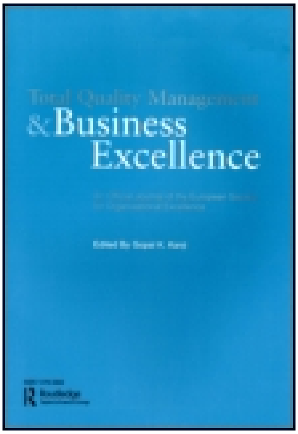

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CARFAX

Multi-response robust design by principal component analysis

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Abstract *Most previous Taguchi method applications have only addressed a single-response problem. However, more than one correlated response normally occurs in a manufactured product. The multi-response problem has received only limited attention. In this work, we propose an effective procedure on the basis of principal component analysis (PCA) to optimize the multi-response problems in the Taguchi method. With the PCA, a set of original responses can be transformed into a set of uncorrelated components. Therefore, the conflict for determining the optimal settings of the design parameters for the multi-response problems can be reduced. Two case studies are evaluated, indicating that the proposed procedure yields a satisfactory result.*

Introduction

Robust design is an engineering method of quality improvement that seeks to obtain a lowest cost solution to the product design specification based on the customer's requirements. The Taguchi method, which combines the experimental design techniques with quality loss considerations, is the conventional approach to achieve robustness. This method can only be used in a single-response case; it cannot be used to optimize a multi-response problem. However, a customer normally considers more than one quality characteristic in most manufactured products and the quality characteristics are usually correlated. Engineering judgement has, up until now, been used primarily to optimize the multi-response problem in the Taguchi method (Phadke, 1989). Unfortunately, an engineer's judgement increases the uncertainty during the decision-making process. Another approach to solve this problem entails assigning a weight for each response (Hung, 1990; Shiau, 1990; Tai *et al.*, 1992). Nevertheless, determining a definite weight for each response in an actual case still remains difficult. Another method employs the regression technique (Logothetis & Haigh, 1988; Pignatiello, 1993). However, such an approach increases the computational process complexity, and the possible correlations among the responses may still not be considered. In addition, a factor which is significant in a single-response case may not be significant when considered in a multi-response case. Therefore, a more effective approach is required to solve this complicated problem.

In this work, we propose a systematic procedure via principal component analysis (PCA) to optimize the multi-response production process. By using PCA, a set of original responses is transformed into a set of uncorrelated components so that the optimal factor/level combination can be found. The proposed procedure includes a series of steps capable of

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decreasing the uncertainty in engineering judgement when the Taguchi method is applied. This work addresses only the static quality characteristic problem, in which the desired response value is fixed.

PCA

Pearson and Hotelling (1933) first introduced PCA. PCA is used to explain the variance–covariance structure through the linear combinations of the original variables. Assume that there are p components to represent the system variability. By using PCA, this variability can be explained by a small number, $k (k \leqslant p)$, of the principal components, i.e. k principal components will account for most of the variance in the original p variables. Let $Y_1, Y_2, \ldots, Y_p$ be a set of variables. Through the PCA, we have the following uncorrelated linear combinations:

$$\begin{aligned} \Omega_1 &= a_{11} Y_1 + a_{12} Y_2 + \ldots + a_{1p} Y_p \\ \Omega_2 &= a_{21} Y_1 + a_{22} Y_2 + \ldots + a_{2p} Y_p \\ &\vdots \\ \Omega_k &= a_{k1} Y_1 + a_{k2} Y_2 + \ldots + a_{kp} Y_p \end{aligned}$$

where $a_{k1}^2 + a_{k2}^2 + \ldots + a_{kp}^2 = 1$. Further, Ω_1 is called the first principal component, Ω_2 is called the second principal component and so on. The coefficients of the kth component are the elements of the eigenvector corresponding to the kth largest eigenvalues. The option of performing the PCA is available on SAS and SPSSX.

More than one correlated quality characteristic is usually considered in a manufactured product. PCA is an effective means of determining a small number of constructs which account for the main sources of variation in such a set of correlated quality characteristics.

Proposed procedure

In this section, an effective procedure is developed to transform a set of responses into a set of uncorrelated components such that the optimal conditions in the parameter design stage for the multi-response problem can be determined. Assume that we have p responses. The proposed procedure is described in the following.

Step 1: Compute the quality loss for each response. Let L_{ij} be the quality loss for ith response at jth trial. L_{ij} can be computed on the basis of Taguchi's loss function.

Step 2: Normalize L_{ij}. To reduce the variability, the scale of the quality loss for each response is normalized. L_{ij} is transformed into Y_{ij} $(0 \leqslant Y_{ij} \leqslant 1)$ by using the following formula:

$$Y_{ij} = \frac{L_i^+ - L_{ij}}{L_i^+ - L_i^-}$$

where Y_{ij} = the normalized quality loss for ith response at jth trial; $L_i^+ = \max\{L_{i1}, L_{i2}, \ldots, L_{ij}\}$; $L_i^- = \min\{L_{i1}, L_{i2}, \ldots, L_{ij}\}$.

Step 3: Perform the PCA on the basis of the computed data, Y_{ij}.

Step 4: Determine the number of principal components, k, and compute

$$\Omega_{kj} = \sum_{i=1}^{p} a_{ki} Y_{ij}$$

where $a_{k1}, a_{k2}, \ldots, a_{kp}$ are the elements of the eigenvector corresponding to the kth largest eigenvalue. $\boldsymbol{\Omega}$ can be considered as a multi-response performance index, which can be used to determine the optimal conditions. Based on Kasier's (1960) study, the components with eigenvalue greater than 1 are chosen to replace the original responses for further analysis.

Step 5: Determine the optimal factor/level combination. The larger the $\boldsymbol{\Omega}$ value implies the better the product quality. If $k = 1$, the factor effects can be estimated and the optimal control factors and their levels determined on the basis of a single $\boldsymbol{\Omega}$ value. If $k \geqslant 1$, trade-offs might be necessary to select a feasible solution.

Illustrations

This section demonstrates the effectiveness of the proposed procedure by using two case studies.

Case study 1

This case study involves improving a hard disk drive's quality. The Industrial Technology Research Institute, Taiwan, performed the case study. In this study, an experiment was performed to determine the effects of design parameters on the responses. Optimal settings could, it was hoped, be found such that a low variability for the responses could be achieved. The four desired responses are:

PW: 50% pulse width (smaller-the-better);
HFA: high-frequency amplitude (larger-the-better);
OW: over write (larger-the-better);
PS: peak shift (smaller-the-better).

In the experiment, five controllable factors were selected for optimization. Table 1 lists these factors and their alternative levels. The standard array L_{18} was selected for the experiment. Table 2 summarizes the data for 18 experiments.

When the proposed procedure is applied in this case study, the quality loss for each response is first computed and normalized, as shown in Table 3. Next, the PCA is performed on these normalized data using SAS. Table 4 lists the eigenvalues. Based on Kasier's criterion, the first principal component is chosen to represent the original four responses. The eigenvector for the first largest eigenvalue is [0.59716, 0.50583, −0.3296, 0.52883]. Consequently, we have

$$\boldsymbol{\Omega}_{1j} = 0.59716 Y_{1j} + 0.50583 Y_{2j} - 0.3296 Y_{3j} + 0.52883 Y_{4j}$$

Table 1. *Factors and their levels (case study 1)*

Factors	Level 1	Level 2	Level 3
A: Disk writability	*8000*	10 000	—
B: Magnetization width	*2.5*	3.0	3.5
C: Gap length	0.3	*0.4*	0.5
D: Coercivity of media	1200	*1400*	1600
E: Rotational speed	3000	*3500*	4000
N (noise): Flying height	2	4.0	4.5

Starting levels are identified by italics.

Table 2. *Data summary by experiment (case study 1)*

Exp. no.	Factors								PW		HFA		OW		PS	
	A	B	C	D	E	F	G	H	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2
1	1	1	1	1	1	1	1	1	63.5	66.0	286.7	257.6	−32.2	−30.1	10.9	12.0
2	1	1	2	2	2	2	2	2	64.2	66.0	343.0	310.6	−34.8	−33.3	11.6	13.0
3	1	1	3	3	3	3	3	3	65.6	67.0	381.1	354.4	−36.2	−35.3	13.6	14.7
4	1	2	1	1	2	2	3	3	54.5	56.6	328.1	295.4	−33.5	−31.5	9.2	10.8
5	1	2	2	2	3	3	1	1	56.2	57.8	368.3	333.0	−36.2	−34.9	10.2	11.2
6	1	2	3	3	1	1	2	2	87.5	89.3	234.3	213.5	−40.0	−38.4	17.8	19.1
7	1	3	1	2	1	3	2	3	63.6	66.1	288.0	259.2	−31.7	−29.5	10.1	11.8
8	1	3	2	3	2	1	3	1	64.3	66.1	335.8	304.9	−35.2	−33.9	10.7	12.1
9	1	3	3	1	3	2	1	2	65.6	66.9	312.7	282.8	−43.7	−46.5	14.4	15.4
10	2	1	1	3	3	2	2	1	47.7	49.5	451.0	393.8	−15.6	−22.3	11.0	11.8
11	2	1	2	1	1	3	3	2	74.9	77.0	291.6	263.0	−33.6	−32.6	16.3	17.9
12	2	1	3	2	2	1	1	3	74.9	76.5	346.8	312.4	−35.1	−33.8	16.9	18.6
13	2	2	1	2	3	1	3	2	47.7	49.5	447.9	393.8	−25.8	−22.3	10.0	11.6
14	2	2	2	3	1	2	1	3	75.0	77.0	312.8	280.5	−29.7	−28.9	14.6	16.5
15	2	2	3	1	2	3	2	1	74.9	76.5	271.9	245.4	−38.4	−38.9	18.0	19.2
16	2	3	1	3	2	3	1	2	54.5	56.6	385.2	336.7	−20.4	−17.2	11.6	13.4
17	2	3	2	1	3	1	2	3	56.2	57.8	378.7	341.5	−35.6	−34.6	12.1	13.4
18	2	3	3	2	1	2	3	1	87.4	89.3	270.6	244.6	−38.5	−37.0	19.4	21.3

Table 3. *Normalized data and Ω values (case study 1)*

Exp. no.	L_{18}								PW	HFA	OW	PS	Ω
	A	B	C	D	E	F	G	G					
1	1	1	1	1	1	1	1	1	0.66413	0.44870	0.78924	0.90214	0.84032
2	1	1	2	2	2	2	2	2	0.65594	0.73942	0.85558	0.83733	0.92640
3	1	1	3	3	3	3	3	3	0.62708	0.87899	0.88714	0.68228	0.88745
4	1	2	1	1	2	2	3	3	0.86720	0.67398	0.82224	1.00000	1.11638
5	1	2	2	2	3	3	1	1	0.83735	0.82543	0.88352	0.95515	1.13128
6	1	2	3	3	1	1	2	2	0.00000	0.00000	0.93825	0.23602	0.18423
7	1	3	1	2	1	3	2	3	0.66176	0.45898	0.77444	0.93643	0.86709
8	1	3	2	3	2	1	3	1	0.65355	0.71303	0.86540	0.90518	0.94423
9	1	3	3	1	3	2	1	2	0.62830	0.60521	1.00000	0.61315	0.67602
10	2	1	1	3	3	2	2	1	1.00000	1.00000	0.00000	0.90623	1.58178
11	2	1	2	1	1	3	3	2	0.37521	0.48406	0.83626	0.38798	0.39854
12	2	1	3	2	2	1	1	3	0.38225	0.74998	0.86349	0.31566	0.49009
13	2	2	1	2	3	1	3	2	1.00000	0.99757	0.50777	0.94707	1.43494
14	2	2	2	3	1	2	1	3	0.37384	0.59826	0.73805	0.54812	0.57244
15	2	2	3	1	2	3	2	1	0.38225	0.34811	0.93128	0.21854	0.21317
16	2	3	1	3	2	3	1	2	0.86720	0.85425	0.06560	0.82054	1.36188
17	2	3	2	1	3	1	2	3	0.83735	0.85467	0.80169	0.80169	1.06758
18	2	3	3	2	1	2	3	1	0.00161	0.33984	0.91838	0.00000	−0.12953

where Y_{j1}, Y_{j2}, Y_{j3} and Y_{j4} represent the normalized quality loss for the responses PW, HFA, OW and PS at jth trial respectively. The Ω values are computed and listed in the last column of Table 3. Table 5 summarizes the main effects on Ω and Fig. 1 plots their corresponding factor effects. The controllable factors on Ω value in order of significance are: C, E, D, B and A. The larger the Ω value implies the better the quality. Consequently, the optimal condition can be set as $A_1B_1C_1D_3E_3$.

Table 4. *Eigenvalues for the principal components (case study 1)*

Principal component	Eigenvalue
First	2.93478
Second	0.68014
Third	0.33771
Fourth	0.04736

Table 5. *Main effects on Ω (case study 1)*

Factor	Level 1	Level 2	Level 3	Max − min
A	0.5950	0.5926	—	0.0024
B	0.6623	0.5163	0.6027	0.1460
C	1.0108	0.6380	0.1325	0.8783
D	0.5147	0.5900	0.6771	0.1624
E	0.2029	0.6474	0.9311	0.7282

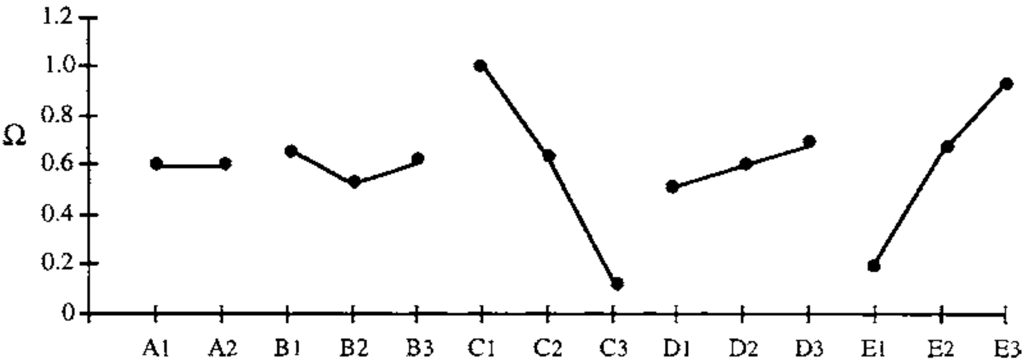

Figure 1. *Factor effects on Ω (case study 1).*

This case study is also analyzed by the Taguchi's approach. The tentative optimum setting can be separately made in the following:

PW: $A_2B_1C_1D_1E_3$
HFA: $A_2B_1C_1D_3E_3$
OW: $A_1B_2C_3D_1E_1$
PS: $A_1B_3C_1D_2E_3$

These results demonstrate that different levels of the same factor can be optimum for different responses. As a result, the decision-making process is difficult. Based on the significance of the factor effects and the engineering judgement, the optimal condition is set as $A_1B_1C_1D_1E_3$.

To predict the anticipated improvements under the chosen optimum conditions, the signal to noise (SN) ratios for these four responses are predicted using the additive model. Table 6 displays the computations. This table reveals that no significant difference arises in the effectiveness between the proposed procedure and Taguchi's approach. However, the proposed procedure can be considered as a more convenient approach than Taguchi's method in the multi-response case, owing to the fact that the decision-making's complexity is reduced.

Table 6. *Prediction of SN ratios using the additive model (case study 1)*

Factor	Starting condition (dB)	Optimum condition (dB)		Anticipated improvement (dB)	
		Taguchi's approach	Proposed procedure	Taguchi's approach	Proposed procedure
PW	− 36.277	− 33.733	− 33.738	2.544	2.539
HFA	50.465	51.422	52.225	0.957	1.760
OW	31.510	29.816	27.710	− 1.694	− 3.80
PS	− 21.480	− 19.366	− 19.366	2.114	2.114

Case study 2

Phadke (1989) considered a case study to improve a polysilicon deposition process. This study was conducted by Peter Hey in 1984. Six controllable factors were identified: deposition temperature (A), deposition pressure (B), nitrogen flow (C), silane flow (D), setting time (E) and cleaning method (F). All the factors were studied at three levels each. The L_{18} orthogonal array was used and factors A–F were assigned to columns 2, 3, 4, 5, 6 and 8 respectively. The quality characteristics of interest were the surface defects (smaller-the-better), the thickness (nominal-the-best) and the deposition rate (larger-the-better). The target value in the study for the thickness of polysilicon layer was 3600 Å. Nine observations were taken for each trial run. The starting condition was set as $A_2B_2C_1D_3E_1F_1$. The optimum condition chosen from the experimental data by Phadke was $A_1B_2C_1D_3E_2F_2$.

The above case is analyzed again by the proposed procedure. The quality loss for each response is computed and normalized as shown in Table 7. Next, the PCA is performed on these normalized data using SAS. Table 8 lists the eigenvalues. The first principal component

Table 7. *Normalized data and Ω values (case study 2)*

Exp. no.	L_{18} A	B	C	D	E	F	G	H	Surface defects	Thickness	Deposition rate	Ω
1	1	1	1	1	1	1	1	1	1.00000	0.98227	1.00000	1.72010
2	1	1	2	2	2	2	2	2	0.99966	0.98472	0.91315	1.67160
3	1	1	3	3	3	3	3	3	0.99793	0.98580	0.88463	1.65466
4	1	2	1	1	2	2	3	3	0.99998	0.99908	0.91596	1.68119
5	1	2	2	2	3	3	1	1	0.88676	0.50158	0.60600	1.16437
6	1	2	3	3	1	1	2	2	0.89456	0.96755	0.82783	1.54883
7	1	3	1	2	1	3	2	3	0.93862	0.49694	0.56539	1.17057
8	1	3	2	3	2	1	3	1	0.06889	0.64065	0.12499	0.46156
9	1	3	3	1	3	2	1	2	0.58790	0.94250	0.00000	0.87348
10	2	1	1	3	3	2	2	1	1.00000	0.85922	0.96887	1.63552
11	2	1	2	1	1	3	3	2	1.00000	0.99404	0.98542	1.71815
12	2	1	3	2	2	1	1	3	0.98072	0.99233	0.89924	1.65612
13	2	2	1	2	3	1	3	2	0.99453	0.95994	0.79942	1.59001
14	2	2	2	3	1	2	1	3	0.99972	1.00000	0.85550	1.64698
15	2	2	3	1	2	3	2	1	0.83480	0.99066	0.78605	1.50072
16	2	3	1	3	2	3	1	2	0.99997	0.95780	0.56227	1.45670
17	2	3	2	1	3	1	2	3	0.10669	0.56430	0.16314	1.46518
18	2	3	3	2	1	2	3	1	0.00000	0.00000	0.37354	0.21342

Table 8. *Eigenvalues for the principal components (case study 2)*

Principal component	Eigenvalue
First	2.37892
Second	0.46894
Third	0.15215

is chosen to represent the original three responses. The eigenvector for the first largest eigenvalue is [0.61559, 0.54279, 0.57134]. As a result, we have

$$\Omega_{1j} = 0.61559 Y_{1j} + 0.54279 Y_{2j} + 0.57134 Y_{3j}$$

where Y_{j1}, Y_{j2} and Y_{j3} represent the normalized quality loss for the surface defects, thickness and deposition rate at jth trial respectively. The Ω values are computed and listed in the last column of Table 7. The factor effects on Ω can be obtained and the optimal conditions can therefore be set as $A_1B_1C_3D_2E_3F_2$.

To predict the anticipated improvements under the chosen optimum conditions, the SN ratios for surface defects, thickness and deposition rate are predicted using the additive mode. Table 9 displays the computations for Phadke's study and proposed analyses. According to this table, an improvement in surface defects for the proposed procedure analysis is equal to $[(-2.29)-(-56.69)] = 54.40$ dB, which is larger than the improvement in Phadke's study of 36.85 dB. Similarly, the improvement in thickness uniformity for the proposed procedure analysis is better than that of Phadke's study. A slight reduction occurs in the deposition rate for the proposed procedure. Consequently, in this case study, the proposed procedure can be considered as a more effective approach than Phadke's study (based on an engineer's judgement) in the multi-response problem.

Conclusions

A procedure has been proposed in this study to achieve the optimization of multi-response problems in the Taguchi method. By using PCA, a set of (correlated) responses is transformed into a set of a small number of uncorrelated components. Principal components reduce the number of dimensions and decrease the complexity of the multi-response problems. Accordingly, based on these uncorrelated components, the optimal conditions in the parameter design stage can be easily chosen in an objective manner. In addition, two case studies demonstrate the effectiveness of the proposed procedure.

Table 9. *Prediction of SN ratios using the additive model (case study 2)*

Factor	Starting condition (dB)	Optimum condition (dB)		Anticipated improvement (dB)	
		Phadke's study	Proposed procedure	Phadke's study	Proposed procedure
Surface defects	−56.69	−19.84	−2.29	36.85	54.40
Thickness	29.95	36.79	41.23	6.84	11.28
Deposition rate	34.97	29.60	27.21	−5.37	−7.76

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