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A new method for tool steel materials selection under fuzzy environment

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Abstract

This paper presents a new method to solve the tool steel materials selection problem under fuzzy environment, where the importance weights of different criteria and the ratings of various alternatives under different criteria are assessed in linguistic terms represented by fuzzy numbers. Because the proposed method uses simple arithmetic operations rather than the complicated arithmetic operations for aggregation and ranking of fuzzy numbers described in Wang and Chang (1995), its execution is much faster than the one presented in Wang and Chang (1995). © 1997 Elsevier Science B.V.

Keywords: Fuzzy number; Tool steel; Multiple criteria; Material selection

1. Introduction

In [17], Wang and Chang pointed out that tool steels can be used for cutting tools, locators, clamps, and gauges. There are various kinds of tool steels, such as air-hardening die steels, high-carbon high-chromium die steels, water-hardening tool steels, oil-hardening tool steels, hot-work die steels, shock-resisting tool steels, etc. [1]. In recent years, some studies on the problem of effective tool steel material selection have begun [3, 16, 17]. In [16], Tang presented an analytical model of material selection based on material cost and material quality. In [3], Chen et al. presented a model for raw material selection, where two decision factors were considered in the model, i.e., raw material cost and additional material cost for the inappropriateness of raw material quality. In [17], Wang et al. pointed out that the methods presented in [3, 16] tend to use analytical approach to determine the most suitable material, and they also pointed out that it is difficult to make a suitable tool material selection by the conventional quantitative analysis due to the fact that the characteristics of different materials stated in engineering design handbooks [1, 2] are multi-dimensional and qualitative, and the assessments of these attributes by experts are given in linguistic terms. Thus, in [17], Wang et al. introduced a fuzzy set multiple criteria decision-making approach to help the designer to select the most suitable steel materials. The method presented in [17] allows the importance weights of different criteria assigned by the designers and the material evaluation data under different criteria

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from the design handbooks to be assessed in linguistic terms represented by fuzzy numbers. However, the method presented in [17] is not efficient enough due to the fact that it needs to perform the complicated aggregation and ranking operations of fuzzy numbers to determine the most suitable tool steel material. Thus, it is necessary to develop a more efficient method to help the designer to select the most suitable tool steel materials systematically.

In this paper, we propose a new method to solve the tool steel materials selection problem under fuzzy environment, where the importance weights of different criteria and the ratings of various alternatives under different criteria are assessed in linguistic terms represented by fuzzy numbers. Because the proposed method uses simple arithmetic operations rather than the complicated arithmetic operations for aggregation and ranking [11, 14, 15] of fuzzy numbers presented in [17], its execution is much faster than that presented in [17].

The rest of this paper is organized as follows. In Section 2, we briefly review the theory of fuzzy sets from [4–10, 12, 13, 18]. In Section 3, we briefly review the Wang and Chang [17] method for tool steel materials selection. In Section 4, we present a new method to solve the tool steel materials selection problem under fuzzy environment. In Section 5, we use an example to illustrate the tool steel materials selection process. The conclusions are discussed in Section 6.

2. Fuzzy set theory

In [18], Zadeh proposed the theory of fuzzy sets. Roughly speaking, a fuzzy set is a class with fuzzy boundaries. Let U be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$. A fuzzy set A of U is a set of ordered pairs $\{(u_1, f_A(u_1)), (u_2, f_A(u_2)), \ldots, (u_n, f_A(u_n))\}$, where f_A is the membership function of A, $f_A: U \to [0, 1]$, and $f_A(u_i)$ indicates the grade of membership of u_i in A. A fuzzy set A of the universe of discourse U is convex if and only if for all u_1, u_2 in U,

$$f_A(\lambda u_1 + (1 - \lambda)u_2) \geqslant \mathrm{Min}(f_A(u_1), f_A(u_2)), \tag{1}$$

where $\lambda \in [0, 1]$. A fuzzy set A of the universe of discourse U is called a normal fuzzy set implying that

$$\exists u_i \in U, \quad f_A(u_i) = 1. \tag{2}$$

A fuzzy number is a fuzzy subset in the universe of discourse U that is both convex and normal. According to [13], a fuzzy number M of the universe of discourse U may be characterized by a trapezoidal distribution parametrized by (a, b, c, d) shown in Fig. 1.

In the following, we introduce a defuzzification method of trapezoidal fuzzy numbers [6, 7, 13]. Let us consider a trapezoidal fuzzy number M shown in Fig. 2, where e is a defuzzification value of the

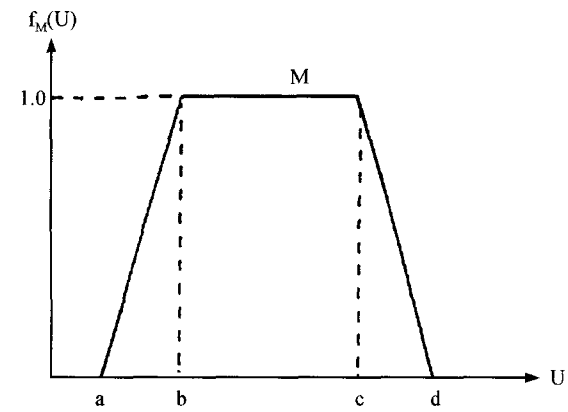

Fig. 1. A trapezoidal fuzzy number M.

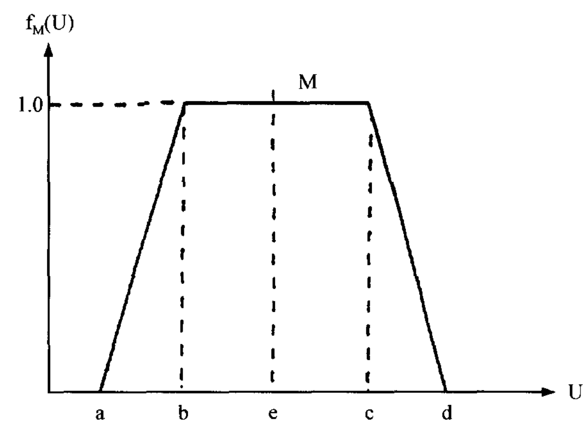

Fig. 2. Defuzzification of a fuzzy number M.

trapezoidal fuzzy number M. From Fig. 2, we can see that

$$(e-b)(1)+\tfrac{1}{2}(b-a)(1)=(c-e)(1)+\tfrac{1}{2}(d-c)(1)$$
$$\Rightarrow (e-b)+\tfrac{1}{2}(b-a)=(c-e)+\tfrac{1}{2}(d-c)$$
$$\Rightarrow (e-b)-(c-e)=\tfrac{1}{2}(d-c)-\tfrac{1}{2}(b-a)$$
$$\Rightarrow 2e=\frac{d-c-b+a}{2}+\frac{2b+2c}{2}$$
$$\Rightarrow 2e=\frac{a+b+c+d}{2}$$
$$\Rightarrow e=\frac{a+b+c+d}{4}. \quad (3)$$

3. A review of the Wang and Chang method for tool steel materials selection

In [17], Wang and Chang presented a fuzzy set approach to solve the tool steel materials selection problem under fuzzy environment, where the importance weights of the criteria and the performance ratings of criteria assessed by the decision makers are described in fuzzy terms represented by fuzzy numbers. The criteria considered in [17] include:

(1) nondeforming properties for materials
(2) safety in hardening for materials
(3) toughness for materials
(4) resistance to softening effect of heat for materials
(5) wear resistance for materials
(6) machinability for materials
(7) cost for materials

where the importance weights of the criteria are assessed in linguistic terms represented by fuzzy numbers, such as very low (VL), low (L), medium (M), high (H), and very high (VH), and the membership functions of the five linguistic terms are shown in Fig. 3.

In [17], Wang and Chang also assumed that the decision-makers can assign the ratings of different tool steels under different selection criteria using linguistic terms represented by fuzzy numbers, such as worst (W), poor (P), fair (F), good (G), and best (B), where the membership functions of the five linguistic terms are shown in Fig. 4.

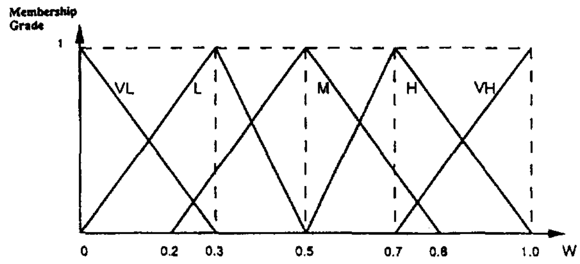

Fig. 3. Membership functions for linguistic weighting values.

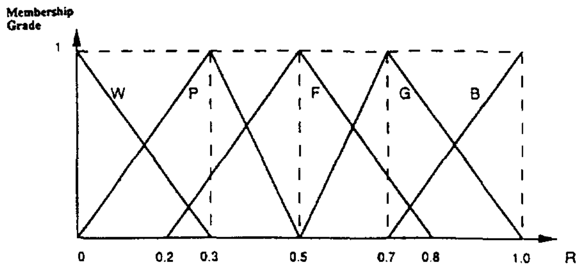

Fig. 4. Membership functions for linguistic rating values.

In [17], Wang and Chang used the aggregation and ranking operations of fuzzy numbers to solve the tool steel materials selection problem. The method is briefly reviewed as follows. Assume that a committee of n decision-makers is formed to conduct the task of tool steel materials selection, then the n decision-makers' opinions can be aggregated by

$$W_t=(1/n)\otimes(W_{t1}\oplus W_{t2}\oplus\cdots\oplus W_{tn}),\quad t=1,2,\ldots,k. \quad (4)$$

where W_t is the aggregated weighting for criterion t, W_{tn} is the importance weighting given by decision-maker n to criterion t, k is the number of criteria, $\oplus$ and $\otimes$ are the addition operator and the multiplication operators of fuzzy numbers, respectively. Furthermore, assume that the performance ratings of different tool steels under different selection criteria are evaluated. If the ratings are assigned by different decision-makers, then their opinions can be aggregated by

$$R_{it}=(1/n)\otimes(R_{it_1}\oplus R_{it_2}\oplus\cdots\oplus R_{it_n}), \quad (5)$$

where $i = 1, 2, \ldots, m$, $t = 1, 2, \ldots, k$, R_{it} is the aggregated rating of alternative i under criterion t, R_{it_n} is the assigned rating of alternative i under criterion t by decision-maker n. After the weights and ratings have been assigned and aggregated, R_{it} can further be weighted by the aggregated weight W_t to obtain the final rating F_i,

$$F_i = (1/k) \otimes [(R_{i1} \otimes W_1) \oplus (R_{i2} \otimes W_2) \oplus \cdots \oplus (R_{ik} \otimes W_k)], \tag{6}$$

where $F_i \approx (Y_i, Q_i, R_i, Z_i)$ is the approximated fuzzy number of the fuzzy suitability index of alternative i. Finally, Wang et al. used Chen's method [11] of maximizing set and minimizing set to rank the final ratings of different materials to obtain the most suitable tool steel material, where the maximizing set is defined as $M = \{(x, f_M(x)) \mid x \in R\}$ with the membership function

$$f_M(x) = \begin{cases} (x - x_1)/(x_2 - x_1), & x_1 \leqslant x \leqslant x_2, \\ 0, & \text{otherwise}, \end{cases} \tag{7}$$

and the minimizing set is defined as $G = \{(x, f_G(x)) \mid x \in R\}$ with the membership function

$$f_G(x) = \begin{cases} (x - x_2)/(x_1 - x_2), & x_1 \leqslant x \leqslant x_2, \\ 0, & \text{otherwise}, \end{cases} \tag{8}$$

where $x_1 = \inf D$, $x_2 = \sup D$, $D = \bigcup_{i=1}^{m} D_i$, $D_i = \{x \mid f_{F_i}(x) > 0\}$, and $i = 1, 2, \ldots, m$. The right utility $U_M(F_i)$ and the left utility $U_G(F_i)$ of each fuzzy rating F_i are defined by

$$U_M(F_i) = \sup_x (f_{F_i}(x) \wedge f_M(x)) \tag{9}$$

and

$$U_G(F_i) = \sup_x (f_{F_i}(x) \wedge f_G(x)), \tag{10}$$

where F_i is the fuzzy suitability index of alternative i, for $i = 1, 2, \ldots, m$. The ranking value $U_T(F_i)$ of F_i is obtained by combining the right and the left utilities,

$$U_T(F_i) = [U_M(F_i) + 1 - U_G(F_i)]/2, \tag{11}$$

for $i = 1, 2, \ldots, m$. The ranking order is determined by the ranking value. The greater the ranking value $U_T(F_i)$, the higher priority the alternative i will be, where $i = 1, 2, \ldots, m$.

However, the Wang and Chang method presented in [17] is not efficient enough due to the fact that it needs a lot of computation time to perform the aggregation and ranking operations of fuzzy numbers. Thus, it is necessary to develop a more efficient method to solve the tool steel materials selection problem.

4. A new method for tool steel materials selection

In this section, we present a new method to solve the tool steel materials selection problem under fuzzy environment. Because the proposed method uses simple arithmetic operations rather than the complicated aggregation and ranking operations of fuzzy numbers mentioned in [17], its execution is much faster than the one presented in [17].

Let us consider the membership functions of the linguistic terms "very low", "low", "medium", "high", and "very high" shown in Fig. 3, where the linguistic terms and their corresponding quadruple representations of fuzzy numbers are shown in Table 1.

By applying formula (3), the defuzzified values of the fuzzy numbers shown in Table 1 can be evaluated as follows.

(1) The defuzzified value of the fuzzy number "very low" is equal to

$$\frac{0 + 0 + 0 + 0.3}{4} = 0.075.$$

(2) The defuzzified value of the fuzzy number "low" is equal to

$$\frac{0 + 0.3 + 0.3 + 0.5}{4} = 0.275.$$

Table 1
Linguistic terms and their corresponding fuzzy numbers

Linguistic terms	Fuzzy numbers
Very low (VL)	(0, 0, 0, 0.3)
Low (L)	(0, 0.3, 0.3, 0.5)
Medium (M)	(0.2, 0.5, 0.5, 0.8)
High (H)	(0.5, 0.7, 0.7, 1)
Very high (VH)	(0.7, 1, 1, 1)

(3) The defuzzified value of the fuzzy number "medium" is equal to

$$\frac{0.2 + 0.5 + 0.5 + 0.8}{4} = 0.5.$$

(4) The defuzzified value of the fuzzy number "high" is equal to

$$\frac{0.5 + 0.7 + 0.7 + 1}{4} = 0.725.$$

(5) The defuzzified value of the fuzzy number "very high" is equal to

$$\frac{0.7 + 1 + 1 + 1}{4} = 0.925.$$

Furthermore, let us consider the membership functions of the linguistic terms, worst (W), poor (P), fair (F), good (G), and best (B) shown in Fig. 4, where the linguistic terms and their corresponding quadruple representations of fuzzy numbers are shown in Table 2.

By applying formula (3), the defuzzified values of these fuzzy numbers can be evaluated as follows.

(1) The defuzzified value of the fuzzy number "worst" is equal to

$$\frac{0 + 0 + 0 + 0.3}{4} = 0.075.$$

(2) The defuzzified value of the fuzzy number "poor" is equal to

$$\frac{0 + 0.3 + 0.3 + 0.5}{4} = 0.275.$$

(3) The defuzzified value of the fuzzy number "fair" is equal to

$$\frac{0.2 + 0.5 + 0.5 + 0.6}{4} = 0.5.$$

Table 2
Linguistic terms and their corresponding fuzzy numbers

Linguistic terms	Fuzzy numbers
Worst (W)	(0, 0, 0, 0.3)
Poor (P)	(0, 0.3, 0.3, 0.5)
Fair (F)	(0.2, 0.5, 0.5, 0.8)
Good (G)	(0.5, 0.7, 0.7, 1)
Best (B)	(0.7, 1, 1, 1)

(4) The defuzzified value of the fuzzy number "good" is equal to

$$\frac{0.5 + 0.7 + 0.7 + 1}{4} = 0.725.$$

(5) The defuzzified value of the fuzzy number "best" is equal to

$$\frac{0.7 + 1 + 1 + 1}{4} = 0.925.$$

Assume that there is a committee of n decision-makers (i.e., $D_1, D_2, \ldots, D_n$) to conduct the tool steel selection, and further assume that there are m candidate tool steel materials (i.e., $A_1, A_2, \ldots, A_m$) to be considered. Furthermore, assume that there are k basic tool steel characteristics evaluation criteria (i.e., $C_1, C_2, \ldots, C_k$). Let W_{tj} be the importance weights given by decision-maker D_j to criterion C_t, where W_{tj} is a linguistic term represented by a fuzzy number parametrized by a quadruple $(a_{tj}, b_{tj}, c_{tj}, d_{tj})$, $1 \leqslant j \leqslant n$ and $1 \leqslant t \leqslant k$, and let R_{it} be the assigned rating of alternative A_i under criterion C_t by the committee of n decision-makers, where R_{it} is a linguistic term represented by a fuzzy number parametrized by a quadruple $(p_{it}, x_{it}, y_{it}, z_{it})$, $1 \leqslant i \leqslant m$, and $1 \leqslant t \leqslant k$. By applying formula (3), the defuzzified values of the fuzzy numbers W_{tj} and R_{it} are w_{tj} and r_{it}, respectively, where

$$w_{tj} = \frac{a_{tj} + b_{tj} + c_{tj} + d_{tj}}{4}, \quad (12)$$

$$r_{it} = \frac{p_{it} + x_{it} + y_{it} + z_{it}}{4}, \quad (13)$$

$1 \leqslant j \leqslant n$, $1 \leqslant t \leqslant k$, and $1 \leqslant i \leqslant m$.

Assume that the importance weight of each criterion assigned by each decision-maker is shown in Table 3, where W_{tj} is a linguistic term represented by a fuzzy number parametrized by a quadruple $(a_{tj}, b_{tj}, c_{tj}, d_{tj})$, $1 \leqslant j \leqslant n$, and $1 \leqslant t \leqslant k$.

By applying formula (12), the fuzzy number W_{tj} shown in Table 3 can be defuzzified into a crisp value w_{tj} shown in Table 4, where $1 \leqslant j \leqslant n$, and $1 \leqslant t \leqslant k$.

Furthermore, the aggregated importance weight of each criterion C_i assessed by the committee of

Table 3
The importance weights of the criteria

Criteria	Decision-makers			
	D_1	D_2	$\cdots$	D_n
C_1	W_{11}	W_{12}	$\cdots$	W_{1n}
C_2	W_{21}	W_{22}	$\cdots$	W_{2n}
.	.	.	$\cdots$	.
.	.	.	$\cdots$	.
.	.	.	$\cdots$	.
C_k	W_{k1}	W_{k2}	$\cdots$	W_{kn}

Table 4
Defuzzified values of the importance weights of the criteria

Criteria	Decision-makers			
	D_1	D_2	$\cdots$	D_n
C_1	w_{11}	w_{12}	$\cdots$	w_{1n}
C_2	w_{21}	w_{22}	$\cdots$	w_{2n}
.	.	.	$\cdots$	.
.	.	.	$\cdots$	.
.	.	.	$\cdots$	.
C_k	w_{k1}	w_{k2}	$\cdots$	w_{kn}

n decision-makers can be evaluated by the function T:

$$T(C_i) = \frac{w_{i1} + w_{i2} + \cdots + w_{in}}{n}, \tag{14}$$

where $1 \leqslant i \leqslant k$.

According to [17], the evaluation criteria can be distinguished into two categories:

(1) Subjective criteria (e.g., $C_1, C_2, \ldots, C_{k-1}$).

(2) Objective criteria (e.g., C_k).

The ratings of different tool steels under different subjective criteria can be assessed by the decision-makers based on the available assessment information given in the design handbook. For example, assume that the ratings of each candidate tool steel material under the subjective criteria (i.e., $C_1, C_2, \ldots,$ and C_{k-1}) assigned by the committee of n decision-makers are shown in Table 5, where R_{it} is a linguistic term represented by a fuzzy number parametrized by a quadruple $(p_{it}, x_{it}, y_{it}, z_{it})$, $1 \leqslant i \leqslant m$, and $1 \leqslant t \leqslant k-1$.

Table 5
Ratings of the tool steel materials under the subjective criteria

Alternatives	Subjective criteria			
	C_1	C_2	$\cdots$	C_{k-1}
A_1	R_{11}	R_{12}	$\cdots$	$R_{1(k-1)}$
A_2	R_{21}	R_{22}	$\cdots$	$R_{2(k-1)}$
.	.	.	$\cdots$	.
.	.	.	$\cdots$	.
.	.	.	$\cdots$	.
A_m	R_{m1}	R_{m2}	$\cdots$	$R_{m(k-1)}$

By applying formula (13), the fuzzy number R_{it} shown in Table 5 can be defuzzified into a crisp real value r_{it} shown in Table 6, where $1 \leqslant i \leqslant m$, and $1 \leqslant t \leqslant k-1$.

The rating of the objective criteria C_k (e.g., material cost) can be assessed by the following method. Assume that the cost of each tool steel material is represented by fuzzy numbers parametrized by quadruples shown in Table 7.

By applying formula (3), the defuzzified values of the fuzzy numbers shown in Table 7 can be obtained and are also shown in Table 7, where

$$v_i = \frac{a_{i1} + a_{i2} + a_{i3} + a_{i4}}{4}, \tag{15}$$

and $1 \leqslant i \leqslant m$. Then, based on the converting function F [17], the converted rating $F(A_i)$ of the alternative A_i under the objective criteria C_k (i.e., "material cost") can be evaluated as follows:

$$F(A_i) = \frac{1}{v_i * \left(\frac{1}{v_1} + \frac{1}{v_2} + \cdots + \frac{1}{v_m}\right)}, \tag{16}$$

where $F(A_i) \in [0, 1]$, $1 \leqslant i \leqslant m$, and $\sum_{i=1}^{m} F(A_i) = 1$.

Based on Tables 4, 6, 7, and formulas (14)–(16), the ranking values of the tool steel material alternatives $A_1, A_2, \ldots, A_m$ can be evaluated by the function R,

$$R(A_i) = r_{i1} * T(C_1) + r_{i2} * T(C_2) + \cdots + r_{i(k-1)} * T(C_{k-1}) + F(A_i) * T(C_k), \tag{17}$$

Table 6
Defuzzified values of the ratings of the tool steel materials under the subjective criteria

Alternatives	Subjective criteria			
	C_1	C_2	$\cdots$	C_{k-1}
A_1	r_{11}	r_{12}	$\cdots$	$r_{1(k-1)}$
A_2	r_{21}	r_{22}	$\cdots$	$r_{2(k-1)}$
.	.	.	$\cdots$	.
.	.	.	$\cdots$	.
.	.	.	$\cdots$	.
A_m	r_{m1}	r_{m2}	$\cdots$	$r_{m(k-1)}$

Table 7
Cost of the tool steel materials

Alternatives	Material cost (per piece)	Defuzzified values
A_1	$(a_{11}, a_{12}, a_{13}, a_{14})$	v_1
A_2	$(a_{21}, a_{22}, a_{23}, a_{24})$	v_2
.	.	.
.	.	.
.	.	.
A_m	$(a_{m1}, a_{m2}, a_{m3}, a_{m4})$	v_m

where $C_1, C_2, \ldots, C_{k-1}$ are subjective criteria; C_k is an objective criteria; $T(C_j)$ is the aggregated importance weight of criterion C_j, where $1 \leqslant j \leqslant k$; $r_{i1}, r_{i2}, \ldots, r_{i(k-1)}$ are the defuzzified values of the ratings of the alternative A_i under the subjective criteria $C_1, C_2, \ldots, C_{k-1}$, respectively; $F(A_i)$ is the converted rating of the alternative A_i under the objective criterion C_k, and $1 \leqslant i \leqslant m$. The larger the value of $R(A_i)$, the higher priority the alternative A_i will have. If $R(A_i)$ is the largest value among the values $R(A_1), R(A_2), \ldots, R(A_m)$, then the best selection is A_i, where $1 \leqslant i \leqslant m$.

5. Example

In this section, we use an example to illustrate the proposed method. In order to compare the proposed method with the one shown in [17], we use the example shown in [17] to illustrate the tool steel materials selection process.

Assume that a committee of three decision-makers conduct the task of tool steel materials selection, where the AISI steel numbers are $\mathscr{W}_1$, $\mathscr{A}_2$, $\mathscr{D}_2$, $\mathscr{S}_1$, and $\mathscr{T}_1$, respectively. The tool steel selection criteria are shown in Table 8. The importance weights of the criteria are shown in Table 9. The ratings of the tool steel materials under the six criteria are shown in Table 10. The cost data of different candidate tool steel materials are shown in Table 11.

Based on Table 1, the linguistic terms shown in Table 9 can be represented by fuzzy numbers parametrized by quadruples shown in Table 12. By

Table 8
The tool steel selection criteria

Subjective criteria	Objective criteria
Nondeforming properties (C_1)	Material cost (C_7)
Safety in hardening (C_2)	
Toughness (C_3)	
Resistance to softening effect of heat (C_4)	
Wear resistance (C_5)	
Machinability (C_6)	

Table 9
The importance weights of the seven criteria

Criteria	Decision-makers		
	D_1	D_2	D_3
C_1	H	H	VH
C_2	M	H	M
C_3	VH	VH	H
C_4	H	H	M
C_5	M	M	M
C_6	H	H	VH
C_7	VH	VH	VH

Table 10
Ratings of the tool steel materials under the six criteria

AISI steel number	C_1	C_2	C_3	C_4	C_5	C_6
$\mathscr{W}_1$	P	F	G	P	F	B
$\mathscr{A}_2$	B	B	F	F	G	F
$\mathscr{D}_2$	B	B	F	F	G	P
$\mathscr{S}_1$	F	G	G	F	F	F
$\mathscr{T}_1$	G	G	F	B	G	F

Table 11
Cost of the tool steel materials

AISI steel number	Material cost (per piece)
$\mathscr{W}_1$	(1.5, 1.6, 1.6, 1.7)
$\mathscr{A}_2$	(1.8, 2.0, 2.0, 2.2)
$\mathscr{D}_2$	(1.0, 1.0, 1.0, 1.0)
$\mathscr{S}_1$	(1.2, 1.4, 1.4, 1.6)
$\mathscr{T}_1$	(2.5, 3.0, 3.0, 3.5)

Table 12
The importance weights of the criteria represented by fuzzy numbers

	Decision-makers		
Criteria	D_1	D_2	D_3
C_1	(0.5, 0.7, 0.7, 1.0)	(0.5, 0.7, 0.7, 1.0)	(0.7, 1.0, 1.0, 1.0)
C_2	(0.2, 0.5, 0.5, 0.8)	(0.5, 0.7, 0.7, 1.0)	(0.2, 0.5, 0.5, 0.8)
C_3	(0.7, 1.0, 1.0, 1.0)	(0.7, 1.0, 1.0, 1.0)	(0.5, 0.7, 0.7, 1.0)
C_4	(0.5, 0.7, 0.7, 1.0)	(0.5, 0.7, 0.7, 1.0)	(0.2, 0.5, 0.5, 0.8)
C_5	(0.2, 0.5, 0.5, 0.8)	(0.2, 0.5, 0.5, 0.8)	(0.2, 0.5, 0.5, 0.8)
C_6	(0.5, 0.7, 0.7, 1.0)	(0.5, 0.7, 0.7, 1.0)	(0.7, 1.0, 1.0, 1.0)
C_7	(0.7, 1.0, 1.0, 1.0)	(0.7, 1.0, 1.0, 1.0)	(0.7, 1.0, 1.0, 1.0)

applying formula (12), the defuzzified values of the fuzzy numbers shown in Table 12 can be evaluated as shown in Table 13. By applying formula (14), the aggregated weight for criterion C_i $(i = 1, 2, \ldots, 7)$ is also shown in Table 13.

Based on Table 2, the linguistic terms shown in Table 10 can be represented by fuzzy numbers parametrized by quadruples, and by applying formula (13), the defuzzified values of the fuzzy numbers can be evaluated as shown in Table 14.

By applying formula (3), the defuzzified values of the fuzzy numbers shown in Table 11 can be evaluated as shown in Table 15. Furthermore, by applying formula (16), the converted rating values of the candidate tool steel materials can be evaluated as shown in Table 15.

By applying formula (17), the ranking values of the five candidate tool steel materials can be evaluated as follows:

$$\begin{aligned}R(\mathscr{W}_1) &= 0.275*0.7917 + 0.5*0.575 + 0.725\\&\quad *0.8583 + 0.275*0.65 + 0.5*0.5\\&\quad + 0.925*0.7917 + 0.1970*0.925\\&= 0.2177 + 0.2875 + 0.6223 + 0.1788\\&\quad + 0.25 + 0.7323 + 0.1822\\&= 2.4708,\end{aligned}$$

$$\begin{aligned}R(\mathscr{A}_2) &= 0.925*0.7917 + 0.925*0.575 + 0.5\\&\quad *0.8583 + 0.5*0.65 + 0.725*0.5\\&\quad + 0.5*0.7917 + 0.1576*0.925\\&= 0.7323 + 0.5319 + 0.4292 + 0.325\\&\quad + 0.363 + 0.3959 + 0.1458\\&= 2.9231,\end{aligned}$$

$$\begin{aligned}R(\mathscr{D}_2) &= 0.925*0.7917 + 0.925*0.575 + 0.5\\&\quad *0.8583 + 0.5*0.65 + 0.725*0.5\\&\quad + 0.275*0.7917 + 0.3152*0.925\\&= 0.7323 + 0.5319 + 0.4292 + 0.325\\&\quad + 0.363 + 0.2177 + 0.2916\\&= 2.8907,\end{aligned}$$

$$\begin{aligned}R(\mathscr{S}_1) &= 0.5*0.7917 + 0.725*0.575 + 0.725\\&\quad *0.8583 + 0.5*0.65 + 0.5*0.5 + 0.5\\&\quad *0.7917 + 0.2251*0.7917\\&= 0.3959 + 0.4169 + 0.8447 + 0.325\\&\quad + 0.25 + 0.3959 + 0.1782\\&= 2.8066,\end{aligned}$$

Table 13
Defuzzified values of the importance weights of the criteria and the aggregated importance weight of each criterion

	Decision-makers			
Criteria	D_1	D_2	D_3	Aggregated importance weights
C_1	0.725	0.725	0.925	0.7917
C_2	0.5	0.725	0.5	0.575
C_3	0.925	0.925	0.925	0.8583
C_4	0.725	0.725	0.5	0.65
C_5	0.5	0.5	0.5	0.5
C_6	0.725	0.725	0.925	0.7917
C_7	0.925	0.925	0.925	0.925

Table 14
Defuzzified values of the ratings of the tool steel materials under the six criteria

AISI steel number	C_1	C_2	C_3	C_4	C_5	C_6
$\mathscr{W}_1$	0.275	0.5	0.725	0.275	0.5	0.925
$\mathscr{A}_2$	0.925	0.925	0.5	0.5	0.725	0.5
$\mathscr{D}_2$	0.925	0.925	0.5	0.5	0.725	0.275
$\mathscr{S}_1$	0.5	0.725	0.725	0.5	0.5	0.5
$\mathscr{T}_1$	0.725	0.725	0.5	0.925	0.725	0.5

Table 15
Defuzzified values of the cost of tool steel materials and the converted rating of the tool steel materials

AISI steel number	Defuzzified values of material cost	Converted rating
$\mathscr{W}_1$	1.6	0.1970
$\mathscr{A}_2$	2.0	0.1576
$\mathscr{D}_2$	1.0	0.3152
$\mathscr{S}_1$	1.4	0.2251
$\mathscr{T}_1$	3.0	0.1051

$$\begin{aligned} R(\mathscr{T}_1) &= 0.725 * 0.7917 + 0.725 * 0.575 + 0.5 \\ &\quad * 0.8583 + 0.925 * 0.65 + 0.725 * 0.5 \\ &\quad + 0.5 * 0.7917 + 0.1051 * 0.925 \\ &= 0.5740 + 0.4169 + 0.4292 + 0.6013 \\ &\quad + 0.3625 + 0.3959 + 0.0972 \\ &= 2.877. \end{aligned}$$

From the above calculations, we can see that the ranking order is $\mathscr{A}_2$, $\mathscr{D}_2$, $\mathscr{T}_1$, $\mathscr{S}_1$, and $\mathscr{W}_1$. Thus, the best selection is steel number $\mathscr{A}_2$. It is obvious that this result coincides with the one presented in [17]. We can see that the proposed method is more simple in calculations than the one presented in [17], and it can be executed much faster than the one presented in [17].

6. Conclusions

In this paper, we have presented a new method to solve the tool steel materials selection problem under fuzzy environment. We also use an example to illustrate the tool steel materials selection process. From the example, we can see that the proposed method is more simple in calculations than the one presented in [17], and its execution is much faster than the one presented in [17]. In this paper, we have assumed that the importance weights of different criteria and the ratings of various candidate tool steel materials under different criteria are assessed in linguistic terms represented by trapezoidal fuzzy numbers. However, the proposed method also can be used in the situation that the importance weights and the rating of the candidate alternatives under different criteria are directly described by trapezoidal fuzzy numbers. The proposed method can provide a useful way to solve the tool steel materials selection problem.

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