Modeling Channel Assignment of Small-Scale Cellular Networks

Hui-Nien Hung, Pei-Chun Lee, Yi-Bing Lin, *Fellow, IEEE*, and Nan-Fu Peng

***Abstract*—In a cellular telecommunications network, the call blocking, forced termination, and call incompletion probabilities are major output measures of system performance. Most previous analytic studies assumed that the handover traffic to a cell is a fixed-rate Poisson process. Such assumption may cause significant inaccuracy in modeling. This paper shows that the handover traffic to a cell depends on the workloads of the neighboring cells. Based on this observation, we derive the exact equation for the handover force-termination probability when the mobile station (MS) cell residence times are exponentially distributed. Then, we propose an approximate model with general MS cell residence time distributions. The results are compared with a previously proposed model. Our comparison study indicates that the new model can capture the handover behavior much better than the old one for small-scale cellular networks.**

***Index Terms*—Call duration time, cellular network, channel assignment, handover.**

I. INTRODUCTION

EMERGING cellular telecommunications network technologies have attracted considerable attention in academic research as well as commercial deployment. A cellular network supports telephony services when users are in movement [10]. The cellular phone service area is populated with base stations (BS's). The radio coverage of a BS is referred to as a cell. Customers within a cell can connect to the corresponding BS via mobile stations (MS's) or mobile phones. When a call for a customer occurs, one radio channel of the BS is used for connecting the MS and the BS. If all radio channels are in use when a new call is attempted, the call will be blocked and cleared from the system. If the call is accepted, a radio channel will be occupied until the call is completed, or until the MS moves out of the cell. When a communicating MS moves from one cell to another, the occupied channel in the old cell is released, and an idle channel is acquired in the new cell. During this handover procedure, if no channel is available in the new cell, the call is forced to terminate before its completion. When the call is connected, the call may be completed after several successful handovers, or may be forced to terminate due to a failed handover. The duration of a call connection (if the call is completed) is referred to as the call duration time.

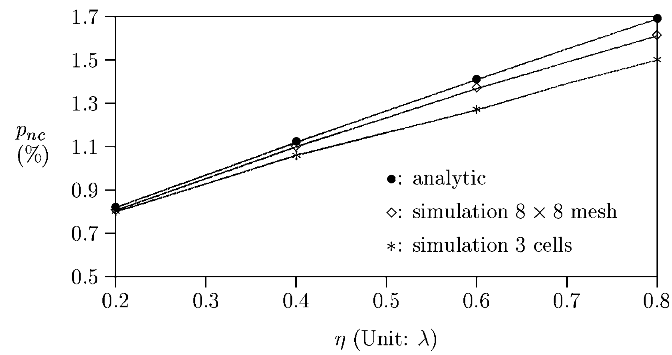

Fig. 1. Effect of network size on the call incompletion probability ($\mu = 0.3\lambda$).

For billing and network planning purposes, the handover behavior and the probability of call completion need to be analyzed. Several analytic studies have contributed to cellular network performance evaluation [1], [3], [4], [6], [12], [13], [15], [17]. Most studies assume that the handover traffic to a cell is a fixed-rate Poisson process. This assumption is reasonable for large-scale cellular networks, or when the networks experience light load traffic [2]. In reality, the handover traffic to a cell depends on the workloads of the neighboring cells. This fact has significant impact on modeling of small-scale cellular networks. Fig. 1 plots the call incompletion probability p_{nc} against the user mobility η and the call arrival rate λ where the number of radio channels in a cell is 9. The "•" curve is generated from a previous analytic model that assumes fixed-rate handover traffic [12]. The "◇" curve is generated from simulation of a 64-cell mesh configuration, and the "∗" curve is generated from simulation of a three-cell configuration (illustrated in Fig. 2). The simulation model [11] actually simulates the MS movement in mesh or hexagonal networks of cells. This simulation model is used throughout the paper. Fig. 1 indicates that the fixed-rate assumption is acceptable when the number of cells is reasonably large, but is inaccurate for small-scale cellular networks. In this paper, we derive the exact equation for the handover force-termination probability when the MS cell residence times are exponentially distributed. Then, we propose an approximate model with general MS cell residence times. The results are compared with the previously proposed model [12]. Our comparison study

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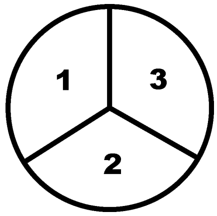

Fig. 2. Three-cell cellular system.

indicates that the new model can capture the handover behavior much better than the old one for small-scale cellular networks.

II. Exact Analytic Model for Exponential MS Cell Residence Times

This section describes an exact analytic solution for exponential MS cell residence times. Consider a cellular system with n cells. For $1 \le i \le n$, let S_i be the index set of cell i's neighbors. That is, cell j is a neighbor of cell i if $j \in S_i$. Let $|S_i|$ be the number of cell i's neighbors. For the illustration purpose, we consider a homogeneous system conforming to the following requirements:

1) *Capacity*: Each cell has c channels.
2) *Movement*: The routing probabilities to the neighboring cells are the same. That is, for an MS at cell i, it moves to each of cell i's neighbors with probability $1/|S_i|$. We further assume that $|S_i| = |S_j| = s$ for $1 \le i, j \le n$. The MS cell residence time distributions are the same for all cells.
3) *Call traffic*: The new call arrival rates to all cells are the same. The call duration time distributions are the same for all calls.

The following input parameters are considered in our model.

1) λ: The new call arrival rate to a cell. The new call arrivals are a Poisson stream [7] and the new call arrivals to each cell are independent.
2) $1/\mu$: The mean call duration time. The call duration times have an exponential distribution.
3) $1/\eta$: The mean MS cell residence time. The MS cell residence times are independent and identically distributed (i.i.d.). This section assumes exponential MS cell residence times. In the next section, we will consider general MS cell residence time distributions.

We assume that the call duration time and the MS cell residence time are independent of each other. Let random variable N_i be the number of busy channels in cell i $(1 \le i \le n)$. The following output measures are evaluated in our study.

1) p_b (the new call blocking probability): The number of new call blockings divided by the number of new calls. Since the system is homogeneous, p_b for all cells are the same and for a cell i, p_b can be expressed as

$$\begin{aligned} p_b &= \Pr[\text{A new call is blocked} \\ &\qquad \mid \text{this new call occurs at cell } i] \\ &= \Pr[N_i = c]. \end{aligned} \tag{1}$$

2) p_f (the forced termination probability): The number of forced terminations divided by the number of handovers
3) p_{nc} (the call incompletion probability; i.e., the probability that a call is either blocked or forced to terminate): The sum of the numbers of new call blockings and forced terminations divided by the number of new calls. Note that $p_{nc} \ne p_b + p_f$.

To derive p_f, we first define five events.

1) *Event* A_i. A call is handed over into cell i. For $1 \le i \le n$, A_i are mutually exclusive events.
2) *Event* B_k. A call is handed over out of a cell with k busy channels. For $1 \le k \le c$, B_k are mutually exclusive events.
3) *Event* C. A handover occurs in the cellular system. The relationship among C, A_i, and B_k is

$$C = \bigcup_{i=1}^{n} A_i = \bigcup_{k=1}^{c} B_k. \tag{2}$$

4) *Event* D_m. A call is handed over out of cell m. For $1 \le m \le n$, D_m are mutually exclusive events, and

$$C = \bigcup_{m=1}^{n} D_m. \tag{3}$$

5) *Event* $E_{j,k}$. A call is handed over out of a specific cell j with k busy channels. For $1 \le j \le n$ and $1 \le k \le c$, $E_{j,k}$ are mutually exclusive events. Note that

$$\begin{aligned} D_m &= \bigcup_{k=1}^{c} E_{m,k} \\ C &= \bigcup_{k=1}^{c} \bigcup_{j=1}^{n} E_{j,k} \\ B_k &= \bigcup_{j=1}^{n} E_{j,k}. \end{aligned} \tag{4}$$

Since the system is homogeneous, p_f for all cells are the same, which can be expressed as

$$p_f = \Pr\left[\{N_i = c\} \cap A_i | A_i\right]. \tag{5}$$

Since $A_i = A_i \cap (\bigcup_{k=1}^{c} B_k)$, (5) can be rewritten as

$$p_f = \Pr\left[\{N_i = c\} \cap A_i \cap \left(\bigcup_{k=1}^{c} B_k\right) \middle| A_i\right]. \tag{6}$$

Equation (6) says that to compute p_f, we need to consider how the flow-in handover traffic behaves. For example, there is no flow-in handover traffic into cell i if $k = 0$ for all cell i's neighbors (i.e., there is no busy channel in any of cell i's neighbors).

Since $B_1, B_2, \ldots, B_c$ are mutually exclusive events, from [16], (6) can be expressed as

$$\begin{aligned} p_f &= \sum_{k=1}^{c} \Pr\left[\{N_i = c\} \cap A_i \cap B_k | A_i\right] \\ &= \sum_{k=1}^{c} \frac{\Pr\left[\{N_i = c\} \cap B_k \cap A_i\right]}{\Pr[A_i]} \\ &= \sum_{k=1}^{c} \frac{\Pr\left[\{N_i = c\} \cap B_k \cap A_i\right]}{\Pr[B_k \cap A_i]} \times \frac{\Pr[B_k \cap A_i]}{\Pr[A_i]} \\ &= \sum_{k=1}^{c} \Pr\left[\{N_i = c\} \cap B_k \cap A_i | B_k \cap A_i\right] \\ &\quad \times \Pr[B_k | A_i]. \end{aligned} \tag{7}$$

In (7), let

$$p_{1,k} = \Pr\left[\{N_i = c\} \cap B_k \cap A_i | B_k \cap A_i\right] \tag{8}$$

which is the probability that all channels in cell i are busy given that a call is handed over from a cell with k busy channels to cell i. From (4), (8) can be rewritten as

$$p_{1,k} = \Pr\left[\{N_i = c\} \cap B_k \cap A_i \cap \left(\bigcup_{l=1}^{n} E_{l,k}\right) \middle| B_k \cap A_i\right]. \tag{9}$$

Since $E_{1,k}, E_{2,k}, \cdots, E_{n,k}$ are mutually exclusive events, from [16], (9) can be expressed as

$$\begin{aligned} p_{1,k} &= \sum_{l=1}^{n} \Pr\left[\{N_i = c\} \cap B_k \cap A_i \cap E_{l,k} | B_k \cap A_i\right] \\ &= \sum_{l=1}^{n} \frac{\Pr\left[\{N_i = c\} \cap E_{l,k} \cap B_k \cap A_i\right]}{\Pr[B_k \cap A_i]} \\ &= \sum_{l=1}^{n} \frac{\Pr\left[\{N_i = c\} \cap E_{l,k} \cap B_k \cap A_i\right]}{\Pr[E_{l,k} \cap B_k \cap A_i]} \times \frac{\Pr[E_{l,k} \cap B_k \cap A_i]}{\Pr[B_k \cap A_i]} \\ &= \sum_{l=1}^{n} \Pr[N_i = c | E_{l,k} \cap B_k \cap A_i] \\ &\quad \times \Pr[E_{l,k} \cap B_k \cap A_i | B_k \cap A_i]. \end{aligned} \tag{10}$$

From (4), we have $E_{l,k} \cap B_k = E_{l,k}$ and (10) is rewritten as

$$p_{1,k} = \sum_{l=1}^{n} \Pr[N_i = c | E_{l,k} \cap A_i] \times \Pr[E_{l,k} \cap A_i | B_k \cap A_i]. \tag{11}$$

In (11), $\Pr[E_{l,k} \cap A_i | B_k \cap A_i]$ represents the routing probability that a call is handed over from cell l with k busy channels to cell i, given that the call is handed over from a cell with k busy channels to cell i. Since we consider homogeneous topology and routing pattern, the MS can only move into cell i from any one of cell i's neighbors with probability $1/s$. Therefore,

$$\Pr[E_{l,k} \cap A_i | B_k \cap A_i] = \begin{cases} \frac{1}{s}, & \text{if } l \in S_i \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

Substitute (12) into (11) to yield

$$p_{1,k} = \sum_{l \in S_i} \Pr[N_i = c | E_{l,k} \cap A_i] \times \frac{1}{s}. \tag{13}$$

Due to homogeneous assumptions, (13) can be rewritten as

$$\begin{aligned} p_{1,k} &= \Pr[N_i = c | E_{l,k} \cap A_i] \\ &= \Pr\left[N_i = c | \{\text{A call is handed over from cell } l \text{ to cell } i\} \cap \{N_l = k\}\right] \end{aligned} \tag{14}$$

where $1 \leq k \leq c$. It is clear that Event $\{N_i = c\}$ and Event {A call is handed over from cell l to cell i} are conditionally independent given Event $\{N_l = k\}$, where $1 \leq k \leq c$. Therefore, from [16], (14) can be expressed as

$$\begin{aligned} p_{1,k} &= \Pr[N_i = c | N_l = k] \\ &= \frac{\Pr\left[\{N_i = c\} \cap \{N_l = k\}\right]}{\Pr[N_l = k]} \\ &= \frac{\Pr[N_l = k | N_i = c] \times \Pr[N_i = c]}{\Pr[N_l = k]} \end{aligned} \tag{15}$$

where $1 \leq k \leq c$. In (7), let

$$p_{2,k} = \Pr[B_k | A_i]. \tag{16}$$

Given that a call is handed over into cell i, $p_{2,k}$ is the probability that the call is handed over out of a cell with k busy channels. From (3) and $B_k = B_k \cap (\bigcup_{l=1}^{n} D_l)$, (16) can be rewritten as

$$p_{2,k} = \Pr\left[B_k \cap \left(\bigcup_{l=1}^{n} D_l\right) \middle| A_i\right]. \tag{17}$$

Since $D_1, D_2, \ldots, D_n$ are mutually exclusive events, from [16], (17) can be expressed as

$$\begin{aligned} p_{2,k} &= \sum_{l=1}^{n} \Pr[B_k \cap D_l | A_i] \\ &= \sum_{l=1}^{n} \frac{\Pr[B_k \cap D_l \cap A_i]}{\Pr[A_i]} \\ &= \sum_{l=1}^{n} \frac{\Pr[B_k \cap D_l \cap A_i]}{\Pr[D_l \cap A_i]} \times \frac{\Pr[D_l \cap A_i]}{\Pr[A_i]} \\ &= \sum_{l=1}^{n} \Pr[B_k | D_l \cap A_i] \times \Pr[D_l | A_i]. \end{aligned} \tag{18}$$

In (18), $\Pr[D_l | A_i]$ is the probability that under the condition that a call is handed over into cell i, it came from cell l. Because of homogeneous topology and routing pattern, the handover call is from any of cell i's neighbors with the same probability. Therefore,

$$\Pr[D_l | A_i] = \begin{cases} \frac{1}{s}, & \text{if } l \in S_i \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

Substitute (19) into (18) to yield

$$p_{2,k} = \sum_{l \in S_i} \Pr[B_k | D_l \cap A_i] \times \frac{1}{s} = \sum_{l \in S_i} \Pr[N_l = k | D_l \cap A_i] \times \frac{1}{s} \quad (20)$$

where $1 \leq k \leq c$. Due to network homogeneity, $\sum_{l \in S_i} \Pr[N_l = k | D_l \cap A_i] = s \times \Pr[N_l = k | D_l \cap A_i]$. Thus, (20) is rewritten as

$$\begin{aligned} p_{2,k} &= s \times \Pr[N_l = k | D_l \cap A_i] \times \frac{1}{s} \\ &= \Pr[N_l = k | D_l \cap A_i] \\ &= \Pr\left[\{N_l = k\} \cap D_l \cap A_i | D_l \cap A_i\right] \end{aligned} \quad (21)$$

where $1 \leq k \leq c$. In (21), Event $D_l \cap A_i$ represents the event that a call is handed over from cell l to cell i, and Event $\{N_l = k\} \cap D_l \cap A_i$ means that a call is handed over from cell l with k busy channels to cell i. Note that the mobility rate of an MS is η. Because of homogeneous topology and routing pattern, if a busy channel in cell l will be released due to MS movement to cell i, then the channel is released at rate η/s, and (21) can be rewritten as

$$\begin{aligned} p_{2,k} &= \frac{\Pr[N_l = k] \times k \times \left(\frac{\eta}{s}\right)}{\sum_{k=1}^{c} \Pr[N_l = k] \times k \times \left(\frac{\eta}{s}\right)} \\ &= \frac{k \Pr[N_l = k]}{E[N_l]}. \end{aligned} \quad (22)$$

Substituting (15) and (22) into (7), we have

$$\begin{aligned} p_f &= \sum_{k=1}^{c} p_{1,k} \times p_{2,k} \\ &= \sum_{k=1}^{c} \frac{\Pr[N_l = k | N_i = c] \times \Pr[N_i = c]}{\Pr[N_l = k]} \times \frac{k \Pr[N_l = k]}{E[N_l]} \\ &= \left\{ \frac{\Pr[N_i = c]}{E[N_l]} \right\} \left\{ \sum_{k=1}^{c} k \Pr[N_l = k | N_i = c] \right\} \\ &= p_b \times \frac{E[N_l | N_i = c]}{E[N_l]}. \end{aligned} \quad (23)$$

The probability p_{nc} was derived in [12], which is expressed as follows:

$$p_{nc} = p_b + \frac{\eta(1 - p_b)\left[1 - f_m^*(\mu)\right] p_f}{\mu\left[1 - (1 - p_f) f_m^*(\mu)\right]} \quad (24)$$

where $f_m^*(s)$ is the Laplace transform of the MS cell residence time distribution. For exponential MS cell residence time distributions, $f_m^*(\mu) = \eta/(\mu + \eta)$, and (24) can be expressed as

$$\begin{aligned} p_{nc} &= p_b + \frac{\eta(1 - p_b)\left(\frac{\mu}{\mu+\eta}\right) p_f}{\mu\left[1 - (1 - p_f)\left(\frac{\eta}{\mu+\eta}\right)\right]} \\ &= \frac{\mu p_b + \eta p_f}{\mu + \eta p_f}. \end{aligned} \quad (25)$$

$\Pr[N_i = c]$, $E[N_l]$ and $E[N_l | N_i = c]$ in (1) and (23) are derived by solving an n-dimensional Markov process, which are then used to compute p_{nc}. We note that our approach is similar to the one proposed in [14], where the model can be extended for heterogeneous network modeling (e.g., the numbers of channels in cells are different).

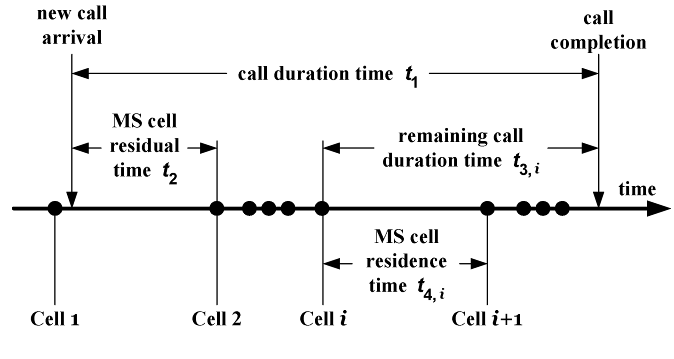

Fig. 3. Timing diagram.

III. Approximate Model for General MS Cell Residence Times

This section proposes an approximate solution for modeling general MS cell residence times. The idea is to adjust the exact analytic solution developed in the previous section. Specifically, we approximate the general MS cell residence time by an exponential distribution with the adjusted rate η^*.

Consider the timing diagram in Fig. 3. In this figure, a call arrives when the MS resides in cell 1. The call duration time is t_1. The MS cell residual time at cell 1 (i.e., the interval between when the call arrives and when the MS moves out of cell 1) is t_2. For $i = 2, 3, \ldots$, if the call is successfully handed over to cell i, then the remaining call duration time is $t_{3,i}$. For $i = 2, 3, \ldots$, the MS cell residence time at cell i (i.e., the interval between when the MS enters cell i and when it moves out of cell i) is $t_{4,i}$. Since the call duration times are exponentially distributed, t_1 and $t_{3,i}$ have the same exponential distribution. Let random variable X be the number of call completions for a connected call. Note that the value of X is either one or zero, depending on whether the call is eventually completed or forced to terminate. Let random variable Y be the number of handovers for a connected call. We derive $E[X]$ and $E[Y]$ as follows. In Fig. 3, let $\alpha_1 = \Pr[t_1 > t_2]$ be the probability that a new call is not completed before the MS moves out of the first cell, and $\alpha_2 = \Pr[t_{3,i} > t_{4,i}]$ be the probability that a handover call is not completed before the MS moves out of cell i, where $i = 2, 3, \ldots$ From [8], we have

$$\alpha_1 = \left(\frac{\eta}{\mu}\right)\left[1 - f_m^*(\mu)\right] \quad \text{and} \quad \alpha_2 = f_m^*(\mu). \quad (26)$$

With (26), $E[X]$ is derived as follows:

$$\begin{aligned} E[X] = & 1 \times \Pr[\text{A connected call is completed}] \\ & + 0 \times \Pr[\text{A connected call is forced to terminate}] \\ = & (1 - \alpha_1) + \sum_{k=1}^{\infty} \left\{ \alpha_1 [(1 - p_f)\alpha_2]^{k-1} (1 - p_f)(1 - \alpha_2) \right\}. \end{aligned} \quad (27)$$

In the right-hand side of (27), $(1-\alpha_1)$ is the probability that a new call is completed before the MS moves out of the cell, and $\{\alpha_1[(1-p_f)\alpha_2]^{k-1}(1-p_f)(1-\alpha_2)\}$ is the probability that a call is successfully handed over for k times and is completed at the $k+1$-st cell, where $k \geq 1$. From (27), we have

$$E[X] = (1-\alpha_1) + \frac{\alpha_1(1-p_f)(1-\alpha_2)}{1-(1-p_f)\alpha_2}. \qquad (28)$$

Substitute (26) into (28) to yield

$$E[X] = 1 - \frac{\eta\,[1-f_m^*(\mu)]\,p_f}{\mu\,[1-(1-p_f)f_m^*(\mu)]}. \qquad (29)$$

Similarly, $E[Y]$ is derived as

$$E[Y] = \sum_{k=1}^{\infty} k\left\{\alpha_1\,[(1-p_f)\alpha_2]^{k-1} \times [(1-p_f)(1-\alpha_2)+p_f]\right\} \qquad (30)$$

$$= \frac{\alpha_1}{1-(1-p_f)\alpha_2}. \qquad (31)$$

In the right-hand side of (30), $\{\alpha_1[(1-p_f)\alpha_2]^{k-1} \times [(1-p_f)(1-\alpha_2)+p_f]\}$ is the probability that a call is successfully handed over for k times before it is completed, or is successfully handed over for $k-1$ times and forced to terminate at the kth handover. Substitute (26) into (31) to yield

$$E[Y] = \frac{\eta\,[1-f_m^*(\mu)]}{\mu\,[1-(1-p_f)f_m^*(\mu)]}. \qquad (32)$$

Define θ as

$$\theta = \frac{E[X]}{E[X+Y]}. \qquad (33)$$

Although X and Y are dependent random variables, we have [5]

$$E[X+Y] = E[X] + E[Y]. \qquad (34)$$

From (34), (33) is rewritten as

$$\theta = \frac{E[X]}{E[X]+E[Y]}. \qquad (35)$$

From (29) and (32), (35) can be expressed as

$$\theta = 1 - \frac{\eta\,[1-f_m^*(\mu)]}{\mu\left\{1-(1-p_f)f_m^*(\mu) + \left(\frac{\eta}{\mu}\right)(1-p_f)\,[1-f_m^*(\mu)]\right\}}. \qquad (36)$$

For the exponential MS cell residence time distribution, $f_m^*(\mu) = \eta/(\mu+\eta)$, and (36) can be simplified as

$$\theta = \frac{\mu}{\mu+\eta}. \qquad (37)$$

To approximate the general MS cell residence time distribution by an exponential distribution, we define η^* as the mobility rate for the approximate exponential distribution. From (37), the approximate mobility rate is

$$\eta^* = \frac{\mu}{\theta} - \mu. \qquad (38)$$

By substituting (36) into (38), we have

$$\eta^* = \frac{\eta\,[1-f_m^*(\mu)]}{1-(1-p_f)f_m^*(\mu) - \left(\frac{\eta}{\mu}\right)[1-f_m^*(\mu)]\,p_f}. \qquad (39)$$

Therefore, for MS cell residence time distribution with Laplace transform $f_m^*(s)$, this distribution can be approximated by an exponential distribution with mobility rate η^* [expressed in (39)] for the channel assignment model. The probabilities p_b, p_f, and p_{nc} are computed as follows:

Input parameters: λ, μ, η, c, and $f_m^*(\mu)$.
Output measures: θ, p_b, p_f, and p_{nc}.
Step 1. Select an initial value for p_f.
Step 2. $p_{f,old} \leftarrow p_f$. Compute η^* by using (39).
Step 3. Use the standard n-dimensional Markov process [14] to solve p_b and p_f with (1) and (23).
Step 4. Let ϵ be a predefined value ($\epsilon = 0.001$ in our example). If $|p_f - p_{f,old}| > \epsilon$, then go to *Step 2*. Otherwise, go to *Step 5*.
Step 5. The values for θ, p_b, and p_f converge. Compute p_{nc} by using (24).

IV. Numerical Examples

This section uses numerical examples to compare the analytic model in Sections II and III (called the "new model") with the model we proposed in [12] (called the "old model"). For the demonstration purpose, we consider a three-cell cellular system (i.e., $n=3$; see Fig. 2), where each cell has nine channels (i.e., $c=9$) and two neighbors (i.e., $s=2$). Such systems have been manufactured and deployed in Asia.[1] For other small-scale cell configurations, similar results are observed, which will not be presented in this paper.

Fig. 4 plots the blocking probabilities against the mobility rate η for the exponential MS cell residence time model, where the call duration times are exponentially distributed with rate $\mu = 0.3\lambda$. The figure indicates that both p_b and p_f decrease as η increases. Since the number of handovers increases as η increases, p_{nc} increases as η increases. The same phenomena were found in [1] and [9], and the reader is referred to these pervious studies for more details. We observe that the new analytic results almost match the simulation results, while the errors between the old analytic model and the simulation experiments can be up to 18%. The figure suggests that the new analytic model is more accurate than the old one. Fig. 4 also indicates that the higher the mobility, the more the inconsistency between the old analytic and the simulation results. Therefore, the advantage of

[1]Widax, http://www.widax.com.tw/

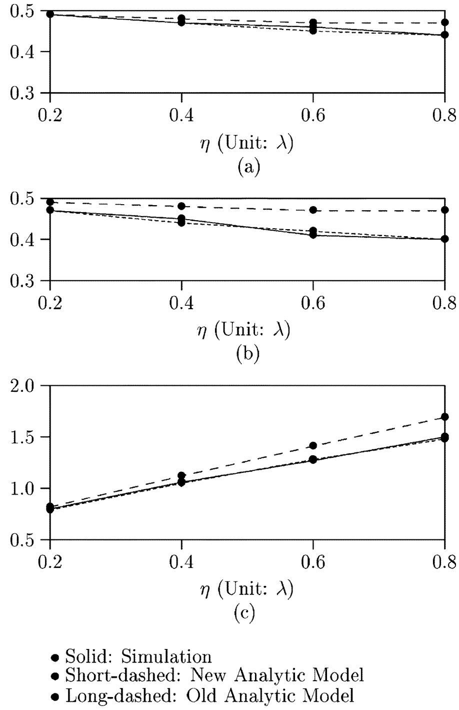

Fig. 4. Results for exponential MS cell residence time model ($\mu = 0.3\lambda$). (a) $p_b(\%)$. (b) $p_f(\%)$. (c) $p_{nc}(\%)$.

TABLE I
p_{nc} VALUES AND ERRORS: ANALYTIC MODELS VERSUS SIMULATION ($\eta = 0.5\lambda$). (a) p_{nc}. (b) ERRORS OF p_{nc}

ν (Unit: $1/\eta^2$)	0.01	0.1	1
$\mu = 0.1\lambda$			
Simulation	39.429%	39.5817%	39.1805%
Old Analytic Model	40.915%	40.7124%	40.4659%
New Analytic Model	39.4335%	39.4122%	39.2026%
$\mu = 0.2\lambda$			
Simulation	8.3426%	8.4134%	8.2144%
Old Analytic Model	9.2013%	9.1963%	9.1491%
New Analytic Model	8.2463%	8.2428%	8.2131%
$\mu = 0.3\lambda$			
Simulation	1.2036%	1.1719%	1.1532%
Old Analytic Model	1.2674%	1.2673%	1.2659%
New Analytic Model	1.1665%	1.1665%	1.1655%

(a)

ν (Unit: $1/\eta^2$)	0.01	0.1	1
$\mu = 0.1\lambda$			
Old Analytic Model	3.77%	2.86%	3.28%
New Analytic Model	0.01%	0.43%	0.06%
$\mu = 0.2\lambda$			
Old Analytic Model	10.29%	9.31%	11.38%
New Analytic Model	1.15%	2.03%	0.02%
$\mu = 0.3\lambda$			
Old Analytic Model	5.3%	8.14%	9.77%
New Analytic Model	3.08%	0.46%	1.07%

(b)

the new analytic model becomes significant when the mobility is high. The old analytic model is not as accurate as the new one because it assumes $p_f = p_b$, while in the new analytic model, we have proven that $p_f = p_b \times (E[N_l|N_i = c]/E[N_l]) \neq p_b$ (see (23)). When the mobility is low, the value of $E[N_l|N_i = c]$ is close to $E[N_l]$. Consequently, $p_f \approx p_b$ and the old analytic model works well.

For a general MS cell residence time distribution, its variance ν may have significant impact on the output measures. For Gamma MS cell residence times, Table I shows the call incompletion probability p_{nc} values and their errors between analytic and simulation models, where the MS cell residence time variances are $\nu = 0.01/\eta^2$, $0.1/\eta^2$, and $1/\eta^2$. In Table I, the mobility rate is $\eta = 0.5\lambda$ and the call completion rates are $\mu = 0.1\lambda$, 0.2λ, and 0.3λ, respectively. The results suggest that for various ν, the p_{nc} values of the new analytic model are much closer to the simulation results than that of the old analytic model.

V. CONCLUSION

Most analytic modeling studies for cellular networks assume that the handover traffic to a cell is a fixed-rate Poisson process. This assumption may introduce significant inaccuracy for modeling small-scale cellular networks. This paper showed that the handover traffic to a cell depends on the workloads of the neighboring cells. We derived the exact equation for the handover force-termination probability when the MS cell residence times are exponentially distributed. Then we proposed an approximate model for general MS cell residence time distributions. The results are compared with a previously proposed model, which indicate that the new model can capture the handover behavior much better than the old one for small-scale cellular networks.

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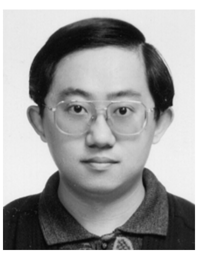

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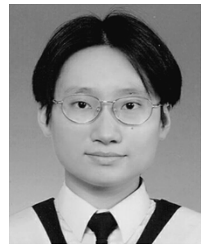

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