Induced Einstein-Kalb-Ramond theory and the black hole

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A four-dimensional induced Einstein-Kalb-Ramond theory with a conformally coupled Kalb-Ramond term is discussed. It is argued that the Kalb-Ramond field is difficult to interpret as an axion field from a dynamical point of view, and hence tends to be unstable. One can, however, apply Routh's method to extract an effective action for the axion field. It is found that the Kalb-Ramond hair will turn the event horizon into a naked singularity. Such an exact solution is found and analyzed carefully.

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The ten-dimensional Einstein-Kalb-Ramond action [1] has attracted a lot of activity lately. The Kalb-Ramond action is $-\frac{1}{6}F^2$ where the Kalb-Ramond field strength F_{abc} is the curvature tensor for the Kalb-Ramond field A_{ab}. Defining a three-form [2] $F \equiv F_{abc}dx^a \wedge dx^b \wedge dx^c$ and a two-form $A \equiv A_{ab}dx^a \wedge dx^b$, the formal relation between F_{abc} and A_{ab} can be read off directly from the definition $F = dA$.

The Kalb-Ramond action has been applied to study its implications to the inflationary process [1, 3] and black hole formation [4]. It has been discussed in its original ten-dimensional model as well as a four-dimensional truncation. The well-known no-hair theorem claims that the only possible hairs outside the event horizon of a black hole are mass, gauge charge, and angular momentum. It is found, however, that an axion hair [5] can exist outside the event horizon of a black hole. A nonminimal coupling of the Kalb-Ramond action inspired by superstring theory also appears to admit an axion hair.

In this paper, we are going to discuss the dynamics induced by the Kalb-Ramond term in four dimensions. Note that, in four dimensions, the Kalb-Ramond field can be written as $F_{abc} = \epsilon_{abcd}T^d$ with the help of the totally skew symmetric type $T(0,4)$ Levi-Civita tensor ϵ_{abcd} and some type $T(1,0)$ contravariant vector T^d due to the fact that F_{abc} is a totally skew symmetric type $T(0,3)$ tensor. Note that F and $T(\equiv T_a dx^a)$ are in fact dual forms, namely, $*F = -6T$ such that all degrees of freedom have been taken into account. By solving the equations of motion, one can show that $T = d\chi$ if the first homology group [6] of the spacetime manifold M is trivial, namely, $H_1(M) = 0$. The χ field has long been considered as an axion field. One will argue that it is in fact difficult to interpret the Kalb-Ramond field solution as an effective pseudoscalar axion field from the dynamical point of view. It can, however, be interpreted as an axion field through Routh's method [7] in a nontrivial way. At any rate, the axion field tends to be unstable and seems to vanish in some models. This point has been discussed previously in some inflationary models. It is speculated that the generic nature of instability tends to tune the Kalb-Ramond contributions in the post-inflationary era into a negligible fate.

On the other hand, scale invariance is one of the key symmetries in obtaining the low energy effective action for the massless string [8] mode. It is also important in many effective field theories such as the nonlinear σ which has been rather successful in describing low energy nucleonic interactions. Some even proposed that the global scale invariance should be gauged [9]. Note that QCD is also known to be scale invariant since the action of the matter fields and gauge fields is scale invariant as all coupling constants are dimensionless by construction.

Therefore, in this paper, we are going to study a four-dimensional induced gravity model with a Kalb-Ramond field conformally coupled to the metric and scalar field. Hopefully, there is a chance that the induced Kalb-Ramond term will be able to generate nontrivial hair for the Kalb-Ramond field and the scalar measuring field in the presence of the black hole event horizon. The action is

$$S = -\int d^4x\sqrt{g}\Big[\frac{1}{2}\epsilon e^{\varphi}R + \frac{1}{8}g^{ab}e^{\varphi}\partial_a\varphi\partial_b\varphi + V(\varphi) + \frac{1}{6}e^{-\varphi}F_{abc}F^{abc}\Big], \quad (1)$$

with conformally coupled Kalb-Ramond field strength. Note that we have written ϕ^2 as e^{φ} for convenience. Here ϕ denotes a real scalar measuring field in a sense that the proper scale invariant metric should be defined as $d\tilde{s}^2 \equiv e^{\varphi}g_{ab}dx^a dx^b$. One notes that the $\phi \to -\phi$ symmetry has been removed in this simplified expression which will reduce unnecessary complications in our discussions. Moreover, ϵ is a dimensionless coupling constant.

Note that the equation of motion for the Kalb-Ramond field can be shown to be

$$\partial_a(\sqrt{g}e^{-\varphi}F^{abc}) = 0, \quad (2)$$

which can be rewritten as $*d(e^{-\varphi}T) = 0$ in the language of differential form. Hence, one can show that the solution to the Kalb-Ramond field is

$$F_{abc} = \frac{1}{2\sqrt{2}}\epsilon_{abcd}e^{\varphi}\partial^d\theta, \quad (3)$$

if $H_1(M) = 0$. Note that Eq. (3) is the solution to the complete Euler-Lagrange equation (2) of A_{ab}. It appears that we are left with one degree of freedom θ undetermined since other equations do not appear to contribute any constraint on θ. One can, however, show that there is a nontrivial Bianchi identity $D_aG^{ab} = 0$ which will add a constraint on θ and resolve the underdetermined puzzle.

It is straightforward to find that the Bianchi identity can also be shown to give

$$D_a(e^{\varphi}\partial^a\theta) = 0, \qquad (4)$$

after some algebra.

Equivalently, the remaining degree of freedom θ can also be thought of as being constrained by the hidden Bianchi identity $dF = 0$ due to the fact that F is an exact three-form, namely, $F = dA$. Hence, one can show that the Bianchi identity on the three-form F can be reduced to the Laplace equation

$$d * (e^{\varphi} d\theta) = 0. \qquad (5)$$

It reads $D_a(e^{\varphi}\partial^a\theta) = 0$ in component form and agrees with earlier results. Hence the Bianchi identity in a differential form $dF = 0$ is a reflection of the on-shell Bianchi identity $D_a(G^{ab} - T^{ab}) = 0$.

One observes, from Eq. (3), that the kinetic energy term for A_{ab} comes from the F_{0ij} component, while the kinetic energy term for θ comes from the F_{ijk} component. In short, relation (3) shows that the dual transformation interchanges the spatial part and the kinetic part of A_{ab} and θ. Therefore their physical behaviors are not quite the same although θ appears to be an effective field embedded in the Kalb-Ramond field. Although θ can be considered as an effective axion field through Routh's method in a nontrivial manner as we will show shortly, one should, however, look more closely at the property of the reduced action for the Kalb-Ramond field before one shows how to apply Routh's method.

To be more specific, it is easy to show that, for example, their gravitational responses are dramatically different due to their completely different gravitational couplings; namely, one has $\partial A \partial A ggg$ coupling while the other one has $\partial\theta\partial\theta g$ coupling. This will turn their associated energy-momentum tensor into completely different forms. The reason is that the following two operations do not commute: (i) varying the Lagrangian with respect to g_{ab} and (ii) substituting the Kalb-Ramond field A_{ab} as a function of θ, i.e., writing $A_{ab} \equiv A_{ab}(\theta)$. More specifically, we have

$$\left[\frac{\delta \mathcal{L}^A}{\delta g_{ab}}\delta g_{ab}\right]^* \neq \frac{\delta\left[\mathcal{L}^A\right]^*}{\delta g_{ab}}\delta g_{ab}. \qquad (6)$$

Here $[\,]^*$ denotes the mapping (ii) mentioned above, i.e., replacing A_{ab} by θ according to relation (3). Moreover, $\mathcal{L}^A$ is the induced Kalb-Ramond Lagrangian. If θ is to be interpreted as an effective axion field, equations of motion associated with the effective Lagrangian $\mathcal{L}^{A*}$ for the effective field variables should be consistent [10] with the original equations of motion associated with the original action. Unfortunately, the θ field fails to meet the consistency unless $\theta' = 0$. Hence some mechanism is needed to make sense of interpreting Kalb-Ramond field as an axion field. Note that above consistency check [11] has been employed to show that there are nontrivial constraints in the compactification program in Kaluza-Klein theory. It is also shown that corrected Friedmann-Robertson-Walker metric will be able to reproduce complete field equations.

Note that the physical meaning of the commuting mapping is that one should have a refined minimum action. Moreover, one does not expect that the substitution of A_{ab} as a functional of θ, namely, $A_{ab} = A_{ab}(\theta)$, is capable of reproducing the θ equation (5). Indeed, one can show that

$$\frac{\delta \mathcal{L}^A_{\text{eff}}(\theta)}{\delta\theta} = \left[\frac{\delta \mathcal{L}^A}{\delta A_{ab}}\right]^* \frac{\delta A_{ab}(\theta)}{\delta\theta}, \qquad (7)$$

if one defines the effective Lagrangian of the Kalb-Ramond term $\mathcal{L}^A_{\text{eff}} \equiv \mathcal{L}^{A*} = \mathcal{L}^A(A_{ab}(\theta))$. Note that the factor $\frac{\delta A_{ab}(\theta)}{\delta\theta}$ will, in general, project out a certain part of the equations of the A_{ab} equation $\left[\frac{\delta\mathcal{L}^A}{\delta A_{ab}}\right]^* \equiv \frac{\delta\mathcal{L}^A(A_{ab})}{\delta A_{ab}}|_{A_{ab}(\theta)} = 0$ which becomes the effective equation for θ, namely, $\frac{\delta\mathcal{L}^A_{\text{eff}}(\theta)}{\delta\theta} = 0$. Hence one does expect that the effective action defined above will, in general, fail to reproduce the constraint hidden in the Bianchi identity unless the projection factor is invertible. Indeed, the projection operator above will throw away some correct information carried by the original equation in this model. Worse, the A_{ab} equation does not offer a complete story of A_{ab} since θ is still an underdetermined leftover field in Eq. (2). The cure is, however, fairly simple as it turns out. The point is that A_{ab} is not a simple functional of θ. It is given indirectly through Eq. (3), obtained earlier. Indeed, one finds

$$\mathcal{L}^A_{\text{eff}}(\theta) = \tfrac{1}{8}e^{\varphi}\partial_a\theta\partial^a\theta. \qquad (8)$$

The variational equation of θ gives exactly Eq. (4). Note, however, that it fails to reproduce correct metric equation and, worse, it carries a negative kinetic energy term. Note that it is easy to show that once the sign is changed, its variational equations for g_{ab}, ϕ, and θ will be correct again all together.

To induce the correction, one needs to apply the well-known Routh's method for eliminating cyclic variables. Routh suggests that one can apply Legendre transformations on cyclic variables and leave all other variables unchanged in the associated Hamiltonian-Lagrangian (or Routhian) system. To be more specific, one can define the Routhian function H_L by

$$H_L(\phi_1,\ldots,\phi_k;\dot\phi_1,\ldots,\dot\phi_k;\pi_{k+1},\ldots,\pi_n) = \sum_{i=k+1}^{n}\pi_i\dot\phi_i - L(\phi_1,\ldots,\phi_k;\dot\phi_1,\ldots,\dot\phi_n) \qquad (9)$$

as the reduced Hamiltonian-Lagrangian for the Legendre transformation on the cyclic variables $\phi_{k+1},\ldots,\phi_n$. Here $L(\phi_1,\ldots,\phi_k;\dot\phi_1,\ldots,\dot\phi_n)$ is the Lagrangian of the system with cyclic variables ϕ_i, $i = k+1,\ldots,n$. Moreover, π_i, $i = k+1,\ldots,n$, are the conjugate momentum of the cyclic variables, and hence constants in time. It is known that the variational equation of $H_L(\phi_1,\ldots,\phi_k;\dot\phi_1,\ldots,\dot\phi_k)$ with respect to the cyclic variables is the same as the variational equation of L upon constraints induced when the cyclic variables are imposed. To get a correct effective Lagrangian, one should define $L_{\rm eff}(\phi_1,\ldots,\phi_k;\dot\phi_1,\ldots,\dot\phi_k)$ as

$$L_{\rm eff}(\phi_1,\ldots,\phi_k;\dot\phi_1,\ldots,\dot\phi_k) = -H_L(\phi_1,\ldots,\phi_k;\dot\phi_1,\ldots,\dot\phi_k;\pi_{k+1},\ldots,\pi_n), \quad (10)$$

after solving all cyclic variables. Field theory can be considered as the continuous limit of lattice particles in a finite box. Hence one can still define an effective Lagrangian $\mathcal{L}_{\rm eff}(\phi_i,\partial_a\phi_i)$ similar to the discrete particle Lagrangian. Indeed, one can show that the effective Lagrangian for noncyclic fields $\phi_1\ldots\phi_k$ is

$$\mathcal{L}_{\rm eff}(\phi_1,\ldots,\phi_k;\partial_a\phi_1,\ldots,\partial_a\phi_k) = -\mathcal{H}_L(\phi_1,\ldots,\phi_k;\partial_a\phi_1,\ldots,\partial_a\phi_k;\pi^a_{k+1},\ldots,\pi^a_n). \quad (11)$$

Similarly, one can show that a similar program works for the Kalb-Ramond field by considering it as a cyclic field. Note that the Kalb-Ramond field is not truly a cyclic field since the Kalb-Ramond equation (2) does not eliminate all four degrees of freedom. But one can show that the method of Routh still works. Indeed, the effective Lagrangian can be defined as

$$\mathcal{L}_{\rm eff}(g_{ab},\phi,\theta) = \mathcal{L}(g_{ab},\phi,A_{ab}(\theta)) - \partial_a\theta\frac{\delta\mathcal{L}(g_{ab},\phi,A_{ab}(\theta))}{\delta\partial_a\theta}. \quad (12)$$

Indeed, one can show that $\mathcal{L}_{\rm eff}(g_{ab},\phi,\theta) = \mathcal{L}^{g,\phi}(g_{ab},\phi,\theta) - \frac{1}{8}e^{\varphi}\partial_a\theta\partial^a\theta$ with $\mathcal{L}^{g,\phi}$ denoting the Lagrangian of the metric and scalar field.

Although we have shown that Routh's method can correctly reproduce all relevant field equations after one solves the incomplete equation that leads to the leftover θ field, the reason is still not obvious since A_{ab} is not a truly cyclic field. The key may have to do with the delicate symmetry in the general covariant gravitational fields that is capable of generating an appropriate constraint implicitly hidden in the Bianchi identity. In short, the leftover θ field will manage itself such that the energy-momentum tensor associated becomes divergentless in order to be consistent with the Bianchi identity. Hence Routh's method still works out. Note that a similar program still works in the original Einstein-Kalb-Ramond theory without conformal couplings.

We will try to study the behavior of an axion field in the presence of a black hole event horizon. For the moment, we will assume the static and spherically symmetric metric defined by

$$ds^2 \equiv -e^{2A}dt^2 + e^{2B}dr^2 + r^2 d\Omega. \quad (13)$$

Here $d\Omega$ is the solid angle.

Therefore, one can obtain the equations of motion

$$2rB' + e^{2B} - 1 = r^2\left\{\varphi'' + \left(1+\frac{1}{8\epsilon}\right)\varphi'^2 - B'\varphi' + \frac{2}{r}\varphi' + \frac{1}{\epsilon}Ve^{2B-\varphi} + \frac{1}{8\epsilon}\theta'^2\right\}, \quad (14)$$

$$(A+B)'\left(\frac{2}{r}+\varphi'\right) = \varphi'' + \left(1+\frac{1}{4\epsilon}\right)\varphi'^2 + \frac{1}{4}\epsilon\theta'^2, \quad (15)$$

$$\varphi'' + \varphi'^2 + (A-B)'\varphi' + \frac{2}{r}\varphi' = \frac{4}{1+6\epsilon}e^{2B-\varphi}(\partial_\varphi V - 2V). \quad (16)$$

Accordingly, with the metric given by (13), Eq. (5) can be shown to give

$$\theta' = \frac{k_2}{r^2}e^{B-A-\varphi}. \quad (17)$$

Here k_2 is an integration constant, one used to conclude, from the regularity of θ, that $\theta' = 0$ (i.e., $k_2 = 0$) since $e^{B-A}|_{r_H} = \infty$. Although the generic physical quantity $F^2 \propto g^{rr}\theta'^2 \propto e^{-2A}$ is in general divergent at r_H since e^{-2A} diverges at r_H in most examples, it is not clear whether the factor e^{-A} really diverges at r_H in accordance with e^{B-A}. One will have to solve the equation of motion in order to find out if the zeros of e^A are identical to the zeros of e^{-B}. Therefore we will try to solve the equations of the system directly. In fact, we will show that there is indeed an asymmetric solution to A and B. We will show, however, that the presence of the axion field will turn the event horizon of the axion free system into a naked singularity. Hence the black hole is not stable against θ hair.

Note that the φ field can be shown to be constant outside the event horizon of a black hole. Indeed, multiplying Eq. (16) by $\int_{r_H}^\infty dr\sqrt{g}e^{\varphi-2B}(\varphi-\varphi_0)$, one obtains

$$\int_{r_H}^\infty drr^2e^{A-B+\varphi}(\varphi')^2 = \frac{4}{1+6\epsilon}\int_{r_H}^\infty drr^2e^{A+B}(\varphi-\varphi_0)(2V-\partial_\varphi V), \quad (18)$$

upon removing all surface terms. Here r_H is the radius of the event horizon. One can show that

$$(\varphi-\varphi_0)(2V-\partial_\varphi V) = -\frac{\lambda v^2}{4}(\varphi-\varphi_0)(e^\varphi - v^2), \quad (19)$$

if $V = \frac{\lambda}{8}(e^\varphi - v^2)^2$. Note that $\varphi \to \varphi_0$ as $r\to\infty$ if $v \neq 0$ as required by the finite action. Therefore it is easy to find that $(\varphi-\varphi_0)(e^\varphi - v^2) \geq 0$. Hence the right hand side of Eq. (18) has to vanish since the left hand side of that equation is always positive. Note also that $2V - \partial_\varphi V = 0$ if $v = 0$. Hence $\varphi' = 0$ in both the scale invariant phase and symmetric broken phase. For later convenience, we will change the metric parametrization as

$$ds^2 \equiv -e^{2A(r)}dt^2 + e^{-2A(r)}dr^2 + e^{2C(r)}d\Omega. \quad (20)$$

After some algebra, one can show that the equations of motion become

$$C'' + C'^2 = -k_3e^{-4(A+C)}, \quad (21)$$

$$C'^2 + 2A'C' - e^{-2(A+C)} = k_3e^{-4(A+C)}. \quad (22)$$

Here we have used the solution of the scalar field $\varphi' = 0$ and $k_3 \equiv \frac{k_2^2}{8\epsilon}e^{-2\varphi_0}$. By adding Eqs. (21) and (22) together, one finds that

$$(e^{2A+2C}C')' = 1. \tag{23}$$

Moreover, Eq. (21) can be rewritten as

$$k_3 \frac{C'^2}{C'' + C'^2} = -(e^{2A+2C}C')^2, \tag{24}$$

which can be integrated to give

$$e^C = r\left(1 - \frac{M + \sqrt{M^2+k_3}}{r}\right)^{\frac{\sqrt{M^2+k_3}-M}{2\sqrt{M^2+k_3}}} \times \left(1 + \frac{\sqrt{M^2+k_3}-M}{r}\right)^{\frac{\sqrt{M^2+k_3}+M}{2\sqrt{M^2+k_3}}} . \tag{25}$$

Hence

$$e^{2A} = \left[\frac{r - (M + \sqrt{M^2+k_3})}{r + (\sqrt{M^2+k_3} - M)}\right]^{\frac{M}{\sqrt{M^2+k_3}}}, \tag{26}$$

with the help of the integrable equation (23). Here M and k_3 denote the ADM mass and the integration constant denoting the strength of the torsion field as indicated in (21). Indeed, $e^{2A} \to 1 - \frac{2M}{r}$ as $r \to \infty$. Note that the r variable in (25) and (26) should be rephrased as $\tilde{r} = r - M - \sqrt{M^2+k_3}$ such that the $\tilde{r}$ variable maps exactly to the usual positive radial variable adopted earlier in this paper. Indeed, one can rewrite C and A as

$$e^C = \tilde{r}^{\frac{\sqrt{M^2+k_3}-M}{2\sqrt{M^2+k_3}}} (\tilde{r} + 2\sqrt{M^2+k_3}\ )^{\frac{\sqrt{M^2+k_3}+M}{2\sqrt{M^2+k_3}}}, \tag{27}$$

$$e^{2A} = \left[\frac{\tilde{r}}{\tilde{r} + 2\sqrt{M^2+k_3}}\right]^{\frac{M}{\sqrt{M^2+k_3}}} . \tag{28}$$

Therefore, the horizon becomes a naked singularity at $\tilde{r} = 0$ (i.e., $r = M + \sqrt{M^2+k_3}$) since $e^C = 0$ at $\tilde{r} = 0$. Note that in the limit $k_3 = 0$ one can reproduce the Schwarzschild solution.

In summary, we have shown how to interpret the Kalb-Ramond solution as an effective axion field. The associated axion field tends to be unstable in some models. For example, we have shown explicitly that a nontrivial Kalb-Ramond field solution exists outside the event horizon associated with a spherically symmetric black hole in an induced Einstein-Kalb-Ramond action. It was shown, however, that the axion field tends to be unstable by turning the event horizon into a naked singularity. Therefore, the no-hair theorem still holds in this model. The instability is also indicated in the inflationary models. Indeed, in the Friedmann-Robertson-Walker spaces, Eq. (5) becomes $\theta_{tt} + 3\alpha_t\theta_t + \theta_t\varphi_t = 0$, where $a(t) \equiv e^{\alpha(t)}$ denotes the scale factor of the Friedmann-Robertson-Walker spaces. This equation can thus be integrated to give $\theta_t = \text{const} \times e^{-3\alpha-\varphi}$. Therefore, the Kalb-Ramond field might have been active during the inflation era even though they are negligible in the post-inflationary era. Indeed, the Kalb-Ramond field tends to vanish in accordance with the huge inflation of the scale factor $a(t)$. Therefore, a more detailed study of the roles played by the Kalb-Ramond field both in the early universe and collapsing stars appears to be very interesting and worth working for. Note that in the normal Einstein-Kalb-Ramond theory without conformal couplings, similar results can also obtained. Inclusion of a Yang-Mills monopole [12] will not affect our results except replacing the Schwarzschild metric with the Reissner-Nordstrom metric in (25) and (26).

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