

## The market size of a city-pair route at an airport

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**Abstract** A model is developed for estimating the size of the market for a city-pair route at an airport from both the demand and supply sides of air transportation. The average airport access cost, average passenger delay cost, and average airline operating cost all either increase or decrease with an increase in the market size of a city-pair route at an airport, so the optimum market size can be determined from trade-offs among these costs. A nonlinear mathematical programming problem is formulated to determine the optimal number of passengers, the local service area of a city-pair market and to perform sensitivity analyses. The results show that long-haul services ought to be concentrated in one large airport, while short-haul services might be dispersed among many small airports. Improvements in the technology of the airport access mode or increases in the average income of the cities served can expand the market size and service area, but at a declining expansion rate. In metropolitan areas with high population density, airlines can operate more efficiently and distribute air services among more airports. City-pair markets with stable passenger demand, or markets served by airlines with efficient scheduling technology are shown to exhibit high cost efficiencies.

### 1. Introduction

Airlines frequently need to estimate passenger demand for city-pair routes at a particular airport in order to plan marketing, scheduling, and routing strategies. Although much research has been done on the development of city-pair air passenger demand models, there are two issues that need to be considered further:

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1. Most city-pair demand models or airport demand models have been investigated by aggregate gravity-type or econometric regression approaches (e.g., Verleger 1972; Morrison and Winston 1985; Kaemmerle 1991; Russon and Riley 1993) or disaggregate utility maximizing approaches (e.g., Harvey 1987; Hansen and Kanafani 1988, 1990) from the perspective of either passengers or airlines. The discussion has focused on models in which the supply characteristics are implicitly assumed to be exogenous. However, in long-term equilibrium, the market for city-pair routes at an airport should be an endogenous variable resulting from interactions between decisions made by air passengers, airlines, and airport authorities.

2. O'Kelly (1986) combined a facility location problem with a model of spatial interaction and described a normative framework which finds the best location for hubs. However, a normative framework which simultaneously finds the optimal passenger number and local market area for the airport market of a city-pair air route has not been investigated. Estimates of the number of passengers in the city-pair market of an airport are useful for many purposes. Estimating the optimal local service area for the city-pair market of an airport could be the key to determining the number of airports needed to serve a multi-airport region or metropolitan area. Moreover, simultaneously determining both the optimal market size and the optimal service area may unveil trade-offs among airline operation-related costs (e.g., air fares, schedule delays) and airport ground access costs.

This paper attempts to develop a unified model to determine both the number of passengers of the optimal market and the optimal local service area for the city-pair route served by an airport. Economies of density exist if unit costs decline as airlines add flights or use larger aircraft with no changes in load factor and stage length within a given city-pair route (Caves et al. 1984). Economies of density are reached within any city-pair market as a response to expanding market size. If more flights are added, passenger delay costs are reduced, and if larger aircraft are used, the unit operating cost declines. Reductions in the above costs result in better quality of service and lower air fares, and thus attract more passengers; consequently, the size of the market for the city-pair route can be expected to expand continuously. However, on the other hand, the airport ground access cost for the city-pair increases with market size, since the increased market size leads to an expanded market area and extended access distance. Consequently, the market size will not expand continuously, because the rising airport ground access cost will reduce passenger demand. Also, an airport congestion cost might arise if passenger flows in an airport approach the airport capacity. We assume in this study that the airport has adequate capacity so that increases in the size of the single city-pair market will not cause airport congestion. The average airline operating cost, passenger delay cost, and airport access cost either decrease or increase with an increase in the size of a city-pair market at an airport, so the optimum market size as well as optimal service area for a city-pair route can be determined from trade-offs among these costs.

The model developed in the rest of the paper addresses the considerations raised above. It explores demand-supply interaction and combines a spatial market area problem with a city-pair model of air travel demand. A nonlinear mathematical programming problem is formulated to determine the optimal number of passengers and the optimal service area of a city-pair market by minimizing the sum of various unit costs, subject to the demand-supply equality. The objective function of the model is formulated on the assumption that both passenger travel costs and airline operating costs should be minimized when planning the city-pair market of an airport.

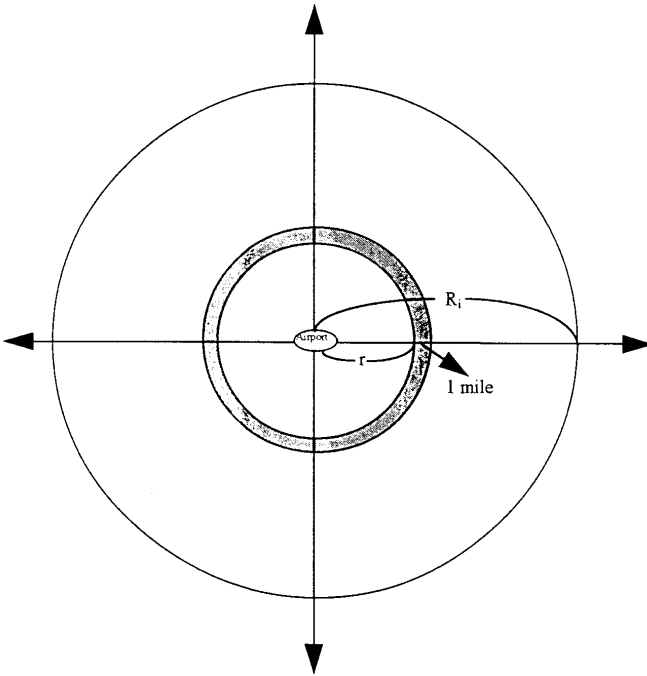
Section 2 describes the city-pair market of an airport, defines its market area and passenger demand, and analyzes demand-supply equilibrium. A variety of unit costs, including airport access cost, passenger delay cost, and airline operating cost, are described in Sect. 3. In Sect. 4, we formulate a nonlinear mathematical program to determine the optimal passenger number and market area, and analyze how the optimal passenger number and market area are affected by changes in the parameters representing stage length, population density, average income, demand variation, and airline technology. Finally, Sect. 5 summarizes the study and presents our conclusions.

## 2. Passenger demand, market area, and demand-supply equilibrium

The market size of a city-pair route at an airport is defined in this paper as including both the total passenger demand and geographical market area served by the city-pair service. In studying the market area, this paper departs from earlier research, which focused only on passenger demand. The market area of the city-pair route of an airport not only defines the number and spatial distribution of airports providing city-pair service in a region, but also influences the average airport access cost, which affects passenger demand in the market in reverse.

Airports are usually located in outlying areas accessible via direct access freeways or transit lines rather than at the center of metropolitan areas restricted by local grid networks, so their market area can be measured by the Euclidean distance. Also, airports and airlines usually exhibit economies of scale, and are suitable for monopoly or oligopoly operation; therefore, this paper assumes that airports have circular market areas (Morrill 1974). For a circular market area with the airport at its center, as shown in Fig. 1, the local passenger demand for a city-pair route  $i$ ,  $J_i$ , can be obtained by integral (1):

$$J_i = \int_0^{R_i} 2\pi r p_o \cdot a_i(r) dr \quad (1)$$



**Fig. 1.** The geographical market area of the airport.  $r$ : Airport access distance,  $R_i$ : radius of market area

where  $a_i(r)$  is a passenger demand density function within the market area of route  $i$  that is a decreasing function of the airport access distance,  $r$ , (trips/person);  $p_o$  is the average population density of the origin city (persons/mile<sup>2</sup>); and  $R_i$  is the radius of the market area (miles).

Factors affecting demand for any city-pair route are assumed to include not only the socioeconomic characteristics of the origin and destination cities, for instance, their total population and average personal income, but also the generalized travel cost of the route, including average fare, in-flight time cost, access cost, and passenger delay cost associated with inconvenience of schedules. We assume the following gravity type function for  $a_i(r)$ :

$$a_i(r) = \delta_0 P_o^{\delta_1} P_i^{\delta_2} G_o^{\delta_3} G_i^{\delta_4} \exp[-\varepsilon(AC_i(r) + DT_i + TT_i + CC_i)] \quad (2)$$

where  $P_o$  and  $P_i$  are the total population of the origin and destination cities;  $G_o$ ,  $G_i$  are the average personal income of the origin and destination cities;  $AC_i(r)$  is access cost for a passenger departing from an origin that lies access distance  $r$  from the airport for city-pair route  $i$ ;  $DT_i$  and  $TT_i$  are, respectively, average passenger delay cost and average in-flight travel time cost for city-pair route  $i$ ;  $CC_i$  is the average fare of city-pair route  $i$ , which is assumed to be the average operating cost per passenger; and  $\delta_0, \delta_1, \delta_2, \delta_3, \delta_4$ , and  $\varepsilon$  are parameters.

In normal situations, higher total population and average personal income entail greater passenger demand, so the values of parameters  $\delta_0, \delta_1, \delta_2, \delta_3, \delta_4$  would be positive.  $AC_i(r), DT_i, TT_i,$  and  $CC_i$  are various travel cost components that constitute the generalized cost of an air trip; these components will be defined in detail in Sect. 3. The higher generalized travel cost will result in less passenger demand, so the parameter  $\varepsilon$  would normally be positive. Though Eq. (2) does not explicitly take into account alternative airports or other modes of transportation, the various cost components of the generalized cost indicate that passenger demand increases as the service level of the city-pair route  $i$  at the airport increases and as the passenger's access distance decreases. The  $AC_i(r)$  in Eq. (2) implies that passengers departing from an origin that lies closer to the airport are more likely to undertake their intercity trips via the airport while those departing from an origin that lies farther from the airport are more likely to undertake their trips via alternative airports or other modes. However, major factors affecting passenger demand are the service level of city-pair route  $i$  at the airport (e.g.,  $DT_i, TT_i,$  and  $CC_i$  in Eq. (2)). Finally, the parameter  $\delta_0$  accounts for other qualitative factors that also affect passenger demand, such as safety and cultural characteristics.

Previous research regarding passenger demand or airline supply strategies usually assumed that either demand or supply was exogenous. In this paper, we assume there are interactions among the market size of city-pair routes, the operating strategies of airlines, and the demand behavior of passengers. Load factor is the ratio of demand to supply, representing the efficiency of resource use and the level of service. It is defined by

$$LF_i = J_i/S_i \tag{3}$$

where  $LF_i$  is the load factor of route  $i$  and  $S_i$  is the total supply of route  $i$ . If airlines take the load factor as a measurement of the service level and set the ideal load factor to be a constant, then an equilibrium supply, the product of aircraft size and flight frequency, can be determined from a given demand by the following equation:

$$S_i = J_i/LF_i^* = B_iF_i \tag{4}$$

where  $LF_i^*$  is the ideal load factor;  $B_i$  is the aircraft size for route  $i$ ; and  $F_i$  is the flight frequency of route  $i$ . The ideal load factors that airlines determine in the short run would change if the reservation process improved in the future. A poor communication and reservation system prevents airlines from making full use of seats. Hence the ideal load factor cannot be very high. In the long run, if computer reservation systems or reconfirmation requirements for passengers holding reservations are made more effective, the ideal load factor could be raised.

In deregulated markets, passenger airlines have the freedom to choose flight schedules, aircraft sizes, and fare prices for city-pair routes. The  $S_i = B_iF_i$  in (4) shows that airlines may either select a strategy of using

larger aircraft and fewer flights or one of using smaller aircraft and more flights so as to maintain the fixed supply,  $S_i$ . The former strategy would lower the airline's unit operating cost and average fare but raise the passenger schedule delay cost; the latter would raise the fare but reduce the delay. Both schedule delay and fare prices affect the airline's ability to attract passengers, as shown by (2). So, there is a trade-off between these two strategies. Moreover, changes in the number of passengers attracted make airlines adjust flight schedules and aircraft sizes, and these adjusted strategies may affect the number of passengers attracted, and so on. So, there are demand-supply interactions; and the equilibrium number of passengers and market area could be determined from these interactions and trade-offs among different travel costs, as discussed below.

### 3. Supply function and passengers' travel costs

The supply function of air transportation for city-pair route  $i$  at the airport is represented by the perceived passenger average cost function. This definition is analogous to the way that Kanafani (1983) defined it. He argues that the supply function in classical microeconomic theory that gives the quantity of a good that a supplier is willing to offer in a market at a given price is not appropriate in transportation, where there are nonmonetary aspects of supply that are as important as the price charged by the supplier. Travel time and delay time associated with schedule inconvenience are very important non-monetary attributes of supply and much of what determines the nature of transportation supply is a result of both traveler and airline behavior. The discussion in this paper is limited to supply analysis only as it relates to supply-demand equilibrium.

The components of an air passenger's perceived cost consist of airport access cost, passenger delay cost, in-flight travel time cost, and the fare. To simplify the analysis, for a specific city-pair route, we assume there is no transfer or connection, thus the cost of the route duration, i.e., the in-flight time cost, is constant if en route flying speed and unit time value are constant. The city-pair market is assumed to be competitive in which the fare is set according to the lowest level offered by any airline in the market and is not at the discretion of any single airline, in principle (Kanafani 1983). We also assume the average operating cost of airlines serving any route represents approximately the average fare for that route. The equilibrium output of the model is assumed to be the total supply of airlines serving the city-pair market without further considering the number of airlines in the market. Therefore, only passenger delay costs, average airline operating cost, and airport access cost will be discussed here.

#### 3.1 Passenger delay cost

The passenger delay cost associated with schedule inconvenience includes schedule delay cost and stochastic delay cost. In reality, in any city-pair

market, there may be passenger demand at any time; though for profit or cost reasons, airlines do not schedule departures whenever there is passenger demand. The cost of departing at a time that differs from the passenger's preferred departure time is called schedule delay cost. We assume passengers will choose the flight scheduled nearest to their preferred departure time. Schedule delay time is related to the headway of scheduled flights and the degree to which an airline's scheduling matches passengers' demand (Teodorovic 1988). We can formulate the average schedule delay cost for city-pair route  $i$ ,  $SC_i$ , as follows:

$$SC_i = 0.5\tau(\gamma T/F_i)r_{dt} \tag{5}$$

where  $T$  is the daily operating time of the airport (hrs),  $F_i$  is weekly flight frequency on the city-pair route,  $(\gamma T/F_i)$  is the average headway between flights serving route  $i$ ,  $r_{dt}$  is the value of time for delay (\$/hr), and parameter  $\tau$  represents the degree of agreement between the airline's scheduling and passengers' demand. In other words, the value of  $\tau$  represents how effectively an airline schedules its flights: when an airline's flight schedules comply more closely with the demand distribution, the value of  $\tau$  is smaller, and the passenger's schedule delay cost is lower, if the other terms in (5) do not change.

The stochastic delay cost appears when the passenger intends to reserve a seat on the flight of his choice, but is refused because there is no room on that flight. The cost of the time difference between the departure time of the flight on which the passenger finds a seat and the departure time of the flight that the passenger first chose is called the stochastic delay cost. Factors affecting the stochastic delay cost include flight frequency, load factor, variability in air traffic demand, airline reservation technology, and the value of time for delay. We follow the stochastic delay function proposed by Swan (1979) and extend the meaning of the parameter  $a$  to represent airline reservation technology so as to obtain the average stochastic delay cost for city-pair route  $i$ ,  $ST_i$ ,

$$ST_i = a(\gamma T/F_i)(LF_i^*)^\beta r_{dt} \tag{6}$$

where  $LF_i^*$  is the ideal load factor; and two parameters,  $a$  and  $\beta$ , represent, respectively, airline reservation technology and variability in air traffic demand. A smaller  $a$  value represents more effective airline reservation technology, i.e., more efficient utilization of aircraft seats. As  $a$  decreases, the average stochastic delay cost determined by (6) decreases, if flight headway and ideal load factor are kept constant. Furthermore, suppose the aircraft size,  $B_i$ , and weekly flight frequency,  $F_i$ , do not change; then an increase in passenger demand,  $J_i$ , would cause an increase in the ideal load factor,  $LF_i^*$ , so as to maintain demand-supply equilibrium shown in (4), and, consequently, result in an increased average stochastic delay cost. However, airlines could alleviate the impact of this increased stochastic delay cost by enhancing their reservation technology, i.e., reducing the value of  $a$ . The

parameter  $\beta$  stands for variability in air traffic demand. The higher the variation in air demand distribution is, i.e., the smaller the value of  $\beta$  is, the less likely it will be for a passenger to find a seat on the flight of his choice; that is, the average stochastic delay cost will increase providing the other variables in (6) do not change.

Assume the daily operating time of the airport,  $T$ , is equal to 18 hours across cities due to so few departure flights scheduled from midnight to 6 am. Then combine (5) and (6), rearrange, and the result yields, the average passenger delay cost for city-pair route  $i$ ,  $DT_i$ ,

$$DT_i = \mu / F_i \quad (7)$$

where

$$\mu = 126r_{dt}(0.5\tau + a(LF_i^*)^\beta) \quad (8)$$

### 3.2 Average operating cost of airlines

The operating costs of airlines serving any city-pair market consist of direct operating costs, including flight operating costs, landing fees, fuel, depreciation, and maintenance costs, and indirect operating costs, including administration, customer service, and marketing costs. Direct operating costs are usually correlated with the number of seats provided, while indirect operating costs are related to the size of the airline and are measured in terms of passengers served.

Unit costs of airlines serving a city-pair market decline markedly as density of service and stage length increase (Caves et al. 1984); therefore, the average total operating cost of providing city-pair service is assumed to be a nonlinear function, which can be expressed as follows:

$$C_i = \psi_0(B_i, Fl) \cdot F_i + \psi_1(B_i, L_i) \cdot L_i B_i F_i \quad (9)$$

where  $C_i$  is the total weekly operating cost for serving the city-pair route  $i$ ,  $\psi_0(B_i, Fl)$  is the landing fee (\$/flight), a function of aircraft size,  $B_i$ , and unit landing fee,  $Fl$  (\$/1000 lb); and  $\psi_1(B_i, L_i)$  is the unit operating cost (\$/seat-mile) excluding the landing fee.  $\psi_0(B_i, Fl)$  is an increasing function of aircraft size,  $B_i$ , as defined by

$$\psi_0(B_i, Fl) = \theta_0 + \theta_1 B_i Fl \quad (10)$$

where  $\theta_0$  and  $\theta_1$  are parameters and  $\theta_1 > 0$ .  $\psi_1(B_i, L_i)$  is a decreasing function of aircraft size,  $B_i$ , and stage length,  $L_i$ , which can be written as follows:

$$\psi_1(B_i, L_i) = \theta_2 + \theta_3 B_i + \theta_4 B_i^2 + \theta_5 L_i \quad (11)$$



where  $\theta_2, \theta_3, \theta_4$ , and  $\theta_5$  are parameters,  $\theta_5 < 0$  and  $d\psi_1(B_i, L_i)/dB_i < 0$ . The average operating cost per passenger for city-pair route  $i$ ,  $CC_i$ , is

$$CC_i = [\psi_0(B_i, Fl) \cdot F_i + \psi_1(B_i, L_i) \cdot L_i B_i F_i] / J_i \tag{12}$$

By substituting (4) into this expression, we obtain an expression that is also a function of the ideal load factor,  $LF_i^*$ :

$$CC_i = \psi_0(B_i, Fl) / (B_i \cdot LF_i^*) + \psi_1(B_i, L_i) \cdot L_i / LF_i^* \tag{13}$$

The equation above shows that when demand-supply equilibrium is maintained, an increase in passenger demand  $J_i$ , would allow airlines to utilize larger aircraft,  $B_i$ , without changing the flight frequency,  $F_i$ , and the ideal load factor,  $LF_i^*$ , thereby yielding a lower operating cost per passenger,  $CC_i$ . If, on the other hand, aircraft size,  $B_i$ , and flight frequency,  $F_i$ , do not change, the increased passenger demand could raise the ideal load factor,  $LF_i^*$ , and reduce the average operating cost per passenger. The economies deriving from the increase in the ideal load factor are comparatively strong in comparison with those deriving from the use of larger aircraft.

### 3.2 Average airport access cost

The access cost for a passenger traveling from his origin to the airport is assumed to include the travel time cost and the travel monetary cost. The access cost for a passenger departing from an origin that lies access distance  $r$  from the airport for city-pair route  $i$ ,  $AC_i(r)$ , is specified by

$$AC_i(r) = (r/v) \cdot r_{ac} + f \cdot r \tag{14}$$

where  $v$  is the average travel speed of access modes (miles/hr),  $f$  is the average monetary cost of access mode per unit distance (\$/mile), and  $r_{ac}$  is the time value of access time. The average access cost per passenger,  $AC_i$ , is obtained by averaging the total access costs for all passengers originating within the market area. From (1), (2), and (14),  $AC_i$  can be formulated as follows:

$$AC_i = \left( \int_0^{R_i} 2\pi r p_o \cdot a_i(r) \cdot AC_i(r) dr \right) / J_i = [(r_{ac}/v) + f] \{ (2/E) - [R_i^2 \exp(-R_i E)] / [1/E - (1/E + R_i) \exp(-R_i E)] \} \tag{15}$$

where  $E = \varepsilon(r_{ac}/v + f)$  and  $R_i$  is the radius of the market area.

#### 4. Formulation of the optimization problem and sensitivity analysis

In this paper, we assume that both the economic efficiencies of airlines and the travel costs of passengers are considered when planners decide the market size of an airport for a city-pair market. The average airport access cost, average passenger delay cost, and average airline operating cost increases or decreases with the size of the city-pair market of an airport. Therefore the optimum equilibrium market size can be determined from trade-offs among these costs. A nonlinear mathematical programming problem is formulated here to determine the optimal market size by minimizing the sum of the various costs, subject to demand-supply equilibrium. The nonlinear optimization problem is formulated as follows:

$$\min_{J_i, R_i, B_i, F_i} TC_i = (AC_i + DT_i + CC_i)/L_i \quad (16)$$

s.t.

$$B_i \cdot F_i = J_i/LF_i^* \quad (17)$$

and

$$J_i, B_i, F_i, R_i > 0$$

The formulations for the three different costs in (16),  $DT_i$ ,  $CC_i$ , and  $AC_i$ , are expressed, respectively, by (7), (13), and (15);  $J_i$  in (17) is expressed by (1); and (17) is an equilibrium condition for demand and supply, as derived by (4) in Sect. 2. This nonlinear optimization model may be solved by means of a variety of algorithms. We solve the optimization problem by using GINO, a computer-modeling program developed by Liebman et al. (1986) based on a generalized reduced gradient algorithm. The outputs for this model include the minimum average unit total cost (\$/passenger-mile), the optimal market size (passengers/week), and the optimal market area (radius of market area, in miles) for the city-pair market and the optimal aircraft size and weekly flight frequency for airlines serving this market. The inputs for the model include the total population, the average personal income of the origin and destination cities, the population density of the origin city, a parameter value representing the scheduling technology of airlines, the average travel speed and momentary cost of access modes, and the time values of different costs, etc.

For a given set of input values, the optimal values for the decision variables can be found by numerically solving the model. Since we are interested in exploring how the market size of the city-pair service at an airport is affected by changes in the stage length of the city-pair route, the average travel speed of access modes, the population density and income of the origin city, and the operating technologies of airlines, we shall present the results of numerical experiments conducted by a set of base values for the model parameters. The base parameter values, as shown in Table 1, are

**Table 1.** Base parameter values for numerical experiments

Parameter	Value	Unit
$T$	18	Hours/day
$L_i$	1 000	Miles
$LF_i^*$	0.65	
$p_o$	10 000	Persons/mile <sup>2</sup>
$P_o, P_i$	10 <sup>6</sup>	Persons
$G_o, G_i$	9 000	\$/year
$\delta_0$	exp(-23.6)	
$\delta_1, \delta_2$	0.5	
$\delta_3, \delta_4$	0.14	
$\varepsilon$	0.018	
$v$	40	Miles/hr
$f$	0.25	\$/mile
$\tau$	0.7	
$a$	2.5	
$\beta$	9	
$r_{ac}$	2.5	\$/hr
$r_{dt}$	1.25	\$/hr
$Fl$	50	Cents/1000 lb
$\theta_0$	61.91	
$\theta_1$	1.97	
$\theta_2$	12.204	
$\theta_3$	-0.07	
$\theta_4$	0.000168	
$\theta_5$	-0.0009	

assumed or found in previous studies (Caves et al. 1984; Teodorovic 1988; Kane 1990; Kling et al. 1991).

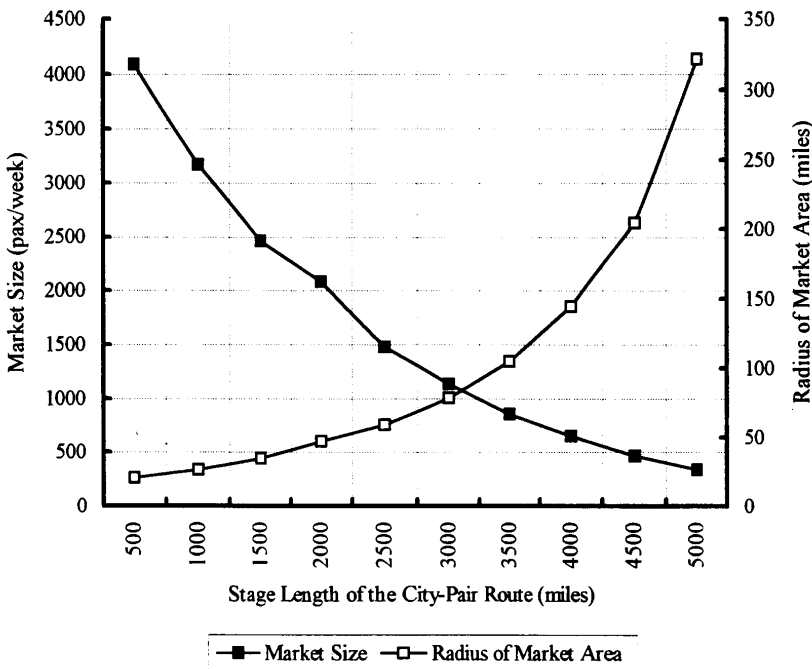
The values of parameters  $\theta_2, \theta_3, \theta_4,$  and  $\theta_5$  in Table 1 are initially assumed on the basis of work by Kling et al. (1991). They collected operating cost data of 305 routes operated by U.S. major airlines and calibrated these parameter values using multiple regression methods. Then, these parameter values were validated using the statistical data of airline and aircraft operating costs listed in Kane (1990). However, all of the aircraft size data shown in Kling et al. (1991) and Kane (1990) is no larger than 412 seats. Thus, the use of these parameter values is noted to be limited to aircraft size,  $B_i$ , less than 416, since  $d\psi_1(B_i, L_i)/dB_i < 0$  cannot be valid when  $B_i > 416$ . The base parameter values listed in Table 1 are only for demonstration purposes. Estimates based on actual data should be used in future applications of the model for specific airports. The numerical experiments are illustrated here to observe the behavior and results of the model. Each numerical experiment was conducted by varying the value of one or a few parameters while holding the others constant.

*Changes in the stage length of the city-pair route*

Analysis of changes in the stage length of the city-pair route should allow us to elaborate the differences between short-haul and long-haul city-pair

**Table 2.** Model results for city-pair routes with different stage length

Stage length (miles)	Objective value (\$/pax-mile)	Market size (pax/week)	Radius of market area (miles)	Aircraft size (seats)	Flights per week
500	0.09086	4088	20.74	205.76	30.56
1000	0.08503	3173	26.70	205.86	23.71
1500	0.08346	2463	34.54	206.15	18.38
2000	0.08305	2084	46.98	206.17	15.55
2500	0.08317	1482	59.08	206.21	11.06
3000	0.08373	1141	78.34	205.59	8.54
3500	0.08469	857	104.58	205.52	6.42
4000	0.08615	650	144.38	205.29	4.87
4500	0.08838	472	204.98	204.57	3.55
5000	0.09201	341	322.04	203.67	2.57



**Fig. 2.** Optimal market size and area vs. stage length

markets. The results of varying the stage length of the city-pair route are shown in Table 2 and Fig. 2. As the stage length of the route increases, the number of passengers of the optimal market declines while the radius of the optimal market area expands. Also, as shown by the results in Table 2, as the stage length increases, airlines should utilize larger aircraft and schedule fewer flights so as to achieve economic efficiencies. These results imply that since the average demand level per person declines as the stage

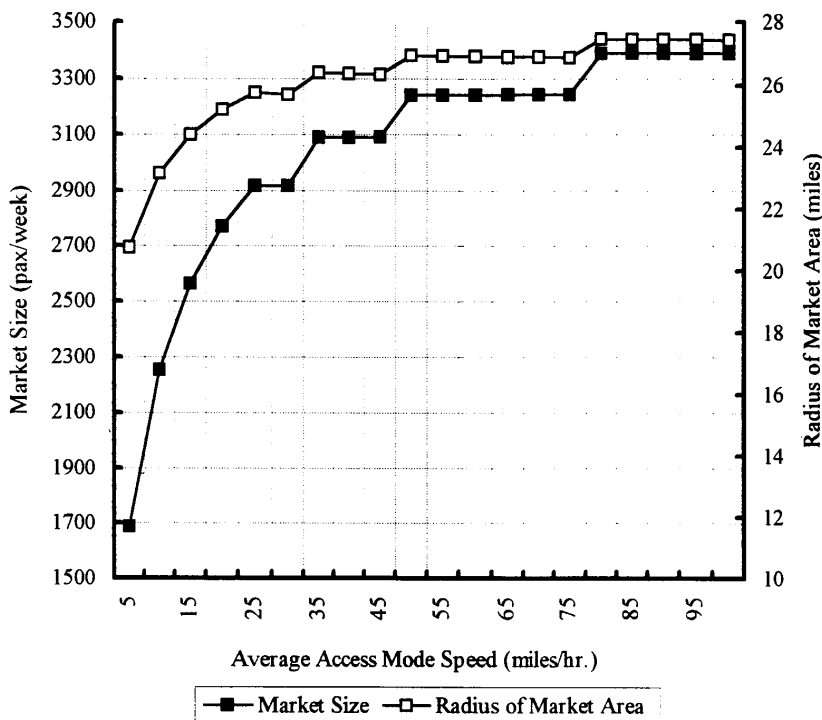


Fig. 3. Optimal market size and area vs. average access mode speed

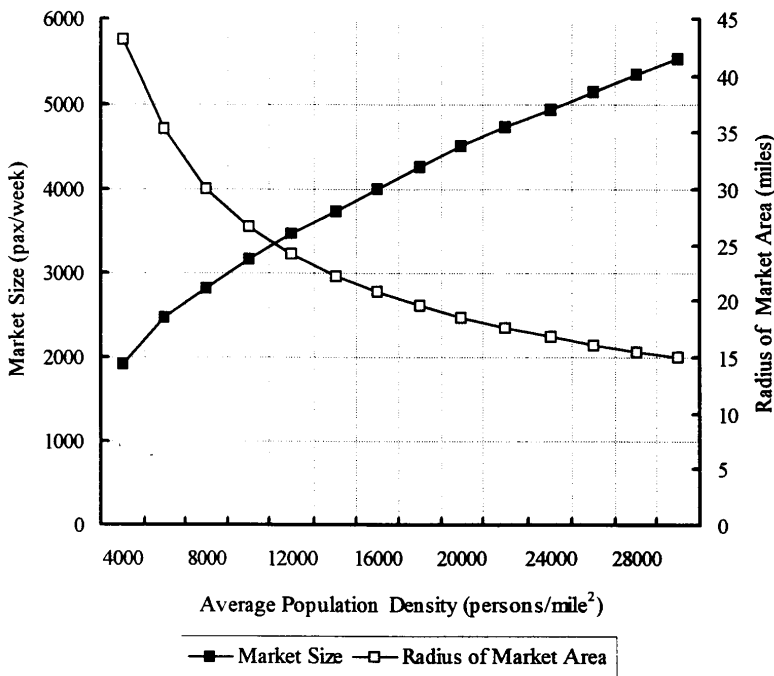
length of the route increases, the market area will tend to expand so as to accumulate more passengers and achieve density economies. Additionally, the ratio of access and delay costs to the total travel cost decreases as the stage length of the route increases, so passengers would be willing to travel longer distances and accept less frequent flights. Scheduling flights less frequently enables airlines to use larger aircraft in long-haul markets. On the other hand, the results also indicate that shorter haul routes with very high passenger demand should be operated by airlines in more airports, who should use slightly smaller aircraft but much more frequent flights. The objective value, average total cost per passenger-mile, for short-haul routes is also higher than that for long-haul routes. For extremely long-haul routes, because of significantly lower passenger demand, the flight frequency should be markedly reduced. Note that the optimal average aircraft size is not significantly different for short-haul and long-haul routes. This might be because the effect of high passenger flows compensates for that of short stage length in determining the aircraft size used for short-haul service.

*Improvements in average travel speed of access modes*

The optimal market size and area increase with increasing average travel speed, but the rate of increase diminishes gradually, as shown in Fig. 3. In

**Table 3.** Optimal objective values, market sizes and areas for different average access mode speeds

Average speed of access modes (miles/hr)	Objective value (\$/pax-mile)	Market size (pax/week)	Radius of market area (miles)
10	0.08822	2254	23.17
20	0.08615	2773	25.21
30	0.08542	2919	25.70
40	0.08504	3091	26.35
50	0.08481	3243	26.94
60	0.08466	3243	26.90
70	0.08455	3244	26.88
80	0.08446	3389	27.45
90	0.08440	3389	27.43
100	0.08434	3389	27.42



**Fig. 4.** Optimal market size and area vs. average population density

other words, improvements in the average travel speed of access modes will reduce the access cost, which will stimulate more passengers to travel from farther areas. This in turn will permit airlines at the airport to achieve economic efficiencies, and passengers will share in the surplus created by those efficiencies. This situation can be accounted for by the objective values in Table 3, which indicates that average total cost per passenger mile decreases as the average travel speed of access modes increases. However,

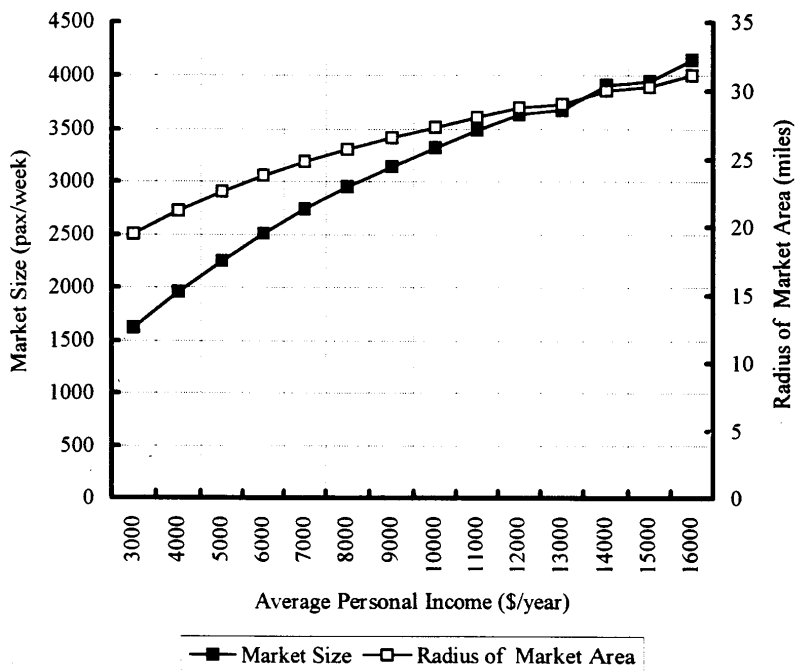


Fig. 5. Optimal market size and area vs. average personal income

the diminishing effect of improvement in airport access modes suggests that other approaches are needed in order to expand the market size for a city-pair route at an airport, especially when the average travel speed of access modes is already high.

*Changes in the average population density of the origin city*

Figure 4 shows that as the average population density of the origin city increases, the number of passengers in the optimal market increases and the radius of the optimal market area decreases. The index of population density roughly represents the extent of urbanization and the scale of a metropolitan area. A higher population density for the origin city means that it is a highly urbanized city with a strong demand for air travel, which creates more density efficiencies for airlines serving city-pair markets that include this city and allows airlines to operate in more airports of this city to share in these markets.

*Changes in the average income of the origin city*

Figure 5 shows that with an increase in the average personal income of the origin city, the optimal market size as well as the optimal area increases, but at a declining rate of increase. The number of passengers in a city-pair

**Table 4.** Optimal objective values, market sizes and areas, and flight frequencies for different variations in air travel demand,  $\beta$ 

$\beta$	Objective value (\$/pax-mile)	Market size (pax/week)	Radius of market area (miles)	Flights per week
5	0.08647	4249	31.30	31.96
6	0.08591	3799	29.45	28.54
7	0.08551	3490	28.13	26.19
8	0.08523	3277	27.18	24.57
9	0.08503	3133	26.53	23.48
10	0.08491	3133	26.50	23.48
11	0.08482	2979	25.82	22.31
12	0.08476	2979	25.81	22.31
13	0.08473	2979	25.80	22.31
14	0.08470	2979	25.80	22.31

market increases with the average income through an income effect, since air transportation is a normal good, and through a substitution effect, because the average income determines the opportunity cost of time. It should be noted that the opportunity cost of time is assumed to be a function of income in the given set of input values. An increasing number of passengers in the optimal market may reduce schedule delay cost and the fare of passengers through economies of schedule and economies of density. However, continued increase in market size would result in an expanded market area and thus raise the average access cost at the airport which in turn would cause the rate of increase in market size to decline. These results imply that, in high income areas, city-pair air services should concentrate on one or a small number of large airports.

#### *Variation in the distribution of passenger demand*

The parameter  $\beta$  in (6) represents variability in air traffic demand. The stochastic delay cost resulting from passengers who can not reserve a seat for the flight of their choice increases with increasing variation in air traffic demand if the flight schedule is fixed. Table 4 and Fig. 6 show that as the variability of air traffic demand decreases, i.e., as the value of  $\beta$  increases, even when the optimal market size and area are smaller, the objective value of the average total travel cost is lower. In other words, economic efficiencies can still be obtained in small markets with stable passenger demand.

#### *Changes in the scheduling technology of airlines*

Parameter  $\tau$  in (5) stands for the scheduling technology that airlines use to match flight schedules with passenger demand. Table 5 indicates that the average total cost per passenger-mile is low for high scheduling technology, symbolized by a low  $\tau$  value, but high for low scheduling technology, even when the number of passengers in the optimal market is low for the



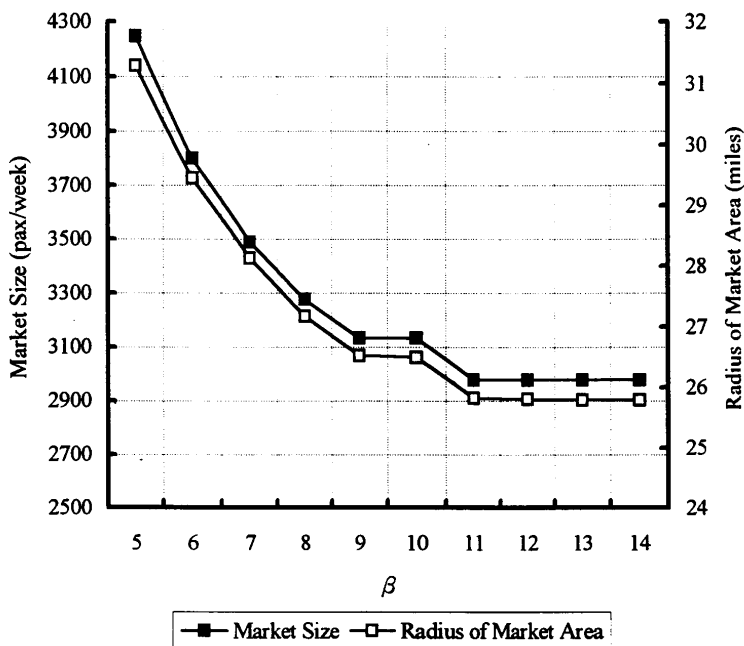


Fig. 6. Optimal market size and area vs. variation in air demand

Table 5. Optimal objective values and market sizes for different scheduling technologies,  $\tau$

$\tau$	Objective value (\$/pax-mile)	Market size (pax/week)
0.1	0.08196	1272
0.2	0.08271	1620
0.3	0.08330	1977
0.4	0.08382	2308
0.5	0.08426	2555
0.6	0.08467	2869
0.7	0.08503	3137
0.8	0.08537	3389
0.9	0.08570	3399
1.0	0.08599	3776
1.1	0.08627	4089
1.2	0.08654	4087

former case. This implies that, even in a small market, cost efficiencies achieved by airlines with high scheduling technology are comparatively strong in comparison to those achieved by airlines with low scheduling technology. However, if there is a single airline in the market, there is no incentive to reduce the value of  $\tau$  since a larger market (always consistent with the model equilibrium condition) compensates for low-scheduling technology, but the average total cost per passenger-mile for high-scheduling technology is still lower than that for low-scheduling technology.

## 5. Conclusion

A model for determining both the number of passengers and local market area for city-pair routes served by an airport has been formulated and employed to analyze how changes in significant parameters affect the optimum market size and service area. A variety of unit costs change in different ways as the market size of a given city-pair route changes. The unit airline operating cost is a decreasing function of aircraft size and load factor: hence, when the market for a given city-pair route expands, airlines can either utilize larger aircraft or raise the load factor to reduce the average operating cost per passenger. Therefore, the average airline operating cost per passenger is a decreasing function of the market size. The schedule delay cost and the stochastic delay cost are primarily related to flight frequency. They are also decreasing functions of the market size, since flight frequency can be raised to serve a large number of passengers if aircraft size and the ideal load factor are fixed. On the other hand, the average access cost of passengers increases with the market size of the airport serving the city-pair route, because a larger market size leads to a larger market area and longer access distance (providing the average air travel demand level of the city-pair market and other variables do not change).

This paper has formulated a nonlinear programming problem to determine the optimal market size and service area for the city-pair route served by an airport by finding an optimal trade-off between the above costs subject to demand-supply equilibrium. Such a trade-off must succeed in both minimizing average total travel cost for passengers and achieving economic efficiencies for airlines. Coupled with numerical experiments, the model provides a basis for examining the effects of variations in stage length, population density and average income of the origin city, average travel speed of access modes, and scheduling technology of airlines. In regard to stage length, the service area for long-haul markets is larger than that for short-haul markets, indicating that long-haul services might be concentrated in one or a small number of large airports, while short-haul services might be dispersed among many small airports. Improving the technology of the airport access mode can expand the market size and service area for a city-pair service at one airport by reducing access cost directly and airline operating cost and delay cost indirectly. In areas with higher population density, which results in higher demand density and a denser market, airlines serving a city-pair market can operate more efficiently through economies of density and distribute air services among more airports. When the cities served have a high personal income, the market size and the market area both increase, but at a declining rate of increase. Finally, city-pair markets with relatively stable passenger demand or markets that are served by airlines with more effective scheduling technology exhibit higher cost efficiencies.

The findings of this research are exploratory, because the input data used in the numerical experiments are only hypothetical. Estimates based on actual data should be used in future applications of the proposed model.

Only local markets have been considered here, so issues regarding transit passengers should also be addressed in future research. Furthermore, future studies should explicitly address issues such as including competition from alternative airports or other modes of transportation in the demand function and should explicitly consider the impact of different types of market structures, pricing, and the regulatory environment on the evolution of the supply function.

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