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# Effect of phonon scattering on free-carrier absorption in quantum well structures

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#### Abstract

The free-carrier absorption has been studied for quantum well structures fabricated from III–V semiconductors such as *n*-type InSb films when the acoustic phonon scattering is dominant. The energy band of carriers in semiconductors is assumed to be nonparabolic. The scattering mechanisms of phonons with carriers in semiconductors are considered for the deformation-potential coupling and the piezoelectric coupling separately. Results show that the free-carrier absorption coefficient in *n*-type InSb films depend upon the polarization of the radiation field relative to the direction normal to the quantum well structures, the photon frequency, the film thickness and the temperature. It is also found that the free-carrier absorption coefficient could be complex due to the interaction of the radiation field and the photon field with carriers in semiconductors. Thus, the index of refraction of semiconducting films could be changed due to this carrier-phonon-photon interaction.

## 1. Introduction

In a magnetic field where the carrier motion is confined to a plane perpendicular to the magnetic field, the free-carrier absorption coefficient  $\alpha$  depends upon the photon polarization relative to the magnetifc field [1]. For carriers confined in a quasi-2D structure, it was found that  $\alpha$  depends upon the polarization of the radiation field relative to the direction normal to the quasi-2D structure [2]. For III-V compound semiconductors, the electron-phonon interaction is dominated by the deformation-potential and piezoelectric couplings [3]. Thus, we investigate the effect of the acoustic phonon on  $\alpha$  in *n*-type InSb films with a quasi-2D structure. The energy band of electrons in semiconductors is assumed to be nonparabolic.

## 2. Theory

For a square well potential along the z-axis with infinitely high barriers at z = 0 and z = d, the electron field operator  $\Psi(\mathbf{r})$  is given [4, 5] by

$$\boldsymbol{\Psi}(\boldsymbol{r}) = \left(\frac{2}{V}\right)^{1/2} \sum_{n=1}^{\infty} \sum_{\boldsymbol{k}} b_{\boldsymbol{k}\boldsymbol{n}} \exp\left(\mathrm{i}\,\boldsymbol{k}\cdot\boldsymbol{x}\right) \sin\left(\frac{n\pi z}{d}\right),\tag{1}$$

where  $\mathbf{r} = (\mathbf{x}, z) = (x, y, z)$ , V = dS is he film volume with the surface area S and the film thickness d,  $\mathbf{k} = (k_x, k_y)$  is the electron wave vector, and  $b_{kn}$  is the electron annihilation operator. The electron energy is given [3] by the relation

$$E_{kn}\left(1+\frac{E_{kn}}{E_g}\right) = \frac{\hbar^2 k^2}{2m^*} + \frac{(\pi\hbar n)^2}{2m^* d^2}, \quad n = 1, 2, 3, \dots,$$
(2)

where  $m^*$  is the effective mass of electron.

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The absorption coefficient for the absorption of photons can be expressed [2] as

$$\alpha = \frac{\varepsilon^{1/2}}{n_0 c} \sum_i W_i f_i, \tag{3}$$

where  $\varepsilon$  is the dielectric constant of material,  $n_0$  is the number of photons in the radiation field,  $f_i$  is the distribution of carriers, and  $W_i$  is the transition probability. From the Born approximation

$$W_{i} = \frac{2\pi}{\hbar} \sum_{f} \left[ |\langle f | M_{+} | i \rangle|^{2} \,\delta(E_{f} - E_{i} - \hbar\Omega - \hbar\omega_{q}) + |\langle f | M_{-} | i \rangle|^{2} \,\delta(E_{f} - E_{i} - \hbar\Omega + \hbar\omega_{q}) \right], \tag{4}$$

where  $E_i$  and  $E_f$  are the initial and final electron energies,  $\hbar\Omega$  is the photon energy, and  $\hbar\omega_q$  is the phonon energy. For interaction between electrons, photons, and phonons, the transition matrix elements  $\langle f|M_{\pm}|i\rangle$  are given by

$$\langle f|M_{\pm}|i\rangle = \sum_{j} \left[ \frac{\langle f|H_{\rm rad}|j\rangle \langle j|V_{\rm s}|i\rangle}{E_{j} - E_{i} \mp \hbar\omega_{q}} + \frac{\langle f|V_{\rm s}|j\rangle \langle j|H_{\rm rad}|i\rangle}{E_{j} - E_{i} - \hbar\Omega} \right],$$
(5)

where  $H_{rad}$  is the electron-photon interaction and  $V_s$  is the scattering potential due to the electron-phonon interaction. The electron-photon interaction is given by

$$H_{\rm rad} = -\frac{e}{m^*} (2\pi\hbar n_o/\varepsilon \Omega V)^{1/2} \,\hat{\varepsilon} \cdot \boldsymbol{p},\tag{6}$$

 $\hat{\varepsilon}$  is the photon polarization vector and p is the electron momentum. Its matrix elements for electrons in the same band are given as follows.

(i) When the photon is polarized parallel to the layer plane,

$$\langle k'n'|H_{\rm rad}|kn\rangle = -\frac{e}{m^*} (2\pi\hbar^3 n_0/\varepsilon \Omega V)^{1/2} \\ \times \hat{\varepsilon} \cdot \mathbf{k} \,\delta_{n',n} \,\delta_{k'_n,k_n}, \,\delta_{k'_n,k_n}.$$
(7a)

(ii) When the photon is polarized perpendicular to the layer plane,

$$\langle \mathbf{k}'n'|H_{\rm rad}|\mathbf{k}n\rangle = -\frac{{\rm i}en}{m^*d}(2\pi\hbar^2 n_o/\varepsilon\Omega V)^{1/2} \\ \times \left\{\frac{1-\cos[\pi(n'+n)]}{n'+n} + \frac{1-\cos[\pi(n'-n)]}{n'-n}\right\}\delta_{k_x,k_x},\,\delta_{k_y,k_y}.$$
(7b)

The distribution function for a quasi-2D nondegenerate electron gas can be expressed as

$$f_{kn} = (n_{\rm e}d/2m^*k_{\rm B}T)^{1/2} \left(\frac{\hbar}{\pi}\right) \left[\sum_{l=1}^{\infty} a_l \exp\left(-\frac{E_{\rm g}a_l}{2k_{\rm B}T}\right)\right]^{-1}$$

$$\times \exp\left[-\frac{E_{g}a_{n}}{2k_{B}T} - \frac{\hbar^{2}k^{2}}{2m^{*}k_{B}T}\right],\tag{8}$$

where  $n_e$  is the concentration of electrons, and

$$a_n = \left[1 + 2(\pi \hbar n)^2 / m^* d^2 E'_g\right]^{1/2}.$$
(9)

There are two dominant electron-phonon interaction mechanisms in *n*-type InSb [6]:

(i) For the deformation-potential coupling,

$$V_{\rm s} = (k_{\rm B}T/2\rho v_{\rm s}^2 V)^{1/2} E_{\rm d} \exp(i\boldsymbol{q}\cdot\boldsymbol{r}), \qquad (10)$$

where  $\rho$  is the density of material,  $v_s$  is the sound velocity, q is the phonon wave vector, and  $E_d$  is the deformation potential.

(ii) For the piezoelectric coupling,

$$V_{\rm s} = (k_{\rm B}T/2\rho v_{\rm s}^2 V)^{1/2} (|e|\beta_{\rm P}/\varepsilon|\boldsymbol{q}|) \exp(i\boldsymbol{q}\cdot\boldsymbol{r}), \qquad (11)$$

where  $\beta_{\rm P}$  is the appropriate piezoelectric constant.

## 3. Numerical results

The relevant values of physical parameters for *n*-type InSb thin films are taken to be:  $n_e = 1.75 \times 10^{14} \text{ cm}^{-3}$ ,  $m^* = 0.013m_0$  ( $m_0$  is the free electron mass),  $\rho = 5.8 \text{ gm/}$ cm<sup>3</sup>,  $\varepsilon = 18$ ,  $E_g = 0.2 \text{ eV}$ ,  $E_d = 4.5 \text{ eV}$ ,  $\beta_P = 1.8 \times 10^4 \text{ esu/}$ cm<sup>2</sup>, and  $v_s = 4 \times 10^5 \text{ cm/s}$ .

In Fig. 1(a), the free-carrier absorption coefficient  $|\alpha|$  is plotted as a function of the photon frequency with  $d = 1 \mu m$  for the radiation field polarized parallel to the layer plane in th case of deformation-potential coupling. It shows that  $|\alpha|$  decreases monotonically with increasing the photon frequency and increases with increasing temperature. In Fig. 1(b),  $\alpha$  is plotted as a function of the photon frequency with  $d = 1 \mu m$  for the radiation field



Fig. 1. Free-carrier absorption coefficient for the deformationpotential coupling as a function of the photon frequency for photons polarized: (a) parallel to the layer plane and (b) perpendicular to the layer plane.



Fig. 2. Free-carrier absorption coefficient for the piezoelectric coupling as a function of the photon frequency for photons polarized (a) parallel to the layer plane and (b) perpendicular to the layer plane.

polarized perpendicular to the layer plane. It can be seen that the changing of  $\alpha$  with temperature appears irregularly in lower frequencies with  $\Omega < 25$  THz. Since Im( $\alpha$ ) is quite small compared to Re( $\alpha$ ), thus  $\alpha \cong |\alpha|$  in this case and  $\alpha$  decreases with increasing the photon frequency. Moreover, the index of refraction of semiconducting films could be changed due to the carrier-phononphoton interaction. In Fig. 2(a),  $|\alpha|$  is plotted as a function of the photon frequency with  $d = 1 \mu m$  for the radiation field polarized parallel to the layer plane in the case of piezoelectric coupling. It shows that  $|\alpha|$  decreases monotonically with increasing the photon frequency and increases with increasing temperature. While for the radiation field polarized perpendicular to the layer plane as shown in Fig. 2(b), it can be seen that  $|\alpha|$  decreases monotonically with increasing the photon frequency, but  $|\alpha|$  decreases with increasing temperature.

## References

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