# Space-charge effects of electrons and ions on the steady states of field-emission-limited diodes

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# Space-charge effects of electrons and ions on the steady states of field-emission-limited diodes

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Space-charge effects of electrons and ions on the steady state of a field-emission-limited diode (FELD) are investigated via a self-consistent approach. The field-emission process is described quantum mechanically by the Fowler–Nordheim equation. The cathode plasma and surface properties are considered within the framework of the effective work function approximation. Ionization effects at the anode as well as electron space-charge effects are described by Poisson's equation. The numerical calculations are carried out self-consistently to yield the steady states of the bipolar field-emission-limited flow of the FELD. We found that the stationary state of the diode exhibits a cut-off voltage. The electric field on the cathode surface is found to be saturated in the high voltage regime and is determined by the effective work function approximately. In addition, the ion current included in the Poisson's equation has been treated as a tuning parameter. The analytical formula of the electron current density has been derived. The field-emission currents in the presence of saturated ion currents can be enhanced to be nearly 1.8 times of the case with no ion current.
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## I. INTRODUCTION

In vacuum electronic devices employing electron beams, diodes are usually used to accelerate the beam. The basic features of diodes can be illustrated with an idealized model, the Child–Langmuir diode in a parallel planar geometry.[1] This expression, Child–Langmuir law, is based on the assumption that the cathode can supply enough current to be space-charge limited, in which case the electric field at the cathode surface is zero and the current is a maximum. The emission is said to be current limited if the cathode cannot supply enough current to be space-charge limited. Jory and Trivelpiece[2] considered the problem of current-limited emission in a one-dimensional planar diode including the relativistic corrections in the equation of motion of the electrons. They obtained an exact relativistic solution for the one-dimensional planar diode, both with current limited and with space-charge limited.[2] While electron emission can result from any of several processes, including thermionic emission,[3] photoemission, secondary emission, field emission, and explosive emission,[4] the dominant mechanism of concern in this article is the field emission. The phenomenon of field emission from a cold metal can be described as a quantum mechanical tunneling of conduction electrons through the potential barrier at the surface of the metal.[5–12]

In recent years, field emission diodes have been extensively investigated theoretically and experimentally. Although field emission can be microscopically enhanced by a field enhancement factor due to protrusions, contamination, oxide layers, dielectric inclusions, grain boundaries, or adsorbates, the current density is usually not enough to be space-charge limited macroscopically. In this case, the current is field-emission limited.

In this work, field-emission-limited diodes (FELDs) are investigated. The field emission process is described quantum mechanically by the Fowler–Nordheim equation. The cathode plasma and surface properties are considered within the framework of the effective work function approximation.[13–15] Ionic effects are investigated by including the upstream ion current in the Poisson's equation.[16,17] The steady states of the bipolar flow of the FELD can be obtained numerically via the following self-consistent approach.[18]

## II. FORMULATION

To analyze the field emission of electrons in a planar diode, the following self-consistent approach is adopted.

### A. Field emission

The phenomenon of field emission from a cold metal can be described as a quantum mechanical tunneling of conduction electrons through the potential barrier at the surface of the metal. The basic field emission process is described by the Fowler–Nordheim equation[5–12]

$$J = \frac{AE_s^2}{\phi t(y)^2}\exp\left(\frac{-B\nu(y)\phi^{3/2}}{E_s}\right), \tag{1}$$

where $A$ and $B$ are the Fowler–Nordheim constants, and $\phi$ is the effective work function assumed to be a constant allowed dependence on material and surface roughness.[13,14,18] One should note that the "effective work function" here is different from the work function as usually defined. The effective work function can be affected by the local electric fields, i.e.,

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the Schottky effect.[13,14] The field enhancement due to the geometry such as cathode surface roughness can also be included in the effective work function approximation. The normal electric field at the cathode surface $E_s$ will be obtained from the Fowler–Nordheim equation to serve as a boundary condition for Poisson's equation. The functions $t(y)$ and $\nu(y)$ were introduced by Spindt *et al.*,[15] and approximated by

$$t(y)^2 = 1.1, \tag{2}$$

$$\nu(y) = 0.95 - y^2, \tag{3}$$

$$y = 3.79 \times 10^{-5} E_s^{1/2}/\phi, \tag{4}$$

where $y$ is the Schottky lowering of the effective work function barrier.

### B. Poisson's equation

To analyze the flow of electrons in a planar diode, we solve Poisson's equation coupled with the law of energy conservation. The steady-state Poisson's equation for the space-charge fields due to both the electrons and ions in the diode region is expressed as

$$\nabla^2 V(z) = -\frac{1}{\varepsilon_0}\left[\rho_e(z) + \rho_i(z)\right], \tag{5}$$

where $V(z)$ is the electrostatic potential, $\varepsilon_0$ is the permittivity of the free space, and the electron and ion charge densities are defined by

$$\rho_e(z) = -J_e/|\nu_e(z)|, \tag{6}$$

and

$$\rho_i(z) = J_i/|\nu_i(z)|, \tag{7}$$

respectively.[16,17] Here $\nu_e(z)$ and $\nu_i(z)$ are the electron and ion velocities, respectively, at axial coordinate $z$, and $J_e$ and $J_i$ are the electron and ion currents, respectively. The boundary conditions for Eq. (5) are given by

$$V(z=0) = 0, \quad V(z=d) = V_0. \tag{8}$$

Since the field-emission-limited condition is assumed for the electron at the cathode, we have an additional boundary condition given by

$$\left(\frac{dV}{dx}\right)_{z=0} = E_s. \tag{9}$$

To consider the law of energy conservation, the kinetic energy of electrons and ions can be related to the potentials

$$\tfrac{1}{2} m_e \nu_e^2(z) = eV(z), \tag{10}$$

$$\tfrac{1}{2} m_i \nu_i^2(z) = eZ\left[V_0 - V(z)\right], \tag{11}$$

where $m_e$ and $m_i$ are the electron and ion rest masses, respectively, and $Z$ is the ion charge state.

Combining Eqs. (5)–(7) yields

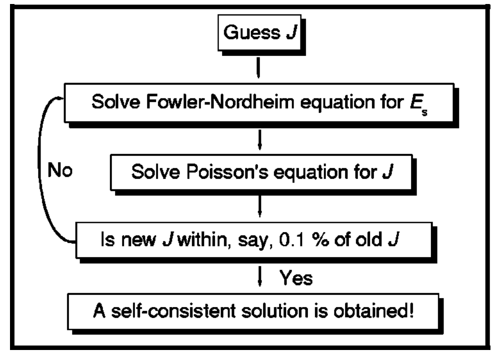


FIG. 1. Iterative solution of the Fowler–Nordheim equation and Poisson's equation.

$$\frac{d^2V}{dz^2} = \frac{J_e}{\varepsilon_0}\sqrt{\frac{m_e}{2e}}V^{-1/2} - \frac{J_i}{\varepsilon_0}\sqrt{\frac{m_i}{2eZ}}(V_0 - V)^{-1/2}. \tag{12}$$

This equation can be integrated to give

$$\frac{dV}{dz} = \left\{\frac{4J_e}{\varepsilon_0}\sqrt{\frac{m_e}{2e}}\left[V^{1/2} + q(V_0 - V)^{1/2} - qV_0^{1/2}\right] + E_s^2\right\}^{1/2}, \tag{13}$$

where the parameter $q$ is defined by

$$q = (J_i/J_e)(m_i/Zm_e)^{1/2}. \tag{14}$$

The constant of integration $E_s$ is the electric field at the cathode surface where $z=0$ and $V=0$. Integrating from cathode surface $z=0$ to anode surface $z=d$, Eq. (13) becomes

$$J_e = \frac{\varepsilon_0}{4d^2}\sqrt{\frac{2e}{m_e}}\left\{\int_0^{V_0}\left[V^{1/2} + q(V_0 - V)^{1/2} - qV_0^{1/2} + \frac{\varepsilon_0}{4J_e}\sqrt{\frac{2e}{m_e}}E_s^2\right]^{-1/2} dV\right\}^2, \tag{15}$$

where $V_0$ is the applied diode voltage.

In a typical high-current field-emission diode, the anode plasma is created from the ionization of the anode material by beam electrons. Thus the parameter $q$ increases from zero to a saturated value

$$q = q_s = 1 + \frac{\varepsilon_0}{4J_e}\sqrt{\frac{2e}{m_e}}\frac{E_s^2}{V_0^{1/2}}. \tag{16}$$

### C. Self-consistent approach

With an initial guess of the current density $J$, an initial approximation of the surface electric field $E_s$ can be determined from the Fowler–Nordheim equation. This $E_s$ then serves as a boundary condition for the Poisson's equation to solve for a better approximation of $J$. Thus, the Fowler–Nordheim equation and Poisson's equation, i.e., Eqs. (1) and (15), respectively, are solved iteratively until we arrive at a self-consistent solution,[18] as shown in Fig. 1.

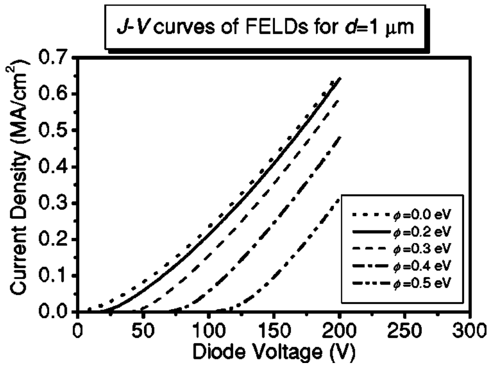


FIG. 2. $J$–$V$ curves (without ion effects) of the FELDs for $d=1$ $\mu$m and $\phi=(0, 0.2, 0.3, 0.4, 0.5)$ eV. The curve corresponding to $\phi=0$ eV represents space-charge limited.

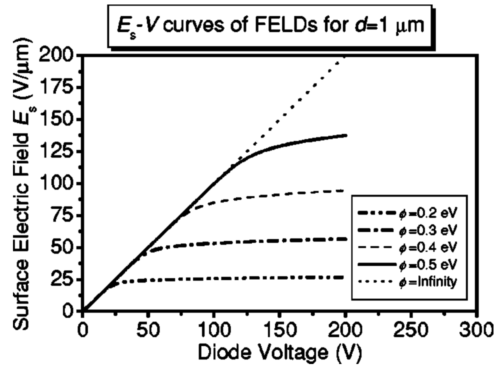


FIG. 3. $E_s$–$V$ curves (without ion effects) of the FELDs for $d=1$ $\mu$m and $\phi=(0.2, 0.3, 0.4, 0.5, \text{infinity})$ eV. The curve corresponding to $\phi$=infinity represents nonemission case.

## III. RESULTS AND DISCUSSIONS

In the first part, we consider the space-charge effects of the electrons only, i.e., ignore ion effects and simply set ion effect parameter $q=0$. In the second part, we include the ion effects by treating upstream ion current as a tuning parameter, i.e., set $q$ equal to certain numbers.

### A. Space-charge effects of electrons (without ion effects)

To consider the space-charge effects of electrons on the FELD, the current density has been calculated as a function of voltage via the self-consistent approach mentioned above by turning off the ion effects, i.e., set $q=0$. The value of the effective work function is usually much lower than that of the "true" work function. This might be due to the high electric fields applied and the local field enhancements of geometry, i.e., cathode surface roughness.[13,14,18] Not only for illustration but our own interests in the applications of the field emission cathodes with low effective work functions, the value considered here ranges from 0.2 to 0.5 eV. Figure 2 shows our calculated $J$–$V$ curves of the FELDs for gap length $d=1$ $\mu$m and effective work function $\phi=(0, 0.2, 0.3, 0.4, 0.5)$ eV. The $J$–$V$ curve corresponding to $\phi=0$ eV approaches the correct limit to be space-charge limited as following the Child–Langmuir law,[1] and the other curves are field-emission limited. There exist cut-off voltages in the $J$–$V$ curves of the field-emission-limited diodes. That is different from space-charge-limited diodes. The surface electric field plays an important role in the field-emission mechanism. Figure 3 shows the surface electric field as a function of voltage, $E_s$–$V$ curves, of the FELDs for $d=1$ $\mu$m and $\phi=(0.2, 0.3, 0.4, 0.5, \text{infinity})$ eV. The $E_s$–$V$ curve corresponding to $\phi$=infinity approaches the correct limit to be a nonemission case. This curve and the former one corresponding to $\phi=0$ eV in Fig. 2 give the correct limits to verify the validity of our theory and numerical calculations. For the other four cases $\phi=(0.2, 0.3, 0.4, 0.5)$ eV in Fig. 3, the surface electric fields of FELDs increase more slowly due to the space-charge effects when the diode voltages are higher than the cut-off voltages, roughly (25, 50, 75, 100) V, respectively. In much higher voltage regime, saturation of surface electric fields is quickly achieved. Due to the space-charge effects of the electrons, the electric field at the cathode surface can be reduced to one-tenth of the applied electric field. Figure 4 shows the $E_s$–$\phi$ curves of the FELDs for $V=200$ V and $d=(0.8, 1.0, 1.2)$ $\mu$m. The surface electric fields can be approximately assumed to be dependent on the effective work function only, independent of the gap length.

### B. Ion effects

The upstream ion current occurs when considerable electrons bombard on the anode. For illustration of the ion effects, the effective work function is selected as 0.2 eV, as higher electron currents can cause higher ion currents. The ion current included in the Poisson's equation has been treated as a tuning parameter. The $J_e$–$V$ and $E_s$–$V$ curves are plotted for variant ion current. Figure 5 shows our calculated $J_e$–$V$ curves of the FELD with gap length $d=1$ $\mu$m and effective work function $\phi=0.2$ eV for $q=(5, 10, 20, 30, 40, q_s)$. As one can see from Fig. 5, the emergence of an upstream ion current would enhance the field-emission-limiting current. The field-emission currents in the presence of saturated ion currents can be enhanced to be nearly 1.8 times of the case with no ion current. Figure 6 shows the corresponding $E_s$–$V$ curves of the FELD. The surface electric fields of

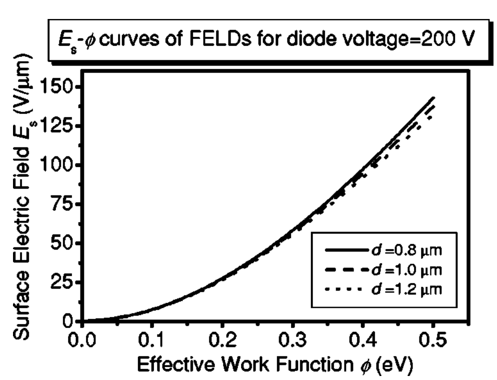


FIG. 4. Surface electric fields as functions of effective work function, $E_s$–$\phi$ curves, of the FELDs for $V=200$ V and $d=(0.8, 1.0, 1.2)$ $\mu$m.

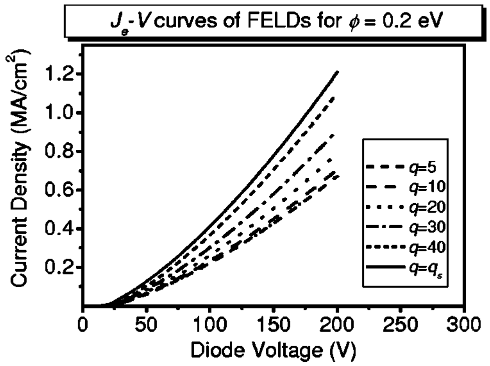


FIG. 5. Field emission current densities as functions of voltage, $J_e$–$V$ curves, of the FELDs for $d=1$ $\mu$m, $\phi=0.2$ eV, and $q=(5, 10, 20, 30, 40, q_s)$.

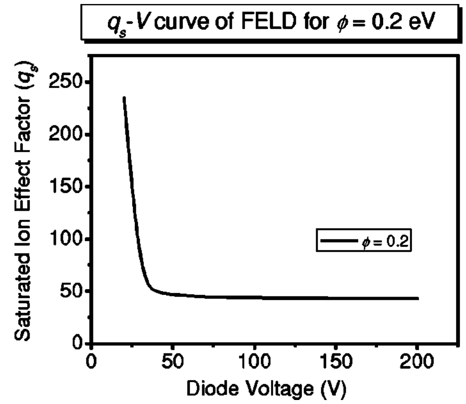


FIG. 7. Saturated ion effect factor as a function of voltage, $q_s$–$V$ curve, of the FELD for $d=1$ $\mu$m and $\phi=0.2$ eV.

the FELD get saturation in a higher voltage regime due to the space-charge effects of the electrons. The saturated surface electric fields seem unaffected by the ion effects. Figure 7 shows the $q_s$–$V$ curves of the FELD. The saturated ion effect parameter, the $q_s$ factor, is nearly constant except for the cases at low voltage limit.

## IV. CONCLUSION

We have investigated the steady states of the bipolar flows of field-emission-limited diodes via the self-consistent approach. One of the differences between FELDs and space-charge-limited diodes is that the former case exhibits a cutoff voltage. The other is that the surface electric fields of FELDs are not zero. The surface electric field is found to be reduced seriously due to the space-charge effects of electrons and is approximately determined by the effective work function. The influence of ion effects on field-emission-limited diodes has been investigated by tuning the ion effect parameter $q$. The emergence of an upstream ion current would enhance the field-emission-limiting current. The field-emission currents in the presence of saturated ion currents $q_s$ can be enhanced to be nearly 1.8 times of the case without ion current. However, the saturated surface electric fields seem unaffected by the ion effects.

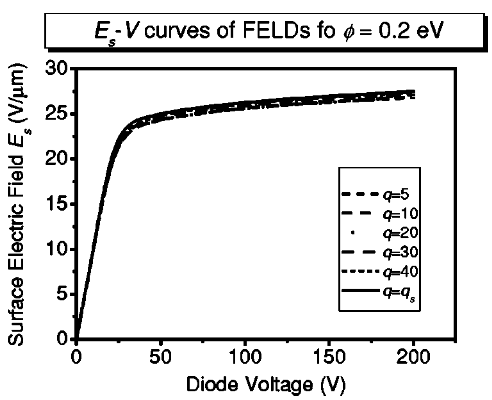


FIG. 6. Surface electric fields as functions of voltage, $E_s$–$V$ curves, of the FELDs for $d=1$ $\mu$m, $\phi=0.2$ eV, and $q=(5, 10, 20, 30, 40, q_s)$.

## ACKNOWLEDGMENT

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[1]C. D. Child, Phys. Rev. **32**, 492 (1911); I. Langmuir, *ibid.* **2**, 450 (1913).
[2]H. R. Jory and A. W. Trivelpiece, J. Appl. Phys. **40**, 3924 (1969).
[3]S. Dushman, Phys. Rev. **21**, 623 (1923).
[4]R. B. Miller, *An Introduction to the Physics of Intense Charged Particle Beams* (Plenum Press, New York, 1982).
[5]R. H. Fowler and L. W. Nordheim, Proc. R. Soc. London, Ser. A **119**, 173 (1929).
[6]L. W Nordheim, Proc. R. Soc. London, Ser. A **121**, 626 (1928).
[7]L. W. Nordheim, Z. Phys. **30**, 177 (1929).
[8]W. Schottky, Z. Phys. **14**, 63 (1923).
[9]N. H. Frank and L. A. Young, Phys. Rev. **38**, 80 (1931).
[10]E. Guth and C. J. Mullin, Phys. Rev. **61**, 339 (1942).
[11]D. V. Gogate and D. S. Kothari, Phys. Rev. **61**, 349 (1942).
[12]S. Gasiorowicz, *Quantum Physics*, 2nd edition (Wiley, New York, 1996).
[13]C. Kim and B. Kim, Phys. Rev. B **65**, 165418 (2002).
[14]J. A. Nation *et al.*, Proc. IEEE **87**, 865 (1999).
[15]C. A. Spindt, I. Brodie, L. Humphrey, and E. R. Westerberg, J. Appl. Phys. **47**, 5248 (1976).
[16]S. A. Goldstein and R. Lee, Phys. Rev. Lett. **35**, 1079 (1975).
[17]E. H. Choi, H. M. Shin, and D. I. Choi, J. Appl. Phys. **61**, 2160 (1987).
[18]M. C. Lin and D. S. Chuu, Appl. Phys. Lett. **80**, 4262 (2002).