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Slow light in photonic crystals

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Abstract

In this work we demonstrate an interesting opportunity to decrease drastically the group velocity of light in two-dimensional photonic crystals constructed form materials with large dielectric constant without dispersion ($\varepsilon_d \sim 100$). The group velocity of light in the crystals can be lowered up to 10^{-5}–$10^{-4}c$ (c is velocity of light in free space) in a wide range of the Brillouin zone. We propose a simple and general explanation why that should appear in photonic crystals with large dielectric constants.

Keywords: Photonic crystals; Slow light

Mainly due to advances in nano-technology now we can control photons in photonic crystals in the same way as we control electrons in semiconductor nano-structures [1]. The principal goal of recent studies in this field is to manipulate, direct, and confine light in the same manner like we can do that with electrons. If parameters and symmetry of the photonic crystals are chosen properly, the dispersion relation for allowed photonic bands and photonic band gaps can be precisely controlled. The structures can be built from different composite materials and this opens up a wide range of possible applications for the photonic crystals: enhancement of the spontaneous emission, waveguide bend's fabrication, couplers, filters and polarizes [2,3].

Recently, photonic crystals built from polar materials have raised a great interest [4,5]. One of the interesting phenomena is the flattening of photonic bands (a drastic decreasing of the light group velocity u_g) below phononic resonance frequency. This effect lies in the opportunity to obtain a large and negative dielectric function in this frequency region. Huang et al. [6] studied extensively this effect and proposed a theoretical model of the localized electromagnetic field. They also investigated an effect of anti-crossing for transverse TE photonic modes in two-dimensional structures due to interplay between metallo-dielectric bands and resonant modes. The introduction of the metallo-dielectric bands comes from the negative dielectric function in polaritonic materials.

In this work we demonstrate another interesting opportunity to design flat bands and anti-crossing photonic bands in two-dimensional semiconductor photonic crystals using large dielectric constants, where the dynamical phononic polarization are not considered. This can decrease drastically the group velocity of the light in two-dimensional photonic crystals. We stress that the effect was not expected for the photonic crystals constructed from materials with dielectric functions without dispersion. Therefore, we propose a general model for the flat and anti-crossing bands in photonic crystals with large dielectric constants.

We consider the light propagation in two-dimensional periodic structures of the square symmetry, when a dielectric rod with a large dielectric constant (ε_d) is inserted in the center of the unit cell of the crystal (see insert in Fig. 1a). Our system consists of materials with frequency independent dielectric functions and the plane-wave expansion method is well suited for the problem. From the Maxwell's equations at the fixed frequency ω one can obtain well known equations for the magnetic (**H**) and electric field (**E**) in the structure

$$\begin{aligned} &\nabla \times \frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) = \left(\frac{\omega}{c}\right)^2 \mathbf{H}(\mathbf{r}) \\ &\frac{1}{\varepsilon(\mathbf{r})} \nabla \times \nabla \times \mathbf{E}(\mathbf{r}) = \left(\frac{\omega}{c}\right)^2 \mathbf{E}(\mathbf{r}), \end{aligned} \tag{1}$$

where

$$\varepsilon(\mathbf{r}) = \begin{cases} 1, & \text{out of the rod} \\ \varepsilon_d, & \text{in the rod} \end{cases},$$

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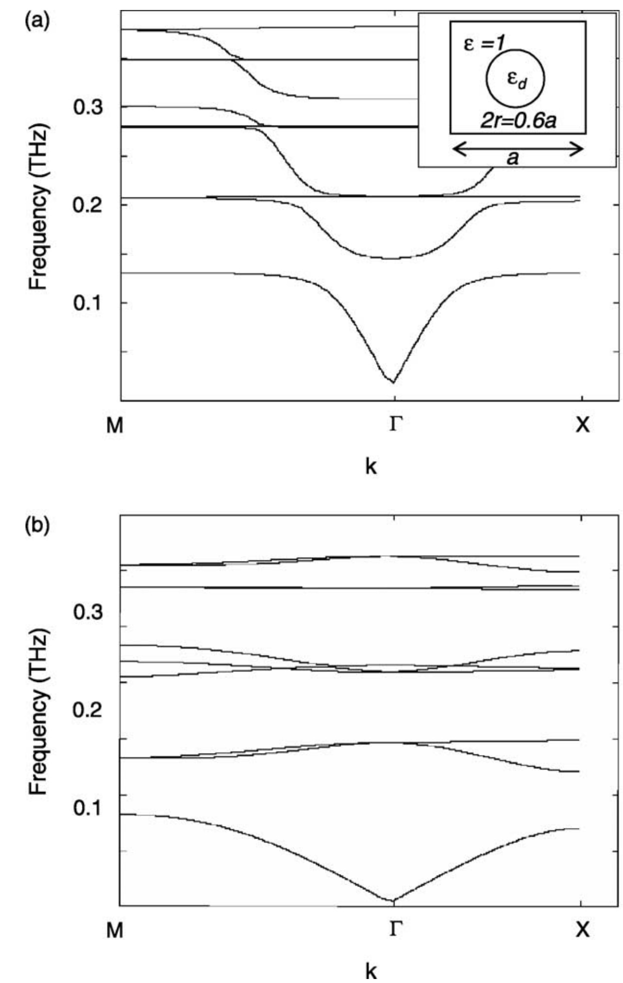

Fig. 1. Band structure of the photonic crystal investigated ($\varepsilon_d = 100$): (a) TE bands; (b) TM bands. In insert: the unit cell of the crystal.

and c is speed of light in free space. For the two-dimensional photonic crystal we consider infinitely long in z-direction dielectric rods. In the crystal we can divided the fields into two polarizations by symmetry: TE (transverse electric, with magnetic field directed along z-axis) and TM (transverse magnetic, with electric field directed along z-axis). Eq. (1) for the modes introduced above can be presented in scalar form

$$\begin{aligned} &\left[\frac{\partial}{\partial x}\frac{1}{\varepsilon(\boldsymbol{\rho})}\frac{\partial}{\partial y}+\frac{\partial}{\partial y}\frac{1}{\varepsilon(\boldsymbol{\rho})}\frac{\partial}{\partial x}\right]H_z(\boldsymbol{\rho}) = \left(\tfrac{\omega}{c}\right)^2 H_z(\boldsymbol{\rho}) \\ &\frac{1}{\varepsilon(\boldsymbol{\rho})}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]E_z(\boldsymbol{\rho}) = \left(\frac{\omega}{c}\right)^2 E_z(\boldsymbol{\rho}), \end{aligned} \quad (2)$$

where $\boldsymbol{\rho} = \{x, y\}$ is in-plane radius vector.

For the periodic structures we consider dielectric function: $\varepsilon(\boldsymbol{\rho}) = \varepsilon(\boldsymbol{\rho} + \mathbf{a}_i)$, where $\mathbf{a}_i$ is the two-dimensional primitive lattice vector of the crystal. Using the Fourier transform of the dielectric function we obtain

$$\frac{1}{\varepsilon(\boldsymbol{\rho})} = \sum_{\mathbf{G}} \kappa(\mathbf{G})\exp(\mathrm{i}\mathbf{G}\boldsymbol{\rho}),$$

where $\mathbf{G}$ is vectors in the reciprocal lattice. Applying the Bloch's theorem to $H_z(\boldsymbol{\rho})$ and $E_z(\boldsymbol{\rho})$ ($\mathbf{k}$ stands for the wave vector)

$$H_z^{n,\mathbf{k}}(\boldsymbol{\rho}) = \exp(\mathrm{i}\mathbf{k}\boldsymbol{\rho})\sum_{\mathbf{G}} H^{n,\mathbf{k}}(\mathbf{G})\exp(\mathrm{i}\mathbf{G}\boldsymbol{\rho}),$$

for TE polarization;

$$E_z^{n,\mathbf{k}}(\boldsymbol{\rho}) = \exp(\mathrm{i}\mathbf{k}\boldsymbol{\rho})\sum_{\mathbf{G}} E^{n,\mathbf{k}}(\mathbf{G})\exp(\mathrm{i}\mathbf{G}\boldsymbol{\rho}),$$

for TM polarization.

we can present the Eq. (2) as the following

$$\begin{aligned} &\textstyle\sum_{\mathbf{G}} M^{\mathrm{TE}}(\mathbf{G},\mathbf{G}')H^{n,\mathbf{k}}(\mathbf{G}') = \left(\frac{\omega}{c}\right)^2 H^{n,\mathbf{k}}(\mathbf{G}) \\ &\textstyle\sum_{\mathbf{G}} M^{\mathrm{TM}}(\mathbf{G},\mathbf{G}')E^{n,\mathbf{k}}(\mathbf{G}') = \left(\frac{\omega}{c}\right)^2 E^{n,\mathbf{k}}(\mathbf{G}) \end{aligned}, \quad (3)$$

where

$$\begin{aligned} &M^{\mathrm{TE}}(\mathbf{G},\mathbf{G}') = \kappa(\mathbf{G}-\mathbf{G}')(\mathbf{k}+\mathbf{G})(\mathbf{k}+\mathbf{G}') \\ &M^{\mathrm{TM}}(\mathbf{G},\mathbf{G}') = \kappa(\mathbf{G},\mathbf{G}')(\mathbf{k}+\mathbf{G}')(\mathbf{k}+\mathbf{G}') \end{aligned}.$$

In our computational experience, stable (well converging) solutions of Eq. (3) for photonic crystals with large ε_d (about or more than 100) can be obtained when we use up to 1700 Bloch waves.

In Fig. 1, we present calculated photonic band structures for a square array of dielectric rods with $\varepsilon_d = 100$. The band structures are almost isotropic and one can find the flattened bands (where $u_g/c \ll 1$) for both polarizations. The corresponding group velocity of light is presented in Fig. 2. This is not surprising since the electromagnetic fields are almost confined in the dielectric rods [7]. But, TE modes demonstrate even less dispersion than TM modes. The most unexpected is that TE anti-crossing bands appear in our theory, where the dielectric function is independent on frequency.

To explain the results above we stress that the system investigated is actually an array of dielectric resonators those have been studied many years ago [8]. It was

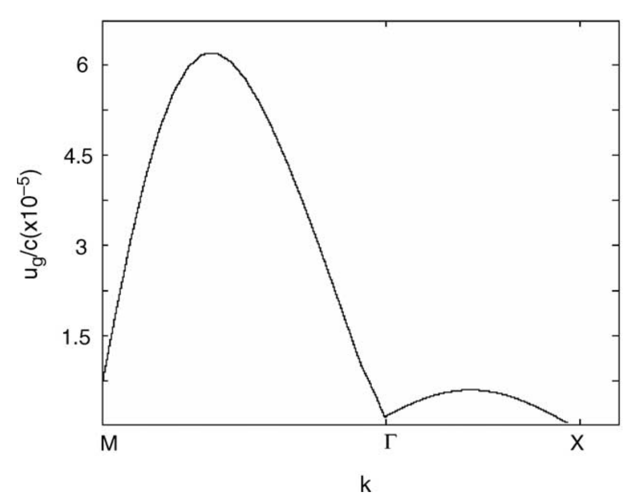

Fig. 2. Relative group velocity of light in the third TE band within the first Brillouin zone for a two-dimensional photonic crystal ($\varepsilon_d = 100$).

suggested that the perfect magnetic conductor boundary conditions (PMCBCs) are applicable when the dielectric constant goes to infinity inside the resonators (at the boundaries of the dielectric rods in our photonic crystal). The PMCBCs assure the tangential magnetic field and perpendicular electric field have to be zero at the boundaries. This is just the dual condition of metallic resonators and we can directly use the metallic resonators solutions by replacing TE modes by TM modes and vice versa. Using that we have effectively for *mn*th resonance mode in the rods

$$H_z^{mn}(\boldsymbol{\rho}) \approx H_0 J_m\left(\omega_{mn}\frac{\sqrt{\varepsilon_{\rm d}}}{c}\rho\right)\exp(\mathrm{i}m\phi) \quad \text{for TE modes;}$$

$$E_z^{mn}(\boldsymbol{\rho}) \approx E_0 J_m\left(\omega_{mn}\frac{\sqrt{\varepsilon_{\rm d}}}{c}\rho\right)\exp(\mathrm{i}m\phi) \quad \text{for TM modes,}$$

where J_m is the Bessel function of the first kind and ω_{mn} is the resonance frequency. Since the tangential magnetic field vanishes at the boundary under PMCBCs, the TE modes are greatly localized inside the rods [7].

To explain the anti-crossing phenomenon we assume that the constitutive equation for the rod polarization in *mn*th mode ($\mathbf{P}_{mn}$) can be present as

$$\frac{\partial^2}{\partial t^2}\mathbf{P}_{mn} = -\omega_{mn}^2\mathbf{P}_{mn} + \omega_{mn}^2\gamma_{mn}\mathbf{E}, \tag{4}$$

where the unit-less coefficient γ_{mn} presents coupling between polarization of the rod and the electromagnetic field $\mathbf{E}$ outside the rod (when $\varepsilon_{\rm d}$ is large but not infinite γ_{mn} can be small but not zero). In the same time for the transverse mode combining

$$-\frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2} = \nabla\times\nabla\times E = \nabla\nabla\cdot E - \Delta E$$

and

$$\mathbf{D} = \mathbf{E} + \chi_{mn}\mathbf{P}_{mn},$$

(χ_{mn} is another coupling parameter for electromagnetic field and polarization) we obtain for amplitudes of the field $\mathbf{E}(t)=\mathbf{E}^0\exp(\mathrm{i}\omega t)$ and polarization $\mathbf{P}(t)=\mathbf{P}^0\exp(\mathrm{i}\omega t)$:

$$\mathbf{E}^0 = \frac{\omega^2\chi_{mn}}{c^2k^2-\omega^2}\mathbf{P}_{mn}^0. \tag{5}$$

Substituting (5) into (4) we arrive to the following expression for the frequency of the transverse mode

$$\omega^2 = \tfrac{1}{2}\left\{\omega_{mn}^2(1+\gamma_{mn}\chi_{mn}) + c^2k^2 \pm\sqrt{[\omega_{mn}^2(1+\gamma_{mn}\chi_{mn})+c^2k^2]^2 - 4\omega_{mn}^2c^2k^2}\right\},$$

which obviously describes the anti-crossing bands. The combination $\gamma_{mn}\chi_{mn}$ characterizes a small (but not zero) coupling between localized modes in the dielectric rods and the electromagnetic field. It is clear now that the appearance of the flat (slow light) bands and anti-crossing is inherent to the large dielectric constant and large polarization in the rods.

In conclusion, our calculation results suggest a drastic difference between photonic modes with TE and TM polarizations in two-dimensional photonic crystals with large dielectric constant. While TM modes demonstrate well-know properties, TE modes correspond to flat bands and the anti-crossing effect in the band structure the photonic crystals. The group velocity of light in the crystal can be lowered up to 10^{-5}–$10^{-4}c$ in a wide range of the Brillouin zone. We propose a general model for photonic crystal with large dielectric constants. The model explains the anti-crossing bands those are caused by the dielectric rod's resonance coupled to the electromagnetic field.

The model and the effect can be verified by experiment since one can use for example zirconium–tin–titanate ($\varepsilon_{\rm d}=90$ up to 1 THz [9]). For a photonic crystal with lattice constant $a=300$ μm, our theory predicts the effects mentioned above in the region below 1 THz. In addition there is a prediction [7] that the effective permeability in the lowest bends appears to be negative. Clearly, new meta-materials built from dielectrics with large dielectric constants remain of a great interest.

Acknowledgements

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