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Constraining the mass scale of a Lorentz-violating Hamiltonian with the measurement of astrophysical neutrino-flavor composition

Kwang-Chang Lai,[1,*] Wei-Hao Lai,[2,†] and Guey-Lin Lin[2,‡]

[1]*Center for General Education, Chang Gung University, Kwei-Shan, Taoyuan 333, Taiwan*
[2]*Institute of Physics, National Chiao Tung University, Hsinchu 300, Taiwan*

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We study Lorentz violation effects on flavor transitions of high energy astrophysical neutrinos. It is shown that the appearance of a Lorentz-violating Hamiltonian can drastically change the flavor transition probabilities of astrophysical neutrinos. Predictions of Lorentz-violation effects on flavor compositions of astrophysical neutrinos arriving on Earth are compared with IceCube flavor composition measurement which analyzes astrophysical neutrino events in the energy range between 25 TeV and 2.8 PeV. Such a comparison indicates that the future IceCube-Gen2 will be able to place stringent constraints on a Lorentz-violating Hamiltonian in the neutrino sector. We work out the expected sensitivities by IceCube-Gen2 on dimension-3 CPT-odd and dimension-4 CPT-even operators in a Lorentz-violating Hamiltonian. The expected sensitivities can improve on the current constraints obtained from other types of experiments by more than two orders of magnitudes for certain ranges of the parameter space.

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I. INTRODUCTION

Although physical laws are believed to be invariant under Lorentz transformation, violations of Lorentz symmetry might arise in string theory as discussed in [1,2]. It is possible to incorporate Lorentz-violation (LV) effects in an observer-independent effective field theory, the so-called standard model extension (SME) [3,4], which encompasses all the features of standard model particle physics and general relativity plus all possible LV operators [5–7]. While LV signatures are suppressed by the ratio $\Lambda_{\mathrm{EW}}/m_{\mathrm{P}}$ with Λ_{EW} being the electroweak energy scale and m_{P} the Planck scale, experimental techniques have been developed for probing such signatures [8,9]. The effects of LV on neutrino oscillations were pointed out in [10–12]. One can categorize LV effects on neutrino flavor transitions into three aspects: the modifications to energy dependencies of neutrino oscillation probabilities, the directional dependencies of oscillation probabilities, and the modifications to neutrino mixing angles and phases. In the standard vacuum oscillations of neutrinos, the oscillatory behavior of flavor-transition probability is determined by the dimensionless variable $\Delta m^2 L/E$ with Δm^2 the neutrino mass-squared difference, L the neutrino propagation distance, and E the neutrino energy. This dependence results from the Hamiltonian $H_{\mathrm{SM}} = UM^2U^\dagger/2E$ with $M^2_{ij} = \delta_{ij}(m_j^2 - m_1^2)$. The extra terms in Lorentz-violating Hamiltonian H_{LV} introduces L and LE dependencies into the oscillation probability, in addition to the standard L/E dependence. The directional dependence of oscillation probability is due to the violation of rotation symmetry in H_{LV}. The coefficients of LV operators change periodically as the Earth rotates daily about its axis. This induces temporal variations of neutrino oscillation probability at multiples of sidereal frequency $\omega_\oplus \approx 2\pi/(23\,\mathrm{h}\,56\ \mathrm{min})$. Finally, the full Hamiltonian $H \equiv H_{\mathrm{SM}} + H_{\mathrm{LV}}$ is diagonalized by the unitary matrix V, which differs from U due to the appearance of H_{LV}. Hence, the values of neutrino mixing angles and phases associated with V deviate from those associated with U. Such deviations increase with neutrino energies since H_{SM} is $\mathcal{O}(E^{-1})$, while H_{LV} contains $\mathcal{O}(E^0)$ and $\mathcal{O}(E)$ terms.

Experimentally, effects of Lorentz violation on neutrino oscillations have been investigated in short-baseline neutrino beams [13–16], in long-baseline neutrino beams [17,18], in reactor neutrinos at Double Chooz [19,20], and in atmospheric neutrinos at IceCube [21] and Super-Kamiokande [22]. These experiments probe either the spectral anomalies of the oscillated neutrino flux or the sidereal variations of neutrino oscillation probabilities. In this paper, we shall focus on LV effects on neutrino mixing angles and phases. As mentioned before, these effects grow with neutrino energies. Thus, it is ideal to probe such effects through the flavor transitions of high-energy astrophysical neutrinos [23]. For simplicity, we only consider isotropic LV effects.

The observation of high-energy astrophysical neutrinos by IceCube [24–27] is a significant progress in neutrino astronomy and provides new possibilities for testing neutrino properties. The first result by IceCube on the flavor composition of observed astrophysical neutrinos has been published in [28], and was updated in [29] by a combined-likelihood analysis taking into account more statistics. Meanwhile, independent efforts have been made to determine neutrino flavor compositions from IceCube

*kcl@mail.cgu.edu.tw
†s9927525.py99g@g2.nctu.edu.tw
‡glin@mail.nctu.edu.tw

data [30–34]. As we shall see in latter sections, the flavor measurement in [29] is not yet able to constrain $H_{\rm LV}$ more stringently than the previous experiments. Fortunately, there is an active plan for extending the current IceCube detector to a larger volume, which is referred to as IceCube-Gen2 [35,36]. This extension shall increase the effective area of the current 86-string detector up to a factor of 5. The expected improvement on neutrino flavor discrimination by IceCube-Gen2 has been studied in [37]. Using this result, we shall study sensitivities of IceCube-Gen2 to the parameters of $H_{\rm LV}$.

Astrophysical neutrinos are commonly produced by either pp or $p\gamma$ collisions at astrophysical sources. For sufficiently high energies, pp collisions produce equal number of π^+ and π^-, which decay to neutrinos through $\pi^+ \to \mu^+ + \nu_\mu \to e^+ + \nu_\mu + \nu_e + \bar{\nu}_\mu$ and $\pi^- \to \mu^- + \bar{\nu}_\mu \to e^- + \bar{\nu}_\mu + \bar{\nu}_e + \nu_\mu$. This leads approximately to the flux ratio $\Phi^0(\nu_e):\Phi^0(\nu_\mu):\Phi^0(\nu_\tau) = 1/3:2/3:0$ for both neutrinos and antineutrinos. Here $\Phi^0(\nu_\alpha)$ denotes generically the flux of neutrino or antineutrino of flavor α. This type of source is referred to as the pion source. A more detailed study on the neutrino flavor fraction with the consideration of neutrino spectral index is given in [38]. For an E^{-2} spectrum, the neutrino flavor fraction at the source is $(f_e^0, f_\mu^0, f_\tau^0) = (0.35, 0.65, 0)$, where $f_\alpha^0 \equiv \Phi^0(\nu_\alpha)/(\Phi^0(\nu_e) + \Phi^0(\nu_\mu) + \Phi^0(\nu_\tau))$. However, for the purpose of this work, it suffices to take $(f_e^0, f_\mu^0, f_\tau^0) = (1/3, 2/3, 0)$. We note that the secondary muons in some astrophysical objects can lose energy quickly by synchrotron cooling in magnetic fields or interactions with matter before their decays. Hence, the neutrino flavor fraction at the source becomes (0, 1, 0). This type of source is referred to as the muon-damped source [39–41]. In fact, there are also cases in which the flavor fraction of astrophysical neutrinos at the source is energy dependent. For example, the flavor fraction of neutrinos can gradually change from $(1/3, 2/3, 0)$ at lower energies to (0, 1, 0) at high energies. Such a phenomenon has been discussed in [39,40] and investigated systematically in [41]. The latter work also discusses sources with flavor fractions different from those of the pion source and muon-damped source. While a general study should consider the energy dependence of neutrino flavor fraction and variations of neutrino flavor fractions among different sources, we shall only focus on the simplified scenario where all sources of astrophysical neutrinos arising from pp collisions possess an energy-independent flavor fraction for neutrinos at $(1/3, 2/3, 0)$.

The production mechanism of astrophysical neutrinos with $p\gamma$ collisions is more complicated. The leading process of this category is $p\gamma \to n\pi^+$, which gives rise to the flavor fraction $(1/2, 1/2, 0)$ for neutrinos and (0, 1, 0) for antineutrinos. The subleading process is $p\gamma \to p\pi^+\pi^-$, which is non-negligible when the spectral index β of the target photon is harder than 1 [42,43]. This process produces equal numbers of neutrinos and antineutrinos with a common flavor fraction $(1/3, 2/3, 0)$. Since the flavor fraction of neutrinos produced by $p\gamma$ collisions is relatively uncertain, we will not consider astrophysical neutrinos produced by such a mechanism.

We note that effects of a new physics Hamiltonian (with Lorentz violation as a special case), parametrized as $(E_\nu/\Lambda_n)^n U_n O_n U_n^\dagger$, on the flavor transitions of astrophysical neutrinos were discussed in [44,45] for $n = 0$ and 1 (similar discussions were also given in [46–49]), and comparisons with earlier IceCube flavor measurement [28] were made. The authors scan all possible structures of the mixing matrix U_n for given new physics scales Λ_n and O_n and determine the allowed range of astrophysical neutrino flavor fractions on Earth resulting from the full Hamiltonian $H = H_{\rm SM} + (E_\nu/\Lambda_n)^n U_n O_n U_n^\dagger$. In our work, we shall focus on LV effects which are parametrized in a different form from the above new physics Hamiltonian. We shall discuss current and future constraints on LV effects by comparing the predicted neutrino flavor fraction with the range of flavor fraction measured by the current IceCube detector [29] and that expected [37] in the future IceCube-Gen2 detector. Our results can be directly compared with the previously most stringent constraints obtained by Super-Kamiokande [22].

This paper is organized as follows. In Sec. II, we incorporate LV effects into the full neutrino Hamiltonian in the framework of SME. We then study analytically the flavor transition of astrophysical neutrinos assuming the dominance of $H_{\rm LV}$ over $H_{\rm SM}$. As stated before, such a dominance is possible for high-energy astrophysical neutrinos. We discuss constraints on LV effects by the current IceCube flavor measurement. Such discussions pave the way for detailed numerical studies in the next section. In Sec. III, we study the flavor transitions of astrophysical neutrinos with the full Hamiltonian $H = H_{\rm SM} + H_{\rm LV}$. The expected sensitivities of IceCube-Gen2 to $H_{\rm LV}$ are studied. We conclude in Sec. IV.

II. LORENTZ VIOLATION IN NEUTRINO OSCILLATIONS

LV effects in neutrino oscillations are incorporated by introducing an additional Lorentz-violating term $H_{\rm LV}$ to the full Hamiltonian of the neutrino. Hence.

$$H = H_{\rm SM} + H_{\rm LV}, \tag{1}$$

where $H_{\rm SM} \equiv UM^2U^\dagger/2E$ is the standard model neutrino Hamiltonian in vacuum with M^2 the neutrino mass matrix

$$M^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} \tag{2}$$

and U the PMNS matrix. Here we do not consider matter effects due to neutrino propagations inside the Earth. This is because we only focus on neutrino events with energies higher than a few tens of TeV. In this case the Earth regeneration effect to the neutrino flavor transition is negligible. For neutrinos, the general form of the LV Hamiltonian is given by

$$H^{\nu}_{\mathrm{LV}} = \frac{p_\lambda}{E}\begin{pmatrix} a^{\lambda}_{ee} & a^{\lambda}_{e\mu} & a^{\lambda}_{e\tau} \\ a^{\lambda *}_{e\mu} & a^{\lambda}_{\mu\mu} & a^{\lambda}_{\mu\tau} \\ a^{\lambda *}_{e\tau} & a^{\lambda *}_{\mu\tau} & a^{\lambda}_{\tau\tau} \end{pmatrix} - \frac{p^\rho p^\lambda}{E}\begin{pmatrix} c^{\rho\lambda}_{ee} & c^{\rho\lambda}_{e\mu} & c^{\rho\lambda}_{e\tau} \\ c^{\rho\lambda *}_{e\mu} & c^{\rho\lambda}_{\mu\mu} & c^{\rho\lambda}_{\mu\tau} \\ c^{\rho\lambda *}_{e\tau} & c^{\rho\lambda *}_{\mu\tau} & c^{\rho\lambda}_{\tau\tau} \end{pmatrix}. \tag{3}$$

Since we shall only consider isotropic LV effects, we have the simplified form for H^{ν}_{LV} given by [10]

$$H^{\nu}_{\mathrm{LV}} = \begin{pmatrix} a^{T}_{ee} & a^{T}_{e\mu} & a^{T}_{e\tau} \\ a^{T *}_{e\mu} & a^{T}_{\mu\mu} & a^{T}_{\mu\tau} \\ a^{T *}_{e\tau} & a^{T *}_{\mu\tau} & a^{T}_{\tau\tau} \end{pmatrix} - \frac{4E}{3}\begin{pmatrix} c^{TT}_{ee} & c^{TT}_{e\mu} & c^{TT}_{e\tau} \\ c^{TT *}_{e\mu} & c^{TT}_{\mu\mu} & c^{TT}_{\mu\tau} \\ c^{TT *}_{e\tau} & c^{TT *}_{\mu\tau} & c^{TT}_{\tau\tau} \end{pmatrix}, \tag{4}$$

where T is the time component of a Sun-centered celestial equatorial coordinate (T, X, Y, Z). For antineutrinos, we have

$$H^{\bar{\nu}}_{\mathrm{LV}} = -\begin{pmatrix} a^{T}_{ee} & a^{T}_{e\mu} & a^{T}_{e\tau} \\ a^{T *}_{e\mu} & a^{T}_{\mu\mu} & a^{T}_{\mu\tau} \\ a^{T *}_{e\tau} & a^{T *}_{\mu\tau} & a^{T}_{\tau\tau} \end{pmatrix}^{*} - \frac{4E}{3}\begin{pmatrix} c^{TT}_{ee} & c^{TT}_{e\mu} & c^{TT}_{e\tau} \\ c^{TT *}_{e\mu} & c^{TT}_{\mu\mu} & c^{TT}_{\mu\tau} \\ c^{TT *}_{e\tau} & c^{TT *}_{\mu\tau} & c^{TT}_{\tau\tau} \end{pmatrix}^{*}. \tag{5}$$

The two terms on the right hand side of $H^{\nu,\bar{\nu}}_{\mathrm{LV}}$ are distinguished by their CPT transformation properties and the dimensionality of the operators they are originated from. The first term is CPT-odd and originated from a dimension-3 operator while the second term is CPT-even and originated from a dimension-4 operator. Diagonalizing the full Hamiltonian in Eq. (1) yields a new mass-flavor mixing matrix V. The neutrino flavor transition probability $P_{\alpha\beta} \equiv P(\nu_\beta \to \nu_\alpha)$ is then given by

$$\begin{aligned} P_{\alpha\beta} = \delta_{\alpha\beta} &- 4\sum_{j>i}\Re(V_{\beta j}V^{*}_{\beta i}V^{*}_{\alpha j}V_{\alpha i})\sin^2(L\Delta E_{ji}/2) \\ &+ 2\sum_{j>i}\Im(V_{\beta j}V^{*}_{\beta i}V^{*}_{\alpha j}V_{\alpha i})\sin^2(L\Delta E_{ji}), \end{aligned} \tag{6}$$

where $\Delta E_{ji} \equiv E_j - E_i$ is the difference between the energy eigenvalues. For high-energy astrophysical neutrinos, L is so large that the rapid oscillating terms are averaged out so that

$$P_{\alpha\beta} = \sum_{i=1}^{3}|V_{\alpha i}|^2|V_{\beta i}|^2. \tag{7}$$

Since $P_{\alpha\beta}$ depends only on the elements of V, the neutrino flavor composition observed on Earth for a given astrophysical neutrino source is affected by LV parameters. Therefore, the measurement of neutrino flavor fraction by neutrino telescopes such as IceCube is useful for constraining LV parameters. For convenience in discussions, we shall first concentrate on constraints on $a^{T}_{\alpha\beta}$ by setting $c^{TT}_{\alpha\beta} = 0$. The constraints on $c^{TT}_{\alpha\beta}$ will be commented on later.

Recently, Super-Kamiokande [22] has set upper limits for $|a^{T}_{\alpha\beta}|$, which are of the order 10^{-23} GeV. With $|a^{T}_{\alpha\beta}|$ of this energy scale, it is interesting to note that $\Delta m^2_{ij}/E$ is smaller than $|a^{T}_{\alpha\beta}|$ by more than 3 orders of magnitude for neutrino energies beyond a few tens of TeV. Hence, for neutrino events analyzed in IceCube flavor measurements [29], the LV term H_{LV} dominates over the standard model Hamiltonian $UM^2U^{\dagger}/2E$ if any $a^{T}_{\alpha\beta}$ term is set at the SK limit, $\sim 10^{-23}$ GeV. Therefore, IceCube measurements of flavor ratios should be useful for constraining the LV mass scale.

To illustrate the current IceCube capability of constraining LV parameters, we calculate the accessible ranges of neutrino flavor fractions on Earth resulting from the full Hamiltonian $H^{\nu,\bar{\nu}}_{\mathrm{SM}} + H^{\nu,\bar{\nu}}_{\mathrm{LV}}$ and the astrophysical pion source for neutrinos with the flavor fraction $(1/3, 2/3, 0)$. For an illustrative purpose, we consider special scenarios for $H^{\nu,\bar{\nu}}_{\mathrm{LV}}$ where only one pair of matrix elements in the LV Hamiltonian—for instance, $a^{T}_{\alpha\beta}$ and its complex conjugate $a^{T*}_{\alpha\beta}$—are nonvanishing. We classify these special scenarios as $|a^{T}_{e\mu}| \neq 0$, $|a^{T}_{e\tau}| \neq 0$, $|a^{T}_{\mu\tau}| \neq 0$, and $a^{T}_{\mu\mu,\tau\tau} \neq 0$, respectively. For the last scenario we take $a^{T}_{\tau\tau} = -a^{T}_{\mu\mu}$. In each special scenario for $H^{\nu,\bar{\nu}}_{\mathrm{LV}}$, the magnitude of the relevant matrix element $|a^{T}_{\alpha\beta}|$ is varied from zero to the current Super-Kamiokande 95% C. L. limit, the phase of $a^{T}_{\alpha\beta}$ is varied from 0 to 2π, and the neutrino mixing parameters in $H^{\nu,\bar{\nu}}_{\mathrm{SM}}$ are taken to be their best-fit values [50]. The predicted ranges of flavor fractions on Earth by the full Hamiltonian $H^{\nu,\bar{\nu}}_{\mathrm{SM}} + H^{\nu,\bar{\nu}}_{\mathrm{LV}}$ for all considered scenarios of the LV Hamiltonian are shown in Fig. 1. We stress that $H^{\nu,\bar{\nu}}_{\mathrm{LV}}$ dictates the neutrino flavor fraction when $|a^{T}_{\alpha\beta}|$ is taken at the current SK limit in each special scenario. For comparison, the standard model–predicted neutrino flavor fractions with neutrino mixing angles and CP phase in $H^{\nu,\bar{\nu}}_{\mathrm{SM}}$ varied over 3σ range [49] is also shown as the green area [51] in Fig. 1. It is clear that, except for a tiny piece of area, the predicted ranges of flavor fractions of neutrinos by the full Hamiltonian $H^{\nu,\bar{\nu}}_{\mathrm{SM}} + H^{\nu,\bar{\nu}}_{\mathrm{LV}}$ are all within the current IceCube 3σ contour. Therefore, a stringent constraint on

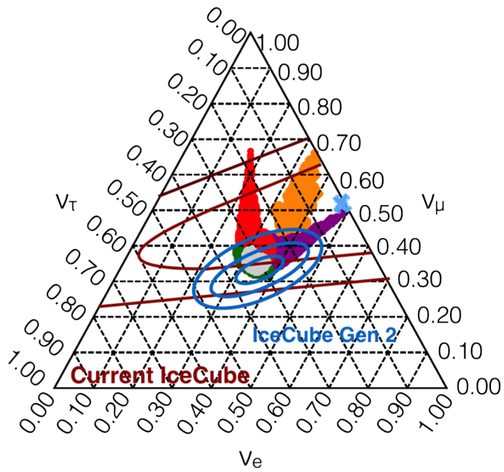

FIG. 1. The flavor fractions of astrophysical neutrinos arriving on Earth. These neutrinos are assumed to come from the astrophysical pion source with the flavor fraction $(1/3, 2/3, 0)$. The predicted ranges of flavor fractions on Earth by the full Hamiltonian $H_{\rm SM}^{\nu,\bar{\nu}} + H_{\rm LV}^{\nu,\bar{\nu}}$ are denoted by purple, red, gray, and orange areas for the special scenarios of $H_{\rm LV}^{\nu,\bar{\nu}}$ with $|a_{e\mu}^T| \neq 0$, $|a_{e\tau}^T| \neq 0$, $|a_{\mu\tau}^T| \neq 0$, and $a_{\mu\mu,\tau\tau}^T \neq 0$, respectively. In each scenario, the magnitude of the relevant matrix element $|a_{\alpha\beta}^T|$ is varied between 0 and the current Super-Kamiokande limit. The green area is the accessible range of neutrino flavor fraction by $H_{\rm SM}^{\nu,\bar{\nu}}$ with neutrino mixing angles and CP phase varied over 3σ range. Regions inside the brown lines are the current IceCube measurements with the blue cross denoting the best fit values [29]. Regions inside the blue curves are the expected IceCube-Gen2 $1\sigma - 3\sigma$ sensitivity regions given in [37].

$H_{\rm LV}^{\nu,\bar{\nu}}$ requires IceCube-Gen2, which is the main target of our study in the next session.

III. THE SENSITIVITY OF ICECUBE-GEN2 TO THE LV PARAMETERS

In this section, we apply the projected flavor discrimination sensitivity of IceCube-Gen2 [37] to estimate the future constraints on LV parameters. In the above projected sensitivity, only the pion source produced by pp collisions is considered. Therefore, we shall only consider this type of source in the following discussions.

Before studying constraints on the most general flavor structure of $H_{\rm LV}^{\nu,\bar{\nu}}$, it is useful to summarize our analysis in the previous section. Let us take $f_\alpha \equiv \Phi(\nu_\alpha)/(\Phi(\nu_e) + \Phi(\nu_\mu) + \Phi(\nu_\tau))$ as the neutrino flavor fraction on the Earth. Since we shall focus on the pion source caused by pp collisions, there are equal numbers of neutrinos and antineutrinos produced with the flavor fraction $(1/3, 2/3, 0)$ at the source for both neutrinos and antineutrinos. Therefore, we have $f_e = P_{ee}/3 + 2P_{e\mu}/3$. Since $P_{\alpha\beta} = P_{\beta\alpha}$ still holds with the addition of the LV Hamiltonian, we thus have $P_{ee} = 1 - P_{\mu e} - P_{\tau e} = 1 - P_{e\mu} - P_{e\tau}$. Hence, $f_e = 1/3 + (P_{e\mu} - P_{e\tau})/3$. Similarly, we can show that $f_\mu = 1/3 + (P_{\mu\mu} - P_{\mu\tau})/3$, and $f_\tau = 1/3 + (P_{\mu\tau} - P_{\tau\tau})/3$. Clearly for astrophysical neutrinos arising from the pion source, the deviation of their flavor fraction on Earth to $(1/3, 1/3, 1/3)$ is due to $\mu - \tau$ symmetry-breaking effects in the transition probability matrix. For the standard model Hamiltonian $H_{\rm SM}$, the $\mu - \tau$ symmetry-breaking effects are small. To leading orders in $\cos 2\theta_{23}$ and $\sin\theta_{13}$, one has $(P_{e\mu} - P_{e\tau}) = 2\epsilon$, $(P_{\mu\mu} - P_{\mu\tau}) = (P_{\mu\tau} - P_{\tau\tau}) = -\epsilon$ with $\epsilon = 2\cos 2\theta_{23}/9 + \sqrt{2}\sin\theta_{13}\cos\delta/9$ (taking $\sin^2\theta_{12} = 1/3$) [52], where δ is the CP violation phase. Hence, LV effects can be detectable provided they introduce sizable $\mu - \tau$ symmetry-breaking effects in the neutrino flavor transition probability matrix.

In the case that only $a_{e\mu}^T$ and $a_{e\mu}^{T*}$ are nonvanishing in $H_{\rm LV}^{\nu,\bar{\nu}}$, $\mu - \tau$ symmetry is clearly broken. If $H_{\rm LV}^{\nu,\bar{\nu}}$ dominates over $H_{\rm SM}^{\nu,\bar{\nu}}$, the flavor transition probability is determined by the LV Hamiltonian and we find $(P_{e\mu} - P_{e\tau}) = (P_{\mu\mu} - P_{\mu\tau}) = 1/2$ and $(P_{\mu\tau} - P_{\tau\tau}) = -1$ in this limit. Consequently, the flavor fraction of astrophysical neutrinos arriving on Earth deviates significantly from $(1/3, 1/3, 1/3)$. This corresponds to the tip of the purple area in Fig. 1, which represents the flavor fraction $(1/2, 1/2, 0)$. Similarly, large $\mu - \tau$ symmetry breaking occurs in the scenarios $|a_{e\tau}^T| \neq 0$ and $a_{\mu\mu,\tau\tau}^T \neq 0$ ($a_{\mu\mu}^T \neq a_{\tau\tau}^T$). On the other hand, $\mu - \tau$ symmetry is preserved in the scenario $|a_{\mu\tau}^T| \neq 0$.

We have just seen that the $\mu - \tau$ symmetry-breaking effect in $H_{\rm LV}^{\nu,\bar{\nu}}$ can be probed with the pion source produced by pp collisions. Since we have assumed that all astrophysical neutrinos come from the pion source, it is essential to quantify the $\mu - \tau$ symmetry-breaking effect in $H_{\rm LV}^{\nu,\bar{\nu}}$. To do that, it is useful to write $H_{\rm LV}^{\nu} = H_1^{\nu} + H_2^{\nu}$ with

$$H_1^\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a_{\mu\mu}^T & a_{\mu\tau}^T \\ 0 & a_{\mu\tau}^{T*} & a_{\tau\tau}^T \end{pmatrix}, \tag{8}$$

and

$$H_2^\nu = \begin{pmatrix} 0 & a_{e\mu}^T & a_{e\tau}^T \\ a_{e\mu}^{T*} & 0 & 0 \\ a_{e\tau}^{T*} & 0 & 0 \end{pmatrix}. \tag{9}$$

Similar decomposition can be applied to $H_{\rm LV}^{\bar{\nu}}$.

We note that the simplified structure H_1^ν has been considered as the LV coupling between dark energy and neutrinos, and the measurement of astrophysical ν_μ and ν_τ event difference was proposed to constrain H_1^ν in the future [53]. Here we shall begin with simplified scenarios that $H_{\rm LV}^{\nu,\bar{\nu}} = H_1^{\nu,\bar{\nu}}$ and $H_{\rm LV}^{\nu,\bar{\nu}} = H_2^{\nu,\bar{\nu}}$. We then proceed to discuss the general case with $H_{\rm LV}^{\nu,\bar{\nu}} = H_1^{\nu,\bar{\nu}} + H_2^{\nu,\bar{\nu}}$. We shall study the sensitivities of IceCube-Gen2 to these Hamiltonians.

A. $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_1^{\nu,\bar{\nu}}$

For $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_1^{\nu,\bar{\nu}}$, we can write

$$H_1^\nu = \left(\frac{a_{\mu\mu}^T + a_{\tau\tau}^T}{2}\right)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a_{\mu\mu}^T + a_{\tau\tau}^T & 0 & 0 \\ 0 & a_{\tau\tau}^T - a_{\mu\mu}^T & -2a_{\mu\tau}^T \\ 0 & -2a_{\mu\tau}^{T*} & a_{\mu\mu}^T - a_{\tau\tau}^T \end{pmatrix}. \quad (10)$$

The first term of H_1^ν is proportional to the identity matrix and does not affect the neutrino flavor transition probability. One can ignore this term and rewrite H_1^ν as

$$H_1^\nu = -M\begin{pmatrix} \gamma & 0 & 0 \\ 0 & \cos 2\alpha & -e^{i\beta}\sin 2\alpha \\ 0 & -e^{-i\beta}\sin 2\alpha & -\cos 2\alpha \end{pmatrix}, \quad (11)$$

where $M = \sqrt{(a_{\tau\tau}^T - a_{\mu\mu}^T)^2 + 4a_{\mu\tau}^T a_{\mu\tau}^{T*}}/2$, $\gamma = (a_{\mu\mu}^T + a_{\tau\tau}^T)/\sqrt{(a_{\tau\tau}^T - a_{\mu\mu}^T)^2 + 4a_{\mu\tau}^T a_{\mu\tau}^{T*}}$, $\cos 2\alpha = (a_{\tau\tau}^T - a_{\mu\mu}^T)/\sqrt{(a_{\tau\tau}^T - a_{\mu\mu}^T)^2 + 4a_{\mu\tau}^T a_{\mu\tau}^{T*}}$, $\sin 2\alpha = 2|a_{\mu\tau}^T|/\sqrt{(a_{\tau\tau}^T - a_{\mu\mu}^T)^2 + 4a_{\mu\tau}^T a_{\mu\tau}^{T*}}$, and β is the phase of $a_{\mu\tau}^T$. Since $\sin 2\alpha$ is positive by definition, α varies between 0 and $\pi/2$. The Hamiltonian $H_1^{\bar{\nu}}$ can be inferred from H_1^ν by the replacements $-M \to M$ and $\beta \to -\beta$. Taking into account the total Hamiltonian, $H^{\nu,\bar{\nu}} = H_{\mathrm{SM}}^{\nu,\bar{\nu}} + H_1^{\nu,\bar{\nu}}$, one can predict the neutrino flavor fraction on Earth assuming the initial neutrino flavor fraction at the source to be $(1/3, 2/3, 0)$. We note that the neutrino energy appearing in $H_{\mathrm{SM}}^{\nu,\bar{\nu}}$ should, in principle, follow the $E^{-2.2}$ distribution with the threshold at 100 TeV according to Ref. [37]. However, for simplicity, we fix $E = 100$ TeV. This is a conservative choice that makes $H_{\mathrm{SM}}^{\nu,\bar{\nu}}$ less suppressed in comparison to the dominant $H_1^{\nu,\bar{\nu}}$.

Given the IceCube-Gen2 sensitivity shown in Fig. 1, we obtain the expected constraints on the LV mass scale M as a function of mixing angle α with the phase β varied between 0 and 2π and the ratio γ of the order of unity. The expected constraints on M are shown in the part of Fig. 2 labeled by $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_1^{\nu,\bar{\nu}}$. To derive the expected constraints on M, we first fix the $\mu-\tau$ symmetry-breaking parameter $S_{\mu\tau} \equiv \sin^2 2\alpha$ while allowing the parameters β and γ to vary. We then identify the critical value of M such that the resulting neutrino flavor fraction on Earth reaches the boundary of the IceCube-Gen2 3σ C.L. contour. In this way we obtain an expected constraint on M for a specific $\sin^2 2\alpha$. We repeat the above procedure for different values of $\sin^2 2\alpha$ so that the entire sensitivity curve is obtained. The parameter range above the sensitivity curve will be ruled out at 3σ if no deviation to the standard neutrino flavor transition mechanism is observed.

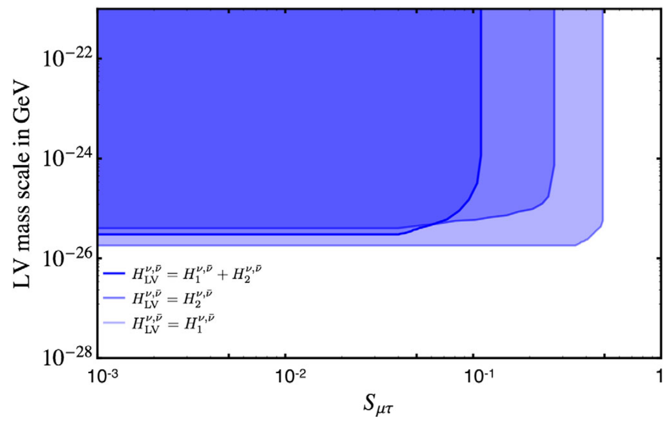

FIG. 2. The sensitivity of IceCube-Gen2 to the LV mass scale as a function of $\mu-\tau$ symmetry-breaking parameter $S_{\mu\tau}$. The parameter range above each sensitivity curve will be ruled out at 3σ if no deviation to the standard flavor transition of neutrinos is observed. These excluded ranges are obtained assuming the flavor fraction of astrophysical neutrinos from each source is $(1/3, 2/3, 0)$ for all neutrino energies beyond 100 TeV threshold. The LV mass scales for $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_1^{\nu,\bar{\nu}}$ and $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_2^{\nu,\bar{\nu}}$ are M and M' defined in Eqs. (11) and (12), respectively, while $S_{\mu\tau}$ for these two cases are $\sin^2 2\alpha$ and $\sin^2 2\rho$, respectively. The LV mass scale for $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_1^{\nu,\bar{\nu}} + H_2^{\nu,\bar{\nu}}$ is M under the assumption $M = M'$, and $S_{\mu\tau}$ for this case is $\sin 2\alpha \times \sin 2\rho$.

We note that the $\mu-\tau$ symmetry limit in $H_1^{\nu,\bar{\nu}}$ corresponds to $\sin^2 2\alpha = 1$ while the maximum breaking corresponds to $\sin^2 2\alpha = 0$. This can be seen from the matrix structure given by Eq. (11) or the neutrino flavor transition probabilities resulting from the Hamiltonian $H_1^{\nu,\bar{\nu}}$. For the latter we found $(P_{e\mu} - P_{e\tau}) = 0$, $(P_{\mu\mu} - P_{\mu\tau}) = 1 - \sin^2 2\alpha$, and $(P_{\mu\tau} - P_{\tau\tau}) = -1 + \sin^2 2\alpha$. It is clear that $\sin^2 2\alpha$ indeed determines the above $\mu-\tau$ symmetry breaking effects in neutrino flavor transition probabilities. For $0 \le \sin^2 2\alpha \le 0.35$, the sensitivity of IceCube-Gen2 to M is about 2×10^{-26} GeV. The sensitivity to M diminishes for $\sin^2 2\alpha > 0.46$ ($\sin 2\alpha > 0.68$). In our numerical studies, the neutrino mixing parameters in $H_{\mathrm{SM}}^{\nu,\bar{\nu}}$ are taken as the best fit values given in [50]. This will be our choice for neutrino mixing parameters throughout the rest of the paper. We also vary each neutrino mixing parameter over 1σ range to see the effect. No appreciable effect in the sensitivity to M is found. We note that the current SK 95% C.L. limits on the related matrix elements are $\mathrm{Re}(a_{\mu\tau}^T) < 6.5 \times 10^{-24}$ GeV and $\mathrm{Im}(a_{\mu\tau}^T) < 5.1 \times 10^{-24}$ GeV [22]. It is clear that the expected bounds by IceCube-Gen2 shall improve the current bounds by more than two orders of magnitudes provided $\sin 2\alpha < 0.68$. Particularly, the IceCube Gen2 sensitivity presented here is at 3σ C.L.

B. $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_2^{\nu,\bar{\nu}}$

For $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_2^{\nu,\bar{\nu}}$, we can write

$$H_2^{\nu} = M' \begin{pmatrix} 0 & e^{i\sigma}\cos\rho & e^{i\lambda}\sin\rho \\ e^{-i\sigma}\cos\rho & 0 & 0 \\ e^{-i\lambda}\sin\rho & 0 & 0 \end{pmatrix}, \quad (12)$$

where $M' = \sqrt{a_{e\mu}^T a_{e\mu}^{T*} + a_{e\tau}^T a_{e\tau}^{T*}}$, $\cos\rho = |a_{e\mu}^T|/M'$, $\sin\rho = |a_{e\tau}^T|/M'$, and σ and λ are phases of $a_{e\mu}^T$ and $a_{e\tau}^T$, respectively. The Hamiltonian $H_2^{\bar{\nu}}$ can be inferred from H_2^{ν} by the replacements $M' \to -M'$, $\sigma \to -\sigma$, and $\lambda \to -\lambda$. Since both $\cos\rho$ and $\sin\rho$ are positive by definition, the angle ρ is between 0 and $\pi/2$. Taking into account the total Hamiltonian, $H^{\nu,\bar{\nu}} = H_{\mathrm{SM}}^{\nu,\bar{\nu}} + H_2^{\nu,\bar{\nu}}$, one can predict the neutrino flavor fraction on Earth assuming the initial neutrino flavor fraction at the source is $(1/3, 2/3, 0)$.

Given the IceCube-Gen2 sensitivity shown in Fig. 1, we obtain the expected constraints on the LV mass scale M' as a function of mixing angle ρ with the phases σ and λ varied between 0 and 2π. The sensitivity to M' is shown in that part of Fig. 2 labeled by $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_2^{\nu,\bar{\nu}}$. We have varied each neutrino mixing parameter over 1σ range and no appreciable effect on the sensitivity to M' is found. The parameter $S_{\mu\tau}$ that characterizes the degree of $\mu-\tau$ symmetry breaking in $H_2^{\nu,\bar{\nu}}$ is $\sin^2 2\rho$. The $\mu-\tau$ symmetry limit corresponds to $\sin^2 2\rho = 1$, i.e., $\rho = \pi/4$. On the other hand, the maximum breaking corresponds to $\sin^2 2\rho = 0$, i.e., $\rho = 0$ or $\pi/2$. This is seen from the matrix structure given by Eq. (12) or the neutrino flavor transition probabilities resulting from the Hamiltonian $H_2^{\nu,\bar{\nu}}$. For the latter one can show that the neutrino flavor transition probabilities depend on both $\sin 2\rho$ and $\cos 2\rho$. Hence a specific value of $S_{\mu\tau} \equiv \sin^2 2\rho$ corresponds to two different neutrino flavor transition probabilities distinguished by the sign of $\cos 2\rho$. In principle there are two sensitivity points for each $S_{\mu\tau}$ but we have chosen the more conservative one to plot the sensitivity curve.

For $0 \leq \sin^2 2\rho \leq 0.2$, the sensitivity of IceCube-Gen2 to M' varies slowly from 4×10^{-26} GeV to 7×10^{-26} GeV. In comparison, the current SK 95% C.L. limits on related matrix elements are $\mathrm{Re}(a_{e\mu}^T) < 1.8 \times 10^{-23}$ GeV, $\mathrm{Im}(a_{e\mu}^T) < 1.8 \times 10^{-23}$ GeV, $\mathrm{Re}(a_{e\tau}^T) < 4.1 \times 10^{-23}$ GeV and $\mathrm{Im}(a_{e\tau}^T) < 2.8 \times 10^{-23}$ GeV [22]. One can see that the expected bounds by IceCube-Gen2 shall improve the current bounds by more than two orders of magnitude provided $\sin^2 2\rho \leq 0.2$. The sensitivity to M' diminishes for $\sin^2 2\rho > 0.27$ ($\sin 2\rho > 0.52$).

C. $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_1^{\nu,\bar{\nu}} + H_2^{\nu,\bar{\nu}}$

For the general case with $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_1^{\nu,\bar{\nu}} + H_2^{\nu,\bar{\nu}}$, the mass scales M and M' of $H_1^{\nu,\bar{\nu}}$ and $H_2^{\nu,\bar{\nu}}$, respectively, are independent parameters. These two scales can be comparable or one of the scales is suppressed in comparison to the other. Since the latter scenario has already been discussed, we only focus on the former case. To simplify our discussions, we take $M = M'$. The sensitivity of IceCube-Gen2 to M is shown in the part of Fig. 2 labeled by $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = H_1^{\nu,\bar{\nu}} + H_2^{\nu,\bar{\nu}}$. The parameter $S_{\mu\tau}$ that characterizes the degree of $\mu-\tau$ symmetry breaking is $\sin 2\alpha \times \sin 2\rho$. For $\sin 2\alpha \times \sin 2\rho = 1$, one must have both $\sin 2\alpha$ and $\sin 2\rho$ equal to unity, i.e., the $\mu-\tau$ symmetry is respected in both $H_1^{\nu,\bar{\nu}}$ and $H_2^{\nu,\bar{\nu}}$. For $\sin 2\alpha \times \sin 2\rho = 0$, either $H_1^{\nu,\bar{\nu}}$ or $H_2^{\nu,\bar{\nu}}$ (or both) breaks $\mu-\tau$ symmetry maximally. The sensitivity of IceCube-Gen2 to M is 3×10^{-26} GeV for $0 \leq \sin 2\alpha \times \sin 2\rho \leq 0.04$. The sensitivity becomes 10^{-25} GeV for $\sin 2\alpha \times \sin 2\rho = 0.08$. All these sensitivities improve significantly from the current SK bounds. The sensitivity of IceCube-Gen2 to M diminishes for $\sin 2\alpha \times \sin 2\rho > 0.11$. We also vary the neutrino mixing parameter in 1σ range, and no appreciable effect on the sensitivity to M is found.

D. Sensitivities to $c_{\alpha\beta}^{TT}$

So far we have only discussed IceCube-Gen2 sensitivities to $a_{\alpha\beta}^T$. One can also study the sensitivities to parameters $c_{\alpha\beta}^{TT}$ by turning off $a_{\alpha\beta}^T$. Clearly $-4Ec_{\alpha\beta}^{TT}/3$ replaces $a_{\alpha\beta}^T$ when the latter is turned off. It should, however, be noted that, for the antineutrino case, $c_{\alpha\beta}^{TT}$ is changed into $c_{\alpha\beta}^{TT*}$, while $a_{\alpha\beta}^T$ is turned into $-a_{\alpha\beta}^{T*}$.

Following the previous treatment, one can also decompose the dimension-4, CPT-even LV Hamiltonian into two terms such that

$$\tilde{H}_1^{\nu} = -\frac{4E}{3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_{\mu\mu}^{TT} & c_{\mu\tau}^{TT} \\ 0 & c_{\mu\tau}^{TT*} & c_{\tau\tau}^{TT} \end{pmatrix}, \quad (13)$$

and

$$\tilde{H}_2^{\nu} = -\frac{4E}{3} \begin{pmatrix} 0 & c_{e\mu}^{TT} & c_{e\tau}^{TT} \\ c_{e\mu}^{TT*} & 0 & 0 \\ c_{e\tau}^{TT*} & 0 & 0 \end{pmatrix}, \quad (14)$$

where we have used $\tilde{H}_{1,2}^{\nu,\bar{\nu}}$ to denote the CPT-even LV Hamiltonian. The LV Hamiltonian for antineutrinos can be obtained by taking complex conjugates. Analogous to our definitions of M and M' from $a_{\alpha\beta}^T$, we can define dimensionless parameters $W \equiv \sqrt{(c_{\tau\tau}^{TT} - c_{\mu\mu}^{TT})^2 + 4c_{\mu\tau}^{TT} c_{\mu\tau}^{TT*}}/2$ and $W' \equiv \sqrt{c_{e\mu}^{TT} c_{e\mu}^{TT*} + c_{e\tau}^{TT} c_{e\tau}^{TT*}}$, respectively. Let us consider the full LV Hamiltonian $H_{\mathrm{LV}}^{\nu,\bar{\nu}} = \tilde{H}_1^{\nu,\bar{\nu}} + \tilde{H}_2^{\nu,\bar{\nu}}$ and take $W = W'$. The sensitivity of IceCube-Gen2 to W is shown in Fig. 3. We have taken $\sin 2\eta \times \sin 2\xi$ as the parameter to characterize the degree of $\mu-\tau$ symmetry breaking, with

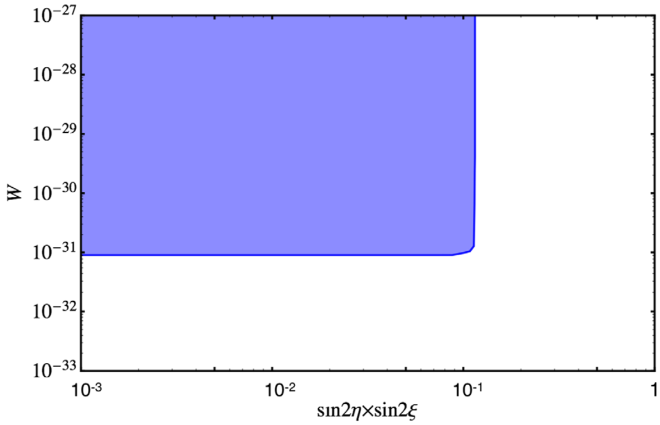

FIG. 3. The sensitivity of IceCube-Gen2 to the LV scale W ($W = W'$) of the Hamiltonian $\tilde{H}_1^{\nu,\bar{\nu}} + \tilde{H}_2^{\nu,\bar{\nu}}$ as a function of $\mu - \tau$ symmetry-breaking parameter $\sin 2\eta \times \sin 2\xi$. This sensitivity is obtained assuming the flavor fraction of astrophysical neutrinos from each source is $(1/3, 2/3, 0)$ for all neutrino energies beyond 100 TeV threshold. The parameter range above the sensitivity curve will be ruled out at 3σ if no deviation to the standard flavor transition of neutrinos is observed.

$\sin 2\eta = 2|c_{\mu\tau}^{TT}|/\sqrt{(c_{\tau\tau}^{TT} - c_{\mu\mu}^{TT})^2 + 4c_{\mu\tau}^{TT} c_{\mu\tau}^{TT*}}$ and $\sin\xi = |c_{e\tau}^{TT}|/W'$. Furthermore, we also take $E = 100$ TeV in $H_{\mathrm{LV}}^{\nu,\bar{\nu}}$ for simplicity. The sensitivity of IceCube-Gen2 to W is about 10^{-31} for $0 \leq \sin 2\eta \times \sin 2\xi \leq 0.12$. Such a sensitivity shall improve significantly from the current SK 95% C.L. limits, $\mathrm{Re}(c_{\mu\tau}^{TT}) < 4.4 \times 10^{-27}$, $\mathrm{Im}(c_{\mu\tau}^{TT}) < 4.2 \times 10^{-27}$, $\mathrm{Re}(c_{e\mu}^{TT}) < 8.0 \times 10^{-27}$, $\mathrm{Im}(c_{e\mu}^{TT}) < 8.0 \times 10^{-27}$, and much less stringent constraints on $c_{e\tau}^{TT}$. The sensitivity curve rises up immediately for $\sin 2\eta \times \sin 2\xi > 0.12$. This behavior is quite distinct from the behavior of the sensitivity curve in Fig. 2, which rises mildly in the range $0.04 \leq \sin 2\alpha \times \sin 2\rho \leq 0.08$ before its sharp rise at $\sin 2\alpha \times \sin 2\rho = 0.11$. We attribute the shape difference between the two sensitivity curves to the sign difference between the $a_{\alpha\beta}^T$ and $c_{\alpha\beta}^{TT}$ terms. To see this, we change the sign of the $c_{\alpha\beta}^{TT}$ ($c_{\alpha\beta}^{TT*}$) terms in the neutrino sector while keeping the sign of $c_{\alpha\beta}^{TT*}$ ($c_{\alpha\beta}^{TT}$) in the antineutrino sector unchanged. It is found that the shape ofthe sensitivity curve in Fig. 3 is completely identical to the shape of the sensitivity curve in Fig. 2, as it should be according to Eqs. (4) and (5).

IV. DISCUSSIONS AND CONCLUSIONS

In this paper, we discuss the sensitivities of future IceCube-Gen2 to Lorentz-violation parameters in the neutrino sector. We consider the effects of a Lorentz-violating Hamiltonian on the flavor transitions of astrophysical neutrinos coming from the pion source produced by pp collisions. In such a case, there are equal numbers of neutrinos and antineutrinos produced with the flavor fraction $(1/3, 2/3, 0)$ at the source for both neutrinos and antineutrinos. We have shown that the flavor fraction of such neutrinos as they arrive at Earth is $(1/3, 1/3, 1/3)$ if the neutrino Hamiltonian respects $\mu - \tau$ symmetry. The deviation to such a flavor fraction is therefore controlled by the breaking of $\mu - \tau$ symmetry in the neutrino Hamiltonian. For both a CPT-odd and a CPT-even LV Hamiltonian, we decompose the LV Hamiltonian into two matrix structures as shown in Eqs. (8), (9), (13), and (14). For each matrix structure we define the parameter that characterizes the degree of $\mu - \tau$ symmetry breaking and the scale of the matrix to be probed by the measurement of astrophysical neutrino flavor fractions.

Since the neutrino Hamiltonian in the standard model is approximately $\mu - \tau$ symmetric, the effect from the new physics Hamiltonian is important only when this Hamiltonian significantly breaks the $\mu - \tau$ symmetry. Taking Fig. 2 as an example, the LV Hamiltonian $H_1^{\nu,\bar{\nu}}$ breaks the $\mu - \tau$ symmetry significantly for $\sin^2 2\alpha \leq 0.46$ ($\sin 2\alpha \leq 0.68$) such that the expected constraint on the LV mass scale M by IceCube-Gen2 is stringent. It is of interest to see how restricted the parameter range $0 \leq \sin 2\alpha \leq 0.68$ is. Without specific preference to the detailed structure of $H_1^{\nu,\bar{\nu}}$, one can assume the angle α to be uniformly distributed from 0 to $\pi/2$ for a fixed LV mass scale M. The condition $0 \leq \sin 2\alpha \leq 0.68$ requires either $0 \leq 2\alpha \leq 0.75$ or $\pi - 0.75 \leq 2\alpha \leq \pi$. Such a range for α occupies $1.5/\pi \equiv 48\%$ of the total parameter space for α. For $H_2^{\nu,\bar{\nu}}$, the LV mass scale M' is testable for the parameter range $0 \leq \sin 2\rho \leq 0.52$. Assuming ρ is uniformly distributed between 0 and $\pi/2$, the range for ρ required by the above condition occupies about 35% of the total parameter space for ρ. Finally, for the case of the full LV Hamiltonian with $M = M'$, the LV mass scale M is testable in the parameter range $\sin 2\alpha \times \sin 2\rho \leq 0.11$. This is 21% of the total parameter space of α and ρ evaluated by a simple Monte Carlo. In the case of the CPT-even LV Hamiltonian, the dimensionless LV scale W ($W = W'$) of $\tilde{H}_1^{\nu,\bar{\nu}} + \tilde{H}_2^{\nu,\bar{\nu}}$ is testable for $\sin 2\eta \times \sin 2\xi \leq 0.12$. Clearly, the percentage of total parameter space of η and ξ that satisfies this condition is also around 20%.

In summary, we have taken a phenomenological approach that incorporates all LV effects in the neutrino sector with a set of local operators [3–7]. We only focus on the isotropic LV effects [10] so that the structure of the LV Hamiltonian is given by Eqs. (4) and (5). We have worked out the sensitivities of IceCube-Gen2 to the CPT-odd LV parameter $a_{\alpha\beta}^T$ originated from a dimension-3 operator and the CPT-even LV parameters $c_{\alpha\beta}^{TT}$ originated from dimension-4 operators. We have shown that the expected IceCube-Gen2 sensitivities to LV mass scales can improve the current SK bounds [22] by at least two orders of magnitude for sufficiently large $\mu - \tau$ symmetry-breaking effects in the LV Hamiltonian. We reiterate that our results are based upon the assumption that all sources of astrophysical neutrinos have an energy-independent flavor fraction for neutrinos at $(1/3, 2/3, 0)$. It is worthwhile to pursue further studies with both the energy dependence of the neutrino flavor fraction and the variations of neutrino flavor fractions among different sources taken into account.

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Note added.—Recently, we became aware of the newest IceCube analysis on Lorentz-violation effects in the neutrino sector using atmospheric neutrino data [54], which sets 99% C.L. bounds on $\mathrm{Re}(a^T_{\mu\tau})$ and $\mathrm{Im}(a^T_{\mu\tau})$ at 2.9×10^{-24} GeV and 99% C.L. bounds on $\mathrm{Re}(c^{TT}_{\mu\tau})$ and $\mathrm{Im}(c^{TT}_{\mu\tau})$ at 3.9×10^{-28}.

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