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Control, anticontrol and synchronization of chaos for an autonomous rotational machine system with time-delay

Zheng-Ming Ge *, Ching-I Lee

Department of Mechanical Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 300, Taiwan, Republic of China

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Abstract

Chaos, control, anticontrol and synchronization of chaos for an autonomous rotational machine system with a hexagonal centrifugal governor and spring for which time-delay effect is considered are studied in the paper. By applying numerical results, phase diagram and power spectrum are presented to observe periodic and chaotic motions. Linear feedback control and adaptive control algorithm are used to control chaos effectively. Linear and nonlinear feedback synchronization and phase synchronization for the coupled systems are presented. Finally, anticontrol of chaos for this system is also studied.

1. Introduction

During the past two decades, chaos of a large number of nonautonomous systems have been observed and studied [1–3]. In comparison, the number of chaotic autonomous systems discovered is far fewer than the former. The centrifugal governor is a device that automatically controls the speed of an engine and prevents damage caused by a sudden change of load torque. It plays an important role in many rotational machines such as the diesel engine, steam engine and so on. When the parameter of an engine system is varied, the speed of the engine will change. In order to decrease the change of engine speed, and to avoid chaotic motion emerging in the operational process of the engine, in this paper the regular and chaotic dynamics and chaos control of an autonomous rotational machine system with a hexagonal centrifugal governor are studied in detail. If the chaotic dynamics of this system is used to some chaos application (like secure communication), anticontrol and synchronization of chaos are also presented in this paper.

For the mass-spring system shown in Fig. 1, suppose m is not a particle. It has a length $P_2 - P_1$. When the spring force acts on m at P_1 in some instant t, m does not move immediately. Because, when a force acts on the surface P_1 of m, the stress waves propagate inside m. Usually the stress waves start from P_1 and are reflected at P_2. After crisscross of stress waves inside m, the acceleration of m becomes uniform, and m actually begins to move at this time. Let the time interval between the instant of exertion of the force on m and the instant at which m actually moves be τ, the equation of motion of the time-delay system is given as $m\ddot{x}(t) + kx(t - \tau) = 0$. This kind of time-delay is considered in this paper.

* Corresponding author. Tel.: +886 35712121; fax: +886 35720634.
E-mail address: zmg@cc.nctu.edu.tw (Z.-M. Ge).

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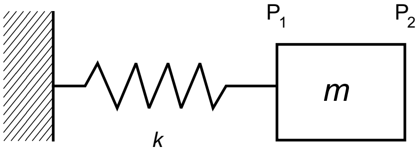

Fig. 1. A mass-spring system.

The first aim is to present the detailed dynamics of this autonomous mechanical system. Many modern techniques are used in analyzing deterministic nonlinear system behavior. Computational methods, such as phase diagrams, time history and power spectrum, are employed to obtain the characteristics of the nonlinear system. Since chaos is often undesirable in mechanical systems, two methods are used to control chaos. Later in the papers, attention is shifted to chaos synchronization and anticontrol of chaos. The chaos synchronization of coupled systems is an important topic for chaos study because of its possible application to secure communications [4–9]. The linear and nonlinear feedback based approaches are discussed. Both complete synchronization and phase synchronization of two coupled chaotic systems are studied by the linear and nonlinear feedback based approaches. Anticontrol of chaos is also studied.

2. Equations of motion

The rotational machine with centrifugal governor is depicted in Fig. 2. Some basic assumptions for the system are

(1) neglect the mass of the rods and the sleeve;
(2) viscous damping in rod bearing of the fly-ball is represented by the damping constant c.

From Fig. 2, the system considered can be modeled by the following delay differential equations:

$$\begin{aligned}
\dot{\phi} &= \varphi \\
\dot{\varphi} &= \frac{r}{l}\omega^2 \cos\phi + \omega^2 \sin\phi\cos\phi - \frac{2k}{m}(1-\cos\phi_{t-\tau})\sin\phi - \frac{g}{l}\sin\phi - c\varphi \\
\dot{\omega} &= q\cos\phi - F
\end{aligned} \tag{2.1}$$

where l, m, r and ϕ represent respectively, the length of the rod, the mass of fly-ball, the distance between the rotational axis and the suspension joint, and the angle between the rotational axis and the rod. ω is the angular velocity of the governor, $q > 0$ is the proportionality and F is an equivalent torque of the load [10].

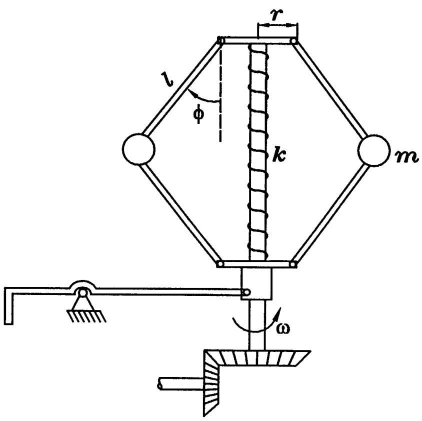

Fig. 2. Physical model of the system.

3. Regular and chaotic dynamics of time-delay system

In nonlinear dynamical systems, variation of system parameters may cause sudden change in the qualitative behavior of their state. The state change is referred to as a bifurcation and the parameter value at which the bifurcation occurs is called the bifurcation value. Denoting $\phi = x$, $\dot{\phi} = y$, $\omega = z$, Eq. (2.1) is rewritten in the form

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= \frac{r}{l} z^2 \cos x + z^2 \sin x \cos x - \frac{2k}{m}(1 - \cos x_{t-\tau}) \sin x - \frac{g}{l} \sin x - cy \\ \dot{z} &= q \cos x - F \end{aligned} \tag{3.1}$$

Here q is considered as the control parameter to be varied when the values of parameters r, l, k, m, F, c are given as 0.4, 2, 20, 2, 1.942, 0.4, respectively.

The phase portrait is the evolution of a set of trajectories emanating from various initial conditions in the state space. When the solution reaches steady state, the transient behavior disappears. By numerical integration, the phase portrait of the system, Eq. (3.1), is plotted in Fig. 3(a) for $q = 3$. Clearly, the motion is periodic. But Fig. 3(b) for $q = 5.5$ shows the chaotic state. Furthermore the power spectrum is a continuous broad-band as shown in Fig. 4(b) for $q = 5.5$. The noise-like spectrum is characteristic of a chaotic dynamical system.

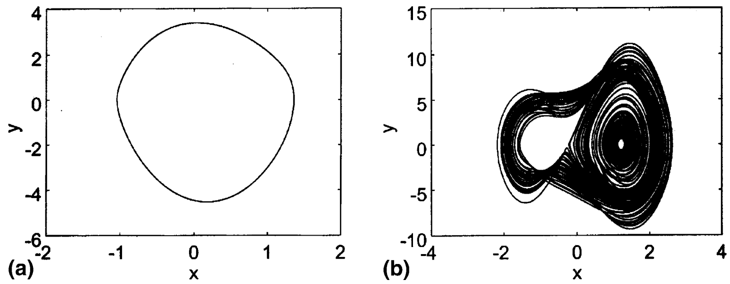

Fig. 3. (a) Phase portrait for $q = 3$ (b) $q = 5.5$.

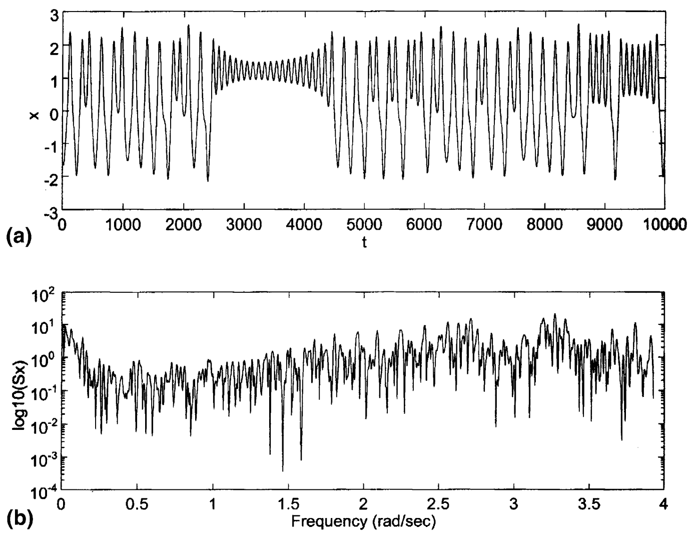

Fig. 4. (a) Time history for $q = 5.5$, (b) power spectrum for $q = 5.5$.

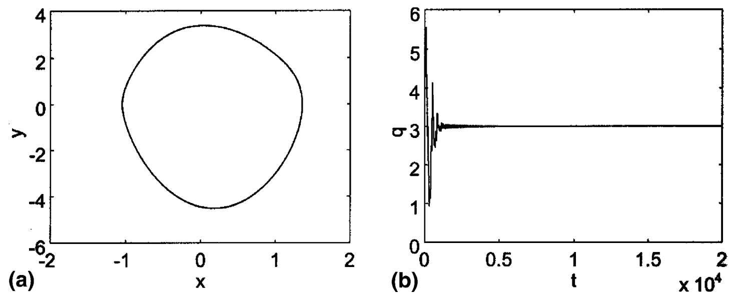

Fig. 5. Controlled system via adaptive feedback.

4. Controlling chaos

Several interesting nonlinear dynamic behavior characteristics of the system have been discussed in previous sections. It has been shown that the time-delay autonomous system exhibits both regular and chaotic motion. Usually chaos is unwanted or undesirable. In order to improve the performance of a dynamic system or to avoid the chaotic phenomena, we need to control chaotic motion to become a periodic motion which is beneficial for working at a specific condition. It is thus of great practical importance to develop suitable control methods. Much interest has been focused on this type of problem—controlling chaos [11]. For this purpose, two methods are used to control our system from chaos to order.

4.1. Controlling chaos by an adaptive control algorithm (ACA)

A simple and effective adaptive control algorithm is suggested [12], which utilizes an error signal proportional to the difference between the goal output and actual output of the system. The error signal governs the change of parameters of the system, which readjusts so as to reduce the error to zero. This method can be explained briefly. The system motion is set back to a desired state X_s by adding dynamics on the control parameter P through the evolution equation,

$$\dot{P} = \varepsilon G(X - X_s) \tag{4.1}$$

where the function G depends on the difference between X_s and the actual output X, and ε indicates the stiffness of the control. The function G could be either linear or nonlinear. In order to convert the dynamics of system (3.1) from chaotic motion to the desired periodic motion X_s, the chosen parameter q is perturbed as

$$\dot{q} = K_1(X - X_S) \tag{4.2}$$

If $K_1 = 0.5$, the system can reach the period-1 trajectory easily as shown Fig. 5. It is clear that the desired periodic motion can be reached by the adaptive control algorithm.

4.2. Controlling chaos by linear feedback control

A linear feedback control with a special form is used in this method. It is assumed that the input signal $u(t)$ disturbs only the third equation in (3.1). and

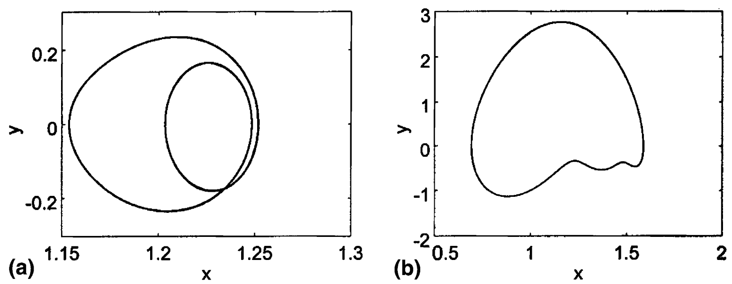

Fig. 6. (a) Phase portrait of controlled system via linear feedback control for $K_2 = 0.5$, (b) $K_2 = 5$.

$$u(t) = K_2[y_i(t) - y(t)] = K_2D(t) \tag{4.3}$$

Here, $y(t)$ is the chaotic output signal, $y_i(t)$ is the periodic motion of system. The difference $D(t)$ between the signal $y_i(t)$ and $y(t)$ is used as a control signal. Here K_2 is an adjustable weight of the feedback. By selecting the weight K_2, we can convert chaotic behavior to periodic motion. We can control the chaotic behavior to period-1 and period-2 motion by choosing $K_2 = 5$ and 0.5, respectively, as shown in Fig. 6.

5. Chaos synchronization

A characteristic property of chaotic dynamics is its sensitive dependence on initial condition. Different initial conditions will cause different trajectories for the system. However, Pecora and Carroll [13] showed that synchronization can be achieved for chaotic systems. This interesting phenomenon plays a significant role in the chaotic dynamics of communication signals and may be applied to the real-time recovery of signals that have been masked in a strange attractor and thus to encode communication. Other applications of synchronization of chaos also have expectant potential [14]. A natural way to develop synchronization for chaotic systems is through system decomposition. The chaotic system is decomposed into two subsystems as follows:

Drive system:

$$\begin{aligned}
\dot{x}_1 &= y_1 \\
\dot{y}_1 &= \frac{r}{l}z_1^2\cos x_1 + z_1^2\sin x_1\cos x_1 - \frac{2k}{m}(1-\cos x_{1_{t-\tau}})\sin x_1 - \frac{g}{l}\sin x_1 - cy_1 \\
\dot{z}_1 &= q\cos x_1 - F
\end{aligned} \tag{5.1}$$

Response system:

$$\begin{aligned}
\dot{x}_2 &= y_2 \\
\dot{y}_2 &= \frac{r}{l}z_2^2\cos x_2 + z_2^2\sin x_2\cos x_2 - \frac{2k}{m}(1-\cos x_{2_{t-\tau}})\sin x_2 - \frac{g}{l}\sin x_2 - cy_2 \\
\dot{z}_2 &= q\cos x_2 - F
\end{aligned} \tag{5.2}$$

In the following, linear and nonlinear feedback based approaches are discussed.

5.1. Linear feedback synchronization

In this approach, the error between the output of the identical drive and response is used as the control signal. For the unidirectional case, where only the first equation of response (5.2) is combined with a linear feedback, while the equations of drive remain the same [13].

$$\begin{aligned}
\dot{x}_2 &= y_2 + K(x_1 - x_2) \\
\dot{y}_2 &= \frac{r}{l}z_2^2\cos x_2 + z_2^2\sin x_2\cos x_2 - \frac{2k}{m}(1-\cos x_{2_{t-\tau}})\sin x_2 - \frac{g}{l}\sin x_2 - cy_2 \\
\dot{z}_2 &= q\cos x_2 - F
\end{aligned} \tag{5.3}$$

where K is the constant feedback gain. With $K = 3$, the trajectories of subsystems and the synchronization errors, $e_x = x_2 - x_1$, $e_y = y_2 - y_1$, and $e_z = z_2 - z_1$, are shown in Fig. 7. In this case, $K = 2.75$ is a critical value, below which no synchronization occurs.

Then we consider the two identical systems which are coupled together. The mutually coupled chaotic systems by adding linear coupling term are written as

Drive system:

$$\begin{aligned}
\dot{x}_1 &= y_1 + K(x_2 - x_1) \\
\dot{y}_1 &= \frac{r}{l}z_1^2\cos x_1 + z_1^2\sin x_1\cos x_1 - \frac{2k}{m}(1-\cos x_{1_{t-\tau}})\sin x_1 - \frac{g}{l}\sin x_1 - cy_1 \\
\dot{z}_1 &= q\cos x_1 - F
\end{aligned} \tag{5.4}$$

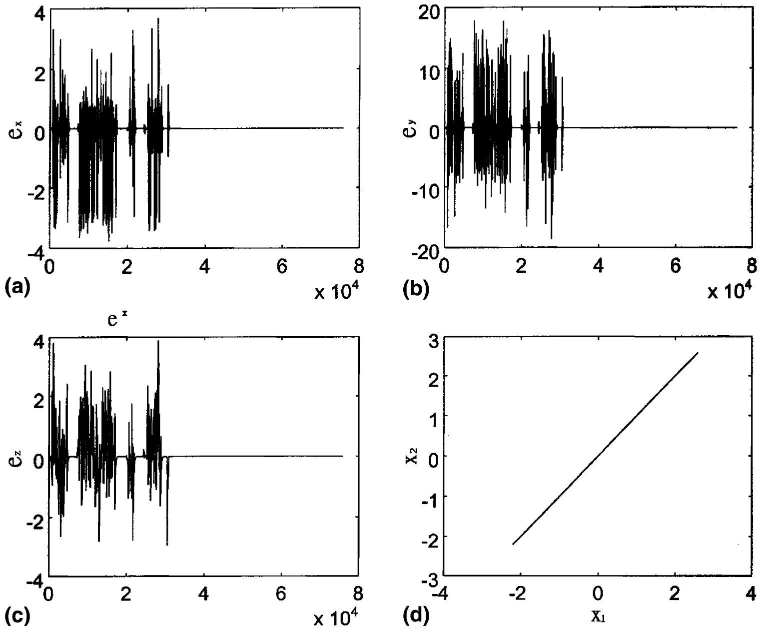

Fig. 7. Chaos synchronization via a unidirectional linear feedback approach for $K = 3$.

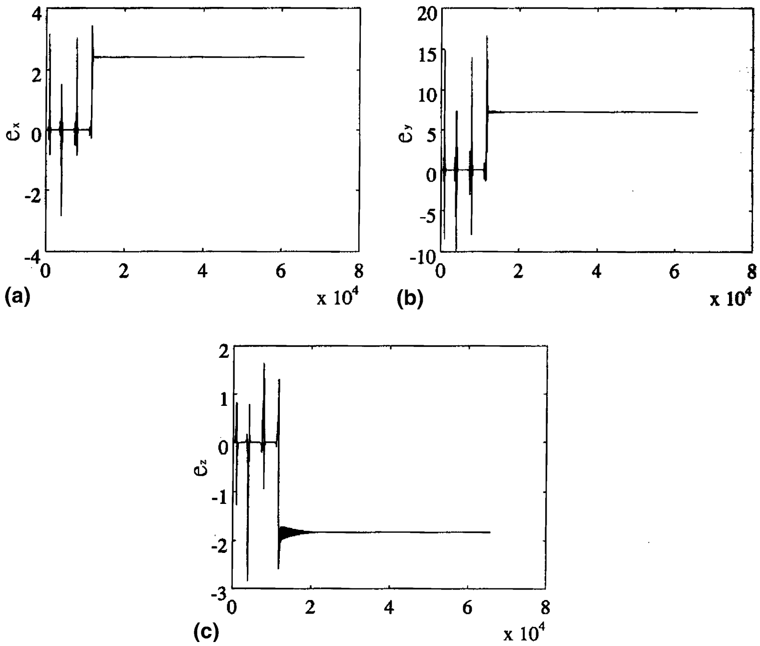

Fig. 8. Chaos synchronization via a mutual linear feedback approach for $K = 1.5$.

Response system:

$$\begin{aligned}
&\dot{x}_2 = y_2 + K(x_1 - x_2)\\
&\dot{y}_2 = \frac{r}{l} z_2^2 \cos x_2 + z_2^2 \sin x_2 \cos x_2 - \frac{2k}{m}(1 - \cos x_{2_{t-\tau}}) \sin x_2 - \frac{g}{l} \sin x_2 - c y_2 \qquad (5.5)\\
&\dot{z}_2 = q \cos x_2 - F
\end{aligned}$$

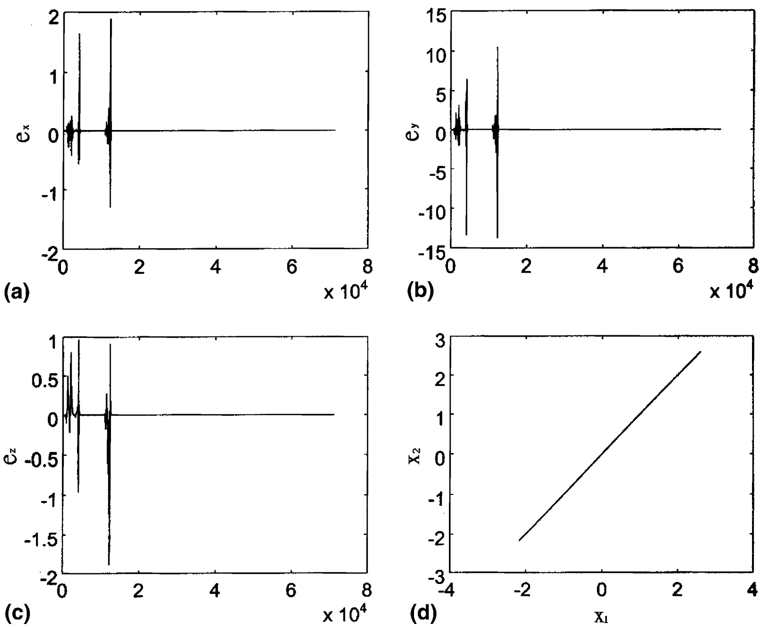

Fig. 9. Chaos synchronization via a mutual linear feedback approach for $K = 3$.

The simulation result show that generalized synchronization occurs when $K = 0.8$–2.75, and two chaotic systems are in complete synchronization when $K > 3.75$. With $K = 1.5$ and 3, the synchronization errors are shown in Figs. 8 and 9, respectively.

5.2. Nonlinear feedback synchronization

The mutually coupled chaotic systems by adding nonlinear coupling term are written as
Drive system:

$$\begin{aligned}
\dot{x}_1 &= y_1 + K\sin(x_2 - x_1)\\
\dot{y}_1 &= \frac{r}{l}z_1^2\cos x_1 + z_1^2\sin x_1\cos x_1 - \frac{2k}{m}(1-\cos x_{1_{t-\tau}})\sin x_1 - \frac{g}{l}\sin x_1 - cy_1\\
\dot{z}_1 &= q\cos x_1 - F
\end{aligned} \tag{5.6}$$

Response system:

$$\begin{aligned}
\dot{x}_2 &= y_2 + K\sin(x_1 - x_2)\\
\dot{y}_2 &= \frac{r}{l}z_2^2\cos x_2 + z_2^2\sin x_2\cos x_2 - \frac{2k}{m}(1-\cos x_{2_{t-\tau}})\sin x_2 - \frac{g}{l}\sin x_2 - cy_2\\
\dot{z}_2 &= q\cos x_2 - F
\end{aligned} \tag{5.7}$$

With $K = 1.5$, the trajectories of subsystems and the synchronization errors are shown in Fig. 10.

5.3. Phase synchronization

Recently, the concept of phase, as well as phase synchronization, has been discussed in detail for chaotic oscillators [15,16]. If we project the attractor on plane (x, y), it has one rotation center shown in Fig. 3(b). The phase of the system can be defined as:

$$\phi = \arctan\frac{y - y_c}{x - x_c} \tag{5.8}$$

where the point (x_c, y_c) is the rotation center. The phase can be easily calculated for each subsystem, and the mean frequency are defined as [16]:

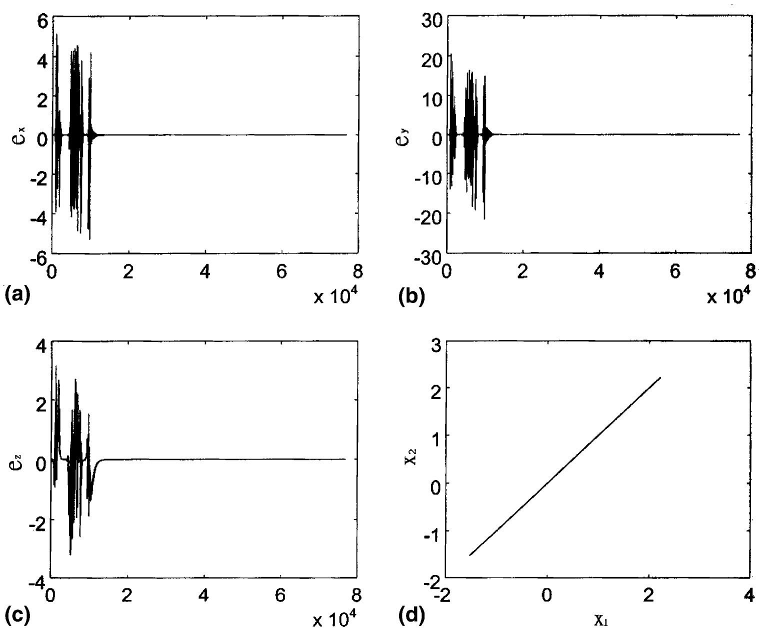

Fig. 10. Chaos synchronization via a mutual nonlinear feedback approach for $K = 1.5$.

$$\Omega = \langle \dot{\phi} \rangle = \lim_{T\to\infty} \frac{\phi(T) - \phi(0)}{T} \tag{5.9}$$

Phase synchronization occurs when the phases of two oscillators have the relationship $\phi_1(t) \approx \phi_2(t)$ with time, i.e., frequency-locking state for $\Delta\Omega = \Omega_1 - \Omega_2 \to 0$.

We study two coupled centrifugal governor systems with strength of coupling K,

$$\begin{aligned}
\dot{x}_{1,2} &= y_{1,2} + K(x_{2,1} - x_{1,2}) \\
\dot{y}_{1,2} &= \frac{r}{l} z_{1,2}^2 \cos x_{1,2} + z_{1,2}^2 \sin x_{1,2} \cos x_{1,2} - \frac{2k}{m}(1 - \cos x_{1,2_{t-\tau}}) \sin x_{1,2} - \frac{g}{l} \sin x_{1,2} - c y_{1,2} \\
\dot{z}_{1,2} &= q \cos x_{1,2} - F
\end{aligned} \tag{5.10}$$

To examine phase synchronization, we modulate the coupling parameter K in Eq. (5.10). Some numerical results are given in Fig. 11. For the unidirectional case, Fig. 11(a) shows the maximum absolute difference of mean frequency between two trajectories. Obviously, the phases of two systems are synchronizing in the region $K > 3.9$. Fig. 11(b) shows the maximum absolute difference of mean frequency between the mutually coupled chaotic systems. When $K > 1.4$, they are phase synchronization. Therefore, phase synchronization is easily achieved in mutually coupled chaotic systems.

6. Anticontrol of chaos

Sometimes, chaos is not only useful but actually important. Besides secure communication and information processing, chaos is desirable in many applications of liquid mixing while the required energy is minimized. For this purpose, making a non-chaotic dynamical system chaotic is called "anticontrol of chaos" [17,18]. For our system (3.1), it is periodic motion with q = 3. The feedback controller used is a simple triangle function shown in Fig. 12(a), the periodic motion becomes chaotic shown in Fig. 12(b).

7. Conclusions

The autonomous rotational machine system with a hexagonal centrifugal governor and spring with time-delay exhibits a rich variety of nonlinear behaviors as certain parameters are varied. Due to the effect of nonlinearity, regular

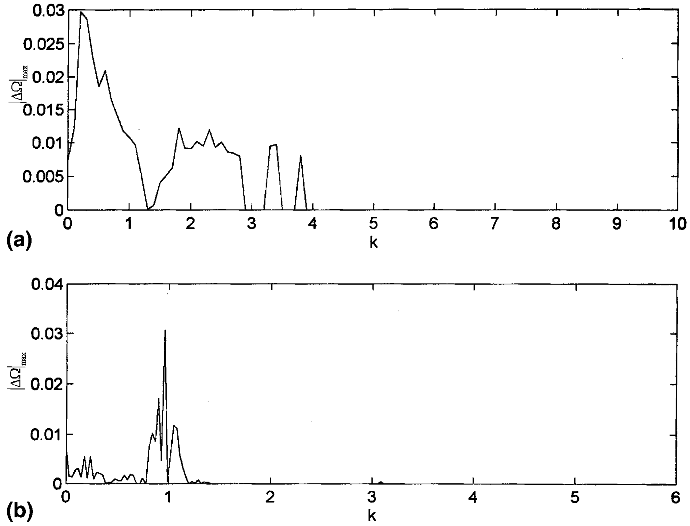

Fig. 11. The maximum absolute difference of mean frequency between two chaotic subsystems.

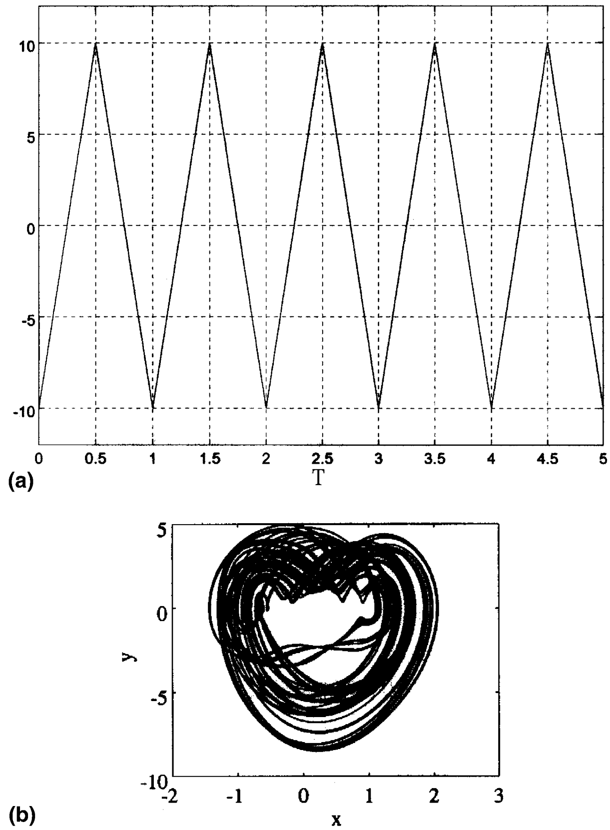

Fig. 12. (a) A sawtooth function, (b) phase portrait of controlled system.

or chaotic motion may occur. In this paper, the periodic and chaotic motion of the autonomous system with time-delay are obtained by the numerical methods such as phase trajectory, time history and power spectrum. The changes of parameters play a major role for the nonlinear system.

In order to improve the performance of a dynamical system or avoid the chaotic phenomena, two methods, adaptive control algorithm and linear feedback control, are used to control the chaotic motion to periodic motion effectively. Synchronization of two chaotic oscillators is studied in this paper. For two chaotic systems, increase of coupling strength leads to the occurrence of complete synchronization and phase synchronization. Finally, anticontrol of chaos is also studied.

Acknowledgment

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