An Empirical Analysis of the Relationship Between the Hedge Ratio and Hedging Horizon: A Simultaneous Estimation of the Short- and Long-Run Hedge Ratios

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This article analyzes the effects of the length of hedging horizon on the optimal hedge ratio and hedging effectiveness using 9 different hedging horizons and 25 different commodities. We discuss the concept of short- and long-run hedge ratios and propose a technique to simultaneously

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estimate them. The empirical results indicate that the short-run hedge ratios are significantly less than 1 and increase with the length of hedging horizon. We also find that hedging effectiveness increases with the length of hedging horizon. However, the long-run hedge ratio is found to be close to the naïve hedge ratio of unity. This implies that, if the hedging horizon is long, then the naïve hedge ratio is close to the optimum hedge ratio.
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INTRODUCTION

One of the best uses of derivative securities, such as futures contracts, is in hedging. In the past, both academicians and practitioners have shown great interest in the issue of hedging with futures, which is evident from a large number of articles written in this area. Most of the studies on the hedge ratio deal with either (a) the derivation of the optimal hedge ratio based on different objective functions, or (b) the empirical estimation of the optimal hedge ratio. Some of the derivations of the optimal hedge ratio are based on the minimization of return variance or maximization of the expected utility. Other derivations of the optimal hedge ratio are based on the mean-Gini coefficient and generalized semivariance. A brief discussion on this can be found in Chen, Lee, and Shrestha (2001).

Studies that deal with the empirical estimation of the optimal hedge ratio employ many different techniques, ranging from simple to complex ones. For example, some of the studies use such a simple method as the ordinary least-squares (OLS) technique (e.g., see Benet, 1992; Ederington, 1979; Malliaris & Urrutia, 1991). However, others use more complex methods such as the conditional heteroscedastic (ARCH or GARCH) method (e.g., see Baillie & Myers, 1991; Cecchetti, Cumby, & Figlewski, 1988; Sephton, 1993), the random coefficient method (e.g., see Grammatikos & Saunders, 1983), the co-integration method (e.g., see Chou, Fan, & Lee, 1996; Geppert, 1995; Ghosh, 1993; Lien & Luo, 1993), and the co-integration–heteroscedastic method (e.g., see Kroner & Sultan, 1993).

Most of the empirical studies, however, ignore the effect of hedging horizon length on the optimal hedge ratio and hedging effectiveness.[1] A few studies that consider the effect of the length of hedging horizon include Ederington (1979), Hill and Schneeweis (1982), Malliaris and Urrutia (1991), Benet (1992), and Geppert (1995). These studies find

[1]Lee and Leuthold (1983) found that return characteristics depend on the investment horizon. Therefore, we expect that the investment horizon will have an impact on the optimal hedge ratio as well.

that in-sample hedging effectiveness tends to increase as the investment horizon lengthens. However, all of these studies, except Geppert (1995), consider 2–3 different hedging horizon lengths, whereas only Geppert (1995) considers 12 different hedging horizon lengths.[2]

From the estimates of the optimal hedge ratios reported in these studies, we can see that the optimal hedge ratio tends to increase with the length of hedging horizon. It is interesting to note that, even though all of these studies report the optimal hedge ratios for different hedging horizons, only Geppert (1995) analyzes the relationship between the optimal hedge ratio and the length of hedging horizon. It is important to note that all these studies consider the minimum variance (MV) hedge ratio, instead of the other hedge ratios based on expected utility, extended mean-Gini coefficient, and generalized semivariance.

This article examines the effects of hedging horizon length on the optimal hedge ratio and hedging effectiveness in greater detail, using 25 different futures contracts and 9 different hedging horizons.[3] Based on the relationship between the hedge ratio and the length of hedging horizon, we discuss the concepts of short- and long-run hedge ratios and propose a method that can be used to simultaneously estimate the short- and long-run hedge ratios. We consider only the MV hedge ratio based on the considerations as discussed below. First, as mentioned above, most of the existing studies analyze the MV hedge ratio. In order to compare our results with those in existing studies, we also consider the MV hedge ratio. Second, the MV hedge ratio is the most heavily used, analyzed, and discussed hedge ratio. Finally, it can be shown that, under some normality and martingale conditions, most of the hedge ratios based on other criteria (e.g., expected utility, extended mean-Gini coefficient, and generalized semivariance) converge to the MV hedge ratio.[4]

It is important to note that our analysis differs from the one done by Geppert (1995) (hereafter referred to as JG), in that the model used by JG requires both the spot and futures prices to have a single unit root. Therefore, the hedge ratio derived by JG is valid only if the unit-root condition is satisfied. However, we find that 14 out of 25 different

[2]Actually, the analytical expression for the optimal hedge ratio [equation (8) in Geppert (1995)] can be used to obtain the optimal hedge ratio for any hedging horizon length once the required parameters are estimated. We will describe the methodology proposed by Geppert (1995) later on in the article.

[3]In the empirical analysis, we use nearest-to-maturity futures contracts (with a rollover on the first day of the contract month) and ignore the impact of maturity on the estimate of the hedge ratio. However, Lee, Budnys, and Lin (1987) find that the optimal hedge ratio increases as the futures contracts approach maturity.

[4]See Chen, Lee, and Shrestha (2001) for a discussion on this issue.

commodities considered in this article do not satisfy this condition. The method used in the article, on the other hand, is valid when the prices are unit-root processes, as well as when the prices are stationary processes. Therefore, the empirical results obtained in this article, which are consistent with the results obtained by JG, should complement the results obtained by JG.

It is also important to note that it is possible, in an empirical analysis, to find one of the futures price and spot price series to be a unit-root series and the other series to be a stationary series. In this case there cannot be a stationary relationship between the two series, which essentially implies that one cannot use one series (e.g., futures prices) to hedge the risk associated with the other series (e.g., spot prices). However, because the futures and spot prices are related through a no-arbitrage condition, such a situation cannot be theoretically acceptable. Occurrences of such a situation in empirical analyses could result from the low power of the conventional unit-root tests. If such situations do occur, then one is advised to use different unit-root tests with more power.[5]

In this study we discover that, for almost all of the 25 futures contracts, the MV hedge ratios are less than 1 (i.e., the naïve hedge ratio) and also that the hedge ratios increase with the length of hedging horizon.[6] Furthermore, it is found that in-sample hedging effectiveness increases with the length of hedging horizon, which is consistent with existing findings. For most of the 25 futures contracts, the long-run hedge ratios are close to the naïve hedge ratio, also consistent with the results obtained by JG, who analyzes 5 futures contracts (SP500, German mark, Japanese yen, Swiss franc, and Municipal Bond Index). This suggests that, if the hedging horizon is long, then there is no need to estimate the optimal hedge ratio, because the naïve hedge ratio of 1 will be optimal.[7] This implication is very important, because the naïve hedge ratio does not require any data collection and estimation.

[5]We would like to thank an anonymous referee for pointing out this situation to us. For detailed information on different types of unit-root tests and the issues regarding the low power of the unit-root tests, see Maddala and Kim (1998).

[6]There are eight contracts for which the hedge ratios are greater than one for some of the hedging horizons considered. For almost all of these contracts, the hedge ratios decrease toward 1 except for soybean oil and silver.

[7]The optimal long-run hedge ratios are estimated based on the coefficients of level of prices instead of the coefficients of changes in prices, which are closely related to coefficients of the error-correction term. Therefore, the long-run hedge ratio does not correspond to any specific hedging horizon. Instead, we can only say that the longer the hedging horizon is, the more appropriate will be the use of the long-run hedge ratios. Because most of the short-run hedge ratios (for a hedging horizon up to 8 weeks) are less than the long-run hedge ratios and are approaching 1, we can empirically say that the long run is longer than 8 weeks.

The remainder of this article is organized as follows. In the next section the methodology is discussed. We then present the empirical results. The article concludes in the final section.

METHODOLOGY

The basic concept of hedging is to eliminate (or reduce) fluctuations in the value of a spot position by including futures contracts in the portfolio. Specifically, consider a portfolio consisting of C_s units of a long spot position and C_f units of a short futures position. Let S_t and F_t denote the spot and futures prices at the end of period t, respectively. The change in the value of the hedged portfolio over the period ΔV_H is then given by

$$\Delta V_H = C_s\,\Delta S_t - C_f\,\Delta F_t \tag{1}$$

where $\Delta S_t = S_t - S_{t-1}$ and $\Delta F_t = F_t - F_{t-1}$.

The MV hedge ratio is obtained by minimizing the variance of ΔV_H and is given by

$$H = \frac{C_f}{C_s} = \frac{\mathrm{Cov}(\Delta S, \Delta F)}{\mathrm{Var}(\Delta F)} \tag{2}$$

The conventional approach to estimate the MV hedge ratio involves an estimation of the regression of the changes in spot prices on the changes in futures prices with the use of the OLS regression technique. Specifically, the regression equation can be written as

$$\Delta S_t = \alpha + \beta\,\Delta F_t + e_t \tag{3}$$

where the estimate of the MV hedge ratio H is given by the estimate of β.

Before we estimate Equation (3), we need to decide on the differencing interval or data frequency. For example, if we use weekly data, then the differencing interval is 1 week. In this case, ΔS_t and ΔF_t represent weekly spot and futures price changes, respectively. Because in hedging we are concerned with the change in the value of the portfolio from the beginning to the end of the hedging horizon, the differencing interval should be equal to the length of the hedging horizon. For example, if the length of the hedging horizon is 1 week, then the differencing interval should be 1 week; that is, weekly data should be used.

The issue of appropriate data frequency or differencing interval for a given length of hedging horizon can be discussed in terms of the way that the MV hedge ratio is derived. In order to simplify the discussion, we assume that one period is equal to 1 week and the length of the hedging horizon (hedging period) consists of an integer number of periods (weeks). As discussed earlier, the MV hedge ratio is obtained by minimizing the variance of the change in the hedged portfolio's value, where the change in the portfolio's value is given by the difference between the values of the hedged portfolio at the beginning and end of the hedging period. For example, if the length of the hedging horizon (or hedging period) is k periods (or k weeks), then the objective should be to minimize the variance of the k-period price change. Similarly, in the optimal hedge ratio formula given by Equation (2), the price changes (ΔS_t and ΔF_t) should involve k-period differencing for a k-period hedging horizon.

Therefore, when using the regression Equation (3) to estimate the optimal hedge ratio, we need to use k-period differencing for a k-period hedging horizon. In other words, the frequency of data used must correspond to the length of the hedging horizon. In order to explain the effect of a mismatch between the hedging period and differencing period, suppose that we estimate the hedge ratio using one-period differencing where the length of the hedging horizon is four periods. As described above, since the hedging horizon is four periods, the objective is to minimize the variance of the change in the value of the portfolio over the 4-week period. The change in the portfolio's value over the 4-week period is given by

$$\Delta_4 V_H = C_s \Delta_4 S_t + C_f \Delta_4 F_t \tag{4}$$

where $\Delta_4 S_t = (1 - L^4)S_t = S_t - S_{t-4}$ and $\Delta_4 F_t = (1 - L^4)F_t = F_t - F_{t-4}$. In this case the optimal hedge ratio is given by [see Equation (2)]

$$H = \frac{C_f}{C_s} = \frac{\mathrm{Cov}(\Delta_4 S_t, \Delta_4 F_t)}{\mathrm{Var}(\Delta_4 F_t)} = \frac{\mathrm{Cov}(S_t - S_{t-4}, F_t - F_{t-4})}{\mathrm{Var}(F_t - F_{t-4})} \tag{5}$$

which can be obtained by estimating regression Equation (3) with the use of four-period (4-week) differencing. However, if one-period (1-week) differencing is used in the regression, then the regression will provide the estimate for the following parameter:

$$a = \frac{\mathrm{Cov}(\Delta S_t, \Delta F_t)}{\mathrm{Var}(\Delta F_t)} = \frac{\mathrm{Cov}(S_t - S_{t-1}, F_t - F_{t-1})}{\mathrm{Var}(F_t - F_{t-1})} \tag{6}$$

instead of the hedge ratio given by Equation (5). Note that Equation (6) is the optimal hedge ratio if the hedging horizon is one period.

In general, the two hedge ratios estimated by Equations (5) and (6) will not be the same. However, the conditions under which the two hedge ratios will be the same can be established. The four-period price changes can be expressed in terms of one-period price changes as follows:

$$\Delta_4 S_t = \Delta S_t + \Delta S_{t-1} + \Delta S_{t-2} + \Delta S_{t-3} \quad \text{and}$$
$$\Delta_4 F_t = \Delta F_t + \Delta F_{t-1} + \Delta F_{t-2} + \Delta F_{t-3}$$

Therefore, if the one-period price changes are serially independent and stationary, then we have

$$\text{Cov}(\Delta_4 S_t, \Delta_4 F_t) = 4\ \text{Cov}(\Delta S_t, \Delta F_t) \quad \text{and} \quad \text{Var}(\Delta_4 F_t) = 4\ \text{Var}(\Delta F_t) \quad (7)$$

In this case, the two hedge ratios will be the same.[8] However, if the conditions of serial independence and stationarity are not satisfied, then the two hedge ratios will be different. Therefore, in general it is inappropriate to use a one-period price change (1-week differencing) to estimate the optimal hedge ratio for a four-period hedging horizon.

The regression method used by JG is consistent with our discussion of the relationship between the length of hedging period and data frequency used in the estimation of the optimal hedge ratio. For example, JG uses *k*-period differencing for a *k*-period hedging horizon in estimating the regression-based MV hedge ratio. Because JG uses approximately 13 months of data for estimating the hedge ratio, he employs overlapping differencing in order to minimize the reduction in sample size caused by multiperiod differencing. However, this will lead to correlated observations, instead of independent observations, and requires the use of regression with autocorrelated errors in the estimation of the hedge ratio. Because our sample size is quite large (with a minimum sample size of 1783 and a maximum sample size of 4956), we use nonoverlapping differencing in the article.

Instead of using overlapping differencing, we can solve the problem of reduction in the sample size associated with a longer hedging horizon by assuming a specific data-generating process where the hedge ratio for various hedging horizons can be expressed in terms of a few parameters that can easily be estimated. One such technique is suggested by JG and we briefly describe his method. Suppose that the spot and futures prices are both unit-root processes. The market efficiency then implies that the two series would be co-integrated. In this case, the futures and spot

[8]This result can be extended to show that, in general, if the price changes are serially independent and stationary, then the frequency of data is irrelevant. In other words, if these conditions hold, then the frequency of data does not need to match the length of the hedging horizon.

prices can be described by the following processes (see Hylleberg & Mizon, 1989; Stock & Watson, 1988):

$$S_t = A_1 P_t + A_2 \tau_t \tag{8a}$$

$$F_t = B_1 P_t + B_2 \tau_t \tag{8b}$$

$$P_t = P_{t-1} + w_t \tag{8c}$$

$$\tau_t = \alpha_1 \tau_{t-1} + \nu_t, \qquad 0 \leq |\alpha_1| < 1 \tag{8d}$$

where P_t and τ_t are permanent and transitory factors that drive the spot and futures prices, and w_t and ν_t are white-noise processes. Note that P_t follows a pure random walk process and τ_t follows a stationary process. The MV hedge ratio for a k-period hedging horizon is hence given by [see Geppert (1995)].

$$H = \frac{A_1 B_1 k \sigma_w^2 + 2A_2 B_2[(1 - \alpha^k)/(1 - \alpha^2)]\sigma_\nu^2}{B_1^2 k \sigma_w^2 + 2B_2^2[(1 - \alpha^k)/(1 - \alpha^2)]\sigma_\nu^2} \tag{9}$$

Based on Equation (9), the long-run hedge ratio (the hedge ratio is where the length of hedging horizon goes to infinity) is given by A_1/B_1. It is important to note that the long-run hedge ratio will be equal to the naïve hedge ratio if $A_1 = B_1$. Therefore, whether the long-run hedge ratio is equal to the naïve hedge ratio is an empirical question. One of the advantages of using Equation (9), instead of a regression, is that it avoids the problem of reduction in the sample size associated with non-overlapping differencing.

It is important to note that the derivation of the hedge ratio given by Equation (9) makes some assumptions. First, the spot and futures prices must be co-integrated, which also require that both prices be unit-root processes. Second, the model implicitly assumes that both the spot and futures price changes have zero expected value, which may not be true in some cases, especially in the case of stock indices. Finally, such a model-based hedge ratio provides a good estimate of the hedge ratio so long as the model [system of Equations (8)] represents the true underlying data-generating process.

In this article we re-examine the regression Equation (3) and modify the equation so that the estimation can be improved. Note that regression Equation (3) represents a relationship between the changes in the series (ΔS_t and ΔF_t), rather than the series themselves (S_t and F_t). Let us consider the following relationship between the series themselves:

$$S_t = a + bF_t + u_t \tag{10}$$

As discussed in the Appendix, Equation (3) represents a short-run relationship between the spot and futures prices, whereas Equation (10) represents a long-run relationship. Therefore, we can estimate Equation (3) to obtain the short-run hedge ratio and Equation (10) to obtain the long-run hedge ratio.[9] Rather than running two separate regressions to estimate the short- and long-run hedge ratios, we suggest a joint estimation of the two hedge ratios using the following regression:

$$\Delta S_t = \alpha_1 + \alpha_2 S_{t-1} + \alpha_3 F_{t-1} + \beta \Delta F_t + \varepsilon_t \qquad (11)$$

It is important to note that Equation (11) is based on the simultaneous equation models considered by Hsiao (1997) and Pesaran (1997).[10] Furthermore, it is different from the two-step method used by Chou, Fan, and Lee (1996) in the sense that the estimation of Equation (11) is a single-step process. From Equation (11), the long-run hedge ratio is given by $-\alpha_3/\alpha_2$ and the short-run hedge ratio is given by β. The long-run hedge ratio given by $-\alpha_3/\alpha_2$ corresponds to the long-run hedge ratio of A_1/B_1 derived by JG. We expect the long-run hedge ratio to remain constant and the short-run hedge ratio to change and approach the long-run hedge ratio as we increase the length of hedging period. It would be interesting to see if the long-run hedge ratio is equal to the naïve hedge ratio of unity. It is important to note that Equation (11) is valid if both the spot and futures price series are stationary. It is also valid if both the series are unit-root processes and are co-integrated.

EMPIRICAL ANALYSIS

This article analyzes 25 different futures contracts where the futures prices are associated with nearest-to-maturity contracts. A list of the futures contracts, sample periods, and sample sizes are given in Table I. The data are obtained from Datastream. The futures contracts used in the study are nearest-to-maturity contracts. The futures contract is rolled over to the next contract on the first day of the contract month. In order to see the impact of the length of hedging horizon, various data

[9]Note that there are different short-run hedge ratios associated with different lengths of hedging horizons. Different short-run hedge ratios can be estimated with the use of data frequency that matches the length of hedging horizon.

[10]Equation (11) is a version of Equation (8) in Pesaran (1997) and of Equation (2.7) in Hsiao (1997). It is parametrized so as to be closely associated with the error-correction models (ECM) encountered in the vector autoregressive (VAR) models with co-integration. However, it is important to note that Equation (11) is different from the ECM models in that ΔF_t, instead of ΔF_{t-1}, appears on the right-hand side. Furthermore, in ECM there are two equations like Equation (11) (one with ΔS_t as the left-hand-side variable and the other with ΔF_t as the left-hand-side variable), whereas here we do not have two different equations.

TABLE I
Summary of 25 Futures Contracts

	Commodity	*Sample Period*	*Sample Size*
1	SP500	June 1, 1982–December 31, 1997	4066
2	TSE35	March 1, 1991–December 31, 1997	1783
3	Nikkei 225	September 5, 1988–December 31, 1997	2432
4	TOPIX	September 5, 1988–December 31, 1997	2432
5	FTSE100	May 3, 1984–December 31, 1997	3564
6	CAC40	March 1, 1989–December 31, 1997	2305
7	All ordinary	January 3, 1984–December 31, 1997	3651
8	Soybean oil	January 2, 1979–December 31, 1997	4956
9	Soybean	January 2, 1979–December 31, 1997	4956
10	Soy meal	January 2, 1979–December 31, 1997	4956
11	Corn	January 2, 1979–December 31, 1997	4956
12	Wheat	March 30, 1982–December 31, 1997	4111
13	Cotton	January 3, 1980–December 31, 1997	4694
14	Cocoa	November 1, 1983–December 31, 1997	3696
15	Coffee	January 2, 1979–December 31, 1997	4956
16	Pork belly	January 2, 1979–December 31, 1997	4956
17	Hogs	March 30, 1982–December 31, 1997	4111
18	Crude oil	April 4, 1983–December 31, 1997	3847
19	Silver	January 2, 1979–December 31, 1997	4956
20	Gold	January 2, 1979–December 31, 1997	4956
21	Japanese yen	January 2, 1986–December 31, 1997	3129
22	Deutsche mark	January 2, 1986–December 31, 1997	3129
23	Swiss franc	January 2, 1986–December 31, 1997	3129
24	British pound	January 2, 1986–December 31, 1997	3129
25	Canadian dollar	November 30, 1987–December 31, 1997	2632

Note. This table lists the commodities, sample periods, and sample sizes for the 25 different futures contracts used for empirical analyses in this study. The data are obtained from Datastream.

frequencies (ranging from daily to 8 week) are examined. The empirical results are presented below.

In-Sample Analysis

Because the method suggested by JG is applicable only if the futures and spot prices consist of a unit root, it would be interesting to see how many of the 25 commodities satisfy the unit-root condition. In order to simplify computation, we perform the Phillips–Perron (1988) unit-root test with weekly data at the 5% significance level. The results are summarized in Table II. It is important to note that for some contracts (such as Wheat, Coffee, and British pound), both the futures price and spot price series may not be significant at the 5% level. We consider the unit-root condition to be violated if either (a) the t tests for both series are negative and significant at the 5% level or (b) the t test for one of the series is negative and significant at the 5% level and the t test for the other series is

TABLE II
Results of the Unit-Root Tests on Futures and Spot Prices

		Phillips–Perron t Test	
Commodity	*Sample Size*	*Futures Price*	*Spot Price*
SP500	785	2.945**	3.024**
TSE35	343	1.013	0.957
Nikkei 225	486	−1.119	−1.141
TOPIX	486	−1.158	−1.172
FTSE100	713	0.819	0.974
CAC40	461	−0.841	−0.714
All ordinary	730	−1.195	−1.138
Soybean oil	991	−3.245**	−3.106**
Soybean	991	−3.412**	−3.422**
Soy meal	991	−3.178**	−3.088**
Corn	991	−3.101**	−3.123**
Wheat	822	−2.730*	−3.272**
Cotton	939	−3.540**	−3.379**
Cocoa	739	−1.592	−1.688
Coffee	991	−3.033**	−2.687*
Pork belly	991	−3.546**	−4.377**
Hogs	822	−4.114**	−3.985**
Crude oil	769	−3.115**	−3.140**
Silver	991	−2.928**	−3.051**
Gold	991	−3.326**	−3.309**
Japanese yen	626	−2.276	−2.285
Deutsche mark	626	−2.917**	−2.911**
Swiss franc	626	−2.838*	−2.851*
British pound	626	−2.909**	−2.837*
Canadian dollar	526	−0.403	−0.306

Note. This table lists the results of the Phillips–Perron unit-root tests on the futures and spot prices based on the weekly data. The critical values at the 10% and 5% significance levels are −2.57 and −2.87, respectively. Double asterisks and asterisks represent 5% and 10% significance levels, respectively.

negative and significant at the 10% level.[11] Based on this criterion, 14 out of 25 commodities do not satisfy the unit-root condition.[12] Therefore, we cannot apply JG's method for these commodities. This shows that JG's unit-root model is not universally acceptable. However, the method used in this article will be applicable to all the commodities considered in the article.

The results for the OLS estimates of the MV hedge ratios obtained from Equation (3) for various data frequencies are shown in Table III. The hedge ratio changes as the length of return period changes. For

[11]This criterion is justified on the ground that, as explained in the introduction section, the futures and spot prices are expected to be either both stationary or both unit root and co-integrated.
[12]Note that the t test for the S&P 500 is significantly positive at the 5% level, indicating the possibility of a second unit root. We have performed the unit-root test on the first differenced series and find them to be stationary. This confirms that the S&P 500 futures and spot prices have a single unit root.

TABLE III

The OLS Estimates of the Minimum-Variance Hedge Ratios for Different Types of Futures Contracts

		Holding Period								
Commodity		*1 Day*	*1 Week*	*2 Week*	*3 Week*	*4 Week*	*5 Week*	*6 Week*	*7 Week*	*8 Week*
SP500	Hedge ratio	0.8314	0.8978	0.9489	0.9421	0.9880	0.9372	0.9711	0.9754	0.9950
	Adj. R^2	0.9176	0.9546	0.9716	0.9793	0.9792	0.9785	0.9850	0.9839	0.9915
TSE35	Hedge ratio	0.8202	0.9463	0.9636	0.9755	0.9744	0.9674	0.9894	1.0200	0.9910
	Adj. R^2	0.7607	0.9309	0.9385	0.9436	0.9341	0.9726	0.9488	0.9699	0.9821
Nikkei 225	Hedge ratio	0.9198	0.9840	0.9852	0.9593	0.9799	0.9562	0.9644	0.9621	0.9711
	Adj. R^2	0.8374	0.9410	0.9627	0.9703	0.9751	0.9753	0.9814	0.9926	0.9799
TOPIX	Hedge ratio	0.7761	0.9233	0.9565	0.9479	0.9563	0.9536	0.9473	0.9660	0.9898
	Adj. R^2	0.7587	0.9228	0.9543	0.9600	0.9746	0.9705	0.9783	0.9860	0.9831
FTSE100	Hedge ratio	0.7857	0.8650	0.8995	0.9148	0.9230	0.9329	0.9415	0.9175	0.9156
	Adj. R^2	0.8814	0.9520	0.9630	0.9642	0.9658	0.9707	0.9745	0.9751	0.9792
CAC40	Hedge ratio	0.8843	0.9405	0.9581	0.9538	0.9704	0.9518	0.9473	0.9510	0.9749
	Adj. R^2	0.9186	0.9662	0.9765	0.9769	0.9839	0.9825	0.9806	0.9812	0.9827
All ordinary	Hedge ratio	0.2970	0.7744	0.8254	0.8300	0.8687	0.8783	0.8527	0.9605	0.8341
	Adj. R^2	0.1708	0.8307	0.9020	0.9238	0.9443	0.9504	0.9509	0.9597	0.9443
Soybean oil	Hedge ratio	0.8741	0.9324	0.9593	0.9690	0.9990	1.0101	0.9962	1.0344	1.0502
	Adj. R^2	0.6528	0.8329	0.8672	0.8801	0.9118	0.9322	0.9182	0.9451	0.9418
Soybean	Hedge ratio	0.8677	0.9062	0.9089	0.8561	0.8856	0.9031	0.9113	0.9153	0.9208
	Adj. R^2	0.7445	0.8518	0.8566	0.8528	0.8880	0.8995	0.8993	0.9110	0.9269
Soy meal	Hedge ratio	0.8435	0.9052	0.9680	0.9704	0.9364	0.9572	0.9357	0.9535	0.9004
	Adj. R^2	0.6559	0.7860	0.8178	0.8074	0.7757	0.7978	0.8606	0.8488	0.8425
Corn	Hedge ratio	0.6873	0.8047	0.8374	0.7764	0.8397	0.8683	0.7999	0.8598	0.8798
	Adj. R^2	0.4865	0.6097	0.6040	0.6663	0.6817	0.6944	0.6617	0.7470	0.7109
Wheat	Hedge ratio	0.7200	0.8459	0.9264	0.8727	0.9661	0.7723	0.9819	0.7860	1.0625
	Adj. R^2	0.3907	0.6022	0.6936	0.6679	0.6561	0.6270	0.7199	0.6261	0.7235
Cotton	Hedge ratio	0.4720	0.5117	0.5352	0.5807	0.5916	0.5676	0.5681	0.5728	0.8537
	Adj. R^2	0.2669	0.3160	0.3602	0.3943	0.4189	0.3848	0.3790	0.4146	0.7905

Cocoa	Hedge ratio	0.8472	0.9301	0.9284	0.9176	0.9725	0.9036	0.9446	0.9211	0.9733
	Adj. R^2	0.4975	0.7929	0.8028	0.7980	0.8263	0.7799	0.7978	0.8254	0.7886
Coffee	Hedge Ratio	0.5114	0.6301	0.7242	0.8215	0.7568	0.8152	0.9025	0.7856	0.8942
	Adj. R^2	0.3303	0.4903	0.5555	0.7700	0.5618	0.7597	0.8833	0.7294	0.7807
Pork belly	Hedge Ratio	0.3743	0.7222	0.7481	0.6839	0.7966	0.7957	0.6571	0.7063	0.8108
	Adj. R^2	0.0762	0.2477	0.3227	0.3203	0.4800	0.4538	0.3158	0.3973	0.5176
Hogs	Hedge Ratio	0.1253	0.2634	0.3211	0.4120	0.4103	0.4423	0.5111	0.4178	0.3955
	Adj. R^2	0.0204	0.1349	0.2107	0.2869	0.2678	0.3548	0.4108	0.3307	0.3057
Crude oil	Hedge Ratio	0.8362	0.9435	0.9816	1.0047	0.9973	1.0104	1.0294	0.9912	0.9945
	Adj. R^2	0.5693	0.8588	0.9277	0.9422	0.9527	0.9692	0.9812	0.9856	0.9795
Silver	Hedge Ratio	0.4732	0.7725	0.9204	1.0856	0.8849	0.9499	1.1706	0.9151	1.0239
	Adj. R^2	0.1173	0.4144	0.5996	0.6724	0.8273	0.9255	0.9363	0.7173	0.9652
Gold	Hedge Ratio	0.4980	0.8662	0.9275	1.0146	0.9714	0.8586	0.9814	0.9574	0.9451
	Adj. R^2	0.2044	0.7943	0.8854	0.9512	0.9530	0.9513	0.9702	0.9825	0.9745
Japanese	Hedge Ratio	0.9179	0.9575	0.9797	0.9582	0.9704	0.9830	0.9844	0.9705	0.9745
yen	Adj. R^2	0.8781	0.9591	0.9686	0.9725	0.9834	0.9849	0.9830	0.9870	0.9914
Deutsche	Hedge Ratio	0.9317	0.9792	0.9922	0.9957	0.9943	0.9906	1.0096	0.9897	0.9988
mark	Adj. R^2	0.8950	0.9687	0.9798	0.9827	0.9864	0.9851	0.9879	0.9913	0.9929
Swiss franc	Hedge Ratio	0.9286	0.9757	0.9880	0.9841	0.9848	0.9882	0.9989	0.9846	0.9838
	Adj. R^2	0.8972	0.9734	0.9797	0.9835	0.9893	0.9863	0.9904	0.9924	0.9946
British pound	Hedge Ratio	0.8863	0.9240	0.9322	0.9441	0.9800	0.9673	0.9627	0.9783	0.9948
	Adj. R^2	0.8531	0.9176	0.9394	0.9468	0.9826	0.9783	0.9609	0.9851	0.9900
Canadian	Hedge Ratio	0.8293	0.9085	0.9323	0.9558	0.9391	0.9692	0.9272	0.9241	0.9118
dollar	Adj. R^2	0.8005	0.8919	0.9188	0.9273	0.9395	0.9652	0.9451	0.9564	0.9650

Note. This table presents the results for the OLS estimates of the minimum-variance hedge ratios obtained from Equation (3) for various holding-period returns for each of the futures contracts listed in Table I.

most stock index futures, the hedge ratio seems to approach the naïve hedge ratio of unity as the hedge period lengthens. However, for some other commodities (e.g., soy meal, hogs, and gold), there does not seem to be any trend towards the naïve hedge ratio. If we combine all the hedge ratios for all of the 25 contracts and 9 hedging horizons, then the mean hedge ratio equals 0.8748, with a standard deviation of 0.1649. This indicates that the short-run hedge ratio is significantly less than the naïve hedge ratio (with a *t* ratio equal to −11.385).

In order to test for the impact of the length of hedging horizon on the hedge ratio as well as hedging effectiveness, the following regressions are estimated using the hedge ratios and R^2 obtained from the estimation of regression Equation (3) for all 25 contracts and all 9 hedging horizons:[13]

$$\beta_i = \phi_0 + \phi_1 T_i + \varepsilon_i \tag{12}$$

$$R_i^2 = \gamma_0 + \gamma_1 T_i + \nu_i \tag{13}$$

where T_i represents the length of hedging horizon expressed in weeks. The results of the regression are presented in Panel A of Table IV. Because both the estimates of ϕ_1 and γ_1 are highly significant and positive, this implies that both the hedge ratio and hedging effectiveness increase with the length of hedging horizon.

The regression Equations (12) and (13), however, do not capture the long-run trend in the sense that positive values of ϕ_1 and γ_1 imply that the hedge ratio as well as hedging effectiveness do not converge to finite values as the length of hedging horizon approaches infinity. Therefore, the following two regressions are also estimated with the use of the same estimated hedge ratios and R^2:

$$\beta_i = \mu_0 + \frac{\mu_1 e^{\mu_2 T_i}}{1 + e^{\mu_2 T_i}} + \text{error} \tag{14}$$

$$R_i^2 = \varpi_0 + \frac{\varpi_1 e^{\varpi_2 T_i}}{1 + e^{\varpi_2 T_i}} + \text{error} \tag{15}$$

The results of the regression are presented in Panel B of Table IV. The long-run beta is obtained by letting T_i approach infinity in Equation (14), and thus the long-run beta (hedge ratio) is estimated by

[13]It is important to note that, for Equations (12) and (13) to be valid, and later for Equations (14) and (15), too, we need to assume a homogeneous behavior across the commodities with respect to the hedging effectiveness and hedge ratio. Such a strong assumption is required in order to increase the sample size by combining all the commodities.

TABLE IV
Effect of Hedging Horizon Length on the Hedge Ratio and Hedging Effectiveness

Panel A: Equations (12) and (13)				
Equation (12)	ϕ_0		ϕ_1	*Adjusted* R^2
	0.8032 (40.58)		0.0178 (4.28)	0.0760
Equation (13)	γ_0		γ_1	*Adjusted* R^2
	0.6957 (24.89)		0.0268 (4.56)	0.0852
Panel B: Equations (14) and (15)				
Equation (14)	μ_0	μ_1	μ_2	*Adjusted* R^2
	0.4552 (5.05)	0.4494 (4.93)	1.7551 (2.67)	0.1195
Equation (15)	ϖ_0	ϖ_1	ϖ_2	*Adjusted* R^2
	0.2379 (1.99)	0.6107 (5.03)	1.5042 (2.71)	0.1188

Note. This table presents the results for the impact of hedging horizon's length on the hedge ratio and hedging effectiveness. Panel A presents the results for regression Equations (12) and (13), and Panel B presents the results for regression Equations (14) and (15). The *t* values are in parentheses.

$\mu_0 + \mu_1$. The estimate of $\mu_0 + \mu_1$ is equal to 0.9046, and the standard error of $\mu_0 + \mu_1$ is estimated as 0.0127. Therefore, the long-run hedge ratio seems to be significantly less than the naïve hedge ratio. However, we will present a better way of estimating the long-run hedge ratio later.

In much the same way, we can estimate the long-run effectiveness of the hedge ratio using the estimate of $\varpi_0 + \varpi_1$. The estimate of $\varpi_0 + \varpi_1$ is equal to 0.8486, and its standard error is equal to 0.0187. This provides some evidence that, even in the long run, the effectiveness of the long-run hedge ratio will not approach one. This result is different from the analytical result shown by JG, where the degree of hedging effectiveness approaches unity as the hedging horizon approaches infinity. The difference in results could be due to the fact that the majority of the commodities considered here do not satisfy the unit-root condition assumed by JG.

It is important to note that Equation (15) is bounded below by $\varpi_0 + 0.5\varpi_1$ and bounded above by $\varpi_0 + \varpi_1$. Therefore, if the hedging horizon approaches 0 (instantaneous hedging), then the hedging effectiveness will be 54.33% ($\varpi_0 + 0.5\varpi_1 = 0.5433$). However, Equation (15) does not impose the restriction that the function is bounded by 0 and 1. Fortunately, in our empirical analysis, the estimates of the upper bound and the lower bound lie between 0 and 1. If such a condition is violated in the empirical analysis, one needs to look for another functional form that satisfies such a restriction.[14] We choose the function, because it is an improvement over Equation (13).

The empirical results so far are based on the estimation of Equation (3). As mentioned earlier, it is better to estimate Equation (11), instead of Equation (3), in order to estimate the optimal hedge ratio. If Equation (11) is a better specification compared to Equation (3), then the adjusted R^2 associated with Equation (11) should be higher compared to the one associated with Equation (3). The results of the estimation of Equation (11) are presented in Table V. Out of 225 adjusted R^2 (25 commodities with 9 hedging horizons), 211 adjusted R^2 reported in Table V are higher than the corresponding ones reported in Table III. The average adjusted R^2 associated with Equation (11) is 84.88 percent, whereas the average adjusted R^2 associated with Equation (3) is 80.33%. This clearly indicates that Equation (11) is a better way of estimating the short-run hedge ratio.

It is also important to note that, for each futures contract, the short-run hedge ratios are different for different hedging horizons. However, the estimates of the long-run hedge ratios should be close to each other, because we are estimating the same long-run hedge ratio regardless of the data frequencies (differencing period) being used. The long-run hedge ratios from Table V seem to be close to 1, with the average equal to 1.0073 and a standard deviation equal to 0.0608. Therefore, the estimates of the long-run hedge ratios are not significantly different from the naïve hedge ratio. This result on the long-run hedge ratio is consistent with the empirical result obtained by JG, indicating that if the hedging horizon is long, then the naïve technique can be quite effective. As for the short-run hedge ratios estimated from Equation (11) and reported in Table V, they are found to be significantly less than 1, with the average hedge ratio of 0.8875 and a standard deviation of 0.1618, which results in a t ratio of -11.433.

Even though the short-run hedge ratios are found to be significantly less than 1, there are some commodities for which the estimated

[14]This issue is similar to the one we face when estimating the variance or standard deviation parameter. It is better if we get the estimate of the variance parameter to be positive without having to impose the restriction in the estimation procedure that will guarantee the estimate to be positive.

TABLE V

Simultaneous Estimation of the Short- and Long-Run Hedge Ratios for Different Types of Futures Contracts

		Holding Period								
Commodity		*1 Day*	*1 Week*	*2 Week*	*3 Week*	*4 Week*	*5 Week*	*6 Week*	*7 Week*	*8 Week*
SP500	β	0.839	0.908	0.952	0.947	0.983	0.954	0.973	0.969	0.978
		(0.004)	(0.006)	(0.007)	(0.007)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)
	$-\alpha_3/\alpha_2$	0.995	0.995	0.995	0.996	0.995	0.997	0.996	0.996	0.995
	Adj. R^2	0.928	0.965	0.979	0.987	0.987	0.988	0.992	0.991	0.995
TSE35	β	0.841	0.961	0.983	0.996	1.002	0.988	1.000	0.993	0.967
		(0.010)	(0.012)	(0.015)	(0.016)	(0.018)	(0.014)	(0.021)	(0.019)	(0.012)
	$-\alpha_3/\alpha_2$	1.003	1.002	1.002	1.003	1.000	1.001	1.003	1.005	1.004
	Adj. R^2	0.799	0.949	0.963	0.970	0.972	0.987	0.976	0.986	0.995
Nikkei 225	β	0.926	0.986	0.988	0.972	0.991	0.974	0.974	0.967	1.003
		(0.008)	(0.010)	(0.011)	(0.011)	(0.011)	(0.012)	(0.012)	(0.009)	(0.014)
	$-\alpha_3/\alpha_2$	0.979	0.977	0.977	0.977	0.975	0.978	0.975	0.979	0.977
	Adj. R^2	0.856	0.954	0.972	0.981	0.985	0.986	0.988	0.994	0.989
TOPIX	β	0.793	0.937	0.965	0.967	0.968	0.968	0.964	0.970	1.002
		(0.008)	(0.011)	(0.012)	(0.013)	(0.012)	(0.013)	(0.013)	(0.012)	(0.014)
	$-\alpha_3/\alpha_2$	0.978	0.976	0.976	0.976	0.975	0.977	0.973	0.979	0.976
	Adj. R^2	0.791	0.940	0.965	0.974	0.983	0.983	0.986	0.990	0.989
FTSE100	β	0.789	0.874	0.911	0.927	0.936	0.956	0.949	0.937	0.937
		(0.005)	(0.007)	(0.009)	(0.010)	(0.011)	(0.012)	(0.012)	(0.012)	(0.013)
	$-\alpha_3/\alpha_2$	0.999	1.000	0.999	0.999	1.000	0.999	0.997	1.001	1.003
	Adj. R^2	0.890	0.958	0.969	0.972	0.975	0.980	0.981	0.984	0.985
CAC40	β	0.888	0.945	0.962	0.959	0.978	0.964	0.963	0.969	0.995
		(0.005)	(0.008)	(0.009)	(0.011)	(0.011)	(0.012)	(0.013)	(0.014)	(0.015)
	$-\alpha_3/\alpha_2$	1.010	1.010	1.012	1.013	1.010	1.011	1.014	1.019	1.004
	Adj. R^2	0.923	0.969	0.980	0.981	0.987	0.986	0.986	0.988	0.989

(*Continued*)

TABLE V

Simultaneous Estimation of the Short- and Long-Run Hedge Ratios for Different Types of Futures Contracts (*Continued*)

		Holding Period								
Commodity		*1 Day*	*1 Week*	*2 Week*	*3 Week*	*4 Week*	*5 Week*	*6 Week*	*7 Week*	*8 Week*
All ordinary	β	0.320	0.784	0.822	0.836	0.865	0.883	0.872	0.964	0.877
		(0.009)	(0.011)	(0.012)	(0.012)	(0.012)	(0.013)	(0.014)	(0.014)	(0.015)
	$-\alpha_3/\alpha_2$	0.987	0.987	0.989	0.989	0.988	0.988	0.992	0.990	0.988
	Adj. R^2	0.427	0.875	0.933	0.951	0.966	0.972	0.969	0.980	0.974
Soybean oil	β	0.876	0.936	0.965	0.976	1.004	1.016	1.005	1.040	1.053
		(0.009)	(0.013)	(0.017)	(0.019)	(0.020)	(0.019)	(0.023)	(0.021)	(0.023)
	$-\alpha_3/\alpha_2$	1.092	1.098	1.094	1.102	1.114	1.099	1.106	1.111	1.122
	Adj. R^2	0.658	0.839	0.874	0.889	0.918	0.938	0.928	0.951	0.948
Soybean	β	0.868	0.900	0.894	0.845	0.870	0.891	0.891	0.897	0.896
		(0.007)	(0.012)	(0.016)	(0.017)	(0.018)	(0.020)	(0.021)	(0.021)	(0.021)
	$-\alpha_3/\alpha_2$	0.968	0.979	0.989	0.980	0.987	0.983	0.994	0.984	0.986
	Adj. R^2	0.752	0.865	0.877	0.888	0.915	0.922	0.929	0.939	0.948
Soy meal	β	0.843	0.906	0.966	0.967	0.936	0.960	0.940	0.955	0.916
		(0.009)	(0.015)	(0.020)	(0.025)	(0.031)	(0.033)	(0.029)	(0.033)	(0.036)
	$-\alpha_3/\alpha_2$	1.041	1.048	1.051	1.047	1.041	1.026	1.038	1.032	1.150
	Adj. R^2	0.660	0.793	0.827	0.823	0.799	0.821	0.876	0.868	0.849
Corn	β	0.688	0.799	0.806	0.758	0.815	0.831	0.782	0.838	0.823
		(0.010)	(0.020)	(0.029)	(0.027)	(0.032)	(0.035)	(0.034)	(0.033)	(0.036)
	$-\alpha_3/\alpha_2$	1.070	1.083	1.082	1.077	1.067	1.092	1.067	1.073	1.076
	Adj. R^2	0.496	0.636	0.664	0.745	0.769	0.787	0.817	0.856	0.866
Wheat	β	0.723	0.844	0.914	0.854	0.912	0.766	0.913	0.754	0.926
		(0.014)	(0.024)	(0.030)	(0.036)	(0.047)	(0.043)	(0.050)	(0.050)	(0.064)
	$-\alpha_3/\alpha_2$	0.932	0.946	0.973	0.960	0.982	0.953	0.992	0.942	0.993
	Adj. R^2	0.401	0.622	0.719	0.710	0.711	0.707	0.777	0.731	0.796
Cotton	β	0.474	0.517	0.547	0.589	0.610	0.585	0.596	0.621	0.869
		(0.011)	(0.023)	(0.029)	(0.036)	(0.037)	(0.040)	(0.044)	(0.045)	(0.034)
	$-\alpha_3/\alpha_2$	1.043	1.047	1.052	1.047	1.045	1.063	1.044	1.046	1.074
	Adj. R^2	0.284	0.389	0.502	0.547	0.647	0.665	0.674	0.696	0.870

Cocoa	β	0.849	0.932	0.931	0.922	0.975	0.911	0.953	0.924	0.975
		(0.014)	(0.017)	(0.024)	(0.029)	(0.033)	(0.039)	(0.043)	(0.041)	(0.052)
	$-\alpha_3/\alpha_2$	0.998	0.994	0.992	1.001	1.001	1.011	0.982	1.017	1.003
	Adj. R^2	0.503	0.795	0.807	0.804	0.832	0.790	0.808	0.833	0.805
Coffee	β	0.513	0.638	0.727	0.839	0.780	0.858	0.926	0.824	0.937
		(0.010)	(0.020)	(0.027)	(0.024)	(0.038)	(0.031)	(0.025)	(0.038)	(0.041)
	$-\alpha_3/\alpha_2$	1.145	1.147	1.151	1.152	1.141	1.132	1.154	1.156	1.173
	Adj. R^2	0.349	0.535	0.630	0.798	0.673	0.805	0.899	0.784	0.821
Pork belly	β	0.385	0.764	0.804	0.754	0.851	0.866	0.805	0.831	0.884
		(0.018)	(0.038)	(0.046)	(0.050)	(0.048)	(0.055)	(0.064)	(0.063)	(0.057)
	$-\alpha_3/\alpha_2$	1.059	1.017	1.035	1.032	1.063	1.051	1.051	1.044	1.098
	Adj. R^2	0.102	0.330	0.435	0.465	0.605	0.615	0.570	0.619	0.726
Hogs	β	0.125	0.270	0.329	0.421	0.415	0.465	0.513	0.463	0.459
		(0.013)	(0.022)	(0.028)	(0.035)	(0.042)	(0.041)	(0.045)	(0.046)	(0.048)
	$-\alpha_3/\alpha_2$	0.803	0.826	0.836	0.812	0.809	0.810	0.814	0.798	0.806
	Adj. R^2	0.059	0.229	0.360	0.462	0.485	0.562	0.637	0.612	0.626
Crude oil	β	0.862	0.969	1.006	1.008	1.004	0.997	1.017	0.996	0.996
		(0.010)	(0.010)	(0.011)	(0.011)	(0.013)	(0.011)	(0.010)	(0.008)	(0.011)
	$-\alpha_3/\alpha_2$	1.005	1.005	1.006	1.006	1.003	1.006	1.006	0.998	1.006
	Adj. R^2	0.703	0.929	0.959	0.971	0.970	0.985	0.989	0.993	0.990
Silver	β	0.617	1.024	0.969	1.262	0.879	0.979	1.155	1.180	1.052
		(0.018)	(0.025)	(0.021)	(0.027)	(0.017)	(0.013)	(0.014)	(0.029)	(0.009)
	$-\alpha_3/\alpha_2$	0.986	0.965	0.987	0.980	0.954	0.953	1.041	0.909	0.965
	Adj. R^2	0.214	0.634	0.848	0.875	0.931	0.968	0.980	0.930	0.992
Gold	β	0.572	0.889	0.933	0.993	0.947	0.888	0.976	0.959	0.949
		(0.010)	(0.011)	(0.011)	(0.010)	(0.011)	(0.010)	(0.010)	(0.008)	(0.010)
	$-\alpha_3/\alpha_2$	0.979	0.968	0.975	0.965	0.978	0.974	0.969	0.960	0.977
	Adj. R^2	0.624	0.884	0.938	0.971	0.973	0.977	0.985	0.991	0.987
Japanese yen	β	0.922	0.964	0.985	0.972	0.978	0.995	0.994	0.984	0.981
		(0.006)	(0.008)	(0.009)	(0.010)	(0.009)	(0.010)	(0.011)	(0.011)	(0.009)
	$-\alpha_3/\alpha_2$	0.986	0.985	0.985	0.984	0.985	0.985	0.984	0.985	0.986
	Adj. R^2	0.887	0.963	0.974	0.978	0.987	0.989	0.989	0.990	0.994

(*Continued*)

TABLE V

Simultaneous Estimation of the Short- and Long-Run Hedge Ratios for Different Types of Futures Contracts (*Continued*)

		Holding Period								
Commodity		*1 Day*	*1 Week*	*2 Week*	*3 Week*	*4 Week*	*5 Week*	*6 Week*	*7 Week*	*8 Week*
Deutsche mark	β	0.934	0.981	0.994	0.997	0.997	0.992	1.010	0.992	1.001
		(0.006)	(0.007)	(0.008)	(0.009)	(0.009)	(0.011)	(0.011)	(0.010)	(0.010)
	$-\alpha_3/\alpha_2$	1.008	1.007	1.006	1.006	1.006	1.006	1.006	1.001	1.003
	Adj. R^2	0.899	0.970	0.981	0.984	0.988	0.987	0.989	0.992	0.994
Swiss franc	β	0.931	0.977	0.990	0.986	0.987	0.989	0.999	0.987	0.986
		(0.005)	(0.006)	(0.008)	(0.008)	(0.008)	(0.010)	(0.009)	(0.009)	(0.008)
	$-\alpha_3/\alpha_2$	0.992	0.991	0.990	0.989	0.988	0.989	0.987	0.982	0.983
	Adj. R^2	0.903	0.975	0.982	0.986	0.991	0.988	0.992	0.993	0.995
British pound	β	0.894	0.940	0.957	0.974	0.992	0.984	0.998	0.992	1.006
		(0.006)	(0.010)	(0.012)	(0.013)	(0.009)	(0.011)	(0.016)	(0.012)	(0.011)
	$-\alpha_3/\alpha_2$	1.021	1.020	1.022	1.020	1.024	1.023	1.023	1.029	1.018
	Adj. R^2	0.865	0.933	0.956	0.965	0.987	0.985	0.977	0.989	0.993
Canadian dollar	β	0.839	0.930	0.961	0.989	0.978	0.997	0.969	0.959	0.952
		(0.008)	(0.013)	(0.016)	(0.019)	(0.019)	(0.017)	(0.022)	(0.021)	(0.022)
	$-\alpha_3/\alpha_2$	1.039	1.040	1.040	1.040	1.038	1.041	1.039	1.041	1.040
	Adj. R^2	0.808	0.904	0.931	0.943	0.953	0.973	0.961	0.968	0.971

Note. This table presents the results for the simultaneous estimation of the short- and long-run hedge ratios obtained from Equation (11) with the use of various holding-period returns for each of the futures contracts listed in Table I. The short-run hedge ratios are given under the row heading β, and the long-run hedge ratios are given under the row heading $-\alpha_3/\alpha_2$. Standard errors are reported in parentheses.

short-run hedge ratios are greater than 1. They include TSE35 (4 and 6 week), Nikkei 225 (8 week), TOPIX (8 week), Silver (1, 3, 6, 7, and 8 week), Crude Oil (6 week), deutsche mark (6 and 8 week), British pound (8 week), and soybean oil (4, 5, 6, 7, and 8 week). However, all of these hedge ratios move back to within 1% of 1 as the hedging horizon lengthens, except for soybean oil and silver, which do not seem to converge to 1. These two commodities are interesting cases and need further analysis, and we intend to do this in the future.

Out-of-Sample Analysis

So far we have discussed the results based on the in-sample analysis. It will be interesting to see the out-of-sample effectiveness of the different hedge ratios as we increase the hedging horizon. Here, we consider the out-of-sample effectiveness of three hedging strategies. The first hedging strategy involves the estimation of the short-run hedge ratio using weekly data, where the hedge ratio is kept the same as the hedging horizon extends from 1 week to 8 weeks. We call this hedge ratio the 1-week hedge ratio. The second strategy involves the estimation of the hedge ratio using the data such that the data frequency matches the hedging horizon. For example, we use i-week differencing to estimate the short-run hedge ratio for an i-week hedging horizon. Such hedge ratios will be referred to as the horizon-adjusted hedge ratio. Finally, the third hedging strategy uses the naïve hedge ratio of unity. We analyze the hedging effectiveness of each of these three hedge ratios, where the hedging effectiveness is computed as follows:

$$\text{Hedging effectiveness} = 1 - \frac{\text{Var}(\Delta V_H)}{\text{Var}(\Delta S)} \tag{16}$$

In the analysis of the out-of-sample performance, the post sample represents the integer number of years' worth of data, which approximately covers half of the total sample size.[15] The first part of the data, which excludes the post sample period, is used to compute the optimal hedge ratios. The out-of-sample hedging effectiveness for the three hedge ratios are summarized in Table VI. The out-of-sample analysis is performed for one equity contract (All Ordinary), one commodity contract (Cotton), and one currency contract (Canadian dollar). It would be

[15]For example, for the All Ordinary index, there are 730 weeks' (14 years and 2 weeks) worth of data. Therefore, the out-of-sample period includes exactly 7 years' worth of data.

TABLE VI

Out-of-Sample Hedging Effectiveness

	Investment Horizon							
	1 Week	*2 Week*	*3 Week*	*4 Week*	*5 Week*	*6 Week*	*7 Week*	*8 Week*
Panel A: All Ordinary Stock Index								
One-week hedge ratio	0.903	0.912	0.922	0.932	0.929	0.936	0.934	0.929
Horizon-adjusted hedge ratio	0.903	0.924	0.943	0.951	0.964	0.966	0.975	0.966
Naïve hedge ratio	0.884	0.889	0.928	0.922	0.957	0.957	0.971	0.969
Relative effectiveness of 1-week hedge ratio	1.000	0.986	0.978	0.980	0.964	0.968	0.958	0.962
Relative effectiveness of horizon-adjusted hedge ratio	1.021	1.040	1.016	1.031	1.007	1.009	1.004	0.997
Panel B: Cotton								
One-week hedge ratio	0.459	0.504	0.599	0.572	0.601	0.638	0.627	0.560
Horizon-adjusted hedge ratio	0.459	0.513	0.615	0.584	0.616	0.689	0.654	0.669
Naïve hedge ratio	0.215	0.341	0.504	0.506	0.517	0.708	0.659	0.653
Relative effectiveness of 1-week hedge ratio	1.000	0.982	0.974	0.979	0.975	0.926	0.959	0.837
Relative effectiveness of horizon-adjusted hedge ratio	2.139	1.506	1.220	1.154	1.191	0.973	0.992	1.025
Panel C: Canadian Dollar								
One-week hedge ratio	0.924	0.958	0.969	0.967	0.979	0.968	0.979	0.975
Horizon-adjusted hedge ratio	0.924	0.959	0.969	0.968	0.976	0.966	0.978	0.980
Naïve hedge ratio	0.918	0.956	0.968	0.967	0.976	0.962	0.972	0.983
Relative effectiveness of 1-week hedge ratio	1.000	1.000	1.000	0.999	1.003	1.001	1.001	0.995
Relative effectiveness of horizon-adjusted hedge ratio	1.006	1.003	1.001	1.001	1.000	1.004	1.005	0.997

Note. This table presents the out-of-sample hedging effectiveness of three different hedge ratios for three futures contracts. The relative hedging effectiveness of the 1-week hedge ratio measures the effectiveness of the 1-week hedge ratio relative to the effectiveness of the horizon-adjusted hedge ratio and is computed as the ratio of the hedging effectiveness of the 1-week hedge ratio to the hedging effectiveness of the horizon-adjusted hedge ratio. Similarly, the relative hedging effectiveness of the horizon-adjusted hedge ratio is computed as the ratio of the hedging effectiveness of the horizon-adjusted hedge ratio to the effectiveness of the naïve hedge ratio.

interesting to see the performance of the 1-week hedge ratio in relation to the effectiveness of the horizon-adjusted hedge ratio, and, similarly, the relative performance of the horizon-adjusted hedge ratio vis-à-vis the naïve hedge ratio. The relative performance of the 1-week hedge ratio is computed as the ratio of the effectiveness of the 1-week hedge ratio to the effectiveness of the horizon-adjusted hedge ratio. We expect the relative performance of the 1-week hedge ratio to decrease as the hedging horizon lengthens. Similarly, the relative performance of the horizon-adjusted hedge ratio is computed as the ratio of the effectiveness of the horizon-adjusted hedge ratio to the effectiveness of the naïve hedge ratio. Again, we expect the relative performance of the horizon-adjusted hedge ratio to decrease as the hedging horizon is extended. Table VI also summarizes the relative hedging effectiveness for the three contracts. The relative effectiveness is also plotted in Figure 1.

It is clear from Table VI and Figure 1 that the relative performances of both the 1-week and horizon-adjusted hedge ratios decline as the hedging horizon increases. However, the decreasing trend is not as clear for the Canadian dollar as for the other two contracts considered. If we assume the trend to be the same for all three contracts and run a regression of the relative performance on a constant and time trend [similar to Equation (12), where β_i is replaced by the relative hedging performance], then the t statistics, for the coefficients of the time trend, are found to be equal to -2.671 for the relative performance of the 1-week hedge ratio and -2.219 for the relative performance of the horizon-adjusted hedge ratio. This supports our earlier finding that the long-run hedge ratio is close to the naïve hedge ratio.

CONCLUSIONS

In this article we have estimated the minimum variance hedge ratios for nine different hedging horizons as well as for 25 different commodities. Furthermore, we analyzed the effects of the length of hedging horizon on the optimal hedge ratio and hedging effectiveness. The empirical results indicate that the hedge ratios are significantly less than one and increase with the length of hedging horizon. It is also found that hedging effectiveness increases with the length of hedging horizon. However, the degree of hedging effectiveness does not approach 1.

We also find that for 14 out of 25 different commodities, the unit-root condition is rejected. This implies that the unit-root model used by Geppert (1995) cannot be universally acceptable.

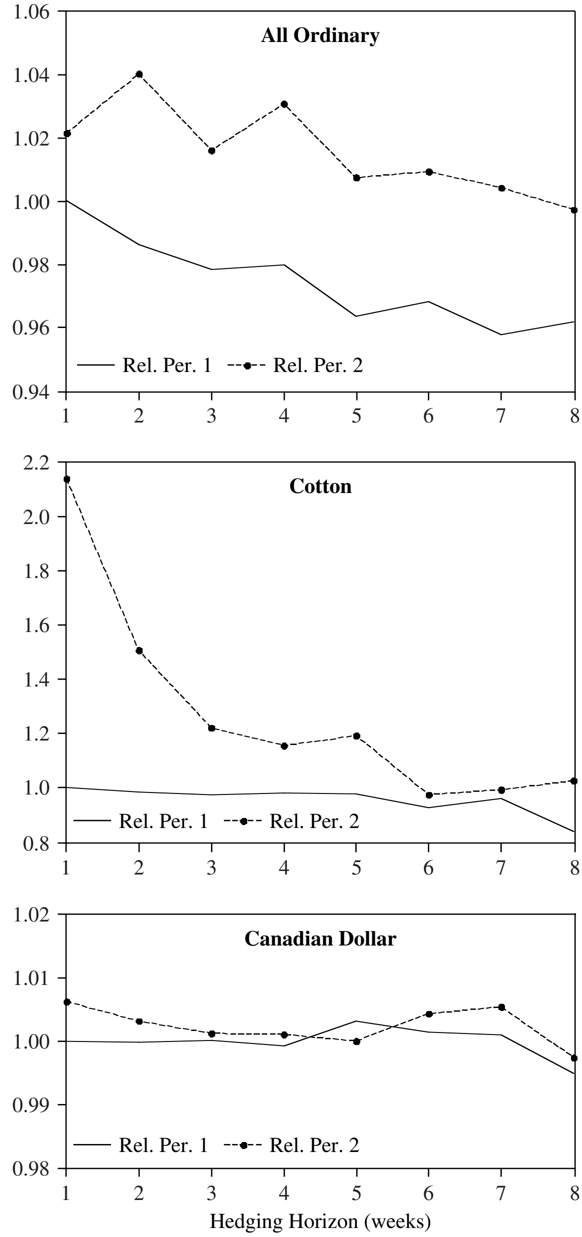


FIGURE 1

Out-of-sample relative hedging effectiveness of the 1-week and horizon-adjusted hedge ratios. "Rel. Per. 1" denotes the relative performance of the 1-week hedge ratio and "Rel. Per. 2" denotes the relative performance of the horizon-adjusted hedge ratio.

This article further presents a model that can be used to simultaneously estimate the short- and long-run hedge ratios. This model is found to be more suitable compared to the conventional regression model. The long-run hedge ratio is found to be close to the naïve hedge ratio of unity.

This implies that if the hedging horizon is long, then the naïve hedge ratio, which does not need any estimation, will be close to the minimum variance hedge ratio. This result is similar to that obtained by Geppert (1995), who imposes restrictions like unit-root and co-integration in the data-generating process. However, no such restriction is imposed in the model used in the article, and hence the results obtained in the article complement the results of Geppert (1995). This implies that the equality of the long-run hedge ratio to the naïve hedge ratio seems to be a more general phenomenon. This inference is also supported by the out-of-sample analysis. By contrast, the short-run hedge ratios (estimated using the new model) are found to be significantly less than 1.

APPENDIX

Consider two time series y_t and x_t. We can estimate the relationship between the two series using the following two equations:

$$y_t = \alpha_0 + \alpha_1 x_t + u_t \tag{A1}$$

and

$$\Delta y_t = \beta_0 + \beta_1 \Delta x_t + e_t \tag{A2}$$

where

$$\Delta y_t = (1 - L)y_{t+1} = y_{t+1} - y_t \quad \text{and} \quad \Delta x_t = (1 - L)x_{t+1} = x_{t+1} - x_t$$

In this appendix we will explain why Equation (A1) can be considered as a long-run relationship and Equation (A2) can be considered as a short-run relationship. There are three different possible explanations for the long- and short-run relationships, respectively represented by Equations (A1) and (A2). The first explanation is related to the frequency-domain approach, the second one to the distributed-lag model, and the third one to the concept of co-integration.

Frequency-Domain Explanation

In the frequency-domain analysis, it is well known that the differenced series represent the high-frequency component of the original series, which can be shown by using the spectral density of each series. Let $f_x(w)$ and $f_y(w)$, respectively, denote the spectral density of the series x_t and y_t. The spectral densities of the first differenced series Δx_t and Δy_t are then given by

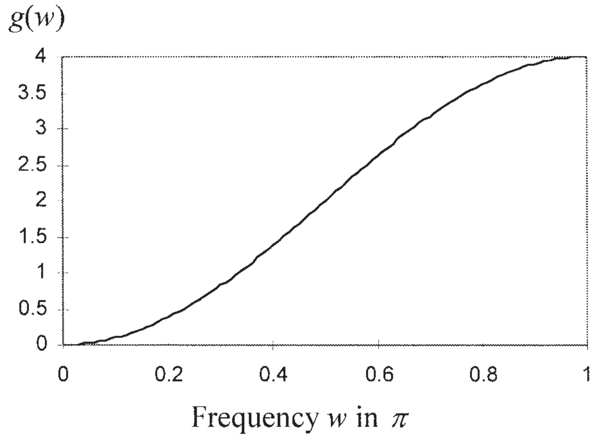
$$f_{\Delta x}(w) = f_x(w)g(w) \tag{A3}$$

$$f_{\Delta y}(w) = f_y(w)g(w) \tag{A4}$$

and

$$g(w) = |1 - e^{-iw}|^2 = 2(1 - \cos(w)) \quad \text{(A5)}$$

where w represents the frequency. In other words, the spectral density of the differenced series is given by the product of the spectral density of the original series and the function $g(w)$, which represents the differencing operator in the frequency domain. The dependence of function $g(w)$ on w is shown below:



It is important to note that function $g(w)$ approaches zero as the frequency approaches zero. This implies that the spectral density of the differenced series is equal to the spectral density of the original series, where lower-frequency components are eliminated or substantially reduced. Therefore, the differenced series represent the high-frequency component of the original series, and the relationship between the differenced series [Equation (A2)] represents a high-frequency relationship. Because the low-frequency components consist of long-term movements and the high-frequency components consist of short-term movements, Equation (A2) can be regarded as a short-run relationship and Equation (A1) can be regarded as a long-run relationship.

Distributed Lag Explanation

Another explanation of the short- and long-run relationships can be provided by looking at the following distributed lag model:

$$y_t = \gamma_0 + \gamma_1 x_t + \gamma_2 y_{t-1} + u_t \quad \text{(A6)}$$

In this case, the long-run (steady-state) relationship between y_t and x_t is represented by the coefficient $\frac{\gamma_1}{(1 - \gamma_2)}$, which is obtained from (after dropping the time subscript):

$$y = \frac{\gamma_0}{(1 - \gamma_2)} + \frac{\gamma_1}{(1 - \gamma_2)} x + \text{error} \tag{A7}$$

On the other hand, the short-run relationship is represented by the coefficient γ_1. We can derive the following equation from Equation (A6):

$$y_t - y_{t-1} = \gamma_1 (x_t - x_{t-1}) + \gamma_2 (y_{t-1} - y_{t-2}) + \text{error}$$

or

$$\Delta y_t = \gamma_1 \Delta x_t + \gamma_2 \Delta y_{t-1} + \text{error} \tag{A8}$$

Comparison of Equation (A8) with Equation (A2), shows clearly that Equation (A2) represents a short-run relationship between the two series x_t and y_t.

Co-Integration Explanation

If the two series x_t and y_t are nonstationary and co-integrated, then Equation (A1) is considered to be a long-run relationship in the sense that it is the stationary relationship between the nonstationary series. In this case, Equation (A1) is called the co-integrating regression. Therefore, from the viewpoint of co-integration analysis, Equation (A1) can be considered as a long-run relationship.

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