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A fuzzy clustering approach to real-time demand-responsive bus dispatching control

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Abstract

Quick response (QR) to passenger needs is a key objective for advanced public transportation systems (APTS), and it has become increasingly important for contemporary metropolitan bus operations to gain a competitive advantage over private transportation. This paper presents a real-time control methodology for demand-responsive bus operations that respond quickly to passenger needs. The proposed method primarily involves two levels of functionality: (1) short-term forecasting of passenger demands using time-series prediction models, and (2) identification of service strategies coupled with the associated bus service segments using fuzzy clustering technologies in response to variances in passenger demand attributes and traffic conditions. The proposed bus operations method identifies the demand-responsive vehicle service strategies primarily according to the predicted up-to-date attributes of passengers' demands, rather than deterministic passenger arrival rates, which were generally used in previous literature. In addition, the variation of traffic conditions along bus lines is considered in the proposed method. Results from numerical studies using real data of passengers' demands, including passenger volume at each bus stop and the passenger origin-destination (O-D) patterns, are presented to demonstrate the effectiveness of the proposed method for real-world applications.

Keywords: Forecasting; Fuzzy clustering; Advanced public transportation systems; Bus dispatching

1. Introduction

Correctly identifying passenger demands and quickly responding to those needs with advanced bus operations control strategies are vital to the development of advanced public transportation systems

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(APTS) in urban areas. Generally, traditional public transportation systems are less convenient than private transportation, which can be quantified to a certain extent by passengers' satisfaction with the total travel time, including walking time, waiting time and in-transit time. To improve the competitiveness of public transportation systems in urban areas, there have recently been increased applications of advanced technologies, such as automatic vehicle location (AVL) systems, wireless communication systems, and electronic fare payment systems for urban transit. However, APTS must also rely on advanced vehicle dispatching strategies. This is especially true since tremendous advances in information and communication technologies have contributed to dramatic changes in passenger demands for public transportation services, and have exerted a profound influence on the operations of fleet management in public transportation systems.

Despite the importance of advanced demand-responsive bus transit operations control in APTS, there have been few related studies in previous literature, and this can be seen in the following two aspects.

First, most research in the field of APTS is devoted to analytical studies and related discussions for the application of new technologies in APTS, such as the assessment of system performance based on only limited case studies. Adoption of various intelligent transportation systems (ITS) technologies in public transportation systems and some local application cases are described in both Khattak et al. [18] and Hansen et al. [12]. Those studies indicate that the utilization of advanced ITS technologies such as global positioning systems (GPS), electronic fare payment systems, and automatic passenger counting systems appears to permit significant improvement in public transportation systems, such as metropolitan bus operations. Similar arguments can also be found in Hickman [13]. In addition, the idea of using historical AVL data to improve bus service, as proposed by Horbury [15], also helps to demonstrate the potential benefits of the emerging ITS technologies to the public transportation operations. Nevertheless, there is a lack of further research to investigate the operational problems of these new technologies in APTS, using the analytical results from previous literature.

Second, although there is some literature on the operations of public transportation, the validity of published methods seems to be subject to specific operational conditions, such as deterministic passenger arrival rates and fixed schedules, which may cause these previous methods to be problematic for real-time operations. In these previous studies, two types of methodology are noteworthy: simulation-based [1,23] and optimization-based [6,8,25,33], both of which are broadly employed in the previous literature. FRABSIM, a fixed-route accessible bus service simulation model proposed by Adesanya [1] was designed to evaluate the performance of bus service under various scenarios of passenger demands. One notable application example of FRABSIM is the estimation of the possible effects of different bus service alternatives on the performance of bus operations when serving persons in wheelchairs. Similarly, Santhakumar et al. [23] developed a simulation model to analyze the effects of diverse bus operational strategies on system performance, including express bus transit service. Given historical demand data collected from four bus lines, in order to improve the performance of local fixed-route bus operations, their numerical results highly recommended the strategy of express bus transit operations coupled with re-configuration of bus stops. In addition, Tsao et al. [33] proposed a nonlinear programming model to determine optimal solutions in terms of the number of bus stops in each zone and the optimal dispatching headway associated with each zone-based bus route, given passenger demand data. In addition to zonal service, short-turn service strategies have also attracted substantial research to address issues of heavy passenger demand, and considerable advances can be found in previous studies, including Furth [8] and Site et al. [25]. One notable generalization obtained from recent related studies is that the effect of variation of passenger distribution on bus routing and scheduling has been given increasingly attention.

The specific real-time deadheading operational problem is elaborately investigated in Eberlein et al. [6], which assumes that such demand-related parameters as passenger arrival rates, the passenger alighting proportion at each transit station, and the boarding and alighting times per passenger are all constant. Nevertheless, the dynamics of such demand-related parameters as the parameters mentioned above in Eberlein et al. [6], and the effect caused by time-varying traffic conditions along a bus route remain as critical issues in the existing approaches of real-time transit operations control.

Overall, it should be noted that the difficulties in formulating and solving real-time demand-responsive bus dispatching control problems are rooted in the uncertainty and vagueness of short-term changes in time-varying passenger origin-destination (O-D) flows and corresponding time-varying traffic flow conditions in bus routes. Correspondingly, it seems that any advanced bus control methodology must have the capability of identifying time-varying passenger O-D demand patterns coupled with corresponding traffic flow conditions so as to implement appropriate bus service strategies for quick response.

Considering the aforementioned uncertain effects on bus operations induced by variations in passenger O-D flow patterns as well as time-varying traffic conditions, we propose a real-time demand-responsive bus operations control method. This method utilizes fuzzy-clustering techniques and time-series prediction models for identification of passenger demand patterns and prior prediction of the short-term changes in passenger volumes in the operations of bus routing. In contrast with existing bus operations control approaches, the proposed method has several distinctive features, as follows.

(1) Conceptually, the proposed methodology stems from the concept of demand-oriented business operations, which advocates the primary goal of satisfying customer needs. Following this approach, we propose that the operations of bus routing should respond efficiently and effectively to short-term changes in passenger demand, thus satisfying passenger needs as much as possible.
(2) For real-time operations control, a two-stage QR demand-responsive bus operations control approach is proposed. Herein, we reiterate that the identification of appropriate bus service strategies should be made before bus routing is established in order to respond to short-term changes in passenger demand patterns. Such a pre-trip measure is very important, especially in the APTS operational environment, when real-time information on passenger volumes and their O-D attributes is obtainable using advanced information and communication technologies. Therefore, two sequential procedures, including prior prediction of time-varying passenger demand attributes and real-time identification of bus service strategies, are used in the proposed method.
(3) For methodology, we integrate the fundamentals of fuzzy clustering and time-series prediction approaches to address the real-time demand-responsive bus operations control problem. In the scenario of prior prediction of short-term passenger demands, a time-series prediction model is developed to forecast the short-term changes in passenger demands in the process of bus routing. This is done using measured real-time passenger-related information together with historical data. Fuzzy clustering techniques are then utilized to identify appropriate bus service strategies in response to the variations in time-varying passenger demands and traffic conditions along bus lines. In contrast, previous approaches, both optimization-based and simulation-based, are not used in this study due to their inadequacy in responding to the dynamics of passengers' O-D flows and their inflexibility in dealing with uncertain and complicated time-varying traffic operational conditions for real-time bus operations control. At least, there is a lack of evidence in the previous literature to show that the existing approaches for real-time bus operations control can perform well in a dynamic bus operational environment with no ideal assumptions for either model parameters or input data.

It is worth mentioning that any advanced bus control strategy can be operated dynamically in the spatial (e.g., various bus stop strategies) and/or temporal (e.g., time-varying bus dispatching frequencies) domains to respond to the short-term changes of time-varying passengers' demand patterns. Nevertheless, the scope of our current study is limited to dealing with the dynamic bus operations in the spatial domain. Correspondingly, appropriate bus service strategies are determined in response to the short-term changes of passenger O-D flows subject to constant bus dispatching frequency and given fleet size for bus operations.

Furthermore, it is worth noting that fuzzy clustering is a part of fuzzy data analysis, and can be viewed as an improved clustering technique. Compared to classical clustering techniques, the distinctive feature of fuzzy clustering is that fuzzy clustering utilizes fuzzy partitioning so that a given data point can belong to several groups with the degree of belongingness bounded within the range of 0 and 1. However, utilizing classical clustering techniques, any given data point can be assigned to one and only one group among mutually exclusive data groups. In contrast with classical clustering techniques, this feature of fuzzy clustering techniques can make pattern recognition more flexible for real-time applications, especially in cases where patterns of data attributes change rapidly. For detailed descriptions of properties of fuzzy clustering techniques and related applications, the reader is referred to the early literature [3,10,31] as well as our previous research [16,24].

The rest of the paper is organized as follows. The architecture of the proposed bus operations control method and the methodology development are presented in Section 2. Section 3 depicts numerical results obtained from cases studied. Finally, concluding remarks are made in Section 4.

2. Methodology development

Throughout this paper, the following three basic assumptions are postulated to facilitate development of the proposed methodology.

(1) Bus dispatching frequency is set to be time-invariant. Correspondingly, the prototype of the proposed method is limited to fixed-schedule operations, although it is claimed to be capable of adjusting bus service strategies in real time in response to short-term changes in passenger demands. Such an assumption is postulated to facilitate modeling the proposed discrete-time time-series prediction model in the case of fixed time intervals; however, it implies limitations of the proposed method for the implementation of more advanced operational strategies, such as dynamic headway control.
(2) Real-time passenger demand data are assumed to be collectable via advanced ITS technologies such as automatic passenger counting systems.
(3) The changes of passenger O-D patterns in the time interval between bus dispatching from the terminal and arriving at the origin bus stop are not considered. Correspondingly, the passenger O-D patterns are assumed to be known in a given time interval, and measurable directly from related advanced technologies such as electronic fare payment systems.

Accordingly, a real-time demand-responsive bus operations control methodology is proposed, and Fig. 1 illustrates the framework of the proposed control method. As shown in Fig. 1, there are two primary mechanisms, (1) short-term passenger demand forecasting and (2) identification of bus service strategies coupled with the served bus stops in given bus routes. Here the real-time control of demand-responsive

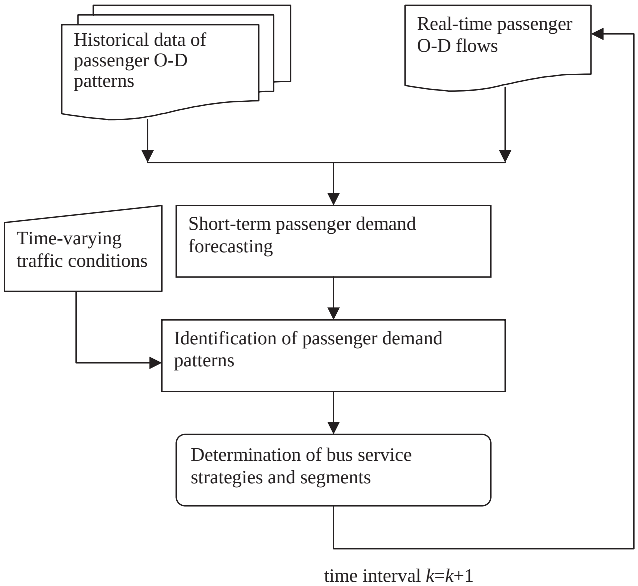

Fig. 1. Framework of the proposed bus operations control method.

bus operations refers to the identification of appropriate bus service strategies each time when a bus is dispatched to serve passengers in a given route. Therefore, such proposed real-time bus dispatching control methodology relies mainly on both the mechanisms of real-time passenger demand data collection and quick response to the changes of corresponding passenger demand patterns for determination of suitable bus service strategies at that moment. However, the highlight of the proposed method is on the latter, i.e., the capability of responding to the changes of passenger demand patterns, which can be found in the corresponding model formulation in the following two subsections. As to the real-time data collection, it is assumed to be available via other ITS-related technologies, as mentioned in the second assumption. The details in the fundamentals of the aforementioned two major functions are described in the following two subsections.

2.1. *Short-term passenger demand forecasting*

This function serves to forecast the time-varying passenger volume at each bus stop in a given time interval k of bus headway. To execute the aforementioned short-term passenger demand forecasting, a time-varying passenger demand variable $\sigma_i(k)$ is introduced, and derived using the fundamentals of exponential smoothing-based prediction approaches, where $\sigma_i(k)$ is defined as the time-varying passenger volume at a given bus stop i in the time interval k.

It is noteworthy that although there have been a certain amount of short-term forecasting techniques, e.g., basic and extended time-series prediction methods, filtering techniques, neural network based approaches and nonparametric regression models, published previously [2,5,7,11,21,22,27–30,36], most of them aim at traffic state prediction (e.g., speed, volume and occupancy) rather than passenger trip generation, as conducted in this study. In addition, the analytical results obtained by comparing the

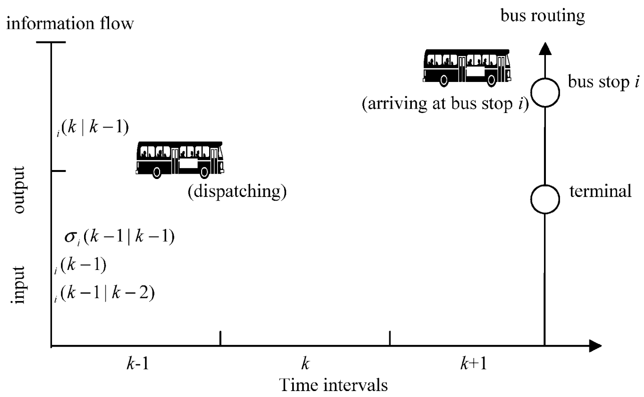

Fig. 2. Relationship of bus dispatching and time-varying passenger demands.

existing typical techniques may vary in different studies due to different study purposes and evaluation criteria [20,26,28,34]. Considering the need of this study and the corresponding requirement in terms of the functionality of passenger demand forecasting performed in the proposed method, a simple exponential smoothing technique is employed for convenience.

In the proposed demand-responsive bus operations control method, $\sigma_i(k)$ should be predicted at the beginning of the time interval k, before bus dispatching, and is derived as follows. As shown in Fig. 2, a bus is dispatched at the start of a given time interval k to serve the passengers at a given bus stop i at the beginning of the next time interval k+1. Here $\Delta_i(k-1)$ is the real increased demand volume of passengers measured from passenger counting systems in a given time interval $k-1$; and $\Delta_i(k-1|k-2)$ is the increased demand volume of passengers predicted at a given bus stop i in the given time interval $k-1$. Utilizing the theories of exponential smoothing methodology, the prior prediction of short-term changes in the passenger volume at the given bus stop i in the time interval k ($\Delta_i(k|k-1)$) can be formulated as

$$\Delta_i(k|k-1) = \alpha \times \Delta_i(k-1) + (1-\alpha) \times \Delta_i(k-1|k-2), \tag{1}$$

where α corresponds to a user-specific weight, which is set to be 0.63 in this study. Conveniently, α is here determined utilizing the SAS statistical analysis package, which searches for the optimal value of α with the objective of minimizing the sum of squared prediction errors. First, we sum up the real passenger volume remaining at a given bus stop i at the end of the time interval $k-1$ $\sigma_i(k-1|k-1)$ and the predicted value $\Delta_i(k|k-1)$, both measured at the beginning of the time interval k. From this, we then have the prior prediction of $\sigma_i(k)$ at the beginning of the given time interval k ($\sigma_i(k|k-1)$) as

$$\sigma_i(k|k-1) = \sigma_i(k-1|k-1) + \alpha \times \Delta_i(k-1) + (1-\alpha) \times \Delta_i(k-1|k-2), \tag{2}$$

Herein, the estimate of $\sigma_i(k|k-1)$ is used as one of determinants in the following fuzzy clustering mechanism to identify appropriate bus dispatching strategies in response to the time-varying passenger demands in the time interval k. In addition, the passenger volume remaining at the given bus stop i at the end of the time interval k ($\sigma_i(k|k)$) should be updated as

$$\sigma_i(k|k) = [\sigma_i(k-1|k-1) + \Delta_i(k)] - \left[\sum_{\forall\theta} f_i^{\theta}(k)\right], \tag{3}$$

where $\Delta_i(k)$ is the real increased demand volume of passengers, as measured by passenger counting systems at the given bus stop i in a given time interval k; and $f_i^{\theta}(k)$ represents the real volume of passengers which are served by a given bus using a given bus dispatching strategy θ at the given bus stop i, as measured by on-board electronic fare payment systems in the given time interval k. It is noteworthy that the updated value of $\sigma_i(k|k)$ is used for recursive estimation in the next time interval k+1. In addition, further details referring to testing the model's validity can be found elsewhere [32].

2.2. *Identification of bus service strategies*

This mechanism serves to identify appropriate bus service strategies which are implemented to satisfy both time-varying passenger demands in a given time interval k. To perform this function in real time, we propose a fuzzy clustering-based algorithm, which analyzes multiple state variables in relation to both time-varying passenger demand attributes and traffic conditions. This algorithm then determines a suitable bus service strategy together with its service segment in the given bus route, in quick response to the time-varying passenger demands under the present traffic conditions along the given bus route. The entire architecture of fuzzy-clustering functionality built in this mechanism is shown in Fig. 3.

Herein, four well-known bus service strategies are considered as candidates for demand-responsive bus service strategies, responding to various conditions of passenger demand patterns, as well as traffic conditions. They are: (1) all-stop service, (2) express service, (3) short-turn service, and (4) zonal service. Fig. 4 graphically characterizes the specific en-route stopping operations of these bus service strategies. Details relevant to their advantages and limitations can be found in previous studies [9,33,35,37]. In the proposed method, all-stop service is regarded as a basic bus service strategy which aims for the case that passengers' O-D trip flows disperse at bus stops on a given bus route, and the proportion of short-distance passenger trips is relatively high in a given time interval. In contrast, the other strategies differ from the all-stop service strategy in terms of the specific patterns of passenger trip flows that they mainly serve. Note that there are no limitations on the number of bus service strategies potentially used in the proposed method. Other sophisticated bus service strategies such as deadheading service and skip-stop service can also be involved in further development of a comprehensive demand-responsive bus operations control system. Nevertheless, this study aims to investigate the feasibility of the proposed approach as well as the primary procedures of model formulation, and thus only certain bus service strategies are illustrated.

To perform real-time bus service strategy identification, four groups of state variables are specified in this study, and are defined as follows.

(1) $\psi_{s_*}(k)$ represents the time-varying degree of passenger trip centralization in a specific short-turn bus service region s_* in a given time interval k, and is denoted by

$$\psi_{s_*}(k) = \frac{\sum\limits_{\forall o_{s_*}} \sum\limits_{\forall d_{s_*}} \left\lfloor v_{o_{s_*},d_{s_*}}(k)/l_{o_{s_*},d_{s_*}} \right\rfloor}{\sum\limits_{\forall o_T} \sum\limits_{\forall d_T} \left\lfloor v_{o_T,d_T}(k)/l_{o_T,d_T} \right\rfloor}, \tag{4}$$

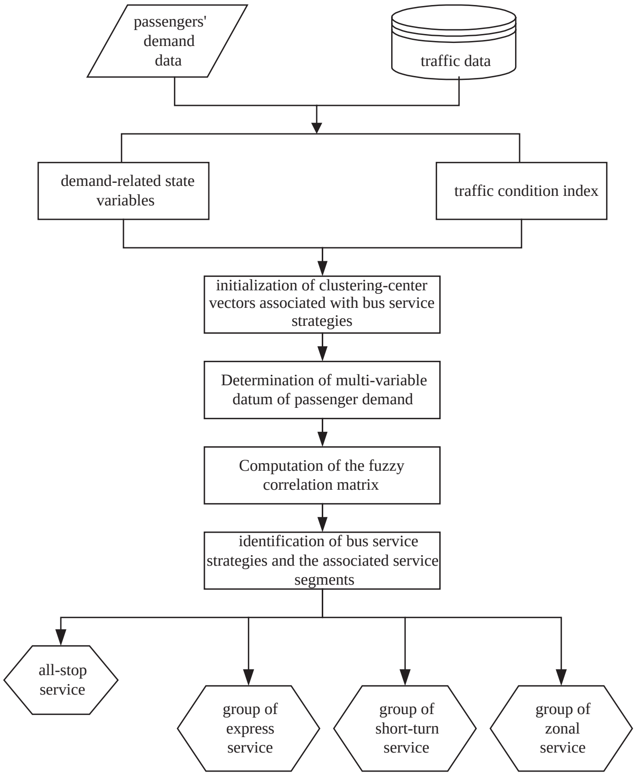

Fig. 3. Functional architecture for the identification of bus service strategies.

where o_{s_*} and d_{s_*} represent a given pair of origin and destination bus stops located in the specific short-turn service region s_*, and $l_{o_{s_*},d_{s_*}}$ is the geographic distance between o_{s_*} and d_{s_*}; similarly, o_T and d_T correspond to a given pair of the origin and destination bus stops served by a given all-stop bus coded with T in a given bus route, and l_{o_T,d_T} is the geographic distance between o_T and d_T; $v_{o_{s_*},d_{s_*}}(k)$ represents the time-varying passengers' O-D trip volume between o_{s_*} and d_{s_*}, estimated in a given time interval k; and similarly, $v_{o_T,d_T}(k)$ represents the time-varying passengers' O-D trip volume between a given pair of the origin o_T and the destination d_{T} in the given bus route T in a given time interval k. Herein, the time-varying passengers' O-D trip volumes $v_{o_{s*},d_{s*}}(k)$ and $v_{o_T,d_T}(k)$ are estimated in real time using the previous mechanism of the proposed system, denoted respectively by

$$v_{o_{s_*},d_{s_*}}(k) = \sigma_{o_{s_*}}(k|k-1) \times p_{o_{s_*},d_{s_*}}(k-1|k-1), \tag{5}$$

$$v_{o_T,d_T}(k) = \sigma_{o_T}(k|k-1) \times p_{o_T,d_T}(k-1|k-1), \tag{6}$$

where the prior predictions of passenger demands ($\sigma_{o_{s_*}}(k|k-1)$ and $\sigma_{o_T}(k|k-1)$) at given bus stops o_{s_*} and o_T are predicted using Eq. (2); $p_{o_{s_*},d_{s_*}}(k-1|k-1)$ represents the time-varying percentage of passengers originating at a given bus stop o_{s_*} and getting off at a given bus stop d_{s_*} measured at time step k−1; and similarly, $p_{o_T,d_T}(k-1|k-1)$ refers to the time-varying percentage of passengers originating at o_T and alighting at d_T. Note that as described previously in assumption (3), both

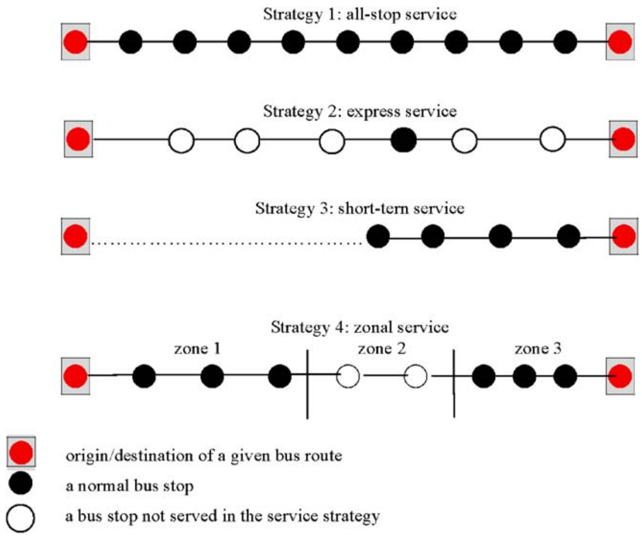

Fig. 4. Illustration of en-route stopping operations for the specified bus service strategies.

$p_{o_{s*},d_{s*}}(k-1|k-1)$ and $p_{o_T,d_T}(k-1|k-1)$ are given, and can be determined employing advanced APTS-related technologies.

(2) $\psi_{e_*}(k)$ denotes the time-varying degree of passenger trip centralization in a specific express service region e_* in a given time interval k, and is given by

$$\psi_{e_*}(k) = \frac{\left[\sum\limits_{\forall o_{e*}} \sum\limits_{\forall d_{e*}} v_{o_{e*},d_{e*}}(k)\right] \Big/ n_{e_*}}{\left[\sum\limits_{\forall i} \sigma_i(k|k-1)\right] \Big/ N}, \tag{7}$$

where o_{e_*} and d_{e_*} represent a given pair of the origin and destination bus stops located in the specific express service region $e*$; n_{e_*} represents the total number of bus stops in the specific express service region e_*; N is the total number of bus stops in a given bus route; and similar to the definition of $v_{o_{s*},d_{s*}}(k)$, $v_{o_{e*},d_{e*}}(k)$ is referred to as the time-varying passengers' O-D trip volume between o_{e_*} and d_{e_*}, estimated in a given time interval k.

(3) $\psi_{z_*}(k)$ represents the time-varying degree of passenger trip centralization in a specific zonal service region z_* in a given time interval k, and is denoted by

$$\psi_{z_*}(k) = \frac{\sum\limits_{\forall o_{z*}} \sum\limits_{\forall d_{z*}} v_{o_{z*},d_{z*}}(k)}{\sum\limits_{\forall i} \sigma_i(k|k-1)}, \tag{8}$$

where o_{z_*} and d_{z_*} represent a given pair of origin and destination bus stops located in the specific zonal service region z_*; and $v_{o_{z*},d_{z*}}(k)$ is referred to as time-varying passengers' O-D trip volume between o_{z_*} and d_{z_*}, as predicted in a given time interval k.

(4) $\delta(k)$ denotes the time-varying congestion index, which is used to quantify the effect of road congestion on the performance of diverse bus service strategies in a given time interval *k*. In addition to the variety of passenger trip patterns, the severity of road traffic congestion along a given bus route is also considered as a critical factor in determining the performance of real-time bus operations control in the study. Correspondingly, the bus service strategies determined in real time by the proposed method may vary with present traffic conditions using the specified traffic state variable. Conveniently employing the advantages and characteristics of *S*-shaped fuzzy membership functions suggested by Zimmermann [38] and Kandel [17], $\delta(k)$ is herein formulated as an *S*-shaped fuzzy membership function with respect to the aggregate road traffic occupancy in the given bus route T $(\bar{u}_T(k))$, and is denoted by

$$\begin{aligned}\delta(k) = F\,(\bar{u}_T(k);\ a, b, c) &= 0, && \text{for} \quad \bar{u}_T(k) \leqslant a\\ &= 2\left(\frac{\bar{u}_T(k)-a}{c-a}\right)^2, && \text{for} \quad a \leqslant \bar{u}_T(k) < b\\ &= 1 - 2\left(\frac{\bar{u}_T(k)-c}{c-a}\right)^2, && \text{for} \quad b \leqslant \bar{u}_T(k) < c\\ &= 1 && \text{for} \quad \bar{u}_T(k) \geqslant c\end{aligned} \tag{9}$$

where $\bar{u}_T(k)$ corresponds to the route-based occupancy value which is determined by averaging the measured occupancies on the links located in the given bus route T in the time interval k; the parameters a, b, and c (termed the primary values in fuzzy theories), represent the pre-set thresholds to characterize the degree of road traffic congestion indicated by the linguistic terms light, medium, and heavy congestion, respectively.

Given the real-time passenger demand data as well as the measured raw traffic data in a given time interval k, we then have an $(n_s + n_e + n_z + 1)\times 1$ time-varying state variable vector $(\boldsymbol{\Psi}(k))$ as:

$$\boldsymbol{\Psi}(k) = \mathbf{Col}\left[\boldsymbol{\psi}_s(k), \boldsymbol{\psi}_e(k), \boldsymbol{\psi}_z(k), \delta(k)\right], \tag{10}$$

where n_s, n_e, and n_z are defined as the numbers of bus stopping strategies associated with the types of short-turning (s for short), express (e for short), and zonal services (z for short), respectively; $\boldsymbol{\psi}_s(k)$, $\boldsymbol{\psi}_e(k)$, and $\boldsymbol{\psi}_z(k)$ are the $(n_s \times 1)$, $(n_e \times 1)$ and $(n_z \times 1)$ state vectors which involve the groups of state variables associated with the types of short-turning, express, and zonal services, respectively, and can be further expressed as

$$\boldsymbol{\psi}_s(k) = [\psi_{s_1}(k), \psi_{s_2}(k), \ldots, \psi_{s_{n_s}}(k)]^{\mathrm{T}}, \tag{11}$$

$$\boldsymbol{\psi}_e(k) = [\psi_{e_1}(k), \psi_{e_2}(k), \ldots, \psi_{e_{n_e}}(k)]^{\mathrm{T}}, \tag{12}$$

$$\boldsymbol{\psi}_z(k) = [\psi_{z_1}(k), \psi_{z_2}(k), \ldots, \psi_{z_{n_z}}(k)]^{\mathrm{T}}. \tag{13}$$

Herein, the elements present in $\boldsymbol{\psi}_s(k)$, $\boldsymbol{\psi}_e(k)$, and $\boldsymbol{\psi}_z(k)$ represent the state variables estimated, respectively, under the bus operational conditions of short-turning (s for short), express (e for short), and zonal services (z for short) associated with corresponding service regions. Accordingly, all the state variables involved in $\boldsymbol{\psi}_s(k)$, $\boldsymbol{\psi}_e(k)$, and $\boldsymbol{\psi}_z(k)$ are estimated using Eqs. (4), (7) and (8), respectively.

Utilizing the time-varying state variables, a fuzzy clustering-based algorithm is proposed to determine if it is necessary to replace the basic all-stop service strategy with another appropriate bus service strategy coupled with specific service regions in response to the time-varying patterns of passengers' trip demands and the present traffic conditions along a given bus route in a given time interval. Otherwise, the conventional all-stop bus service strategy remains in the given time interval. The following summarizes the primary steps executed in this algorithm.

Step 0: Initialize the vector of clustering centers associated with the specified bus service strategies, where each clustering center represents the typical measurements of state variables associated with a given bus service strategy θ and a specific service region θ_*. Conveniently, the mean vector of $\mathbf{\Psi}(k)$, denoted as follows, is employed as the $(n_s + n_e + n_z + 1) \times 1$ clustering-center vector ($\boldsymbol{\zeta}$).

$$\boldsymbol{\zeta} = E[\mathbf{\Psi}(m)] = [E[\boldsymbol{\psi}_s(m)], E[\boldsymbol{\psi}_e(m)], E[\boldsymbol{\psi}_z(m)], E[\delta(m)]]^{\mathrm{T}}, \tag{14}$$

where m denotes any given time interval; and $E[\boldsymbol{\psi}_s(m)]$, $E[\boldsymbol{\psi}_e(m)]$, and $E[\boldsymbol{\psi}_z(m)]$ are given by

$$E[\boldsymbol{\psi}_s(m)] = \left[\bar{u}_{s_1}, \bar{u}_{s_2}, \ldots, \bar{u}_{s_{n_s}}\right]^{\mathrm{T}}, \tag{15}$$

$$E[\boldsymbol{\psi}_e(m)] = \left[\bar{u}_{e_1}, \bar{u}_{e_2}, \ldots, \bar{u}_{e_{n_e}}\right]^{\mathrm{T}}, \tag{16}$$

$$E[\boldsymbol{\psi}_z(m)] = \left[\bar{u}_{z_1}, \bar{u}_{z_2}, \ldots, \bar{u}_{z_{n_z}}\right]^{\mathrm{T}}. \tag{17}$$

The elements of $\boldsymbol{\zeta}$ are time-invariant, and pre-determined using the historical data, which were collected under the implementation of specific bus service strategies.

Step 1: Calculate the time-varying state variable vector $\mathbf{\Psi}(k)$ in a given time interval k. As mentioned previously, these time-varying state variables can be readily estimated in real time using the collected passengers' trip patterns and traffic data, according to the aforementioned definitions of state variables.

Step 2: Compute the fuzzy correlation matrix. In this stage, we attempt to estimate the time-varying $(n_s + n_e + n_z + 1) \times (n_s + n_e + n_z + 1)$ fuzzy correlation matrix ($W(k)$), which is given by

$$\underset{(n_s+n_e+n_z+1)\times(n_s+n_e+n_z+1)}{\mathbf{W}(k)} = \begin{bmatrix} w_{\psi,\psi}(k) & \underset{(1\times n_s)}{\mathbf{W}_{\psi,s}(k)} & \underset{(1\times n_e)}{\mathbf{W}_{\psi,e}(k)} & \underset{(1\times n_z)}{\mathbf{W}_{\psi,z}(k)} \\ \underset{(n_s\times 1)}{\mathbf{W}_{s,\psi}(k)} & \underset{(n_s\times n_s)}{\mathbf{W}_{s,s}(k)} & \underset{(n_s\times n_e)}{\mathbf{W}_{s,e}(k)} & \underset{(n_s\times n_z)}{\mathbf{W}_{s,z}(k)} \\ \underset{(n_e\times 1)}{\mathbf{W}_{e,\psi}(k)} & \underset{(n_e\times n_s)}{\mathbf{W}_{e,s}(k)} & \underset{(n_e\times n_e)}{\mathbf{W}_{e,e}(k)} & \underset{(n_e\times n_z)}{\mathbf{W}_{e,z}(k)} \\ \underset{(n_z\times 1)}{\mathbf{W}_{z,\psi}(k)} & \underset{(n_z\times n_s)}{\mathbf{W}_{z,s}(k)} & \underset{(n_z\times n_e)}{\mathbf{W}_{z,e}(k)} & \underset{(n_z\times n_z)}{\mathbf{W}_{z,z}(k)} \end{bmatrix}. \tag{18}$$

Herein, each given element of $\mathbf{W}(k)$ ($w_{g,h}(k)$) represents the degree of similarity between a given datum pair g and h, as estimated by the following rules:

$$\text{IF } g = h, \quad \text{THEN} \quad w_{g,h}(k) = 1, \tag{19}$$

$$\text{ELSE IF } w_{g,h}(k) \in \mathbf{Row}_\psi(\mathbf{W}(k)) \vee \mathbf{Col}_\psi(\mathbf{W}(k)), \quad \text{THEN}$$

$$w_{g,h}(k) = w_{h,g}(k) = 1 - \frac{1}{\beta}\sqrt{\sum_{j=0}^{3}\{\psi_g(k-j) - \bar{u}_h + \delta(k-j) - E[\delta(k-j)]\}^2}, \tag{20}$$

$$\text{ELSE} \quad w_{g,h}(k) = w_{h,g}(k) = 1 - \frac{2\left|\bar{u}_g - \bar{u}_h\right|}{\beta}, \tag{21}$$

where β is a pre-determined value set for the upper and lower boundaries of $w_{g,h}(k)$, namely 1 and 0, respectively, and here it is set to be 15 in the numerical study. It is also worth noting that according to Eqs. (19)–(21), $\mathbf{W}(k)$ turns out to be a symmetrical matrix.

However, according to the fundamentals of fuzzy clustering techniques [19], the estimated fuzzy correlation matrix $\mathbf{W}(k)$ should be processed through the composition operation such that the following condition, Eq. (22), holds for further use in sample clustering.

$$\tilde{\mathbf{W}}(k) \circ \tilde{\mathbf{W}}(k) = \tilde{\mathbf{W}}(k), \tag{22}$$

where $\tilde{\mathbf{W}}(k)$ represents the processed fuzzy correlation matrix of $\mathbf{W}(k)$. To generate $\tilde{\mathbf{W}}(k)$, we conduct a routine of the max-min composition operation with respect to each given element of $\mathbf{W}(k)$ (e.g., $w_{g,h}(k)$) as:

$$\mathbf{W}(k) \circ \mathbf{W}(k) = \max_{t=1}^{n_s+n_e+n_z+1} \{\min[w_{g,t}(k), w_{t,h}(k)]\} \tag{23}$$

until the condition shown in Eq. (22) is satisfied. Then, we have the processed fuzzy correlation matrix $\tilde{\mathbf{W}}(k)$, which is represented mathematically by

$$\underset{(n_s+n_e+n_z+1)\times(n_s+n_e+n_z+1)}{\tilde{\mathbf{W}}(k)} = \begin{bmatrix} \tilde{w}_{\psi,\psi}(k) & \underset{(1\times n_s)}{\tilde{\mathbf{W}}_{\psi,s}(k)} & \underset{(1\times n_e)}{\tilde{\mathbf{W}}_{\psi,e}(k)} & \underset{(1\times n_z)}{\tilde{\mathbf{W}}_{\psi,z}(k)} \\ \underset{(n_s\times 1)}{\tilde{\mathbf{W}}_{s,\psi}(k)} & \underset{(n_s\times n_s)}{\tilde{\mathbf{W}}_{s,s}(k)} & \underset{(n_s\times n_e)}{\tilde{\mathbf{W}}_{s,e}(k)} & \underset{(n_s\times n_z)}{\tilde{\mathbf{W}}_{s,z}(k)} \\ \underset{(n_e\times 1)}{\tilde{\mathbf{W}}_{e,\psi}(k)} & \underset{(n_e\times n_s)}{\tilde{\mathbf{W}}_{e,s}(k)} & \underset{(n_e\times n_e)}{\tilde{\mathbf{W}}_{e,e}(k)} & \underset{(n_e\times n_z)}{\tilde{\mathbf{W}}_{e,z}(k)} \\ \underset{(n_z\times 1)}{\tilde{\mathbf{W}}_{z,\psi}(k)} & \underset{(n_z\times n_s)}{\tilde{\mathbf{W}}_{z,s}(k)} & \underset{(n_z\times n_e)}{\tilde{\mathbf{W}}_{z,e}(k)} & \underset{(n_z\times n_z)}{\tilde{\mathbf{W}}_{z,z}(k)} \end{bmatrix}. \tag{24}$$

Step 3: Identification of appropriate bus service strategies by the following truncation rules. Let $\tilde{w}_{g,h}(k)$ be a given element of the processed fuzzy correlation matrix $\tilde{\mathbf{W}}(k)$, and be truncated as

$$\begin{aligned} \tilde{w}_{g,h}(k) \quad &= 1, \quad \text{if} \quad \tilde{w}_{g,h}(k) \geqslant \lambda_1, \\ &= 0, \quad \text{otherwise}, \end{aligned} \tag{25}$$

where λ_1 is a pre-determined threshold, which is bounded by the range of 0 and 1. Then, comparing the first column vector (i.e., $\mathbf{Col}_1(\tilde{\mathbf{W}}(k))$) with the other column vectors of $\tilde{\mathbf{W}}(k)$, a given bus service strategy θ associated with a specific service region θ_* is identified for serving passengers in the given time interval k in case of $\mathbf{Col}_1(\tilde{\mathbf{W}}(k)) = \mathbf{Col}_{\theta_*}(\tilde{\mathbf{W}}(k))$; otherwise, an all-stop bus is dispatched in the given time interval k.

Note that the aforementioned threshold λ_1 appears to influence the result of fuzzy clustering in the proposed method, and the effect caused herein by λ_1 may be similar to that caused by a fuzzifier, as discussed in Hoppner et al. [14]. The corresponding effect of λ_1 on the identification of bus service strategies is illustrated in Fig. 5, which implies that different values of λ_1 may result in different sets of bus service strategies. Theoretically, the greater λ_1 is, the faster $\tilde{w}_{g,h}(k)$ becomes 0, which may result in more groups. One extreme case is that the all-stop bus service strategy may remain to be implemented in any time interval if we choose λ_1 to be 1. It is also noteworthy that the identification of multiple bus service strategies is allowed in the proposed method when more than one strategy is identified simultaneously in a given time interval, and thus multiple buses associated with different service strategies and service regions can be dispatched in the given time interval. Nevertheless, such a multi-bus dispatching strategy is not absolutely needed in all practical operational cases because the limited operational resources (e.g., available bus drivers and bus fleet size) and induced operational costs should also be considered, as in

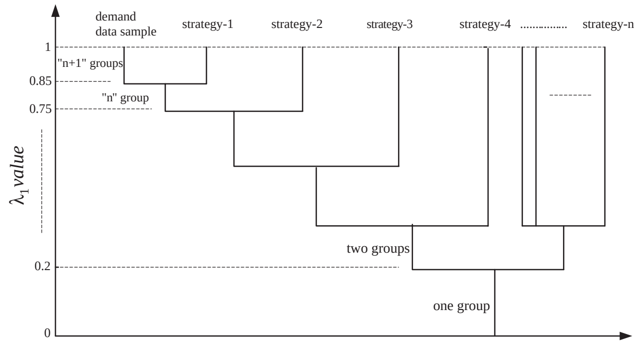

Fig. 5. Illustration of the effect of λ_1 on the determination of bus service strategies.

the following numerical study. It is therefore suggested that λ_1 should be carefully determined using historical data before being used for real-world applications, and here is preset to be 0.96, according to our previous calibration results [32]. Related discussions on the specification of fuzzifiers can also be found in the previous literature [14].

3. Illustrative application

To demonstrate the potential advantages of the proposed methodology for the operations of demand-responsive bus operations control, the model was applied to a hypothetical but realistic case in which the real passenger demand data collected from the Taipei City bus route 255 was used to evaluate the performance of the proposed approach in comparison with the existing all-stop bus service strategy implemented in the given bus route.

Bus route 255 primarily serves the community trips along one radial corridor of Taipei, with one end in the CBD during peak hours, and with a total route length of 30.5 km. The total number of stops on the route is 50, as illustrated in Fig. 6, where stop 25 serves as a transfer stop for the connection with the metropolitan mass transit system. The existing dispatching frequency is one bus per 30 min. To generate a real database used for the numerical study, the 30-min passenger volume profiles in morning peak hours were collected from 06:00 am to 08:00 am over one month at the study site.

The proposed algorithm was then utilized to determine the bus service strategies implemented in the specific four sequential test intervals, namely 06:00–06:30, 06:30–07:00, 07:00–07:30, and 07:30–08:00, in response to the time-varying passenger demand patterns. Note that herein, the processed time-varying passenger volume at each bus stop in a given test time interval is directly used as the data basis for the determination of both bus service strategies and the associated bus stops served. Corresponding model testing for the short-term passenger demand forecasting executed in the first mechanism of the proposed algorithm was completed previous to this study [32], and thus is not presented here.

To quantitatively assess the performance of the proposed method with respect to the improvement of bus operations for a given bus service route, we compared the results obtained from the proposed approach and from the original all-stop service strategy utilizing three proposed measures, defined as follows:

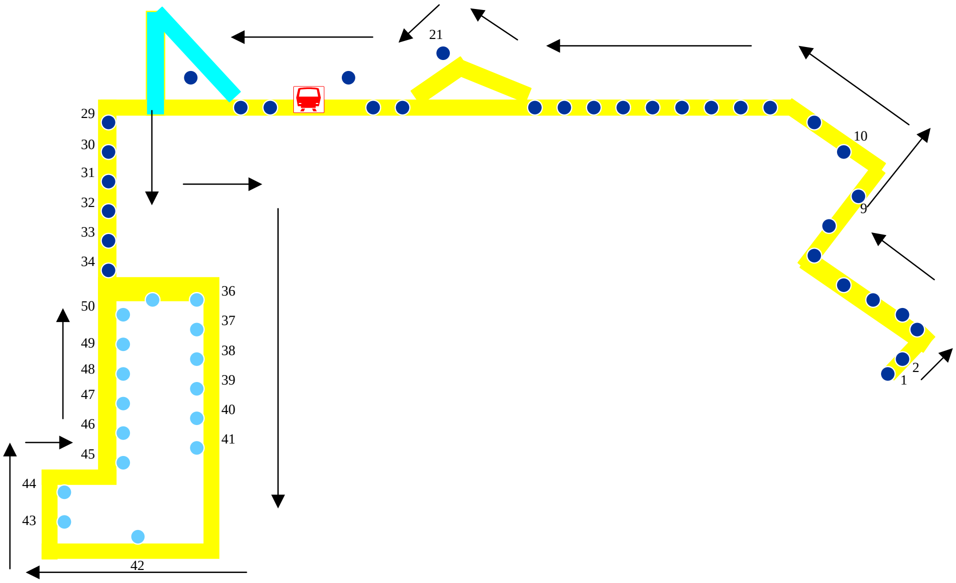

Fig. 6. Illustration of the Taipei City bus route 255 in the study.

(1) The time-varying transportation cost for routing with the bus service strategy θ in a specific service region θ_* in a given time interval k ($TC_{\theta_*}(k)$), which is denoted by

$$TC_{\theta_*}(k) = t_1 \times L_{\theta_*}(k), \tag{26}$$

where $L_{\theta_*}(k)$ is the total distance for a given bus routing with a given bus service strategy θ, in a specific service region θ_* and in a given time interval k; and t_1 is the unit cost for transportation, as determined according to statistics provided by the municipal bus administration of Taipei.

(2) The total monetary value of passengers' in-vehicle travel time under the service associated with a given bus routing with service strategy θ and a specific service region θ_* in a given time interval k ($PC_{\theta_*}(k)$), as given by

$$PC_{\theta_*}(k) = t_2 \times \sum_{\forall o_{\theta_*}} \sum_{\forall d_{\theta_*}} v_{o_{\theta*},d_{\theta*}}(k) \times \frac{l_{o_{\theta*},d_{\theta*}}}{\upsilon_{\theta_*}(k)}, \tag{27}$$

where $v_{o_{\theta_*},d_{\theta_*}}(k)$ represents the time-varying passenger trip volume traveling from a given bus stop o_{θ_*} to a given bus stop d_{θ_*} served by a bus associated with a given bus service strategy θ in a give time interval k; $l_{o_{\theta_*},d_{\theta_*}}(k)$ is the geographical distance between bus stops o_{θ_*} and d_{θ_*}; $\upsilon_{\theta_*}(k)$ is the average running speed of the bus associated with the given bus service strategy θ in the specific service region θ_*; and t_2 is herein defined as the average monetary value of passengers' in-vehicle travel time in the unit of US dollars per hour. Conveniently, t_2 is pre-determined using the analytical results of a related study [4] for the case of Taiwan. It is worth noting that t_2 may vary with the study site, and is adjustable in consideration of the distinction of passengers' perception of time value in different countries.

(3) The total monetary value of passengers' waiting time under the service provided by a given bus with service strategy θ in the specific service region θ_* in time interval k ($\mathbf{W}C_{\theta_*}(k)$), which is given by

$$\mathbf{W}C_{\theta_*}(k) = t_3 \times \left\{ \sum_{\forall o_\theta} \left[\kappa \times \sigma_{o_{\theta_*}}(k|k) + \frac{l_{\Gamma, o_{\theta_*}}}{\upsilon_{*\theta}(k)} \times f^{\theta_*}_{o_{\theta_*}}(k) + \sum_{\forall \vec{o}_{\theta_*}} \varepsilon \times f^{\theta_*}_{\vec{o}_{\theta_*}}(k) \right] \right\}, \tag{28}$$

where o_{θ_*} represents a given bus stop associated with a given bus service strategy θ in the specific service region θ_*; similarly, $\vec{o}_{\theta_*}$ also belongs to the set of bus stops associated with the given bus service strategy θ and the specific service region θ_*, but is located before o_{θ_*}; $\sigma_{o_{\theta_*}}(k|k)$ is the time-varying passenger volume at the given bus stop o_{θ_*} in a given time interval k; $l_{\Gamma, o_{\theta_*}}$ represents the geographical distance from the beginning of the given bus route (Γ) to the bus stop o_{θ_*}; $f^{\theta_*}_{o_{\theta_*}}(k)$ and $f^{\theta_*}_{\vec{o}_{\theta_*}}(k)$ represent the real passenger volumes which are served by a given bus using a given service strategy θ at bus stops o_{θ_*} and $\vec{o}_{\theta_*}$ in a given time interval k, respectively; κ is the length of a given time interval; ε is defined as the average riding time; t_3 is the average monetary value of passengers' waiting time in the unit of US dollars per hour, and for the case of Taiwan, is herein set to be the triple of t_2, according to the above-cited study [4]. Moreover, the numerical results yielded by other strategies are also used to demonstrate the comparative advantages of the bus service strategies determined by the proposed algorithm.

To illustrate the applicability of the proposed method in real-time demand-responsive bus dispatching control, we summarized the main numerical results yielded from the proposed fuzzy clustering-based algorithm in the process of identifying an appropriate bus service strategy for the first test interval (i.e., 06:00–06:30). According to the proposed fuzzy clustering algorithm mentioned above, the vector of clustering centers associated with the specified three bus service strategies, including short-turn, express and zonal service strategies, was generated in the initialization step, as can be seen in Table 1. Then, in Step 1, the time-varying state variable vector $\mathbf{\Psi}(k)$ in the given time interval was estimated using the collected passengers' trip patterns and traffic data. Using the results obtained in the previous steps, the estimated fuzzy correlation matrix $\tilde{\mathbf{W}}(k)$, as presented in Table 2, was measured in Step 2. Table 3 summarizes the output of the strategy identification in the current test interval using the truncation rule mentioned in Step 3 of the proposed algorithm. Accordingly, the short-turn strategy is identified as the suitable bus service strategy for the first test interval.

The comparison results according to the aforementioned criteria are summarized in Table 4, where the strategies identified by the proposed method are highlighted in bold. The generalizations obtained from the numerical results are itemized as follows.

Table 1
Estimated clustering centers for identification of bus service strategies

Type of strategy state variable	Short-turn	Express	Zonal
$\psi_{s*}(k)$	0.67	0.25	0.28
$\psi_{e*}(k)$	0.17	0.89	0.36
$\psi_{z*}(k)$	0.23	0.42	0.73
$\delta(k)$	0.13	0.78	0.44

Table 2
Estimated time-varying fuzzy correlation matrix $\tilde{\mathbf{W}}(k)$

	Data sample	Short-turn	Express	Zonal
Data sample	1	0.968	0.653	0.274
Short-turn	0.968	1	0.217	0.468
Express	0.653	0.217	1	0.639
Zonal	0.274	0.468	0.639	1

Table 3
Identification results for the first test interval 06:00-06:30

	Data sample	Short-turn	Express	Zonal
Data sample	1	1	0	0
Short-turn	1	1	0	0
Express	0	0	1	0
Zonal	0	0	0	1

1. Overall, the numerical results shown in Table 4 reveal that there would be significant improvement in the performance of bus routing by implementing the proposed real-time demand-responsive bus dispatching approach in comparison with the system performance of the existing all-stop bus service strategy. According to the results with respect to the average performance during the four-interval test period, the existing bus operations can be improved by 16.65% by implementing the proposed bus service strategies, and herein, the average passengers' waiting time is particularly improved up to 23.65%, which is a significant improvement for customers. In addition, the supply-oriented transportation cost is reduced by 19.45%, which extends the potential benefit of the proposed method to the supply side, in addition to the demand side.
2. The identified strategies appear superior to the other un-identified bus service strategies in their capability of responding to diverse passenger demand patterns during the test period. This can be seen in that the aggregate cost-based measure associated with the identified strategy is less than that associated with any other strategies in any given time interval. Such a generalization also implies the validity of the proposed method in the identification of appropriate bus strategies in response to time-varying patterns of passenger demands.
3. The measurements shown in Table 4 may also be helpful to analyze the relative performance of the existing bus dispatching operations. For example, there is an interesting finding which can be seen in the results of Table 4 that, under certain patterns of passengers' trip volumes, the existing all-stop bus service strategy proves to be more suitable than some specific strategies, although its overall performance may not satisfy the peak-hour passenger demands for the criterion of waiting time.

4. Concluding remarks

This paper has presented an advanced demand-responsive bus operations control approach in response to the variety of both passenger demands and road traffic conditions. Using the proposed short-term

Table 4
Comparison of system performance

Criteria strategies	$TC_{\theta_*}(k)$	$PC_{\theta_*}(k)$	$WC_{\theta_*}(k)$	Aggregate
$k = 1$ (06:00 am–06:30 am)				
All-stop	49.41	4.25	355.88	409.54
Express	36.60	3.14	344.09	383.83
Short-turn	**40.26**	**3.36**	**287.15**	**330.77**
Zonal	42.09	3.55	346.94	392.58
$k = 2$ (06:30 am–07:00 am)				
All-stop	49.41	4.92	402.62	456.95
Express	36.60	2.83	291.89	331.32
Short-turn	**40.26**	**3.55**	**281.71**	**325.52**
Zonal	42.09	3.78	323.09	368.96
$k = 3$ (07:00 am–07:30 am)				
All-stop	49.41	3.97	365.06	418.44
Express	36.60	3.27	362.04	401.91
Short-turn	40.26	3.17	444.11	487.54
Zonal	**42.09**	**3.36**	**355.49**	**400.94**
$k = 4$ (07:30 am–08:00 am)				
All-stop	49.41	4.61	389.84	443.86
Express	**36.60**	**3.30**	**344.01**	**383.91**
Short-turn	40.26	4.49	389.21	433.96
Zonal	42.09	3.33	347.19	392.61
Average performance (identified strategy vs. all-stop strategy)				
Identified strategy	**39.80**	**3.39**	**317.09**	**360.28**
All-stop strategy	49.41	4.44	378.35	432.20
Relative improvement (%)	19.45	23.65	16.19	16.65

Note: the regions highlighted in bold represent the strategies suggested in given time intervals.

passenger demand forecasting model coupled with advanced APTS-related technologies, the time-varying patterns of passengers' O-D trip volumes are recognized, and then appropriate bus service strategies associated with specific service regions are identified using the proposed fuzzy clustering based algorithm in response to the time-varying passenger demands.

Our numerical study, using the processed data of real passenger demands collected from one city bus route in Taipei, demonstrates the potential advantages of the proposed method over both the existing all-stop bus service strategy and the other specific bus service strategies. Utilizing three specified criteria measures, including one supply-based and two demand-based criteria, the comparison results reveal the applicability of the proposed method for the use of real-time demand-responsive bus operations control.

Nevertheless, there is potential for improving the performance of the proposed method by adding more elaborate bus service strategies such as skip-stop and limited-stop service strategies to the possible bus service strategies in response to the variety of passenger demands. Moreover, more elaborate strategies, such as the adjustment of dispatching headways, can also be implemented for quick response to a growing variety of passengers' trip volumes under conditions of complicated urban transportation networks. Based on the present results, our further research will explore the possibility of integrating the time-varying passengers' demands on multiple bus routes in a given urban area to satisfy the systematic optimization of network-wide bus routing by extending the proposed methodology. Clearly, the acquisition of valid data for model testing and evaluation will be significant for our further research.

More importantly, it is expected that this study can help to demonstrate the applicability of the real-time data collected from advanced APTS-related technologies, and stimulate more research on operational models in the APTS environment. Furthermore, the integration of the proposed method with other public transportation systems, including mass transit systems, also warrants further research in order to improve the competitiveness of urban public transportation systems.

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References

[1] M.F. Adesanya, Fixed route accessible bus service simulation model (FRABSIM), Ph.D. Thesis, Polytechnic University, 1981.
[2] M.S. Ahmed, A.R. Cook, Analysis of freeway traffic time-series data by using Box-Jenkins techniques, Transp. Res. Rec. 722 (1979) 1–9.
[3] J.C. Bezdek, Fuzzy mathematics in pattern classification, Ph.D. Thesis, Cornell University, USA, 1973.
[4] Y.-C. Chang, A study on optimal vehicle capacity of bus system, Master Thesis, National Taiwan University, Taiwan, 1993
[5] H. Chen, S. Grant-Muller, Use of sequential learning for short-term traffic flow forecasting, Transp. Res. C 9 (2001) 319–336.
[6] X.J. Eberlein, N.H.M. Wilson, C. Barnhart, D. Bernstein, The real-time deadheading problem in transit operations control, Transp. Res. B B 32 (2) (1998) 77–100.
[7] L. Fu, L.R. Rilett, Estimation of time-dependent, stochastic route travel times using artificial neural networks, Transp. Plann. Technol. 24 (1) (2000) 25–36.
[8] P.G. Furth, Short turning on transit routes, Transp. Res. Rec. 1108 (1988) 42–52.
[9] P.G. Furth, F.B. Day, Transit routing and scheduling strategies for heavy demand corridors, Transp. Res. Rec. 1011 (1985) 23–26.
[10] L.O. Hall, A.M. Bensaid, L.P. Clarke, R.P. Velthuizen, M.S. Silbiger, J.C. Bezdek, A comparison of neural network and fuzzy clustering techniques in segmenting magnetic resonance images of the brain, IEEE Trans. Neural Networks 3 (5) (1992) 672–682.
[11] M.M. Hamed, H.R. Al-Masaeid, Z.M.B. Said, Short-term prediction of traffic volume in urban arterials, ASCE J. Transp. Eng. 121 (3) (1995) 249–254.
[12] M. Hansen, M. Qureshi, D. Rydzewski, Improving transit performance with advanced public transportation system technologies, PATH Research Report 94-18, Berkeley, CA, 1994.
[13] M. Hickman, T. Day, An assessment of information systems and technologies at California transit agencies, The 75th Annual Meeting of the Transportation Research Board, Washington, DC, 1996.
[14] F. Hoppner, F. Klawonn, R. Kruse, T. Runkler, Fuzzy Cluster Analysis: Methods for Classification, Data Analysis and Image Recognition, Wiley, Singapore, 1999.
[15] A.X. Horbury, Using non-real-time automatic vehicle location data to improve bus service, Transp. Res. B B 33 (8) (1999) 559–579.
[16] T.-L. Hu, J.-B. Sheu, A fuzzy-based customer classification method for advanced demand-responsive logistical distribution operations, Fuzzy Sets and Systems 139 (2003) 431–450.
[17] A. Kandel, Fuzzy Mathematical Techniques with Applications, Addison-Wesley, Reading, MA, 1986.

[18] A. Khattak, H. Noeimi, H. Al-Deek, R. Hall, Advanced public transportation systems: a taxonomy and commercial availability, PATH Research Report UCB-ITS-PRR-93-9, Berkeley, CA, 1993.
[19] G.J. Klir, B. Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice-Hall Publication, New Jersey, 1995.
[20] J.B. Lesort, Prediction of traffic flow, in: M. Papageorgiou (Ed.), Concise Encyclopedia of Traffic and Transportation Systems, Pergamon Press, New York, 1991, pp. 329–331.
[21] H. Nichoison, C.D. Swann, The prediction of traffic flow volumes based on spectral analysis, Transp. Res. 8 (1974) 533–538.
[22] D. Park, L.R. Rilett, Spectral basis neural networks for real-time travel time forecasting, ASCE J Transp. Eng. 125 (6) (1999) 515–523.
[23] S.M. Santhakumar, P. Hariharan, Transportation system management options to improve urban bus route performance using computer simulation, Transp. Res. Rec. 1338 (1991) 22–27.
[24] J.-B. Sheu, A fuzzy clustering based approach to automatic freeway incident detection and characterization, Fuzzy Sets Systems 128 (3) (2002) 93–104.
[25] P.D. Site, F. Filippi, Service optimization for bus corridors with short-turn strategies and variable vehicle size, Transp. Res. A 32 (1) (1998) 19–38.
[26] B.L. Smith, M.J. Demetsky, Traffic flow forecasting: comparison of modeling approaches, ASCE J. Transp. Eng. 123 (4) (1997) 261–266.
[27] B.L. Smith, B.M. Williams, R.K. Oswald, Parametric and nonparametric traffic volume forecasting, Paper Presented at the 2000 Transportation Research Board Annual Meeting, Washington, DC, 2000.
[28] B.L. Smith, B. Williams, R.K. Oswald, Comparison of parametric and nonparametric models for traffic flow forecasting, Transp. Res. C 10 (2002) 303–321.
[29] A. Statholoupos, M.G. Karlaftis, Temporal and spatial variations of real-time traffic data in urban areas, Transp. Res. Rec. 1768 (2001) 135–140.
[30] Y.J. Stephanedes, E. Kwon, P. Michalopoulos, On-line diversion prediction for dynamic control and vehicle guidance in freeway corridors, Transp. Res. Rec. 1287 (1990) 11–19.
[31] M. Sugeno, T. Yasukawa, A fuzzy-logic-based approach to qualitative modeling, IEEE Trans. Syst. 24 (1993) 1279–1284.
[32] S.-H. Tsai, Prototype of a passenger demand forecasting model for real-time bus dispatching, Master Thesis, National Taiwan University, Taiwan, 2001.
[33] S.M. Tsao, P. Schonfeld, Balance transit service: an analysis, J. Transp. Eng. ASCE 110 (1) (1984) 112–128.
[34] B. Van Vrem, H.R. Kirby, M.J.M. Van Der Vlist, J.C. Whittaker, Recent advances in the field of short-term traffic forecasting, Int. J. Forecasting 13 (1997) 1–12.
[35] T.A.S. Vijayaraghavan, K.M. Anantharamaiah, Fleet assignment strategies in urban transportation using express and partial services, Transp. Res. A 29 (1995) 157–171.
[36] J. Whittaker, S. Garside, K. Lindveld, Tracking and predicting a network traffic process, Int. J. Forecasting 13 (1997) 51–61.
[37] N.H.M. Wilson, R.A. Macchi, R.E. Fellows, A.A. Deckoff, Improving service on the MBTA green line through better operations control, Transp. Res. Rec. 1361 (1992).
[38] H.J. Zimmermann, Fuzzy Set Theory and its Applications, Kluwer-Nijhoff, Boston, 1985.