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On the multivariate EGARCH model

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On the multivariate EGARCH model

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In this aticle, the extension of Nelson's (1991) univariate EGARCH model to the multivariate version has been reexamined and compared with the existing one given by Koutmos and Booth (1995). The magnitude and sign of standardized innovations have been constrained in Koutmos and Booth's multivariate EGARCH model, but not in the actual multivariate EGARCH model. The constraints imposed on Koutmos and Booth's EGARCH model may lead to inaccurate parameter estimates. Since the actual multivariate EGARCH model obtained is more general, and can produce more accurate inferential results, we suggest that the actual multivariate EGARCH model be used in future financial empirical studies.

I. Introduction

In the age of globalization, the transmission of price and volatility spillover across international financial markets is an issue of great interest for investors. Nelson (1991) proposed the exponential GARCH (EGARCH) model in an attempt to capture the asymmetric impact of innovations on volatility, based on which many empirical studies have appeared (see, for example, Balaban and Bayar, 2006; Kwek and Koay, 2006; Al-Zoubi and Al-Zu'bi, 2007; Bali and Theodossiou, 2007). Koutmos and Booth (1995) gave a multivariate extension of Nelson's (1991) univariate EGARCH model to facilitate a simultaneous investigation of the asymmetric impact of good news and bad news on the volatility transmission across markets, and found asymmetric volatility spillovers across the New York, Tokyo and London stock markets. With the model, many studies have provided empirical evidence for significant asymmetric volatility transmissions across markets (e.g. Koutmos, 1996; Kanas, 1998; Tse, 1999; Bhar, 2001; Kim et al., 2001; Yang and Doong, 2004; Batten and In, 2006). However, Koutmos and Booth's multivariate EGARCH model was not a direct extension of

Nelson's univariate model because there exist constraints in the function of magnitude and sign of innovation. In this study, the extension of Nelson's univariate EGARCH model to the multivariate version is reexamined and the actual multivariate EGARCH model is obtained. The actual model is more general than Koutmos and Booth's model, and can produce more accurate inferential results.

II. Actual Multivariate EGARCH Model

Univarite EGARCH model

Nelson (1991) proposed a univariate EGARCH model to interpret asymmetric effects between positive and negative asset return innovations. The univariate EGARCH (m, n) model can be represented as

$$\ln(\sigma_t^2) = \alpha_0 + \frac{1 + \beta_1 B + \cdots + \beta_n B^n}{1 - \alpha_1 B - \cdots - \alpha_m B^m} g(\varepsilon_{t-1}),$$
$$a_t = \sigma_t \varepsilon_t, \tag{1}$$

$$g(\varepsilon_t) = \theta \varepsilon_t + \gamma[|\varepsilon_t| - E(\varepsilon_t)], \tag{2}$$

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where a_t is referred to as the shock (or innovation) of an asset return at time t, ε_t is the standardized shock and σ_t is the square root of the volatility σ_t^2, α_0 is a constant, B is the back-shift (or lag) operator such that $B^i g(\varepsilon_t) = g(\varepsilon_{t-i})$, $1 - \sum_{i=1}^{m} \alpha_i B^i$ and $1 + \sum_{i=1}^{n} \beta_i B^i$ are polynomials with zeros outside the circle and have no common factors. $g(\varepsilon_t)$ is a linear combination of ε_t and $|\varepsilon_t| - E(\varepsilon_t)$ with coefficients θ and γ. The term in the bracket measures the magnitude effects and coefficient γ relates lagged standardized innovations to volatility in a symmetric way. The term $\theta\varepsilon_t$ measures the sign effects and coefficient θ relates standardized shocks to volatility in an asymmetric style. $\{g(\varepsilon_t)\}_{t=-\infty,\infty}$ is an *i.i.d.* random sequence with mean zero.

Actual multivariate EGARCH model

The multivariate time series of $\{\mathbf{r}_t\}$ can be expressed as (Tasy, 2005, Chap 10)

$$\mathbf{r}_t = \boldsymbol{\mu}_t + \boldsymbol{a}_t, \tag{3}$$

where $\boldsymbol{\mu}_t = \boldsymbol{E}(\mathbf{r}_t|\boldsymbol{\Omega}_{t-1})$ is the conditional expectation of $\mathbf{r}_t$ given $\boldsymbol{\Omega}_{t-1}$ (the past information set), and $\boldsymbol{a}_t$ is the shock of the series at time t. Process $\mathbf{r}_t$ is assumed to be a multivariate time series such as a vector autoregressive moving average (VARMA) model (e.g. Reinsel, 1993, Chap 1) with conditional expectation $\boldsymbol{\mu}_t$. The $\boldsymbol{\mu}_t$ is presented as:

$$\boldsymbol{\mu}_t = \boldsymbol{\phi}_0 + \sum_{i=1}^{p} \boldsymbol{\Phi}_i \mathbf{r}_{t-i} - \sum_{j=1}^{q} \boldsymbol{\Theta}_j \boldsymbol{a}_{t-j}, \tag{4}$$

where p and q are nonnegative integers, $\boldsymbol{\phi}_0$ is a k-dimensional vector of intercepts, and $\boldsymbol{\Phi}_i$ and $\boldsymbol{\Theta}_j$ are the $k \times k$ matrices of constant parameters. Equation 4 is referred to as the mean equation of $\mathbf{r}_t$. The conditional covariance matrix of $\boldsymbol{a}_t$ in Equation 3 given $\boldsymbol{\Omega}_{t-1}$ is a $k \times k$ positive-definitive matrix $\mathbf{H}_t$ denoted by $\mathbf{H}_t = \text{Cov}(\boldsymbol{a}_t|\boldsymbol{\Omega}_{t-1})$. Multivariate volatility modelling is interesting in the time evolution of $\mathbf{H}_t$. One way to model the heteroscedasticity for capturing asymmetric volatility patterns is to use the multivariate EGARCH model for returns series $\{\mathbf{r}_t\}$. The Nelson's univariate EGARCH model can be directly extended to the multivariate version based on Equation 1 and 2 as follows:

$$\ln(\boldsymbol{\sigma}_t^2) = \boldsymbol{\alpha}_0 + \frac{\boldsymbol{I} + \boldsymbol{\beta}_1 B + \cdots + \boldsymbol{\beta}_n B^n}{\boldsymbol{I} - \boldsymbol{\alpha}_1 B - \cdots - \boldsymbol{\alpha}_m B^m} G(\boldsymbol{\varepsilon}_{t-1}),$$

$$\boldsymbol{a}_t = \mathbf{H}_t^{1/2} \boldsymbol{\varepsilon}_t, \tag{5}$$

$$\boldsymbol{a}_t|\boldsymbol{\Omega}_{t-1} \sim N_k(\boldsymbol{0}, \mathbf{H}_t), \boldsymbol{\varepsilon}_t \sim N_k(\boldsymbol{0}, \boldsymbol{I}),$$

$$G(\boldsymbol{\varepsilon}_t) = \theta\boldsymbol{\varepsilon}_t + \gamma[|\boldsymbol{\varepsilon}_t| - E(|\boldsymbol{\varepsilon}_t|)]. \tag{6}$$

In Equation 5, $\ln(\boldsymbol{\sigma}_t^2)$ represents a vector of univariate $\ln(\sigma_{i,t}^2)$, $i = 1, \ldots, k$, $\boldsymbol{\alpha}_0$ is a vector of constants, $\boldsymbol{\alpha}_j$ and $\boldsymbol{\beta}_l$ are $k \times k$ diagonal matrices for $j = 1, \ldots, m$, $l = 1, \ldots, n$, $\mathbf{I}$ is an identity matrix and $\boldsymbol{\varepsilon}_t$ is a vector of $\varepsilon_{i,t}, i = 1, \ldots, k$. In Equation 6, $\boldsymbol{G}(\boldsymbol{\varepsilon}_t)$ is a k-dimensional random sequence, which is a function of both magnitude and sign of $\boldsymbol{\varepsilon}_t$, and $\boldsymbol{\theta}$ and $\boldsymbol{\gamma}$ are $k \times k$ parameter matrices. Since $(m,n) = (1, 0)$ is a general setup (e.g. McCurdy and Stengos, 1992), Equation 5 becomes

$$\ln(\boldsymbol{\sigma}_t^2) = \boldsymbol{\alpha}_0 + \frac{\boldsymbol{I}}{\boldsymbol{I} - \boldsymbol{\alpha}_1 B} \boldsymbol{G}(\boldsymbol{\varepsilon}_{t-1}), \quad \boldsymbol{a}_t = \boldsymbol{H}_t^{1/2} \boldsymbol{\varepsilon}_t, \tag{7}$$

that is,

$$(\boldsymbol{I} - \boldsymbol{\alpha}_1 B) \ln(\boldsymbol{\sigma}_t^2) = (\boldsymbol{I} - \boldsymbol{\alpha}_1 B)\boldsymbol{\alpha}_0 + \boldsymbol{I}\boldsymbol{G}(\boldsymbol{\varepsilon}_{t-1}).$$

Using the matrix representation, the above equation can be rewritten as

$$\begin{bmatrix} 1 - \alpha_{11}B & 0 & \cdots & 0 \\ 0 & 1 - \alpha_{22}B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 - \alpha_{kk}B \end{bmatrix} \begin{bmatrix} \ln(\sigma_{1,t}^2) \\ \ln(\sigma_{2,t}^2) \\ \vdots \\ \ln(\sigma_{k,t}^2) \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \alpha_{11}B & 0 & \cdots & 0 \\ 0 & 1 - \alpha_{22}B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 - \alpha_{kk}B \end{bmatrix} \begin{bmatrix} \alpha_{10} \\ \alpha_{20} \\ \vdots \\ \alpha_{k0} \end{bmatrix}$$

$$+ \begin{bmatrix} g_1(\boldsymbol{\varepsilon}_{t-1}) \\ g_2(\boldsymbol{\varepsilon}_{t-1}) \\ \vdots \\ g_k(\boldsymbol{\varepsilon}_{t-1}) \end{bmatrix},$$

so

$$\ln(\sigma_{i,t}^2) = \alpha_{i0}^* + \alpha_{ii} \ln(\sigma_{i,t-1}^2) + g_i(\boldsymbol{\varepsilon}_{t-1}), \tag{8}$$

where $\alpha_{i0}^* = (1 - \alpha_{ii})\alpha_{i0}$, $i = 1, 2, \ldots, k$. Moreover, the matrix representation for Equation 6 is given by

$$\begin{bmatrix} g_1(\boldsymbol{\varepsilon}_t) \\ g_2(\boldsymbol{\varepsilon}_t) \\ \vdots \\ g_k(\boldsymbol{\varepsilon}_t) \end{bmatrix} = \begin{bmatrix} \theta_{11} & \theta_{12} & \cdots & \theta_{1k} \\ \theta_{21} & \theta_{22} & \cdots & \theta_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{k1} & \theta_{k2} & \cdots & \theta_{kk} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{k,t} \end{bmatrix}$$

$$+ \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1k} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{k1} & \gamma_{k2} & \cdots & \gamma_{kk} \end{bmatrix} \begin{bmatrix} |\varepsilon_{1,t}| & - & E & (|\varepsilon_{1,t}|) \\ |\varepsilon_{2,t}| & - & E & (|\varepsilon_{2,t}|) \\ \vdots & & & \vdots \\ |\varepsilon_{k,t}| & - & E & (|\varepsilon_{k,t}|) \end{bmatrix}.$$

By using the summation notation, we have

$$g_i(\varepsilon_t) = \sum_{j=1}^{k} \left\{ \theta_{ij}\varepsilon_{j,t} + \gamma_{ij}\left[|\varepsilon_{j,t}| - E(|\varepsilon_{j,t}|)\right] \right\},$$
$$i,j = 1,2,\ldots,k. \qquad (9)$$

The conditional covariance $\sigma_{ij,t}$ of $a_{i,t}$ and $a_{j,t}$ given Ω_{t-1} can be denoted as

$$\sigma_{ij,t} = \rho_{ij}\sigma_{i,t}\sigma_{j,t}, \quad i,j = 1,2,\ldots,k, \quad i \neq j, \qquad (10)$$

where ρ_{ij} is the constant conditional correlation between $a_{i,t}$ and $a_{j,t}$ given Ω_{t-1} (Bollerslev *et al.*, 1992).

Asymmetric effects of standardized innovations on volatility may be measured by partial derivatives for g_i from Equation 9 as follows:

$$\frac{\partial g_i(\varepsilon_t)}{\partial \varepsilon_{j,t}} = \begin{cases} \theta_{ij} + \gamma_{ij}, & \text{if } \varepsilon_{j,t} > 0, \\ \theta_{ij} - \gamma_{ij}, & \text{if } \varepsilon_{j,t} < 0, \end{cases} \quad \text{for } i,j = 1,2,\ldots,k. \qquad (11)$$

Relative asymmetry (or leverage effect) may be measured by the ratio $|-\gamma_{ij} + \theta_{ij}|/(\gamma_{ij} + \theta_{ij})$, which is greater than, equal to or less than 1 for negative asymmetry, symmetry and positive asymmetry, respectively. The total impact of spillover effects from market j to market i is measured by $(\gamma_{ij} + \theta_{ij}), i \neq j$ for a unit increment of positive innovation and $(-\gamma_{ij} + \theta_{ij})$, $i \neq j$ for a unit increment of negative innovation. The terms $\gamma_{ij}\left[|\varepsilon_{j,t}| - E(|\varepsilon_{j,t}|)\right]$ and $\theta_{ij}\varepsilon_{j,t}$ in Equation 9 measure, respectively, the size and sign effects.

Koutmos and Booth's multivariate EGARCH model

Koutmos and Booth's (1995) multivariate EGARCH model with k dimensions is given by

$$\ln(\sigma_{i,t}^2) = \alpha_{i0}^* + \alpha_{ii}\ln(\sigma_{i,t-1}^2) + \sum_{j=1}^{k} \gamma_{ij}f_j(\varepsilon_{j,t-1}),$$
$$i,j = 1,2,\ldots,k \qquad (12)$$

$$f_j(\varepsilon_{j,t}) = \left[|\varepsilon_{j,t}| - E(|\varepsilon_{j,t}|)\right] + \delta_j\varepsilon_{j,t},$$
$$j = 1,2,\ldots,k \qquad (13)$$

$$\sigma_{ij,t} = \rho_{ij}\sigma_{i,t}\sigma_{j,t}, \quad i,j = 1,2,\ldots,k, \quad i \neq j.$$

Equation 12 describes the volatility of the returns for each market, in which the volatility is an asymmetric exponential function of past own and past cross-market standardized innovations. Volatility spillovers across markets are reflected by coefficients

γ_{ij} for $i \neq j$. α_{ij} captures the persistence of volatility. The persistence of volatility may also be quantified by the half-life (HL) (Bhar, 2001) defined by

$$\text{HL} = \frac{\ln(0.5)}{\ln|\alpha_{ii}|}, \qquad (14)$$

which measures the time period required for the shocks to be reduced to the one-half of their original size.

Similarly, $f_j(\varepsilon_{j,t})$ in Equation 13 is an asymmetric function of standardized innovations. $\left[|\varepsilon_{j,t}| - E(|\varepsilon_{j,t}|)\right]$ and $\delta_j\varepsilon_{j,t}$ capture respectively, the size and sign effects. If $|\varepsilon_{j,t}| - E(|\varepsilon_{j,t}|) > 0$ and $\delta_j = 0$, $f_j(\varepsilon_{j,t})$ is positive. If, in addition, $\gamma_{ij} > 0$, volatility is an increasing function of past standardized innovations. $\delta_j\varepsilon_{j,t}$ measures the sign effects and δ_j relates standardized innovations to volatility in an asymmetric manner. For example, for positive γ_{ij}, if $-1 < \delta_j < 0$, then negative innovations have a higher impact on the volatility than positive innovations; if $\delta_j = 0$, then the volatility increased by positive innovations is less than by negative innovations; if $0 < \delta_j < 1$, then positive innovations would decrease volatility but negative innovations increase volatility. The above effects, reflecting that the volatility spillover mechanism is asymmetric, are called the leverage effects (Black, 1976; Nelson, 1991). The partial derivatives for f_j from Equation 13 is given, as for g_i, by

$$\frac{\partial f_j(\varepsilon_{j,t})}{\partial \varepsilon_{j,t}} = \begin{cases} 1 + \delta_j, & \text{if } \varepsilon_{j,t} > 0, \\ -1 + \delta_j, & \text{if } \varepsilon_{j,t} < 0, \end{cases} \quad \text{for } i,j = 1,2,\ldots,k, \qquad (15)$$

with similar properties as mentioned in Equation 11.

As indicated before, the conditional covariance shown in Equation 10 reflects the constant correlation ρ_{ij} across markets, which greatly simplifies the inference for the model. However, a major drawback of the constant-correlation model is that the model overlooks the fact that correlation coefficients tend to change over time (see for example, Engle, 2002; Tse and Tsui, 2002). It is suggested that constant correlation should be tested first. If constant correlation is not rejected, then ρ_{ij} can be used; otherwise time-varying correlation (dynamic conditional correlation, DCC) (Engle, 2002), denoted by $\rho_{ij,t}$, should be used. To test for constant correlation, one may use the method given by Tse (2000).

Model comparison

The relationship between g_i in Equation 9 and f_j in Equation 13 is $g_i(\varepsilon_t) = \sum_{j=1}^{k} \gamma_{ij}f_j(\varepsilon_{j,t})$. It follows that

θ_{ij}, γ_{ij} and δ_j are related by $\delta_j = \theta_{ij}/\gamma_{ij}$ for all i. f_j is a special case of g_i, in which $\delta_j = \theta_{ij}/\gamma_{ij}$. The inferential results for δ_{ij} based on g_i reflect more general information than those for δ_j based on f_j. The different results are due to the constraint that $\delta_{ij} = \delta_{i'j}, \forall i \neq i'$. In other words, the inferential results are identical if the null hypothesis $H_0 : \delta_{ij} = \delta_{i'j}, \forall i \neq i'$ is accepted, which can be tested by using the likelihood ratio test (LRT) (Neyman and Pearson, 1933). If the null hypothesis is rejected, then f_j in Equation 13 may be miss-specified.

To obtain parameter estimates by using the maximum likelihood method, we need the joint log-likelihood function under the distributional assumptions made previously. The log-likelihood function for the multivariate EGARCH model can be written as

$$L(\boldsymbol{\Theta}) = -\left(\frac{1}{2}\right)(kT)\ln(2\pi) - \left(\frac{1}{2}\right)$$
$$\times \sum_{t=1}^{T} \left(\ln |\boldsymbol{H}_t| + \boldsymbol{\varepsilon}_t' \boldsymbol{H}_t^{-1} \boldsymbol{\varepsilon}_t\right), \quad (16)$$

where k is the number of dimensions, T is the number of observations, $\boldsymbol{\Theta}$ is the parameter vector to be estimated. The log-likelihood function is highly non-linear in $\boldsymbol{\Theta}$ and therefore a numerical maximization technique is required. The BHHH algorithm (Berndt et al., 1974) can be used to obtain the solutions.

Furthermore, the numbers of parameters for the two EGARCH models can be compared. For the actual model in Equations 8 and 9, there are $2k + 2k^2$ parameters to be estimated. For Koutmos and Booth's model in Equations 12 and 13, there are $3k + 2k^2$ parameters. The difference of the numbers of parameters is $k(k-1)$. It appears that some information in interpreting market volatility is lost when Koutmos and Booth's model is used (due to the constraints indicated earlier), and this will lead to inaccurate parameter estimates. The actual model reduces to Koutmos and Booth's model when $\delta_{ij} = \delta_{i'j}, \forall i \neq i'$. The actual model is useful for low-dimensionality models, but it may suffer more from estimation problems in high-dimensionality models. For example, when $k = 4$, the model contains 12 more parameters. Specific approaches such as Cholesky decomposition (Tsay, 2005, Chap 10) may be used to deal with some unexpected estimation problems.

The magnitude and sign of standardized innovations have been constrained in Koutmos and Booth's EGARCH model, but not in the actual EGARCH model. Therefore, the parameter estimates resulting from Koutmos and Booth's EGARCH model may not be accurate. The different parameter estimates would lead to different indices such as half life, persistence, spillover, asymmetric effects and so on, and in turn influence subsequent financial decision-making. Since the actual model is a direct extension of Nelson's (1991) univariate EGARCH model and can lead to more accurate inferential results, it should be used instead of Koutmos and Booth's model.

III. Conclusions

In this article, the extension of Nelson's (1991) univariate EGARCH model to the multivariate version has been reexamined and compared with the existing one given by Koutmos and Booth (1995). The actual multivariate EGARCH model obtained is more general. The function $g_i(\varepsilon_t)$ in Equation 9 plays a critical role in the model, in which more general asymmetric effects can be represented. Since the model can produce more accurate inferential results, it is recommended for future financial empirical studies. However, the actual model may suffer more from estimation problems due to the fact that the actual model contains more parameters. The forecasting formulas can be derived based on the actual model obtained as well as Koutmos and Booth's model, and their performance in forecasting can be compared in future studies.

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