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Logistics cost, consumer demand, and retail establishment density*

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Abstract. This article develops models to formulate the optimal density of retail establishments by considering interactions between logistics cost and consumer demand. Commodities are assumed to be distributed from a depot directly or through single intermediate terminal to many retail establishments. Average logistic cost per item, consumer demand, and the interrelationship between them are analyzed. The optimal density of retail establishments and local terminals are determined by minimizing average logistic cost, or maximizing total supply subject to the demand-supply equality. The envelope curves for the optimal configuration strategies corresponding to different values of total market area and terminal cost are derived.

JEL classification: R41

Key words: Logistics cost, consumer demand, retail establishment density, terminal, demand-supply interaction

1 Introduction

In order to achieve cost minimization or large sales, suppliers need to know how to determine the density of retail establishments and the number of intermediate terminals that should be set up in a given region so as to distribute and sell commodities. The commodities are usually assumed to be distributed from a depot directly or through one intermediate, local terminal to many retail establishments. Such physical distribution systems have long been recognized as one-to-many distribution problems and have been extensively investigated in

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many studies (e.g., Blumenfeld et al. 1985; Burns et al. 1985; Campbell 1993; Daganzo 1991; Daganzo and Newell 1986; Hall 1987). However, most of the literature assumed inelastic demand at fixed locations and focused on operating issues such as scheduling, routing, and configuration of physical distribution. In these studies, demand-supply interaction was investigated only at the terminal level (e.g., Hall 1987; Daganzo and Newell 1986; Campbell 1993) but not down at the retail-establishment level where consumers' demands actually occur. On the other hand, the density of retail or public establishments has been discussed by market area literature, but most of studies focused on the trade-off between consumer access time and supplier efficiency (e.g., Stephan 1988; Hsu and Chen 1994). Supplier logistic costs in these studies are usually ignored, or considered not to be an issue, and demand-supply interaction is investigated only at the retail-establishment level.

The present article is an attempt to fill the indicated gaps above. Models for estimating the optimal density of retail establishments and the number of terminals are developed in this article by addressing both physical distribution and market area problems. Instead of assuming inelastic demand at fixed locations, this article considers demand-supply interactions and assumes the density of establishments is endogenous. These interactions and trade-off existing in the ways listed below, need to be further considered and incorporated into the modeling process.

(a) Consumer shopping demand for items in retail establishments usually depends on the access time to the establishment, unit price, and total supply of items at the retail establishment. The density of retail establishment represents the number of establishments per unit area. Other things being equal, increased scale economies imply a few large retail establishments and higher access time for customers. The density of retail establishment affects both the total supply of items at each establishment and average access distance to this establishment. That is, the larger size but more scattered establishment setting leads to larger market areas and longer access distances.

(b) Logistics costs include transportation and inventory cost. The total transportation distance increases with the density of retail establishments providing other conditions do not change (Eilon et al. 1971; Larson and Odoni 1981). Holding the shipment size constant, the shipping frequency also increases with the density of retail establishments. The average transportation cost depends on the transportation distance, shipping frequency, and unit-distance transportation cost, so it will increase with the density of retail establishments. On the other hand, the average inventory cost depends on the average headway, which is the inverse of shipping frequency, so it will decrease with the density of retail establishments. The density of retail establishments plays an important role in the trade-off between average transportation and average inventory cost. Average logistic cost appears as a part of item price, which will consequently affect consumer demand.

(c) Terminal costs are required for terminal shipping and increase with the number of terminals. For terminal shipping, the retail-establishment density affects the average local transportation cost directly and the average terminal cost indirectly. On the other hand, the number of terminals affects directly both line-haul transportation cost and terminal cost. There are trade-offs among line-haul transportation cost, local transportation cost, terminal cost and inventory cost for terminal shipping. The retail establishment density is related to the interaction between consumer demand and establishment supply and the trade-off among different cost components of the average logistic cost for terminal shipping.

This article follows the distribution environment and the approximate analytic models found in previous studies (Blumenfeld et al. 1985; Burns et al. 1985; Campbell 1993; Daganzo 1991; Daganzo and Newell 1986), so the following assumptions are made:

(a) Items are cheap (vehicle operating and depreciation cost per unit item is large compared with the inventory cost per unit time of a full vehicle load) and homogeneous.
(b) Items are demanded and produced at a constant rate.
(c) The density of retail establishments varies slowly across the service region or the market area.

Approximate analytic models are most useful to determine the features of optimum distribution system, not to specify exact facility locations and vehicle routes (Campbell 1993). These models have been extensively found in logistics literature (e.g., Burns et al. 1985; Campbell 1990; Campbell 1993; Daganzo 1984; Daganzo 1991; Daganzo and Newell 1986; Eilon et al. 1971). Average logistics cost per item and consumer demand are formulated in this article by using continuous space modeling. In practical applications it is often more convenient to work with a continuous approximation. Continuous functions are easier to manipulate, and when embedded as a part of more complicated problems, they often lead to simple calculus solutions (Daganzo 1991).

Section 2 will formulate the average logistic cost for both direct and terminal shipping. Consumer demand and nonlinear programming problems are formulated in Sect. 3 to determine the optimal density of retail establishments with two different objectives, i.e., minimizing the average logistic cost and maximizing total supply subject to the demand-supply equality. In Sect. 4, numerical experiments are performed to illustrate the effect of the optimal solution to changes in parameters. Finally, Sect. 5 summarizes the article and presents the conclusion.

2 Logistics cost

There are two types of physical distribution processes discussed in this article. One is distribution of commodities from a depot directly to many retail establishments; the other is distribution of them through one intermediate, local terminal. The former is direct shipment while the latter is terminal shipment. The depot is assumed to be in the market area, but not located at the center of any subregions served by each terminal in the article. There was only one type of distribution process used for terminal shipment. That is, commodities are assumed to be distributed merely from the terminal but not the depot for terminal shipment.

2.1 The logistics cost for direct shipment

Logistic costs for direct shipment include transportation and inventory costs. As we are interested in the relationships among average logistic cost, consumer demand, and retail establishment density, for direct shipping, we formulate the average transportation and inventory costs per item as functions of retail establishment density. The total transportation distance with direct shipping, L, was found by Eilon et al. (1971) to be $K_0(MN')^{1/2}$, where M is the total market area, N' is the total number of retail establishments, and K_0 is a constant that can be assumed to be 0.75. Since $\lambda = N'/M$, where λ is the retail establishment density, therefore $L = K_0 M\lambda^{1/2}$. Assume vehicles are filled to capacity to minimize cost, the direct shipping frequency, F_D, is then the ratio, $M\lambda q_S/S_D$, where S_D is the total shipping capacity of the vehicle fleet, and q_S is the average number of items supplied by each retail establishment. $M\lambda q_S$ represents the total number of items supplied by all retail establishments. Define the unit distance transportation cost of the vehicle fleet for direct shipping, α_D, as a linear function of the total shipping capacity of the vehicle fleet, S_D, that is $\alpha_D = a_D S_D + b_D$, where a_D and b_D are constants representing unit distance variable transportation costs per item and fixed unit distance transportation costs of the vehicle fleet, the total transportation cost for direct shipping, TTC_D, is then:

$$TTC_D = (a_D S_D + b_D)\frac{M\lambda q_S}{S_D}(K_0 M\lambda^{1/2}) = \alpha_D \frac{M\lambda q_S}{S_D}(K_0 M\lambda^{1/2}), \tag{1}$$

and the average transportation cost per item for direct shipping, ATC_D, is then:

$$ATC_D = \frac{\alpha_D}{S_D}(K_0 M\lambda^{1/2}). \tag{2}$$

Equation (2) shows that ATC_D increases with the square root of λ, and decreases with S_D. The assumption of unit distance transportation cost of the vehicle fleet to be $a_D S_D + b_D$ implies the variable unit distance transportation cost for the commodity is the same for all items since items are homogeneous while the fixed unit distance transport cost per item, b_D/S_D decreases with the increase of S_D, thus exhibiting scale economies.

Inventory costs arise in the depot for shipping or the retail establishment for consuming as “stationary inventory”, and in vehicles, as “in-transit inventory”. The stationary inventory cost per item is usually formulated as a function of the headway for direct shipment vehicles, T_D. Since the direct shipping frequency, F_D, is $1/T_D$, T_D is therefore $S_D/M\lambda q_S$. Assume in-transit inventory cost is negligible because items are cheap and in-transit time is short. Then if delays at depot for shipping and delays at retail establishment for consumption are considered, that is the total inventory time is the headway T_D, then average inventory cost per item for direct shipping, AIC_D, is:

$$AIC_D = PR\left(\frac{S_D}{M\lambda q_S}\right), \tag{3}$$

where P is the production cost per item, and R is inventory carrying rate. Combining equations (2) and (3) yields the average logistic cost per item for direct shipping, ALC_D:

$$ALC_D = PR\left(\frac{S_D}{M\lambda q_S}\right) + \frac{\alpha_D}{S_D}(K_0 M\lambda^{1/2}). \tag{4}$$

2.2 The logistics cost for terminal shipment

Differing in only one process involved in direct shipping, terminal shipping involves three processes, so logistics costs can be broken down into the following components: the line-haul transportation cost of shipping commodities from the depot to the terminal, the local transportation cost of shipping commodities from the terminal to retail establishments, the handling cost of transshipment terminals, and the inventory cost.

2.2.1 The line-haul transportation cost

The line-haul transportation cost depends on the shipping frequency, the unit distance transportation cost, and line-haul transportation distance. Many studies have assumed that line-haul vehicles made only one single stop in each delivery tour (e.g., Daganzo and Newell 1986; Campbell 1990). Following this assumption, the average line-haul distance, D_T, was formulated by Eilon et al. (1971) as $D_T = K_0 M^{1/2}$, where M is the total market area of the study, and parameter K_0 as defined by Larson and Odoni (1981) could be assumed to be 0.75 (Campbell 1993). The line-haul shipping frequency, f_1, can be obtained by dividing the total number of items to be shipped from the depot to the terminal over the study period, Q_1, by the total shipment capacity of the line-haul fleet, S_1. That is $f_1 = Q_1/S_1$.

Assume the unit distance transportation cost of the line-haul fleet, α_1, is a linear function of S_1:

$$\alpha_1 = a_1 S_1 + b_1, \tag{5}$$

where a_1 and b_1 are constants for the line-haul fleet; the total line-haul transportation cost to each terminal, TTC_L, is then:

$$TTC_L = (a_1 S_1 + b_1)\frac{Q_1}{S_1} K_0 M^{1/2}, \tag{6}$$

and the average line-haul transportation cost per item, ATC_L, is:

$$ATC_L = (a_1 S_1 + b_1)\frac{Q_1}{S_1} K_0 M^{1/2} \frac{1}{Q_T}, \tag{7}$$

where Q_T is the number of items served by each terminal. Let N denote the number of transhipment terminals, then $Q_1/Q_T = N$.

2.2.2 The local transportation cost

The process of shipping commodities from the transportation terminal to many retail establishments is similar to that of direct shipping, except that the origin of the latter is the depot rather than the terminal. Following the formulation of total transportation cost for direct shipping, TTC_D, in equation (1), the total local transportation cost for each terminal, TTC_{local}, is then:

$$TTC_{local} = \alpha_2 \frac{m_1 \lambda q_S}{S_2} (K_0 m_1 \lambda^{1/2}), \tag{8}$$

and the average local transportation cost per item for each terminal, ATC_{local}, is:

$$ATC_{local} = \frac{\alpha_2}{S_2} (K_0 m_1 \lambda^{1/2}), \tag{9}$$

where the average market area served by each transhipment terminal, m_1, the total shipment capacity of local vehicle fleet, S_2, and the unit distance transportation cost of the local fleet, α_2, $\alpha_2 = a_2 Q_T + b_2$ replace M, S_D, and α_D, $\alpha_D = a_D Q_D + b_D$, respectively, in equations (1) and (2). Similarly, S_2 is the total shipment capacity of the local vehicle fleet, a_2 and b_2 are constants for the local fleet.

2.2.3 The inventory cost of terminal shipment

Based on the formulation of Daganzo (1988), we define the inventory cost per item delivered for terminal shipping, AIC_T, as:

$$AIC_T = PR \max\{T_1, T_2\}, \tag{10}$$

where P is the average production cost per item, R is inventory carrying rate, T_1 is the headway for line-haul vehicles arriving at the terminal, and T_2 is the headway for local vehicles departing from the terminal. Following the definition of f_1 and f_2, we can write:

$$T_1 = S_1/Q_1, \tag{11}$$

$$T_2 = S_2/m_1 \lambda q_S. \tag{12}$$

The formulation of equation (10) also ignores the in-transit inventory cost by assuming that items are cheap and in-transit time is short. The inventory cost depends on the headway as shown in equation (10), and the average waiting time is $\max\{T_1, T_2\}$ because of assumed coordination at the terminal (Daganzo 1988). Coordinating schedules can reduce the wait, and thus the inventory cost. It was also noted in Daganzo (1988), that there were three cases: $T_1 > T_2$, $T_1 < T_2$ and $T_1 = T_2$ when schedules were coordinated. If the headway for line-haul vehicle is larger than that for local vehicle, i.e., $T_1 > T_2$, then operators can reduce inventory costs by increasing local shipment size, thereby yielding smaller shipping frequencies and larger headways for local vehicles. On the other hand, if the headway for line-haul vehicles is smaller than that for local vehicles, i.e., $T_1 < T_2$, then operators can enlarge line-haul shipment sizes, thereby reducing the line-haul shipping frequency and expanding headway for line-haul vehicles. Only when operators distribute with equal local and line-haul headways ($T_1 = T_2$), is there no room for further improving inventory costs by adjusting line-haul or local shipping schedules. We assume that in the long-term, an optimal operating condition exists for terminal shipping, so only will the case in which $T_1 = T_2$ be formulated in the model.

2.2.4 The terminal cost

In addition to transportation and inventory costs, there are terminal costs for distribution with transshipments. The terminal cost includes both fixed costs and variable costs. The fixed terminal cost per item is then f/Q_T, where f is fixed cost per terminal and $Q_T = \lambda m_1 q_S$ is the average total number of items served by each terminal. The variable cost for handling items is the same for all item since items are homogeneous and all items pass through one terminal. Define υa as variable handling cost per item at the terminal, then:

$$T = f/m_1\lambda q_S + \upsilon a, \tag{13}$$

where T is the terminal cost per item.

2.2.5 The logistics cost of terminal shipment

From equations (7), (9), (10) and (13), the average logistic cost per item for terminal shipment, ALC_T, i.e., the sum of the transportation, inventory and terminal costs per item can now be written as:

$$\begin{aligned} ALC_T \quad &= \quad \frac{\alpha_1}{S_1}\left(NK_0M^{1/2}\right) + \frac{\alpha_2}{S_2}\left(K_0m_1\lambda^{1/2}\right) \\ &+ \quad PR\max\{T_1, T_2\} + f/(m_1\lambda q_s) + \upsilon a. \end{aligned} \tag{14}$$

Using Laporte’s notation (Laporte 1988), equation (14) formulates the logistic cost for the 3/R/T situation, where the 3 indicates there are three facility layers (an origin, transshipment terminals, and destinations), the R indicates that line-haul vehicles make single-stop return trips, and the T indicates that local vehicles can operate on multi-stop peddling tours. Origin and destinations in this article are a depot and many retail establishments. Shipping schedules for line-haul and local vehicles in this article are assumed to be coordinated and equivalent so as to minimize inventory costs.

3 Consumers’ demand and formulation of the optimization problem

In previous theoretical models, the level of consumer demand for shopping at a retail establishment was assumed to be a function of access time, the price level, and establishment size (Oppenheim 1990; Hsu and Chen 1994). The average demand level per consumer for the items in an establishment is assumed to be a gravity type function, that is $A = Kq_S^{\sigma}(PRICE)^{-\varepsilon}e^{-(b\,d_e/v)}$, where A is the average demand level per customer, q_s is the average number of items supplied by each retail establishment, $PRICE$ is the unit item price, d_e is the average customers' access distance to the establishment, v is the average travel speed of the customer, and K is a constant. Parameters σ, ε, and b represent, respectively, customers' sensitivity to the number of items supplied by the establishment, the item price, and access time. The values of these parameters would normally be positive. Based on this formulation, the customers' demand level within the market boundary falls as time needed to reach an establishment increases. Then the average total demand of all customers within the market boundary of a retail establishment, q_D, can then be obtained from the formulation by Hsu and Chen (1994) and rewritten as:

$$\begin{aligned} q_D &= 4\int_0^{d_0}\int_0^{d_0-x} A\cdot(POP)dxdy \\ &= 4K(POP)q_S^{\sigma}(PRICE)^{-\varepsilon}\frac{v}{b}\left[\frac{v}{b}\left(1-e^{-b\,d_0/v}\right)-d_0e^{-b\,d_0/v}\right], \qquad (15) \\ &= 2K(POP)q_S^{\sigma}(PRICE)^{-\varepsilon}\frac{v}{b}\left[\frac{v}{b}\left(1-e^{-b\lambda^{-1/2}/v}\right)-\lambda^{-1/2}e^{-b\lambda^{-1/2}/v}\right], \end{aligned}$$

where POP is the average population density, d_o is one half the diagonal length of the market boundary, $d_0 = (2\lambda)^{-1/2}$, and $b'/\sqrt{2} = b$. Assume q_D is the average demand for a retail establishment, then the total demand over the entire market area Q_D, is the summation of demands on all retail establishments in this area. That is:

$$\begin{aligned} Q_D = \; & 2K\mathrm{M}\lambda(POP)q_S^{\sigma}\,(PRICE)^{-\varepsilon} \\ & \times\frac{v}{b}\left[\frac{v}{b}\left(1-e^{-b\lambda^{-1/2}/v}\right)-\lambda^{-1/2}e^{-b\lambda^{-1/2}/v}\right]. \end{aligned} \qquad (16)$$

In this article, we assume suppliers may aim to minimize the average logistics cost or maximize the total supply when planning the density and market area for retail establishments and transshipment terminals. The objective of minimizing average logistics cost is found in most physical distribution literature no matter whether the process is direct or terminal shipping (e.g., Blumenfeld et al. 1985; Burns et al. 1985; Campbell 1993; Daganzo and Newell 1986). On the other hand, suppliers may aim to maximize the total supply, when initially setting up retail establishments so as to obtain as large a market share as possible. Nonlinear programming problems are formulated according to these two objectives subject to the demand-supply equality so as to determine the density of retail establishments and the number of terminals.

From equations (14) and (15), the nonlinear programming problems for minimizing the average logistics cost for terminal shipment are as follows:

$$\begin{aligned} \underset{\lambda,S_1,S_2}{MIN} \quad ALC_T = \; & \frac{\alpha_1}{S_1}\left(NK_0M^{1/2}\right)+\frac{\alpha_2}{S_2}\left(K_0m_1\lambda^{1/2}\right) \\ & +PR\max\{T_1,T_2\}+f/(m_1q_s\lambda)+va \end{aligned} \qquad (17\text{-a})$$

$$
\begin{aligned}
s.t.&\\
q_S \quad - \quad & 2(POP)Kq_S^{\sigma}(PRICE)^{-\varepsilon}\\
& \times\frac{v}{b}\left[\frac{v}{b}\left(1-\mathrm{e}^{-\frac{b}{v}\lambda^{-1/2}}\right)-\lambda^{-1/2}\mathrm{e}^{-\frac{b}{v}\lambda^{-1/2}}\right]=0 \qquad (17\text{-b})
\end{aligned}
$$

$$PRICE = ALC_T + P \qquad (17\text{-c})$$

$$T_1 = T_2 \qquad (17\text{-d})$$

$$\lambda,\ q_S,\ S_1,\ S_2 > 0.$$

To maximize the total supply for terminal shipment, only the objective of Min ALC_T need be replaced by the objective of Max $Q_D = M\lambda q_S$ in the above formulation, while other constraints can be kept the same. The decision variables for two problems are retail establishment density, λ, the shipment capacity of the line-haul fleet, S_1, and the shipment capacity of the local fleet, S_2. Constraint (17-b) represents the assumption of equality between the number of items supplied by each retail establishment and the total demand within the market area of the establishment. Constraint (17-c) defines the price per item as the sum of the average production cost per item, P, and the average logistics cost per item for terminal shipment, ALC_T. The constraint of $T_1 = T_2$ in equation (17-d) represents the optimal coordinated schedules for local vehicles and line-haul vehicles of terminal shipment as explained before. Because of this constraint, the shipment capacity of the local fleet, S_2, would be equal to the shipment capacity of the line-haul fleet, S_1, divided by the number of transshipment terminals, N (i.e., $\because\ T_1 = T_2,\ T_2 = S_2/Q_T$ and $T_1 = S_1/Q_1 = S_1/(NQ_T)\ \therefore\ S_2 = S_1/N$). The optimal number of transshipment terminals, N, is determined by searching for the best objective value obtained from among a variety of experiments which apply different numbers of terminal to the problem under exactly the same conditions.

For direct shipment, from equations (4) and (15), the formulation of the nonlinear programming problem to minimize the average logistics cost is:

$$\underset{\lambda,S_D}{MIN} \quad ALC_D = PR\left(\frac{S_D}{M\lambda q_S}\right) + \frac{\alpha_D}{S_D}(K_0 M\lambda^{1/2}) \qquad (18\text{-a})$$

$$
\begin{aligned}
s.t.&\\
q_S \quad - \quad & 2(POP)Kq_S^{\sigma}(PRICE)^{-\varepsilon}\\
& \times\frac{v}{b}\left[\frac{v}{b}\left(1-\mathrm{e}^{-b\lambda^{-1/2}/v}\right)-\lambda^{-1/2}\mathrm{e}^{-b\lambda^{-1/2}/v}\right]=0 \qquad (18\text{-b})
\end{aligned}
$$

$$PRICE = ALC_D + P \qquad (18\text{-c})$$

$$\lambda,\ q_S,\ S_D > 0.$$

Similarly, without changing any constraints, the objective of Max $Q_D = M\lambda q_S$ should be substituted for the objective of Min ALC_D in the above problem formulation for maximizing total supply for direct shipment. The decision variables for two problems are retail establishment density and fleet shipment capacity. Constraints (18-b) and (18-c) represent demand-supply equality, and item pricing as in the problem formulated for terminal shipment.

In one to many distribution problems, the tradeoffs between transportation and inventory cost components for direct shipment, or the tradeoffs between the transportation, inventory, and terminal cost components for terminal shipment usually determine the optimal values for the shipment size and frequency, the number of terminals, and the distribution cost (e.g., Blumenfeld et al. 1985; Burns et al. 1985; Campbell 1993; Daganzo and Newell 1986). However, in addition to these trade-offs, retail establishment density as well as consumer demand will play important roles in the long run by interacting with these trade-offs in helping to decide on the optimal objective values and distribution strategies. The mathematical problems in this article were formulated to attempt incorporation of these considerations. Therefore, retail establishment density is endogenous in our model, since it affects the tradeoff between different cost components of the average logistics cost, which is part of the item price (equations (17-c) and (18-c)), and influences the extent of the average consumer access distance, and the demand (or supply) at a retail establishment (equations (17-b) and (18-b)) as shown in the problems formulated above.

4 Numerical example and sensitivity analysis

The nonlinear optimization models formulated in the section above may be solved by means of a variety of algorithms. We solved the formulated problem using GINO, a computer modeling program developed by Liebman et al.(1986), and based on a generalized reduced gradient algorithm. This section presents a hypothetical example using a set of parameter values chosen from Campbell (1993) and Hsu and Chen (1994). This set of parameter values shown in Table 1, is also used as the basis for conducting a series of numerical experiments to observe the sensitivity of optimal solutions to changes in parameters.

A total study period of three months is assumed in this example. An inventory carrying rate of 0.1 (1/3 months) for terminal shipment is assumed doubling that of 0.05 (1/3 months) for direct shipment to reasonably account for extra costs which may result from the transshipment process in the real world. This extra cost may arise for terminal shipment due to high damage rates during loading and unloading and/or extra waiting for badly coordinated schedules at transshipment terminals. The unit distance transportation cost for direct shipment is to be assumed the same as that for line-haul transportation of terminal shipment. Both utilize high capacity vehicles to carry large shipment size. Therefore, it is also assumed that local infrastructure constraints do not prevent large vehicles from delivering to retail establishments. The model results are intended to minimize

Table 1. The initial values of base parameters

Parameter	Initial Value	Unit
f	10000	\$
va	1	\$
α_1	$0.0005 \times S_1 + 20$	\$/Km
α_2	$0.00025 \times S_2 + 8$	\$/Km
M	25000	Km^2
P	100	\$/Item
R	0.1	
v	0.42	Km/Min
POP	400	Persons/Km^2
σ	0.05	
ε	1	
K	1	
b	0.5	

average logistics cost and maximize total supply, and are listed in Tables 2 and 3.

The minimum average logistics cost is usually achieved by using the well know economic order quantity (EOQ) principle of calculating the optimum shipping strategy while assuming demand is inelastic. A worthwhile analysis would be to make comparisons to results based on the developed models in this article and those based on *EOQ* principle with inelastic demand. Table 4 summaries these results. The results of the *EOQ* model in the table are calculated by using the model developed in Hall (1986) where $EOQ = (\alpha_D K_0 \lambda^{1/2} Q_D/PR)^{1/2}$ for direct shipment and $EOQ = (\alpha_2 K_0 \lambda^{1/2} Q_T/PR)^{1/2}$ for terminal shipment, where *EOQ* principle is applied only on local shipping process. The average logistics costs of the developed models are shown to be lower than those of *EOQ* models for both direct and terminal shipments. This result implies that the average logistics cost can be reduced if suppliers set up both the optimum retail establishment density and the optimum shipment size by anticipating elastic consumer demand rather than apply merely the optimum shipment size strategy given by a range of retail establishment density values and without the consideration of demand-supply interaction. Restated, the average logistics cost can be reduced if *EOQ* principle is extended by considering elastic demand.

4.1 Sensitivity analysis of minimizing the average logistics cost

As we were interested in understanding how optimal solutions are affected by changes in model parameters such as total market area, M, fixed terminal cost, f, and average travel speed of consumer, v, a series of numerical experiments were conducted in which the value of one or several parameters were varied, while the others were held constant.

4.1.1 Variations in the size of total market area

Suppliers may need to decide on the density of retail establishments and the number of transshipment terminals so as to distribute and sell items in market

Table 2. The optimal result of minimizing average logistics cost[a]

Direct shipment		Terminal shipment				
D			$N=2$	$N=3$	$N=4$	$N=5$
ALC	17.18827	ALC_T	14.75167	14.62 647	14.69826	14.85135
q_S	4.353408	Q_T	13282.1	9912.226	7903.592	6698.364
PRICE	117.1883	λ	0.334432	0.416 449	0.474817	0.542283
S	0.705347	q_S	3.17723	2.856214	2.6 63289	2.470429
λ	0.131246	Q_D	26564.2	29736. 68	31614.37	33491.82
a_D	29.84556	*PRICE*	114.7517	114.62 65	114.6983	114.8514
S_D	19691.12	T_1	0.57999	0.580438	0.578359	0.584539
Frequency	0.725415	a_1	27.70349	28.6 3016	29.14223	29.78864
Q_D	14284.24	S_1	15406.97	17260.3 1	18284.45	19577.29
		a_2	9.925872	9.438359	9.142778	8.97 8864
		S_2	7703.487	5753.437	4571.113	391 5.457
		T_2	0.57999	0.580438	0.578359	0.584 539

[a] *D* for direct shipment, and *N* for the number of terminals.

Table 3. The optimal result of maximizing total supply[a]

Direct shipment		Terminal shipment				
D			$N=7$	$N=8$	$N=9$	$N=10$
QD	48314.92	Q_D	59824.98	59910.61	59961.63	59988.64
qS	0.351059	Q_T	8546.426	7488.826	6 662.403	5998.864
PRICE	141.0809	λ	14.97726	15.07 718	15.37991	15.64833
λ	5.505043	q_S	0.159776	0.1589 44	0.155948	0.153342
ALC	41.08088	*PRICE*	124.5623	124.4343	124.4551	124.5129
a_D	66.10071	ALC_T	24.56232	24.43 43	24.45508	24.51293
S_D	92201.42	T_1	0.987053	0.96700 8	0.976269	0.984015
Frequency	0.524015	a_1	49.52523	48.9 6701	49.26934	49.51487
		S_1	59050.46	57934.01	58538.68	5902 9.74
		a_2	10.10895	9.810438	9.626074	9.47 5743
		S_2	8435.779	7241.751	6504.298	5902 .974
		T_2	0.987053	0.967008	0.976269	0.98 4015

[a] *D* for direct shipment, and *N* for the number of terminals.

areas of different sizes. The average logistics cost initially decreases and then increases with increasing total market area, as shown in Fig. 1a, for a variety of curves standing for different numbers of terminals. The lowest point (cost) of any curve, which stands for a certain number of terminals, indicates the optimal market area that can be served by these terminals. The optimal market area expands with an increase in the number of terminals. In other words, the optimal number of terminals should be set to increase as the size of total market area increases. The envelope curve can be plotted along the lowest points of these curves, which shows the optimal number of terminals required for a variety of market areas of different sizes.

Table 4. The comparison between the optimum results of minimizing average cost and those of EOQ models[a]

Direct shipping			Terminal shipment						
				$N=2$		$N=3$		$N=4$	
	D	*EOQ*		*D*	*EOQ*	*D*	*EOQ*	*D*	*EOQ*
ALC	17.18827	**23.8269**	ALC_T	14.75167	**14.9098**	14.62647	**14.9933**	14.69826	**15.2466**
q_S	4.353408	4.353408	Q_T	13282.1	13282.1	9912.226	9912.226	7903.592	7903.592
λ	0.131246	0.131246	λ	0.334432	0.334432	0.416449	0.416449	0.474817	0.474817
a_D	29.84556	29.84556	q_S	3.17723	3.17723	2.856214	2.856214	2.663289	2.663289
S_D	19691.12	**17017.3**	Q_D	26564.2	26564.2	29736.68	29736.68	31614.37	31614.37
Q_D	0.725415	**0.8394**	T_1	0.57999	0.57999	0.580438	0.580438	0.578359	0.578359
Frequency	14284.24	14284.24	a_1	27.70349	27.70349	28.63016	28.63016	29.14223	29.14223
			S_1	15406.97	15406.97	17260.31	17260.31	18284.45	18284.45
			a_2	9.925872	9.925872	9.438359	9.438359	9.142778	9.142778
			S_2	7703.487	**8454.35**	5753.437	**6142.77**	4571.113	**4831.18**
			T_2	0.57999	**0.63652**	0.580438	**0.61972**	0.578359	**0.61126**

[a] *D* for direct shipment, and *N* for the number of terminals.

The retail establishment density decreases with increasing total market area as shown in Fig. 1b. This result implies that suppliers may set up fewer but larger retail establishments so as to realize economies of scale in larger market areas. In addition, retail establishment density is higher in market areas with the same size but more terminals because as the number of transshipment terminals increases, suppliers may increase the number of retail establishments so as to reduce the average access distance for consumers, thus stimulating demand and maintaining the economical size served by each terminal.

The average logistics cost of direct shipment increases with total market area. With an increase in total market area, the average logistics cost of direct shipment is initially lower than that of terminal shipment and then grows, as illustrated by a comparison of the curves in Fig. 1a. The critical point of market area, as indicated in Fig. 1a, is where for suppliers to decide whether to distribute a commodity through one terminal or directly to a retail establishment. The entire envelope curve can be plotted along this point and the lowest points of all curves. For terminal shipment, the optimal number of shipment terminals for any specific market area can also be found along this envelope.

4.1.2 Variations in fixed terminal cost

Terminal cost plays an important role in deciding whether to use direct or terminal shipment. It is shown in Fig. 2a that raising the fixed terminal cost has the effect of increasing the average logistics cost for terminal shipment, and the extent of this depends on the number of terminals set up. That is a lesser extent for a smaller number of terminals. The entire envelope curve can be plotted along the lowest points of all curves, which shows the optimal shipment type and terminal number corresponding to different fixed terminal cost.

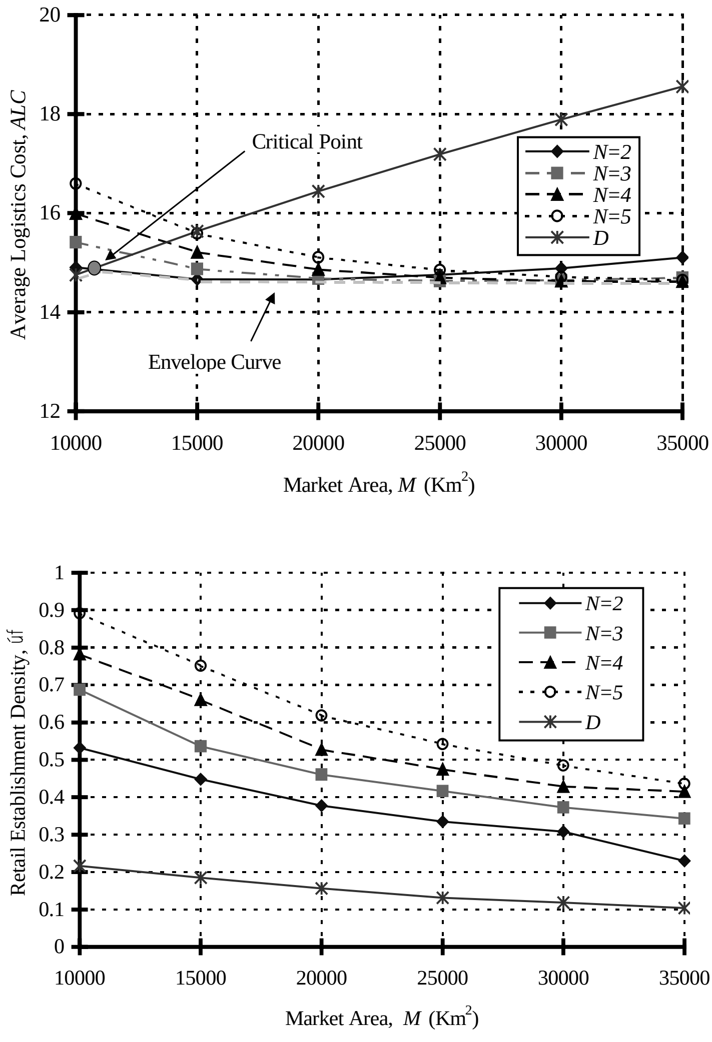
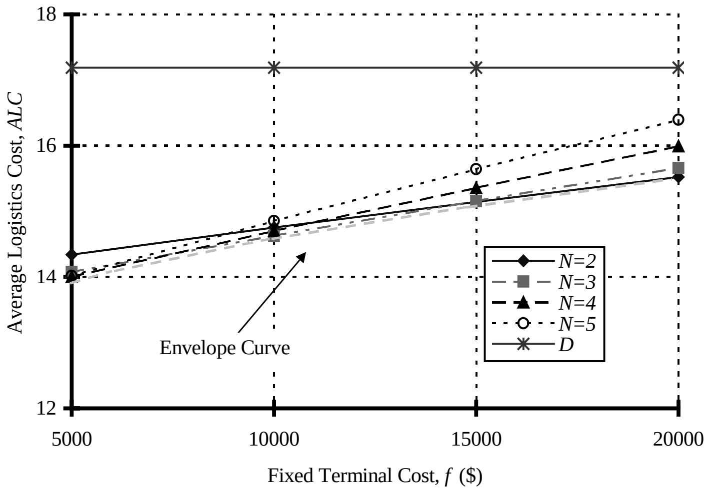


Fig. 1. a Average logistics cost vs. market area in minimizing average logistics cost. **b** Optimal retail establishment density vs. market area in minimizing average logistics cost

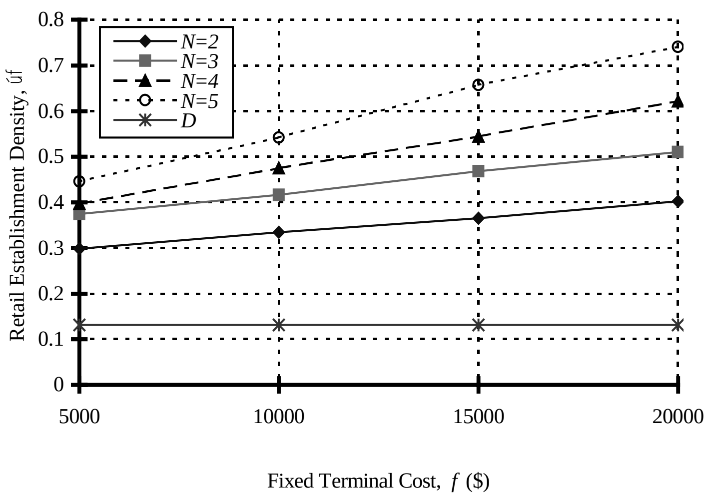
As shown in Fig. 2b, retail establishment density should increase with the raising of fixed terminal cost so as to reduce access distance and stimulate demand to share the rising terminal cost and yield economies of scale for transshipment terminals.

4.1.3 Variations in average travel speed

An increase in average travel speed may result from the improvement of transportation technology used by consumers. Improvements may result from using



a



b

Fig. 2. a Average logistics cost vs. fixed terminal cost in minimizing average logistics cost. **b** Retail establishment density vs. fixed terminal cost in minimizing average logistics cost

automobiles rather than walking or riding buses for shopping. As the average travel speed of consumers increases, the optimal size of retail establishments increases but their density declines as shown in Fig. 3b. It is shown in Fig. 3a that with improved average travel speed, the average logistics cost is reduced. In other words, improvements in transportation technology increase the size and market area of retail establishments, which permits suppliers to obtain scale efficiencies, and consumers to share in the surplus created by those efficiencies.

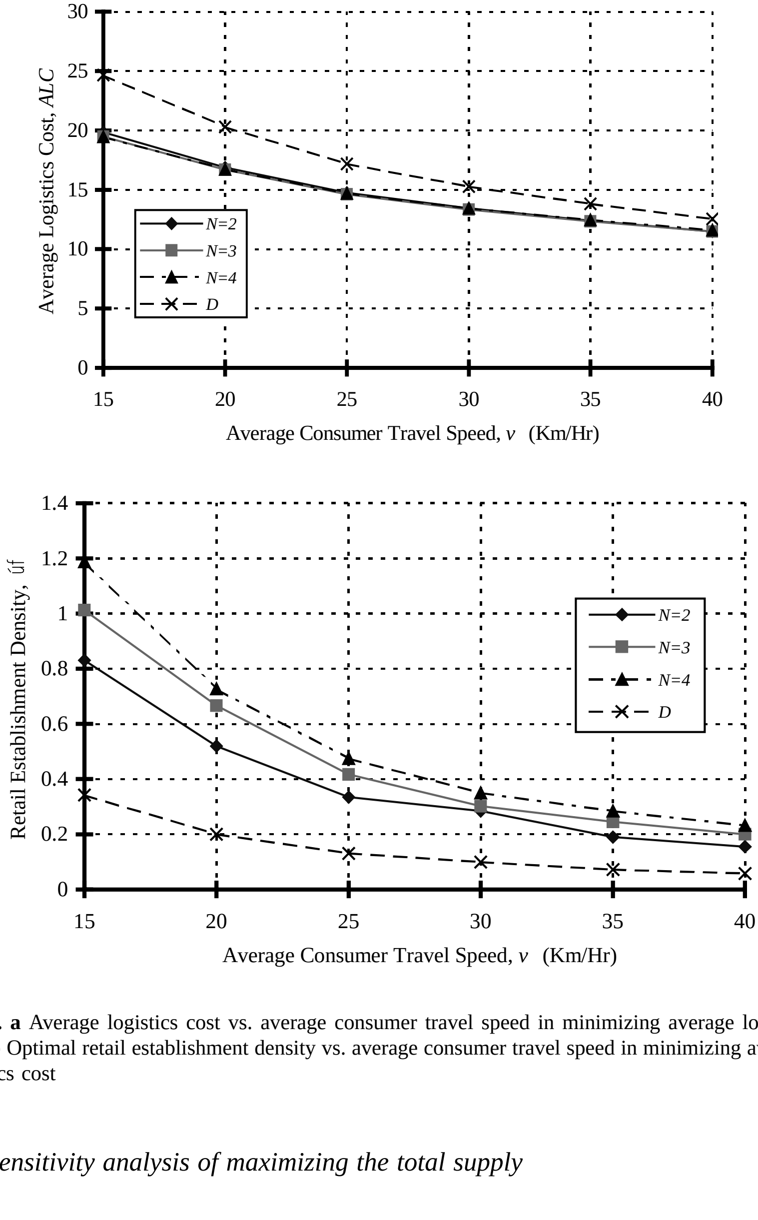
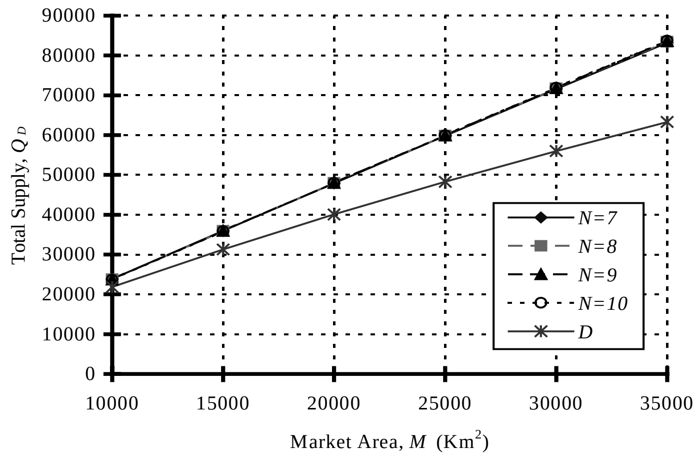


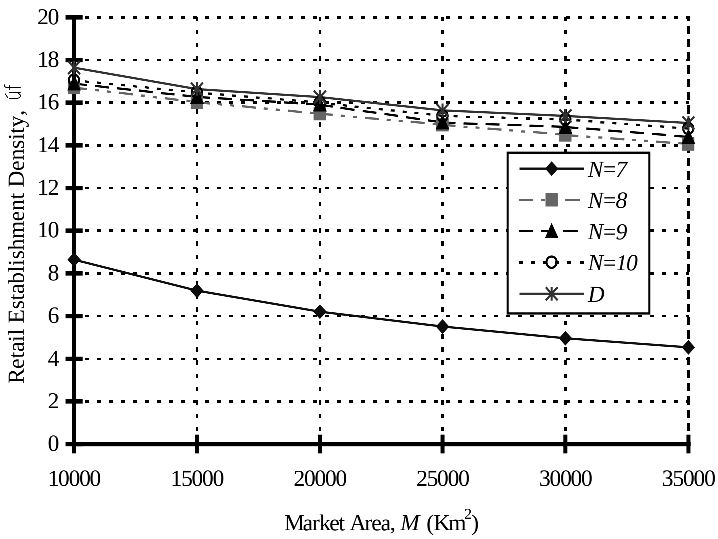
Fig. 3. a Average logistics cost vs. average consumer travel speed in minimizing average logistics cost. **b** Optimal retail establishment density vs. average consumer travel speed in minimizing average logistics cost

4.2 Sensitivity analysis of maximizing the total supply

One numerical experiment on variations in the size of total market area was conducted to compare and observe the behavior of the model with different objective functions, i.e., maximizing total supply. As the total market area increases, the total supply increases as shown in Fig. 4a, while the optimal density of retail establishments declines as shown in Fig. 4b. The extent of these changes both depend on the number of terminals set up. Individual curves are plotted for different market areas of various sizes, as shown in Figs. 5a–e, to determine the optimal number of terminals, respectively, for these market areas. Similarly, the



a



b

Fig. 4. a Total supply vs. market area in maximizing total supply. **b** Optimal retail establishment vs. market area in maximizing total supply

envelope curve in Fig. 5f, which shows the optimal number of terminals suitable for a variety of market areas, can be plotted along the highest points of these individual curves. The optimal number of terminals for any market area is shown to be higher if suppliers aim to maximize total supply as compared with aiming to minimize the average logistics cost as shown in Fig. 1a. This implies that suppliers may not necessarily realize minimized logistics cost by expanding their total supply up to the maximum.

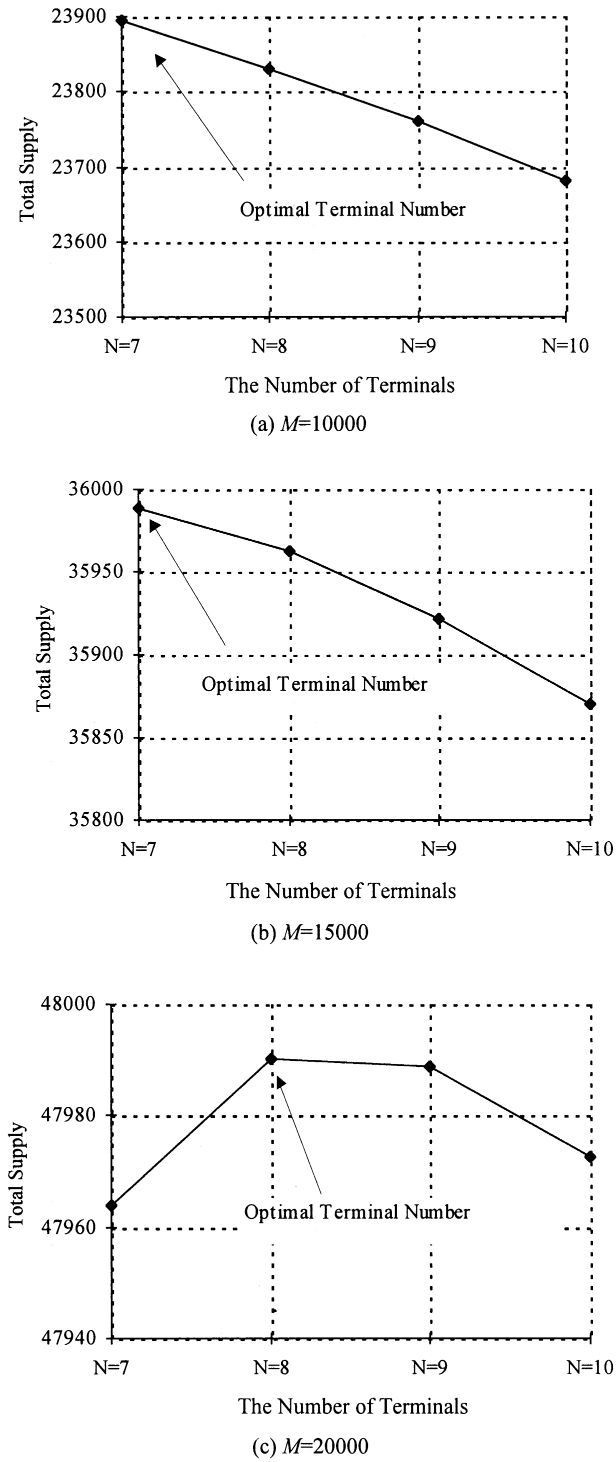
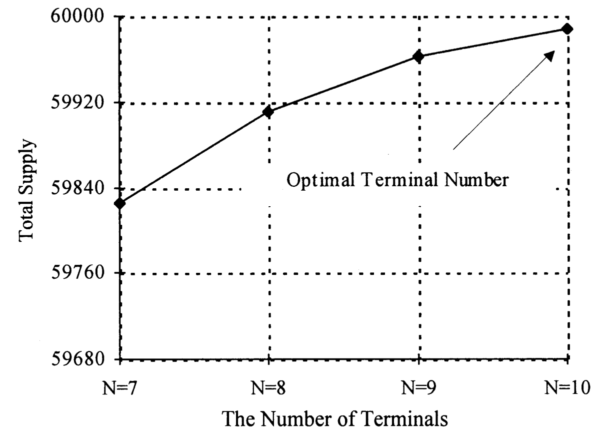
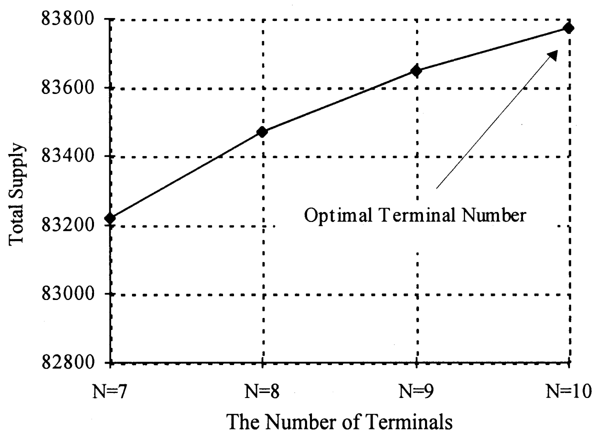

23900
23800
23700
23600
23500
Total Supply
Optimal Terminal Number
N=7
N=8
N=9
N=10
The Number of Terminals
(a) M=10000
36000
35950
35900
35850
35800
Total Supply
Optimal Terminal Number
N=7
N=8
N=9
N=10
The Number of Terminals
(b) M=15000
48000
47980
47960
47940
Total Supply
Optimal Terminal Number
N=7
N=8
N=9
N=10
The Number of Terminals
(c) M=20000

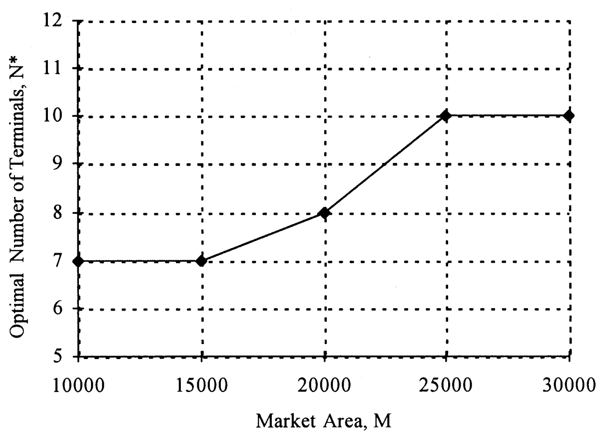
Fig. 5a–c.



(d) M=25000



(e) M=30000



(f) Envelope curve

Fig. 5d–f. Total supply vs. terminal number. **f** Optimal terminal number vs. market area (envelope curve)

5 Summary and conclusions

The article develops analytical models to investigate the optimal density of retail establishments and the optimal number of transshipment terminals which suppliers ought to set up in a region so as to minimize average logistics cost or maximize total supply. The models are developed in such a way that both physical distribution and market area problems are addressed. The commodities are assumed to be distributed from a depot directly or through one intermediate terminal to many retail establishments. In such a distribution system, average logistics cost per item is formulated by combining the transportation and inventory cost components for direct shipment and combining the line-haul transportation, the local transportation, inventory and terminal cost components for terminal shipment. For terminal shipment, schedules for line-haul vehicles and schedules for local vehicles are assumed to be coordinated.

Instead of assuming inelastic demand and fixed demand locations as in most physical distribution literature, this article considers demand-supply interactions and assumes the density of establishments to be endogenous. Consumers' demand is formulated as a function of retail establishment supply, access distance, item price, and population density. Since average logistics cost is also a function of retail establishment supply and density, and a component of item price, therefore demand-supply interactions would occur. Both average logistics cost and consumers' demand are formulated approximately by using continuous-space modeling. Nonlinear programming problems are formulated for two different objectives, i.e., minimizing average logistic cost and maximizing total supply subject to the demand-supply equality so as to determine the optimal density of retail establishment and the optimal number of terminals.

The model provides the basis for performing numerical analyses to determine the sensitivity of the optimal solution to changes in model parameters. The envelope curves for deciding a variety of optimal configuration strategies corresponding to different values of total market area, fixed terminal cost, and average travel speed of consumers are plotted. Critical values for deciding whether to distribute a commodity through one terminal or directly to retail establishments could also be identified. Apart from the results of most physical distribution studies, which focused only on operating issues, the results of this article provide insights into demand-supply interactions when suppliers plan the density of retail establishments and terminals as strategic issues.

The transportation distances in our model are approximated using the formulas in Larson and Odoni (1981) and Daganzo (1984). These formulas are accurate when the density of retail establishments varies slowly over the market area. The retail establishment density may vary over the market area, but should not vary much over the area served by each terminal. The assumption makes the model developed in this article applicable merely for urban areas, where population density varies slowly.

The model developed in this article is the approximate analytic model. Thus, locations of the depot, terminals and retail establishments are not represented.

However, the total transportation distance varies with changes in the number of terminals and the density of retail establishments for the same market area. The average total logistics costs are shown to be sensitive to these variations. These results partially agree with the findings of McCann (1996), which show the variation of total logistics costs with distance, rather than simply that of transportation costs. However, items shipped in the article are assumed cheap and in-transit inventory costs are ignored while terminal costs are assumed to be independent of locations. Future studies could further consider site-location dependent costs such as land and labor costs and formulate inventory costs for high-value items. The model is suitable for analyzing retail establishments distributed with population settlement such as convenience stores and supermarkets. These type of retail establishments are not tend to cluster. For those tend to cluster, e.g. the garment industry, further studies are required to analyze the agglomeration economies and economies of scale in the production of such industry or service.

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