

## Left handed composite materials in the optical range

O. Voskoboynikov<sup>a,\*</sup>, G. Dyankov<sup>a</sup>, C.M.J. Wijers<sup>b</sup>

<sup>a</sup>Department of Electronics Engineering and Institute of Electronics, National Chiao Tung University, 1001 Ta Hsueh Rd, Hsinchu 300, Taiwan, ROC

<sup>b</sup>Faculty of Applied Science, Twente University, P.O.Box 217, 7500 AE Enschede, The Netherlands

Available online 10 March 2005

### Abstract

The purpose of this paper is to show that semiconductor nano-structures built from non-magnetic InAs/GaAs nano-rings can exhibit simultaneously negative effective permittivity and permeability over a certain optical frequency range. The structures are resonant and have this property near the edge of absorption of the nano-rings. This can be particularly interesting in the investigation of the challenging problem of development of left-handed composite materials in the optical range.

© 2005 Elsevier Ltd. All rights reserved.

*Keywords:* Left-handed material; Semiconductor nano-rings; Optics

Left handed composite materials (LHCMs) exhibit simultaneously negative permittivity ( $\epsilon$ ) and permeability ( $\mu$ ) over a certain frequency ( $\omega$ ) range [1–3]. It is acknowledged that LHCMs can have a variety of exciting applications. A particularly important and challenging problem in this field is the realization of LHCMs in the optical frequency range. A negative refractive index was confirmed in the GHz and THz range [3,4] many years after its theoretical prediction [1], but most of magnetic materials at frequencies in the GHz range and above have a magnetic response which is tailing off. That is the reason why it was proposed to realize LHCMs relying upon the idea that inherently nonmagnetic materials can exhibit a magnetic response.

In this work we show theoretically that there is an opportunity to obtain negative permittivity and magnetic permeability simultaneously in the optical range by using nano-structured composite semiconductor materials. Recent advances in the manufacturing of semiconductor nano-rings make it possible to construct arrays of III–V semiconductor nano-scale rings [5]. Just like self-assembled quantum dots, nano-rings possess atom-like optical properties, but at the same time nano-rings are non-simply connected quantum systems, exhibiting

unusual magnetic [6–8] and magneto-optical properties [5,9,10]. One of the problems arising in LHCMs for the optical range is that the size of the structural elements has to be of nanometer scale. Another problem is that the use of conductive elements is inappropriate because of the high losses [11]. Semiconductor nano-rings are ideal building blocks and could meet the requirements mentioned. It is necessary to emphasize that structural elements possessing magnetic response, have sizes much smaller than the operating wavelength and the composite materials made from them can be characterized by effective permittivity  $\epsilon(\omega)$  and magnetic permeability  $\mu(\omega)$  only [1,2,11].

In this study we show that for three-dimensional photonic structures based on an artificial lattice of InAs/GaAs nano-rings [5] the effective permittivity can be negative in the optical range. At the same time the frequency domain with  $\mu(\omega) < 0$  can be tuned by changing the individual capacity of the rings.

To demonstrate theoretically a nano-structured composite material with negative permittivity and permeability in the optical range, we first consider a basic cell of a material made from semiconductor nano-rings (see insert in Fig. 1). Two-dimensional square arrays of these cells (with characteristic lattice constant  $a$ ) then are stacked as a layered meta-structure (with distance between layers  $l$ ) (Fig. 1). This establishes magnetic activity along the direction of stacking ( $z$ -axis in Fig. 1). In this work we consider only TE-polarized light, with the magnetic field parallel to this  $z$ -axis.

\* Corresponding author. Tel.: +886 3 5612121x54174; +886 3 5724361.

E-mail address: vam@faculty.nctu.edu.tw (O. Voskoboynikov).

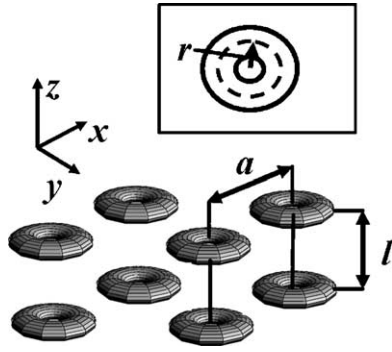


Fig. 1. Schematic diagram of a composite material built from nano-rings. In insert: in-plane cross-section of a single ring with the perimeter determining the effective resistance.

To evaluate the dielectric and magnetic properties of this structure, we first calculate the polarizability tensor  $\hat{\alpha}$  of a single ring. For frequencies close to the energy gap of the nano-ring, the size of the ring is smaller as compared to the wavelength. One can use then the dipole approximation and the Kramers–Heisenberg expression for  $\hat{\alpha}(\omega)$  [12]:

$$\hat{\alpha}(\omega) = \hat{\alpha}_s + \frac{2e^2}{\hbar} \sum_{i,f} \left( \frac{\omega_{fi}}{\omega} \right) \frac{\langle i|\mathbf{r}|f\rangle\langle f|\mathbf{r}|i\rangle^T}{[\omega_{fi} - \omega - i\Gamma]} \quad (1)$$

$\Gamma$  is the damping factor chosen to be independent from frequency near the resonance region. The possible transition energies  $\hbar\omega_{fi} = E_f^h - E_i^c$  are determined by the discrete energies  $E_f^h, E_i^c$  for the electron and hole states respectively. The dynamic part of the polarizability is determined by the optical matrix elements  $\langle i|\mathbf{r}|f\rangle$ , the expectation value of the position vector  $\mathbf{r}=(x,y,z)$  taken over the volume of the nano-ring. The sum over states in (1) is limited to transitions with the lowest  $\hbar\omega_{fi}$ , being near the absorption edge of the system. Using the approach proposed in [6,10] and taken into account the cylindrical symmetry of the ring, we obtain for the optical transition elements [10]

$$\langle i|z|f\rangle = 0 \quad |\langle i|x|f\rangle| = |\langle i|y|f\rangle| = \left| \frac{\langle S|x|X\rangle}{\sqrt{2}} I_{\text{eh}} \right|,$$

where  $I_{\text{eh}} = V_u^{-1} \int \rho d\rho dz \Phi_e(\rho, z) \Phi_h(\rho, z)$  is the electron-hole overlap integral for the envelope wave functions,  $V_u$  is the volume of the conventional bulk unit cell, and  $\rho=(x,y)$ . The matrix elements  $\langle S|x|X\rangle$  can be presented in the conventional notation of the Kane parameter  $P$

$$\langle S|x|X\rangle = \frac{P}{\hbar\omega_{fi}}$$

Previously we have calculated the energy levels and wave functions for electrons and holes confined in InAs/GaAs nano-rings of different sizes and shapes [6] and the static polarizability of these rings. Based on this calculation, we can reconstruct the polarizability tensor and using the Clausius-Mossotti relation we obtain an effective

permittivity for TE-polarized light in the structure

$$\varepsilon(\omega) = \varepsilon^s \frac{1 + \frac{2}{3\varepsilon_0\varepsilon^s} N\alpha_{xx}(\omega)}{1 - \frac{1}{3\varepsilon_0} N\alpha_{xx}(\omega)}, \quad (2)$$

where  $\varepsilon^s = (1-x)\varepsilon_{\text{GaAs}} + x$  is the effective static permittivity of the system,  $x$  is the relative volume occupied by the rings in this structure,  $\varepsilon_{\text{GaAs}}$  is the optical dielectric constant of GaAs,  $N$  is the density of the rings in the structure, and  $\varepsilon_0$  is the permittivity of free space.

We use this effective permittivity of the system to obtain in turn its magnetic permeability. To describe the effective magnetic response of the structure, we follow the approach of [11] and assume that for electromagnetic fields with frequency near the absorption edge of nano-rings, a circular current flow is induced inside the rings. Following [11], we define the effective permeability of this structure in the optical range near the adsorption edge

$$\mu(\omega) = 1 - \frac{F}{1 + \frac{l}{\mu_0\pi r^2\omega^2} \left[ iR_{\text{eff}}\omega - \frac{1}{C_{\text{eff}}} \right]}, \quad (3)$$

where  $F = \pi(r_{\text{out}})^2/a^2$  is the geometrical filling-factor of the basic cell,  $r_{\text{out}}$  is the outer radius of the ring,  $R_{\text{eff}}$  is the resistance of the ring measured over the perimeter of a circle with radius  $r$  (see insert in Fig. 1),  $C_{\text{eff}}$  is the effective capacitance of the ring, and  $\mu_0$  is the permeability of free space. The resistance of the ring we can calculate by the following assumptions

$$R_{\text{eff}} = \frac{1}{\sigma_{\text{eff}}} \frac{2\pi r}{S}, \quad (4)$$

where  $\sigma_{\text{eff}}$  is the effective conductance of the system (note: in [11]  $\sigma$  refers to the resistivity, not to conductance) and  $S$  is the area of the ring cross-section. The conductance is connected to the effective permittivity (2) as follows [13]

$$\varepsilon(\omega) = \varepsilon(0) + i \frac{\sigma_{\text{eff}}}{\varepsilon_0\omega}. \quad (5)$$

Combining (3)–(5) one can find the effective magnetic permeability of the structure in generic form. Detailed calculations give

$$\mu(\omega) \approx 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\gamma^2}, \quad (6)$$

where  $\omega_0$  is the effective resonance frequency, which is a solution of the following equation

$$\omega^2 = \frac{2lc_0^2}{rS} \left\{ \frac{[\varepsilon_r(\omega) - \varepsilon(0)]}{[\varepsilon_r(\omega) - \varepsilon(0)]^2 + [\varepsilon_i(\omega)]^2} + \frac{\varepsilon_0 S}{2\pi r C_{\text{eff}}} \right\}, \quad (7)$$

Table 1  
Dependence of the effective resonance frequency  $\omega_0$  on the capacitance of the structure with  $S_u$

$C_{\text{eff}} (10^{-17} \text{ F})$	30	3	0.3	0.03
$\omega_0 (10^{15} \text{ Hz})$	1.86	1.87	1.92	3.13

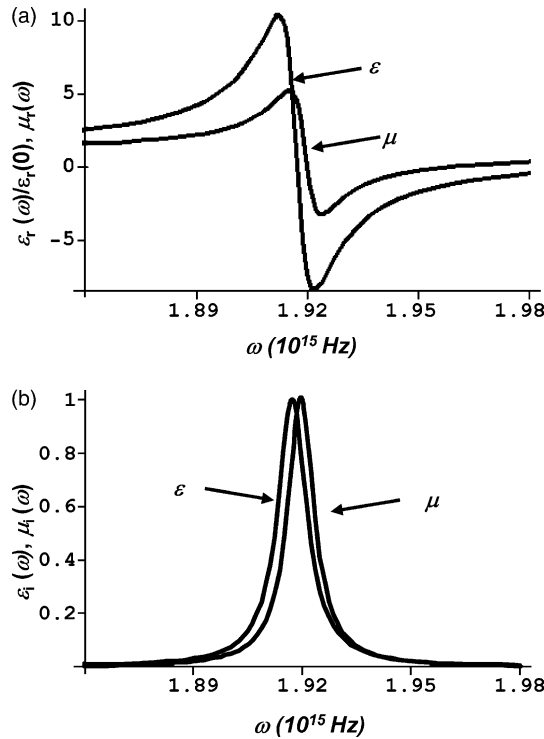


Fig. 2. Effective permittivity and permeability of the system from Fig. 1 with  $S_u = \{33, 5, 10, 4.5, 1 \text{ nm}\}$  and  $C_{\text{eff}} = 0.3 \times 10^{-17} \text{ F}$ : (a) Real parts of  $\varepsilon(\omega)$  and  $\mu(\omega)$ ; (b) normalized imaginary parts of  $\varepsilon(\omega)$  and  $\mu(\omega)$ .

and

$$\gamma = \sqrt{\frac{2lc_0^2}{rS} \frac{\varepsilon_i(\omega_0)}{[\varepsilon_r(\omega_0) - \varepsilon(0)]^2 + [\varepsilon_i(\omega_0)]^2}} \quad (8)$$

is the effective damping factor ( $c_0$  is the speed of light in free space),  $\varepsilon_r(\omega)$  and  $\varepsilon_i(\omega)$  are correspondingly the real and imaginary parts of the effective permittivity.

We will now calculate the dispersion of  $\varepsilon(\omega)$  and  $\mu(\omega)$  for the structures built from the InAs/GaAs nano-rings. The nano-ring shape is generated by an ellipsoidal contour [6]: the cross-section of the ring shows an ellipse with semi-major axis  $r_0$  and semi-minor axis  $z_0$  (semi-height of the ring). We use the values of  $\hat{\alpha}(\omega)$  as calculated previously by us for the nano-rings. Then almost all parameters in (2) and (3) for a structure built from the rings can be easily computed for a certain set of parameters  $S = \{a, l, r, r_0, z_0\}$ . Only the capacitance of the rings remains unknown and should be taken from experiments. But, from literature [9] it is known that the capacitance of semiconductor nano-objects (quantum dots and nano-rings) is in the range from  $10^{-16}$ – $10^{-18} \text{ F}$ . In Table 1 we present results of our calculations of  $\omega_0$  for a few possible values of the capacitance using Eq. (7) and  $S_u = \{33, 5, 10, 4.5, 1 \text{ nm}\}$ .

Obviously the resonance frequency for the effective permeability turns out to be in the range of interest. The capacitance of the ring plays the role of a tuning parameter for the frequency.

The resulting dispersion for the effective permittivity  $\varepsilon(\omega)$  and permeability  $\mu(\omega)$  is presented in Fig. 2. A negative permeability is possible for the system in the same region where the effective permittivity is negative. The region is rather narrow but quite visible. There is a possibility to tune the size of the region and position by varying the ring capacitance like it is shown in Table 1.

We stress that the negative permeability can be obtained only under rigid requirements. One needs to keep the deviations from the dimensions of Fig. 2 lower than about 10%. Larger deviations can crush the frequency region where the permeability is negative. But a strong dispersion (without obtaining negative values) remains for effective permittivity and permeability within a wide range of possible sets  $S$ .

In conclusion, we have demonstrated that dense nano-composite materials built from small nano-rings can exhibit simultaneously negative permittivity and permeability in the optical range. The range is wide enough to be scanned conveniently with lasers having typical spectral width of  $\Delta\nu \approx 10^{13} \text{ Hz}$ . The range can be tuned by changing the ring's capacity. These results could be particularly useful for design of a new class of LHCMS in the optical range.

This work was funded by the National Science Council of Taiwan under Contracts No. NSC 93-2112-M-009-008 and NSC 93-2215-E-009-006.

## References

- [1] V.G. Veselago, Sov. Phys. Usp. 10 (1968) 509.
- [2] J.B. Pendry, Phys. Rev. Lett. 85 (2000) 3966.
- [3] R.A. Shelby, D.R. Smith, S. Schultz, Science 292 (2001) 79.
- [4] T.J. Yen, W.J. Padilla, N. Fang, D.C. Vier, D.R. Smith, J.B. Pendry, D.N. Basov, Z. Zhang, Science 303 (2004) 1494.
- [5] J.M. Garcia, G. Mneiros-Ribeiro, K. Schmidt, T. Ngo, J.L. Feng, A. Lorke, J. Kotthaus, P.M. Petroff, Appl. Phys. Lett. 71 (1997) 2014.
- [6] O. Voskobyonikov, Y. Li, H.M. Lu, C.F. Shih, C.P. Lee, Phys. Rev. B 66 (2002) 155306.
- [7] O. Voskobyonikov, C.P. Lee, Physica E 20 (2004) 278.
- [8] J.I. Climente, J. Planelles, J.L. Movila, Phys. Rev. 70 (2004) 081301.
- [9] A. Emperor, M. Pi, M. Barraco, A. Lorke, Phys. Rev. B 62 (2000) 4573.
- [10] J.I. Climente, J. Planelles, W. Jaskolski, Phys. Rev. 68 (2003) 075307.
- [11] J.B. Pendry, A.J. Holden, D.J. Robbins, W.J. Stewart, IEEE Trans. Microw. Theory Tech. 47 (1999) 2075.
- [12] C.M.J. Wijers, Phys. Rev. A 70 (6) (2004).
- [13] L.D. Landau, E.M. Lifshitz, Electrodynamics of Continues Media, Pergamon, Oxford, 1960.