathcal{T}^{[\mu\nu]}$ and \mathcal{T}^μ were explicitly calculated in [8]. An explicit calculation of their high-energy limits give the kinematic factors of the amplitudes (s - t channel only) $\mathcal{K}_{TTT} = -8E^9 \sin^3 \phi_{CM} = 8\mathcal{K}_{LLT} = -8\mathcal{K}_{(LT)} = -8\mathcal{K}_{[LT]}$, where $s = -(k_1 + k_2)^2$, $t = -(k_2 + k_3)^2$, and $u = -(k_1 + k_3)^2$ are the

Mandelstam variables. Also $\mathcal{T}_{LLL}^6 = \mathcal{T}_{LLT}^5 = 0$ as claimed above. A calculation of subleading order in E shows that the amplitudes are not proportional to each other or the proportional coefficients do depend on the scattering angle ϕ_{CM} . Similar calculations can be done for the third massive level. The result is [14]

$$\begin{aligned} &\mathcal{T}_{TTTT}^4 : \mathcal{T}_{TTLL}^4 : \mathcal{T}_{LLLL}^4 : \mathcal{T}_{TTL}^4 : \mathcal{T}_{LLL}^4 : \tilde{\mathcal{T}}_{LT,T}^4 : \tilde{\mathcal{T}}_{LP,P}^4 \\ &\quad : \mathcal{T}_{LL}^4 : \tilde{\mathcal{T}}_{LL}^4 \\ &= 16 : \frac{4}{3} : \frac{1}{3} : -\frac{4\sqrt{6}}{9} : -\frac{\sqrt{6}}{9} : -\frac{2\sqrt{6}}{3} : 0 : \frac{2}{3} : 0, \end{aligned} \quad (17)$$

where $\mathcal{T}_{\mu\nu,\lambda}$, $\tilde{\mathcal{T}}_{\mu\nu,\lambda}$, $\mathcal{T}_{\mu\nu}$ and $\tilde{\mathcal{T}}_{\mu\nu}$ are amplitudes corresponding to $\alpha_{-1}^{(\mu\nu)}\alpha_{-2}^\lambda$, mixed-symmetric spin-three of $\alpha_{-1}^{\mu\nu}\alpha_{-2}^\lambda$, $\alpha_{-2}^\mu\alpha_{-2}^\nu$ and $\alpha_{-1}^\mu\alpha_{-3}^\nu$, respectively. It is remarkable to discover that both algebraic and sample calculations give exactly the same results Eqs. (16) and (17). In general there is only one independent component of high-energy scattering amplitude at each fixed mass level, and it can be deduced that

$$\mathcal{T}_{n_1 n_2 n_3 n_4}^{TT\dots} = [(-2)E^3 \sin \phi_{CM}]^N \mathcal{T}(N), \quad (18)$$

where n_i is the number of T for the i th particle and

$$\begin{aligned} \mathcal{T}(N) &= \sqrt{\pi}(-1)^{N-1} 2^{-N} E^{-1-2N} \\ &\quad \times \left(\sin \frac{\phi_{CM}}{2}\right)^{-3} \left(\cos \frac{\phi_{CM}}{2}\right)^{5-2N} \\ &\quad \times \exp\left(-\frac{s \ln s + t \ln t - (s+t) \ln(s+t)}{2}\right), \end{aligned}$$

$$N = \sum n_i.$$

As a result, all high-energy string scattering amplitudes can be expressed in terms of those of tachyons. Finally, unlike the saddle point calculation, our algebraic approach is very easy to generalize to closed string case by “doubling the spectrum”. In that case, one has 32 zero-norm state at the second massive level. The nonzero high energy amplitudes can be obtained by doubling Eq. (16), which amounts to 16 nonzero components.

We conclude that the physical origin of the high-energy symmetries and the proportionality constants

in Eq. (16) are from the zero-norm states in the OCFQ spectrum. The most challenging problem remained is the calculation of algebraic structure of these stringy symmetries derived from the complete zero-norm state solutions of Eqs. (2) and (3) with arbitrarily high spins. Presumably, it is a complicated 26D generalization of ω_∞ of the simpler toy 2D string model [13].

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References

- [1] D.J. Gross, J.A. Harvey, E. Martinec, R. Rohm, *Nucl. Phys. B* 256 (1985) 253;
- [2] J.C. Lee, *Phys. Lett. B* 337 (1994) 69.
- [3] D.J. Gross, *Phys. Rev. Lett.* 60 (1988) 1229;
D.J. Gross, *Philos. Trans. R. Soc. London Ser. A* 329 (1989) 401.
- [4] J. Isberg, U. Lindstr, B. Sundborg, *Phys. Lett. B* 293 (1992) 321;
G. Moore, hep-th/9310026;
M. Evans, I. Giannakis, D.V. Nanopoulos, *Phys. Rev. D* 50 (1994) 4022.
- [5] D.J. Gross, P. Mende, *Phys. Lett. B* 197 (1987) 129;
D.J. Gross, P. Mende, *Nucl. Phys. B* 303 (1988) 407.
- [6] D.J. Gross, J.L. Manes, *Nucl. Phys. B* 326 (1989) 73, see Section 6 for details.
- [7] M.B. Green, J.H. Schwarz, E. Witten, *Superstring Theory*, vol. I, Cambridge Univ. Press, Cambridge, 1987.
- [8] J.C. Lee, *Prog. Theor. Phys.* 91 (1994) 353.
- [9] J.C. Lee, *Phys. Lett. B* 241 (1990) 336;
J.C. Lee, *Phys. Rev. Lett.* 64 (1990) 1636;
J.C. Lee, B. Ovrut, *Nucl. Phys. B* 336 (1990) 222.
- [10] H.C. Kao, J.C. Lee, *Phys. Rev. D* 67 (2003) 086003, hep-th/0212196;
J.C. Lee, *Z. Phys. C* 63 (1994) 351.
- [11] J.C. Lee, hep-th/0302123.
- [12] For a review see I.R. Klebanov, A. Pasquinucci, hep-th/9210105, and references therein.
- [13] T.D. Chung, J.C. Lee, *Phys. Lett. B* 350 (1995) 22;
T.D. Chung, J.C. Lee, *Z. Phys. C* 75 (1997) 555.
- [14] C.T. Chan, J.C. Lee, *Nucl. Phys. B* 690 (2004) 3, hep-th/0401133.