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INTEGRATED PRODUCTION AND THE INVESTMENT–UNCERTAINTY RELATIONSHIP

YINGYI TSAI* AND CHING-TANG WU†

Abstract
This paper studies the role of production mode in determining the effects of an increase in uncertainty on the choice of investment outlay. In a continuous-time model of optimal capital investment with innovative *R&D* under demand uncertainty, we show that investments in both capital and innovative research decrease with an increase in uncertainty, and that such investments rise with the level of primary demand. Our result sheds light on the mode of production as a source of the negative investment–uncertainty relationship.
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1. INTRODUCTION

Over the past few decades, economists have directed considerable attention to the topic of optimal investment under uncertainty (Lucas 1967; Craine 1989). The existing theoretical studies have focused, almost exclusively, on the decisions on capital investment by a competitive firm in the presence of future demand uncertainty.[1] In fact, firms often undertake more than a single investment of capital. For example, producers in the technology sector worldwide invest in both new product invention and production capacity. Interestingly, even firms outside the technology sector, such as Mattel, a premier toy brand-producer based in the USA, conducts innovative research on the development of new product models, engaging in capital investment for the manufacture of such new commodities. It is therefore not unreasonable to suggest that investment in *R&D* by firms (whether they be competitive or imperfectly competitive) under uncertainty has, for the most part, been ignored in the literature on optimal (capital) investment under

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* Assistant Professor, Department of Applied Economics, National University of Kaohsiung, Taiwan. Corresponding author: 700 Kaohsiung University Road, Nan-Tze 811, Kaohsiung, Taiwan. Tel: +886-7-591-9189; Fax: +886-7-216-9365; E-mail address: yytsai@nuk.edu.tw
† Associate Professor, Department of Applied Mathematics, National Chiao Tung University, ShinChu, Taiwan. Tel: +886-3-513-1220; Fax: +886-3-572-4679; E-mail address: ctwu@math.nctu.edu.tw
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[1] See, for instance, Hartman (1972), Abel (1983) and Pindyck (1993). The seminal work of McDonald and Siegel (1986) underlies the concept of "option value" of investment under uncertainty, which was later further developed by Dixit and Pindyck (1994) in investigations for the effects of uncertainty on capital investment.

uncertainty. The omission of innovative aspects of investment has resulted in unsatisfactory explanations for the development experiences noted in some Asian countries. On the one hand, do the existing studies have any theoretical bearing on the fact that dedicated technological firms (which are mainly segment-specific and specialised) in South Korea, Taiwan, Singapore and India have become indispensable partners to major technology brand-producers in North America, Japan and Europe within the context of the global value chain? On the other hand, do studies take into account the fact that these dedicated firms face rapid technological progress and compete fiercely with each other in the volatile market of semiconductor, electronics, microcomputer and software? Moreover, why do the majority of the emerging economies continue to cater primarily for the assembly of end products, while some Asian countries are gradually becoming the hubs of research and innovation (Henderson 1994; Gereffi 1995; Macher *et al.*, 2007)?

This paper explores the issue of optimal investment under uncertainty in a modern world economy that is represented by a vertically disintegrated production system relying heavily on investments in both *R&D* and capacity (Hummels *et al.*, 1998; Brown and Linden 2005; Lin and Tsai 2007; Macher *et al.*, 2007). We develop a continuous-time framework of product innovation with capital investment under uncertainty. We consider the choices on innovative *R&D* outlay and capital expenditure of a risk-neutral brand-producer facing an uncertain future demand. Our results show that, all things being equal, a brand-producer raises both the *R&D* outlay and capital spending if the scale of primary market demand is sufficiently high. The rationale for this is as follows: A high level of primary demand corresponds to a brand's large market size. For firms adopting an integrated production mode and utilising in-house facilities for both innovative product design (or development) and end-product manufacture, it is not surprising that the brand-producer raises investments in both *R&D* and in capacity with a greater primary demand, given that innovative *R&D* could contribute positively to the development of a new model and, ultimately, the price.

As far as the interaction between projects competing for investment funding is concerned, it is generally accepted that marginal revenue product per unit input price determines the allocation for investment projects when there are financial constraints. Greater expenditure would therefore be allocated to *R&D* should the marginal revenue product of knowledge stock exceed that of capital stock. For the present purpose of theoretical analysis, we have formalised the Schumpeterian view that large corporations (or the established brand-producer) tend to engages in both *R&D* for new prototype development and capacity installation for commodity manufacture. The model specifically shows that a brand-producer adopting the integrated production mode reduces both investments in both *R&D* and capital if there is an increase in uncertainty. In other words, the integrated production brand-producer's incentive for *R&D* and capital investments is weaker than that of the segment-specific (or specialised) firm with a single investment choice as the degree of uncertainty increases.

An important contribution of this paper is to show that production mode (be it the integrated production or segment-specific specialised) has a decisive role in determining the effects of increased uncertainty on investment expenditure. If the integrated production mode is adopted, meaning that actual production involves more investment projects than mere installation of capacity, then the positive investment–uncertainty relationship, widely documented in the literature, need not hold even in the the presence of convexity effects. (This means that the expected marginal profit from additional investments increases as future demand becomes more uncertain.)

An important implication of the results is that certain policy measures, such as *R&D* subsidy or tax credits for innovation, may, in the end, discourage *R&D* investment by some specialised product design houses. At the same time, theses policies could promote investments by certain integrated production firms, particularly when sales revenue serves as the main criterion for evaluation. Indeed, subsidisation of *R&D* activities lowers the cost of research activities and tends to distort the trade-off between innovative research and capacity investments. For example, capital expenditure by modern corporations often involves both spending on both *R&D* and capacity expansion.[2] It is thus possible for integrated production firms to channel the *R&D* subsidy to various unrelated activities (such as capacity expansion) in the attempt to allow for large-scale production of state-of-the-art products. Taking into account the actual effects of government *R&D* subsidisation programs, the fact that *R&D* investments in some integrated production firms have increased (in response to the implementation of such programs) does not necessarily suggest that these programs promote *R&D* only for the sake of cutting-edge technology (or the latest product model). Moral hazards underlying the possible misappropriation of *R&D* subsidy or scaling down of *R&D* projects for the cutting-edge technologies should be taken into account while assessing the effectiveness of certain *R&D* subsidisation policies.

While there have been other attempts to determine the signs of the investment–uncertainty relationship, and many important insights have resulted, this paper is more closely related to studies by Caballero (1991), even though it differs in a number of ways. Firstly, Caballero sees the decrease in marginal return to capital in imperfect competition as a response to changes in uncertainty, whereas we attribute the return (to capital and knowledge) to choices of production mode by a brand-producer under uncertainty. Secondly, while Caballero considers the single input of labour in imperfect competition, we consider multiple inputs of labour, capital and knowledge to be variables. This is supported by other studies. For example, Brown and Linden (2005) suggest that the employment of production factors in the semiconductor industry illustrating multiple inputs under imperfect competition actually provides a better representation of reality. Thirdly, we consider investments for the accumulation of both capital and knowledge stocks subsequent to the choices of the integrated production mode within the firm's facilities, in contrast to the single investment of capital cited in previous studies by Abel (1983), Pindyck (1982, 1993), and Abel and Eberly (1997).

The remainder of the paper is organised as follows. Section 2 develops a continuous-time model of capital investment with innovative *R&D* under uncertainty. Section 3 discusses the effects of uncertainty on optimal investments of capital and *R&D*. Section 4 discusses the results, and section 5 concludes.

[2] Information of annual capex (shorthand for capital expenditure, consisting of "capacity building/expansion" expenditure and "new invention" research outlay) is treated as confidential materials, at least, in the technology sector. It is, therefore, difficult to identify the exact amount for each spending of different nature. Nevertheless, industry analysts decipher for the estimates by investigating consecutive financial statements, surveying the costs of fixed-asset investment within the industry and making comparisons among firms.

2. THE MODEL

2.1 Market Demand

In the basic model, we assume that the firm faces an inverse demand function of

$$p_t = p(X_t, A_t, Y_t) = X_t A_t^{\gamma} Y_t^{(1-\varepsilon)/\varepsilon},^{3} \quad (1)$$

where the parameters $\gamma\,(0 \leq \gamma < 1)$ and $\varepsilon\,(\varepsilon \geq 1)$ denote the price elasticity of product-specific features A_t, and output, Y_t, respectively. The variable X_t represents an exogenous demand shifter[4] that evolves according to a geometric Brownian motion $dX_t/X_t = \sigma dZ_t$, *i.e.* X_t follows a log-normal distribution of the form of $X_t = x\exp(\sigma Z_t - \sigma^2 t/2)$ with $X_0 = x$ as an arbitrary positive number characterising the initial value of X, the level of primary market demand, σ a constant volatility, and Z follows a standard Brownian motion.

Equation (1) captures the counteracting impacts of quality and quantity on market demand. In fact, a positive effect on the price of quality owing to "product innovation"[5] is evident if we interpret A as knowledge stock accumulated through *R&D* investment, R, that is,

$$dA_t = (R_t - \lambda A_t)\,dt, \quad (2)$$

where λ is the destruction rate for knowledge stock. This setting takes quality to be the outcome of accumulated innovative activities, because high-quality products generates greater profits than do low-quality goods (see, *inter alia*, Gabszewicz and Thisse, 1979; Donnenfeld and Weber, 1995).

Moreover, the "quantitative" aspect capturing the segment of output manufacture suggests that actual production of final output, Y_t, follows a technology of $Y_t = v_t^{\alpha} L_t^{1-\alpha}$, where $\alpha \in (0, 1)$ denotes the share of variable input, v_t, and K_t is the stock of capital accumulated by fixed-asset (or capacity) investment, *i.e.*

$$dK_t = (I_t - \delta K_t)\,dt, \quad (3)$$

with δ denoting the depreciation rate of capital stock.

It is important to note that we employ a Cobb-Douglas technology to capture, in a straightforward manner, the "manufacturing" segment of final goods production, given the brand-producing firm's innovative research for new invention. This representation allows for the study of the impacts on the end-product price of both "quality" and "quantity". However, it must be noted that a direct incorporation into the production technology of quality resulting from *R&D*, as implied by equation (2) not only

[3] A deterministic version of market demand without product quality can be found in Singh and Vives (1984), who derive such demand from solving the consumer's utility maximisation problem. In a similar characterisation of stochastic market demand without research-induced innovation, Tsai and Wu (2006) study the choices of production mode by a brand-producer and its impact on investment–uncertainty relationship.

[4] X consists of variations in consumer taste, changes in technology, and even a changing market environment.

[5] See Levhari and Peles (1973) for a justification of this formulation.

de-generates the insights that could be otherwise result from the separation of quality and quantity but also leads to intractable complexity in the analysis.[6]

2.2 The Value of the Brand-Producing Firm

The brand-producing firm employs the variable input at a fixed cost, ω, the adjustment cost of I_t and R_t is given by $c_I(I_t)$ and $c_R(R_t)$, respectively, where $I_t \geq 0$, $R_t \geq 0$, c_I, and c_R are strictly increasing, continuous functions with $c_I(0) = c_R(0) = 0$. We assume that the firm operates in complete markets, discounts expected future cash flows at a constant rate $r > 0$, and maximises the expected present value of its cash flow. The value of the firm at time t is, therefore, given by

$$V(X_t, K_t, A_t) = \max_{I_s, R_s, v_s} E_t\left[\int_t^\infty [p_s Y_s - \omega v_s - c_I(I_s) - c_R(R_s)] e^{-r(s-t)} ds\right], \tag{4}$$

subject to the constraints stated in equations (2) and (3).

Clearly, the value function in equation (4) satisfies the following Bellman equation:

$$rV(X_t, K_t, A_t)dt = \max_{I_s, R_s, v_s} \{p_s Y_s - \omega v_s - c_I(I_s) - c_R(R_s)\} dt + E_t[dV]. \tag{5}$$

An interpretation of equation (5) suggests that, given a mean rate of return, r, the expected return equals the required mean return. Alternatively, the left-hand side of (5) is the total required return with a mean rate of r over the time interval, while the right-hand side of (5) is the firm's total expected return, consisting of the instantaneous net cash flow, and the expected gain (or loss).

Further, using Ito's lemma and equations (2) and (3), we obtain the value of the firm, V_t, as a function of three state variables, X_t, K_t, and A_t, as

$$dV(X_t, K_t, A_t) = \sigma X_t dZ_t + V_A(R_t - \lambda A_t)dt + V_K(I_t - \delta K_t)dt + \frac{\sigma^2 X_t^2}{2} V_{XX} dt.$$

It follows that equation (5) can now be written as

$$\begin{aligned} rV(X_t, K_t, A_t)dt = \max_{v_t}\{p_t Y_t - \omega v_t\} + \max_{I_t, R_t}\{(V_A R_t - c_R(R_t)) + (V_K I_t - c_I(I_t))\} - \\ \lambda A_t V_A - \delta K_t V_K + \frac{\sigma^2 X_t^2}{2} V_{XX}. \end{aligned} \tag{6}$$

It is evident that $v_t^* = (\alpha/\omega\varepsilon)^\eta X_t^\eta K_t^\mu A_t^\nu > 0$, implying a non-negative optimal use of the variable input, provides the solution to the first part of equation (6), where $\eta \equiv \varepsilon/(\varepsilon - \alpha) > 1$, $\mu \equiv (1 - \alpha)/(\varepsilon - \alpha)$ and $\nu \equiv \gamma\eta = \gamma\varepsilon/(\varepsilon - \alpha)$. Further, the second and the third terms on the right-hand side of equation (6) suggest that optimal research investment and capital investment must satisfy $I_t^* = V_K/c_I'$ and $R_t^* = V_A/c_R'$, respectively. It follows that optimal capital investment, I_t^*, rises with the marginal revenue product of capital, V_K. Similarly, this positive relationship holds for optimal research investment, R_t^*, and the marginal revenue product of knowledge, V_A.

[6] The authors thank the editor and the anonymous referee for highlighting the need for clarification on the "quality"-"quantity" separation treatment.

To complete our analysis for the characterisation of solutions to the brand-producer's problem, as implied by equation (6), section 3 explores the nature of adjustment costs and their impact on optimal investment by investigating both forms of quadratic and linear adjustment costs.

3. ADJUSTMENT COSTS AND OPTIMAL INVESTMENT

3.1 Quadratic Adjustment Costs

We first consider the case of the quadratic adjustment costs of I_t and R_t. Let $c_I(I_t) = \rho I_t^2$, and $c_R(R_t) = \theta R_t^2$, where ρ and θ are factor prices. The solution to the second and the third terms on the right-hand side of equation (6) is given by $I_t^* = V_K/2\rho$ and $R_t^* = V_A/2\theta$, respectively.

Substituting the optimal capital investment and research investment into equation (6), we have

$$rV(X_t, K_t, A_t) = E_1 X_t^{\eta} K_t^{\mu} A_t^{\nu} - \lambda A_t V_A - \delta K_t V_K + \frac{\sigma^2 X_t^2}{2} V_{XX} + \frac{V_A^2}{4\theta} + \frac{V_K^2}{4\rho},$$

where $E_1 \equiv (1 - \alpha/\varepsilon)(\alpha/\omega\varepsilon)^{\frac{\alpha}{(\varepsilon-\alpha)}}$.

It follows immediately that the value function is

$$V(X_t, K_t, A_t) = E_2 X_t^{\eta} K_t^{\mu} A_t^{\nu} + \Phi(X_t, K_t, A_t),$$

where the solution satisfying the partial differential equation Φ is given by

$$\begin{aligned} r\Phi(X_t, K_t, A_t) = {} & -\lambda A_t \Phi_A - \delta K_t \Phi_K + \frac{\sigma^2 X_t^2}{2} \Phi_{XX} + \frac{1}{4\theta}(E_2 \nu X_t^{\eta} K_t^{\mu} A_t^{\nu-1} + \Phi_A)^2 + \\ & \frac{1}{4\rho}(E_2 \mu X_t^{\eta} K_t^{\mu-1} A_t^{\nu} + \Phi_K)^2 \end{aligned} \tag{7}$$

It should be noted, as in equation (7), that Φ is a non-linear partial differential equation and cannot be solved analytically for a closed-form solution (see, Pindyck, 1982; Abel, 1983; Caballero, 1991; Tsai and Wu, 2005).[7]

For the expositional purpose of investigating the complete solutions to equation (6), sub-section 3.2 considers a linear form of adjustment cost as an alternative specification.

3.2 Linear Adjustment Costs

We now consider a scenario in which the adjustment costs are given by $c_I(I_t) = \rho I_t$, and $c_R(R_t) = \theta R_t$. An important interpretation of this formulation can be provided as follows. In the presence of volatile market demand, an incumbent brand-producing firm deals constantly with the issues of capacity installation, knowledge accumulation, and even professional workforce training. It thus becomes more costly to increase the stocks of usable capital and knowledge slowly rather than quickly. Alternatively, a sufficiently brief adjustment period drives the firm to allocate resources that constrain capacity, knowledge

[7] To the best of our knowledge, notable exceptions are Abel (1983) and Abel and Eberly (1997).

and labour. It makes sense, therefore, to describe the brand-producing firm's problem as one with non-negative variable input, $v_t \geq 0$, and investments with financial constraints, *i.e.* $I_t \geq 0$, $R_t \geq 0$ and $I_t + R_t \leq M$. With these assumptions, solutions to the second and the third parts of equation (6), implied by optimal capital investment, I_t^*, and optimal research investment, R_t^*, are now given as

$$I_t^* = \begin{cases} M, & \textit{if } V_K - c_I'(I_t) \geq \max\{0, V_A - c_R'(R_t)\} \\ 0, & \textit{otherwise} \end{cases}, \text{ and} \tag{8}$$

$$I_t^* = \begin{cases} M, & \textit{if } V_A - c_R'(R_t) \geq \max\{0, V_K - c_I'(R_t)\} \\ 0, & \textit{otherwise} \end{cases}, \tag{9}$$

respectively.[8]

Equations (8) and (9) suggest that the properties of adjustment costs of capital stock and knowledge stock have decisive impacts on the optimal choices of investment.

Inserting into equation (6) the results contained in (8) and (9), we obtain

$$\begin{aligned} rV(X_t, K_t, A_t) = {} & E_1 X_t^{\eta} K_t^{\mu} A_t^{\nu} - \lambda A_t V_A - \delta K_t V_K + \frac{\sigma^2 X_t^2}{2} V_{XX} + \\ & (V_A - c_R'(R_t) \mathrm{I}_{\{V_A - c_R'(R_t) \geq \max\{0, V_K - c_I'(I_t)\}\}}) + \\ & M(V_K - c_I'(I_t) \mathrm{I}_{\{V_K - c_I'(I_t) \geq \max\{0, V_A - c_R'(R_t)\}\}}) \end{aligned}, \tag{10}$$

where $E_1 \equiv (1 - \alpha/\varepsilon)(\alpha/\omega\varepsilon)^{\frac{\alpha}{(\varepsilon-\alpha)}}$.

As in equation (7), closed-form solutions cannot be solved using equation (10). Nevertheless, our treatment of linear adjustment cost allows for a closed-form solution to equation (10). In fact, using $c_I(I_t) = \rho I_t$, and $c_R(R_t) = \theta R_t$, it is a straightforward matter to show that the solution to equation (10) now takes the following form

$$V(X_t, K_t, A_t) = E_2 X_t^{\eta} K_t^{\mu} A_t^{\nu} + \Phi(X_t, K_t, A_t) \mathrm{I}_{\{X_t > \min\{\phi^K(K_t, A_t), \phi^A(K_t, A_t)\}\}}, \tag{11}$$

where $E_2 \equiv \dfrac{2(\varepsilon - \alpha)^3}{2r\varepsilon(\varepsilon-\alpha)^2 + 2\varepsilon(1-\alpha)\delta + \lambda\gamma\varepsilon(\varepsilon-\alpha) - \alpha\varepsilon^2\sigma^2} \left(\dfrac{\alpha}{\omega_t \varepsilon}\right)^{\frac{\alpha}{(\varepsilon-\alpha)}}$,

$$\phi^K(K_t, A_t) = \left(\frac{\rho}{E_2} \frac{\varepsilon - \alpha}{1 - \alpha}\right)^{\frac{\varepsilon-\alpha}{\varepsilon}} K_t^{(1-\varepsilon)/\varepsilon} A_t^{\gamma},$$

$$\phi^A(K_t, A_t) = \left(\frac{\theta}{E_2} \frac{\varepsilon - \alpha}{\varepsilon\gamma}\right)^{\frac{\varepsilon-\alpha}{\varepsilon}} K_t^{(1-\alpha)/\varepsilon} A_t^{\gamma-(\varepsilon-\alpha)/\varepsilon},$$

[8] The accumulation process of K_t and A_t is given by $K_t = e^{-\delta t}\left(K_0 + M\int_0^t e^{\delta_s} \mathrm{I}_{\{V_K - c_I'(I_s) \geq \max\{0, V_A - c_R'(R_s)\}\}} ds\right)$ and $A_t = e^{-\delta t}\left(A_0 + M\int_0^t e^{\delta_s} \mathrm{I}_{\{V_A - c_R'(R_s) \geq \max\{0, V_K - c_I'(I_s)\}\}} ds\right)$, respectively.

and the function Φ satisfies the following conditions:

(1) Over the set $\{\varpi \in B^c : \phi^A(K_t, A_t) \le \phi^K(K_t, A_t)\}$, $\Phi(X_t, K_t, A_t)$ satisfies

$$r\Phi(X_t, K_t, A_t) = -\lambda A_t \Phi_A - \delta K_t \Phi_K + \frac{\sigma^2 X_t^2}{2}\Phi_{XX} + M\left(\hat{V}_K + \Phi_K - c_I'(I_t)\right).$$

(2) Over the set $\{\varpi \in B^c : \phi^K(K_t, A_t) \le \phi^A(K_t, A_t)\}$, $\Phi(X_t, K_t, A_t)$ satisfies

$$r\Phi(X_t, K_t, A_t) = -\lambda A_t \Phi_A - \delta K_t \Phi_K + \frac{\sigma^2 X_t^2}{2}\Phi_{XX} + M\left(\hat{V}_A + \Phi_A - c_R'(R_t)\right).$$

(3) The initial condition $\Phi(\min\{\phi^K(K_t, A_t), \phi^A(K_t, A)\}, K_t, A_t) = 0$.

The value of the firm is now a function of capital stock and knowledge stock, as shown in equation (11), in contrast to the capital-dependent-only result noted by Caballero (1991), Pindyck (1993), and Lee and Shin (2000). A close examination of the first term $E_2 X_t^\eta K_t^\mu A_t^\nu$, which equals to the present value of expected marginal revenue products of capital and knowledge, suggests a maximum instantaneous cash flow, sustains when *(i)* $\varepsilon > 1$ (equivalent to $\mu < 1$). This implies a market structure of imperfect competition (a decreasing marginal return to capital); *(ii)* $\nu < 1$ (*i.e.* $\alpha < \varepsilon(1 - \gamma)$, meaning a decreasing marginal return to knowledge (due to linearly homogeneous production technology in the manufacturing segment), and *(iii)* $\gamma < 1 - 1/\varepsilon$, the output elasticity of demand outweighs that of product-specific features. The impact of the changes in new product on the market price is thus less significant than the changes in total quantity. Alternatively, consumers are more attracted to the changes in new commodity features than to the changes in supply.

Further, an examination of the second term in equation (11) (*i.e.* $\Phi(X_t, K_t, A_t)\mathrm{I}_{\{X_t > \min[\phi^K(K_t, A_t), \phi^A(K_t, A)]\}}$), which is the expected gain (or loss) from investments in capital and knowledge stock, shows that the marginal valuation of capital stock relative to that of knowledge stock determines the optimal rate of investment for capital and *R&D*.

3.3 The Effect of Uncertainty on Investments

It is important to note that our treatment of linear adjustment costs allows for a closed-form solution to equation (10), while realising as a general approximation to reality that both capital and knowledge stocks as value to the firm serve as a major source of revenue particularly when both product novelty and manufacturing capacity matter immensely in sectors like electronics and semiconductors. Given the results obtained in 3.2, we are now in a position to examine the effect of uncertainty on investment.

Optimal investments in both capital and *R&D* are now a function of $E_2 X^\eta$. Given $\Phi(X_t, K_t, A_t)\mathrm{I}_{\{X_t > \min[\phi^K(K_t, A_t), \phi^A(K_t, A)]\}}$, we can study the effect of uncertainty on investments by investigating its effect of uncertainty on the range of $\{X_t > \min[\phi^K(K_t, A_t), \phi^A(K_t, A_t)]\}$, that is, $\{\varpi : V_A < c_R'(R_t) \textit{ and } V_K < c_I'(I_t)\}$. The effect of the variance of X_t on the optimal investments is difficult to examine when the variance of X_t is given by $x^2(\exp(\sigma^2 t) - 1)$. We therefore investigate the impact on optimal investments of the volatility σ, as X_t is log-normally distributed and the variance of ln X_t is therefore given by $\sigma^2 t$, implying that the value of σ can be explained as the fluctuation of $\ln X_t$. Moreover, using equations (8) and (9), it is clear that the firm may either invest a maximal amount of M in capacity and knowledge stock outlets or not undertake any investment at all. To facilitate our understanding of the firm's investment behaviour, we are first going to examine the conditions under which no investment is made. The focus is therefore on the relations between the volatility parameter σ and the interval between which both the marginal products of capital and knowledge fall below the input prices, *i.e.* $\{\varpi : V_A < c'(R_t) \text{ and } V_K < c'(I_t)\}$. Using the process of market demand, $dX_t/X_t = \sigma dZ_t$, and equation (11), we have

$$\{\varpi : V_A < c_R'(R_t) \text{ and } V_K < c_I'(I_t)\} = \{X_t < \min\{\phi^K(K_t, A_t), \phi^A(K_t, A_t)\}\}$$
$$= \left\{Z_t < \frac{1}{2}\sigma t + \frac{1}{\sigma}\ln\left(\frac{1}{x}\min\{\phi^K(K_t, A_t), \phi^A(K_t, A_t)\}\right)\right\}$$

It follows that (i) the set $\{\varpi : V_A < c_R'(R_t) \text{ and } V_K < c_I'(I_t)\}$ becomes smaller as X_t increases; and that (ii) given t and X_t, an increase in uncertainty, as measured by σ^2, leads to a decrease in the optimal rate of investment. The set $\{\varpi : V_A < c_R'(R_t) \text{ and } V_K < c_I'(I_t)\}$ becomes larger only if the volatility parameter σ satisfies $\sigma^2 > \frac{2}{t}\ln\left(\frac{1}{x}\min[\phi^K(K_t, A_t), \phi^A(K_t, A_t)]\right)$.

For a sufficiently large size of market demand (as represented by a greater value of X_t), the brand-producing firm raises its investments in the accumulation of stocks of capital and knowledge. As the stocks grow over time, the firm reduces its investment in both capital and research when there is an increase in market volatility (as captured by the parameter σ). A rise of σ expands the range within which the marginal value of capital and knowledge falls below the direct cost, *i.e.* a greater range of $B = \{\varpi : V_A < c_R'(R_t) \text{ and } V_K < c_I'(I_t)\}$.

4. DISCUSSION

Although the representation of capital and knowledge stock in terms of value for the firm is general enough to approximate the actual revenue source in a fairly wide range of practical situations and yet be simple and specific enough to allow exploration of some behavioural implications from the mathematical analysis, the results obtained so far are based upon the specification of linear adjustment costs under stochastic market demand. An important interpretation of this formulation implies, under volatile market demand and rapid technological progress, that an established brand-producer treats as variable inputs of both *R&D* outlay and capital spending. In fact, the practices of merger and acquisition and buyout can justify such representation, even though it may not be true in some of the problems facing the start-ups. To what extent can the results be extended to more general settings in which multiple investments are undertaken with alternative distribution of the random variable, *e.g.* the mean-preserving spread?

The first result, which states that a brand-producer raises both investments in both capital and *R&D* with the primary demand, is based upon the following rationale. For a brand with a sufficiently high primary market demand, the brand-producer may be interpreted as a well-established incumbent in the market and tends to produce a larger quantity. This in turn implies that the benefit of innovative *R&D* for new inventions is greater if the in-house production facilities are large enough to meet the market demand, as the firm can reap the benefit of new invention through economies of scale resulting from mass-production.

The intuition behind the second result holds in a more general sense: a brand-producer who can mitigate problems associated with competing investments and at the same time exploit the benefits arising from economies of scale in production invests less in both *R&D* and capacity than does the segment-specific specialised producer who engages only in *R&D* or capital, given an identical level of volatility. Lastly, assuming that there are no market imperfections, the instantaneous rental of capital, labour hire, and even procurement of technical know-how (through merger and acquisition) are possible. In other words, we envision an environment in which the time period over which the adjustment cost is incurred is so minute that the "positive" adjustment cost facing a brand-producing firm changes instantaneously. This is equivalent to the situation in which "zero" adjustment cost (subsequent to the initial investment) is incurred if the real-time length of each period is sufficiently short, and is in direct contrast to the general characterisation of a discrete-time model in which a firm incurs a quadratic adjustment cost of cI^b, where $b > 1$, in each period subsequent to the initial investment. Hence, the distinction between "positive adjustment costs" and "zero adjustment cost" is therefore one of degree not of nature. Positive adjustment costs are non-negative only when there is a long enough time interval between the initial primary investment and the subsequent investments.

5. CONCLUSIONS

We have analysed the investment decisions on capital and innovative *R&D* by a brand-producer adopting an integrated production mode. It has been demonstrated that there is a negative investment–uncertainty relationship when the firm conducts simultaneous investments in both capital and knowledge, and yet the capital investment and *R&D* investment both increase with an increase in the level of primary demand.

Overall, the results of this paper provide important insights into the debate on whether or not to support the development of certain high-tech industries, if indeed any are to be supported as deserving more active assistance from the public sector than others. In fact, if we approximate the mode of production as representative of the degree of comparative advantage underlying the brand-producer's production (*e.g.* Lin and Tsai, 2004), then our results suggest that interactions among the nature of investment and the subsequent adjustment costs (as a reflection of the destruction rate of the existing knowledge stock), and industry-specific characteristics ought to be duly understood before the public sector can commit itself to the development of certain high-tech industries. These would include semiconductor and information technology, when, *inter alia*, the participation of public funds in *R&D* seems to be inevitable and positive in the context of "non-globalization of innovation" (Macher *et al.*, 2007:9-10).

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