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A model for market share distribution between high-speed and conventional rail services in a transportation corridor

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Abstract. High-speed rail (HSR) lines are usually planned to serve corridors with existing conventional rail (CR) lines, since these corridors typically have large markets concentrated around major cities. This paper formulates a new analytical model to estimate market shares of HSR and CR in a fundamental way, and from an individual behavior point of view. Passengers are divided into those who can take an HSR train directly to their destination stations and those who cannot. Optimal route choices are assumed by minimizing the "generalized total travel time". The relationship among demand-supply attributes such as value of time, train departure time, speed, trip length and fares is explored to identify market boundaries by comparing different routing strategies for each type of passenger. Individual route choices are aggregated by accumulating a transformation probability density function of value of time to estimate the spatial distribution of markets for two types of rail lines. The result estimates detail market distributions for passengers alighting at stations along the corridor. HSRs are shown to best serve medium- to long-trip markets, while CRs are shown to serve best for commuter travel and as feeders for the HSRs.

1. Introduction

The excellent performance of high-speed rail (HSR) lines is a significant improvement over the disadvantage of conventional rail (CR) lines for speed. However, HSRs are usually planned for corridors presently served by CRs, since these corridors typically have large markets around and be-

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tween major cities. Therefore, addressing the problem of how rail passengers will make use of the two rail systems becomes important. This paper develops a new analytical model for exploring this problem in a fundamental way. The problem addressed here involves both mode-choice and route-choice problems. Passengers may choose between the two rail lines for differences in speed and fare, or use both rail services to complete a single intercity trip. For instance, CR may compete with HSR in short to medium range markets to serve passengers whose destinations can be reached directly via CR or HSR, or complement HSR as a collection/distribution mode in medium to long range markets in serving passengers whose destinations can't be reached directly via HSR alone.

The traditional approach to transportation planning for path ridership distribution or transit route choice usually assumes that the traveler's choice is based on the waiting time and the riding time (e.g., Spiess 1983; Jansson and Ridderstolpe 1992). Individual differences in fares and values of time are usually ignored in transit-route-choice problems. However, since the construction cost of HSR line is high and must usually be recovered by expensive fares on travel, and substantial riding time saved due to the high speed of HSR service is the most important factor affecting traveler's choice, a tradeoff between time and fare differences between HSR and CR exists. Furthermore, this tradeoff may be perceived differently by a variety of passengers who have different values of time. Thus, traditional transit-route-choice models are probably not appropriate for solving this problem.

On the other hand, HSR ridership is forecasted intensively in the literature by using multinomial logit or nested logit models that focus on estimating the entire market for HSR, as compared with all other existing modes (e.g., Brand et al. 1992; Mandel et al. 1994). Instead of using a logit model to estimate the entire market, this paper develops a new analytical model for estimating detailed market shares of HSR and CR along different segments of a rail corridor. This model incorporates rail passengers' mode choices as well as route choices into a framework by exploring the relationship among key variables of concern. Passengers are divided into two types according to whether or not they can take an HSR train directly to their destinations. Passengers' optimal route choices are assumed by minimizing their generalized travel time, which is composed of access/egress time, waiting/transfer time, riding time, and fare conversion time. Instead of assuming an "average" value of time, which is commonly used in route choice or zonal design literature for transit or rail corridors (e.g., Wirasinghe and Seneviratne 1986; Furth 1986; Ghoneim and Wirasinghe 1987; Janjsson and Ridderstople 1992), this paper treats an individual's value of time as a random variable and uses it to convert fare into an individual-dependent fare time. Passengers' route choices are formulated as depending on trip distance, value of time, departure time, and fare and service characteristics of HSR and CR services. The relationships among these variables are explored to identify market boundaries by comparing different routing strategies for each type of passenger. Consequently, individual route choices are aggregated so as to estimate the distribution of market shares along different segments of the corridor.

2. Basic assumptions

As in many theoretical modeling studies, this model will initially be formulated as a rather idealized one. However, while intentionally ignoring some factors to obtain mathematical tractability, we have tried to retain enough of the salient features for a reasonable approximation of the real problem. In Sect. 6, we discuss and describe the computational consequences of relaxing some assumptions.

We assume HSR and CR trains travel the full length of the corridor, and make, respectively, all HSR and CR stops. In reality, HSR and CR trains may not stop at all their stations. In Sect. 6.3, we will describe how to relax this assumption. Wherever there are HSR stations they are assumed to be joined to CR stations, so passengers may transfer easily between the two rail services. The passengers are assumed to be well informed about departure times, itineraries and farces associated with all routes, either in advance or after their arrival at departure stations. The riding time for a train traveling between two consecutive stations with a spacing of L, T can be found in studies that use similar assumptions (e.g., Campbell 1993, Cook and Russell 1980, etc.) and is expressed as $T = L/V + V/a$, where $a = V^2/2\,d$. In this expression, the train is assumed to accelerate from the station over a distance d at an average acceleration of a, cruise over a distance of $L - 2\,d$ after reaching cruise speed V, decelerate over a distance d at an average rate of a, and then stop at the next station.

Following the above expression for T, the riding time for traveling in a HSR train between two consecutive HSR stations, t_h, can be expressed as $L/V_h + V_h/\alpha$. Similarly, the riding time for traveling in a CR train between two consecutive CR stations, t_r, is $l/V_r + V_r/\beta$. In these two expressions, L and l are average station spacings, V_h and V_r are cruise speeds, α and β are average rates of acceleration, respectively, for HSR and CR trains. To simplify mathematical expressions, we assume spacings between any two consecutive stations are equivalent. This assumption will be relaxed in Sect. 6 so as to capture the real problem reasonably.

Figure 1 illustrates the station labels used in this paper. Along the HSR corridor, we assume there are $n + 1$ stations, labeled as $i = 0, 1, ..., n$. All of these stations are joint HSR-CR stations. Using these stations as markers, the CR line is divided into n non-overlapping zones. Each zone is labeled with a number which is the same as that of the HSR-CR joint station located at the right end of the zone. Based on the equivalent spacing assumption, there are $n_r^L (= L/l) - 1$ CR stations, labeled as $\mathrm{j} = 1, ..., n_r^L - 1$, in each zone.

Passengers are assumed to select routes so as to minimize their generalized travel times. The generalized travel time includes a variety of time components, i.e. access time, waiting time, riding time, transfer time, and egress time, and fare (converted to time according to a formula of value of time). We classify passengers as Type I passengers who may ride either HSR or CR trains directly to their destination stations from which they then walk or use modes of transportation other than rail services to reach

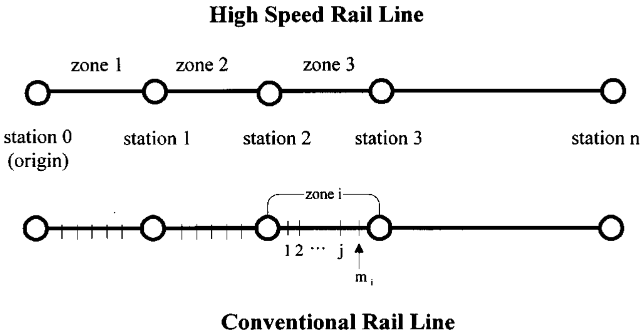

Fig. 1. Labels for station and zones along HSR and CR lines

their final destinations, and Type II passengers who cannot ride HSR trains directly to their destination stations, but who may either take an HSR train to a joint station, then transfer to a CR train that progresses or backtracks to their destination stations, or take a CR train directly to their destination stations. That is, the destinations of Type I passengers have more-or-less direct HSR train services without any transfers between the two rail services, while those of Type II passengers do not.

For a Type I passenger, whose origin and destination station are, respectively, station 0 and station *i*, the generalized travel time for taking an HSR train alone, T_{H}, and the generalized travel time for taking a CR train alone, T_{R}, are given by:

$$T_{\mathrm{H}} = t_a + t_{\mathrm{w}}^h + \mathrm{i}\, t_h + (\mathrm{i} - 1)t_{\mathrm{s}} + t_{\mathrm{e}} + t_{\mathrm{p}}^h \tag{1}$$

$$T_{\mathrm{R}} = t_a + t_{\mathrm{w}}^r + \mathrm{i}\, n_r^L t_r + (\mathrm{i}\, n_r^L - 1)t_{\mathrm{s}} + t_{\mathrm{e}} + t_{\mathrm{p}}^r \tag{2}$$

where t_a and t_{e} are, respectively, access and egress times for origin and destination, which are joint HSR-CR stations, t_{w}^h and t_{w}^r are waiting times for HSR and CR trains, t_{s} is the average stop time for any station, t_h is the riding time in a HSR train between two consecutive joint stations, t_r is the riding time in a CR train between two consecutive CR stations, t_{p}^h and t_{p}^r are fare conversion times for taking HSR and CR trains on this trip.

For a Type II passenger, whose origin and destination stations are, respectively, station 0 and the *j*th CR station in zone *i*, the generalized travel time for taking an HSR train to the *i*–1th station, then transferring to a CR train that progresses to the *j*th station of zone *i*, T_{H1}, can be expressed as

$$\begin{aligned} T_{\mathrm{H1}} = {} & t_a + t_{\mathrm{w}}^h + (\mathrm{i} - 1)t_h + (\mathrm{i} - 2)t_{\mathrm{s}} + t_{\mathrm{p1}}^h + [t_{\mathrm{w(i-1)}}^r + \mathrm{j}\, t_r \\ & + (\mathrm{j} - 1)t_{\mathrm{s}} + t_{\mathrm{p1}}^r] + t_{\mathrm{e}} \end{aligned} \tag{3}$$

Similarly, for the same passenger, the generalized travel time for taking an HSR train to the *i*th station then transfer to a CR train that backtracks to the *j*th station of zone *i*, T_{H2}, is

$$T_{\mathrm{H2}} = t_a + t_{\mathrm{w}}^h + \mathrm{i}\, t_h + (\mathrm{i}-1)t_{\mathrm{s}} + t_{\mathrm{p2}}^h + [t_{\mathrm{wi}}'^r + (n_r^L - \mathrm{j})t_r + (n_r^L - \mathrm{j} - 1)t_{\mathrm{s}} + t_{\mathrm{p2}}^r] + t_{\mathrm{e}} \quad (4)$$

In (3) and (4), $t_{\mathrm{w(i-1)}}^r$ and $t_{\mathrm{wi}}'^r$ are the transfer times at station *i* and station *i*–1, and t_{p1}^r and t_{p2}^r are the fare conversion times, respectively, for traveling in a CR train from joint station *i*–1 to station *j* of zone *i*, and from joint station *i* to station *j* of zone *i*. If this Type II passenger directly takes a CR train to the *j*th station of zone *i*, then the generalized travel time, T_{R}', is

$$T_{\mathrm{R}}' = t_a + t_{\mathrm{w}}^r + [(\mathrm{i}-1)n_r^L + \mathrm{j}]t_r + [(\mathrm{i}-1)n_r^L + \mathrm{j} - 1]t_{\mathrm{s}} + t_{\mathrm{p}}^r + t_{\mathrm{e}} \quad (5)$$

The generalized travel time formulation in (1)–(5) do not assume different weights for various time components in order to simplify mathematical expressions. This assumption is discussed in Sect. 6.

3. Individual route choices and market shares for Type I passengers

We assume that a Type I passenger selects his route so as to minimize the generalized travel time. That is, the choice of traveling in a high-speed train depends on whether or not $T_{\mathrm{H}} \le T_{\mathrm{R}}$. Let $T_{\mathrm{H}} - T_{\mathrm{R}} = 0$, then, from (1) and (2), it is obtained that

$$T_{\mathrm{H}} - T_{\mathrm{R}} = (t_{\mathrm{w}}^h - t_{\mathrm{w}}^r) + \mathrm{i}(t_h + t_{\mathrm{s}}) - \mathrm{i}\, n_r^L(t_r + t_{\mathrm{s}}) + (t_{\mathrm{p}}^h - t_{\mathrm{p}}^r) = 0 \quad (6)$$

Let $t_d = t_{\mathrm{w}}^h - t_{\mathrm{w}}^r$, which stands for the waiting time differences between the latest HSR train and the latest CR train which depart after the passenger's arrival at joint station 0; then, $t_d > 0$ represents the latest departure train is a CR train, $t_d < 0$ represents the latest depature train is a HSR train. Let $t_L^h = t_h + t_{\mathrm{s}}$ and $t_l^r = t_r + t_{\mathrm{s}}$; suppose δ_h and δ_r represent, respectively, the unit distance fares for HSR and CR trains, and γ represents the value of time of an individual passenger, then $t_{\mathrm{p}}^h - t_{\mathrm{p}}^r = (\mathrm{i}\, L\delta_h - \mathrm{i}\, n_r^L l\,\delta_r)/\gamma = \mathrm{i}\, L(\delta_h - \delta_r)/\gamma$. Let $\mathrm{K} = L(\delta_h - \delta_r)/\gamma$, then (6) can be rewritten as:

$$t_d = \mathrm{i}(n_r^L t_l^r - t_L^h - \mathrm{K}) \quad (7)$$

Let $\mathrm{X} = n_r^L t_l^r - t_L^h - \mathrm{K}$, then, if $t_d \le \mathrm{iX}$, i.e. $T_{\mathrm{H}} - T_{\mathrm{R}} \le 0$, so, passengers will choose to travel via HSR, otherwise they will choose to travel via CR. (7) shows that for a passenger with a specific value of time, i.e. a specific *K* value, the travel distance, which is represented by destination label *i*, is the most important factor affecting the choice between two kinds of trains for a given t_d value and given riding times.

As travel distance i increases, the difference in fare conversion time, iK, and the difference in total riding time between these two trains, $\mathrm{i}\,(n_r^L t_l^r - t_L^h)$, will simultaneously expand, while the waiting time difference, t_d, holds, thereby yielding a relatively smaller effect on the passenger's choice. In other words, the extent of t_d's effect on the choice between HSR and CR trains depends on the travel distance. The value of time of an individual passenger is also an important factor affecting the choice providing other variables in (7) are held constant. When passengers' values of time are higher, K is smaller and X is larger, they will then tend to choose HSR trains; otherwise, they will tend to choose CR trains.

In reality, each passenger has a value of time, γ, so γ can be viewed as a random variable by considering the market of interest as a whole. γ usually varies from individual to individual in light of differences in socioeconomic characteristics, such as income and trip purpose. γ reflects how passengers weigh fares with travel time. Normally, γ cannot be negative although its value can be very small. Let γ be a random variable with continuous type, having the probability density function (p.d.f.), $\mathrm{f}_\gamma(\gamma)$, in the space $\{\gamma;\, 0 < \gamma < \infty\}$ and 0 elsewhere. The probability distribution of γ may be different for various markets of interest. For instance, passengers in North America and Japan may have different weights of fares and travel time. The estimation for probability distribution of γ based on actual data is necessary for future application in specific corridor. In the following discussion, we assume γ for all passengers in the same corridor of interest belong to the same probability distribution without loss of generosity.

X is also a random variable transformed from value of time, γ, because all variables other than γ are exogenous in the definition of X. Those Type I passengers who depart from joint station 0 and are confronted with the same t_d will have different probability distributions of iX that vary as the travel distance i increases, as shown in (7) and illustrated in Fig. 2. Let $\mathrm{X}_\mathrm{i}^\mathrm{I} = \mathrm{iX}$, $-\infty < \mathrm{X}_\mathrm{i}^\mathrm{I} < \mathrm{i}(n_r^L t_l^r - t_L^r)$ then the p.d.f. of $\mathrm{X}_\mathrm{i}^\mathrm{I}$, $\mathrm{f}_\mathrm{i}(\mathrm{X}^\mathrm{I})$, can be written as:

$$\mathrm{f}_\mathrm{i}\,(\mathrm{X}^\mathrm{I}) = \mathrm{f}_\gamma\left(\frac{-\mathrm{iC}_2}{\mathrm{X}^\mathrm{I} - \mathrm{iC}_1}\right)\cdot\frac{\mathrm{iC}_2}{(\mathrm{X}^\mathrm{I} - \mathrm{iC}_1)^2} \tag{8}$$

where $\mathrm{C}_1 = n_r^L t_l^r - t_L^h$, and $\mathrm{C}_2 = L\,(\delta_h - \delta_r)$. The derivations of (8) are detailed in Appendix A.

Passengers departing from the same station 0 but alighting at different destinations i, $i = 1, 2, 3$, belong to various curves with different probability density functions as shown in Fig. 2. For a given t_d, passengers whose destinations are station 1 will choose to travel in a HSR train if their $\mathrm{X}_\mathrm{i}^\mathrm{I}$s are larger than t_d, i.e. they belong to the X curve and are beyond t_d as shown in Fig. 2a. On the other hand, if their $\mathrm{X}_\mathrm{i}^\mathrm{I}$s are smaller than t_d, i.e. they belong to the X curve but do not exceed t_d as shown in Fig. 2a, then they will choose to travel in a CR train. Passengers who alight at station i, $i = 2, \ldots, n$, can be analyzed similarly as to their choices.

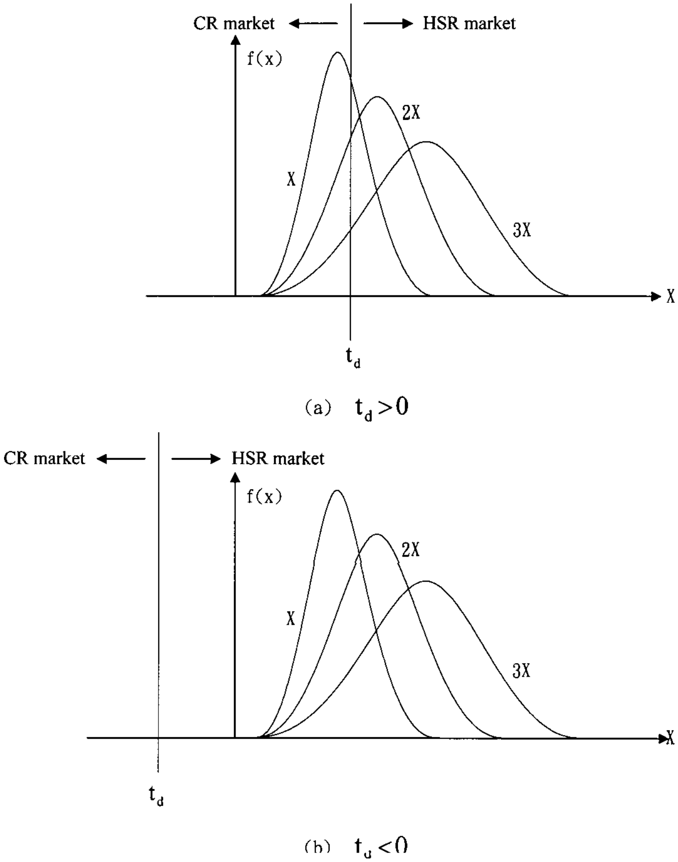

Fig. 2. HSR and CR market segmentation for Type I passengers alighting at station *i*

It is shown in Fig. 2b that when t_d is negative, i.e. the latest departure train is a HSR train, it is very hard for CR to attract passengers. Consequently, for a given t_d, the proportion of Type I passengers, who alight at station, i, $i = 1, ..., n$, and choose a CR train, C^{I_i}, can be obtained from:

$$C^{I_i} = \int_{-\infty}^{t_d} f_i\left(X^I\right) dX^I \quad \text{for} \quad i = 1, \ldots, n \tag{9}$$

and the proportion of Type I passengers who alight at station i and choose an HSR train is $1 - C^{I_i}$.

We can use departure schedules for HSR and CR trains departing from station 0 in a day as cut points and divide a day into many time zones labeled s, $s = 1, ..., 10, ...$, as shown in Fig. 3. All passengers who arrive at station 0 during time zone s are confronted with the same waiting time differ-

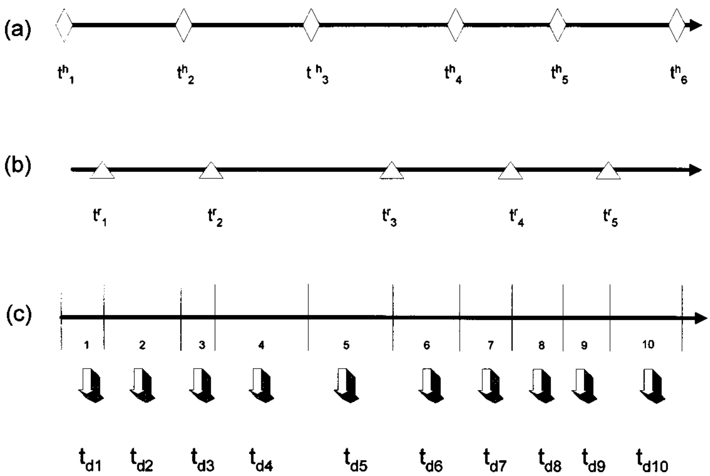

Fig. 3. Time zones divided by mixed departure times of HSR and CR trains

ence t_ds. For example, passengers arriving during time zone 1 are confronted with $t_{d1} = t^h_2 - t^r_1$, and $t_{d1} > 0$; and passengers arriving during time zone 2 are confronted with $t_{d2} = t^h_2 - t^r_2$, and $t_{d2} < 0$, and so on. Consequently, we can calculate C^{I_i} and $1 - C^{I_i}$ for passengers confronted with different t_ds using similar analysis of Fig. 2 and (9).

4. Route choices and market shares for Type II passengers

The destination stations of Type II passengers are served by CR alone, so they may either take CR trains directly or use an HSR train and a CR train that progresses or backtracks to their destination stations. We now examine all combinations of these routes by comparing two routes at a time, then integrate these results so as to arrive at overall market shares for each of the three routes.

4.1. HSR and progressive CR vs. CR alone

Let $T_{H1} - T'_R = 0$, then, from (3) and (5) in Sect. 2, we obtain

$$T_{H1} - T'_R = t_d + (t^h_{pl} + t^r_{pl} - t^r_p) + t^r_{w(i-1)} + (i-1)t^h_L - [(i-1)n^L_r + j]t^r_l - t_s = 0 \tag{10}$$

Since $t^h_{pl} + t^r_{pl} - t^r_p = [(i-1)L\delta_h + j\,l\delta_r - ((i-1)n^L_r - j)l\delta_r]/\gamma = (i-1)K$, (10) can be simplified to:

$$t_d + t^r_{w(i-1)} - t_s = (i-1)(n^L_r t^r_l - t^h_L - K) = (i-1)\,X \tag{11}$$

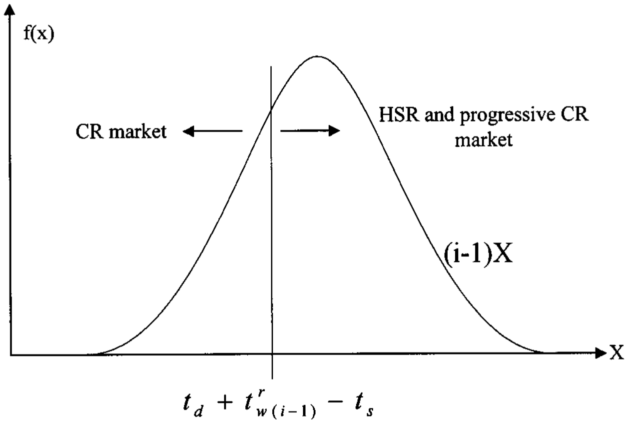

Fig. 4. Market segmentation of CR alone, and HSR and progressive CR

(11) shows that Type II passengers who alight at station j in zone i are confronted with $t^r_{\mathrm{w(i-1)}}$ transfer time at joint station i–1, and with a γ transformed variable distributed with $(i\text{–}1)X$ as shown in Fig. 4, so their route choices will depend on travel distance and transfer time. For any given t_d, the optimal routes for these Type II passengers can be found by comparing their $(i\text{–}1)X$ values to $t_d + t^r_{\mathrm{w(i-1)}} - t_{\mathrm{s}}$. Specifically, those passengers whose $(\mathrm{i}-1)\,\mathrm{X} > t_d + t^r_{\mathrm{w(i-1)}} - t_{\mathrm{s}}$ will choose to take an HSR train to station i–1 and then transfer to a CR train progressing towards their destinations, while those passengers whose $(\mathrm{i}-1)\,\mathrm{X} < t_d + t^r_{\mathrm{w(i-1)}} - t_{\mathrm{s}}$ will take CR trains directly to their destinations.

Normally, the location of $t_d + t^r_{\mathrm{w(i-1)}} - t_{\mathrm{s}}$ will move towards the left with an increase of travel distance i as shown in Fig. 4, suppose $t^r_{\mathrm{w(i-1)}}$ does not vary much with i. That is, for longer trips, the time savings of first choosing an HSR train for long-haul travel and transferring to a CR train to local destinations are comparatively higher than those of choosing conventional trains alone. In these cases, HSR trains are more attractive than CR trains. Assume $\mathrm{X}^{\mathrm{II}}_{\mathrm{i}} = (\mathrm{i}-1)\mathrm{X},\ and\ -\infty < \mathrm{X}^{\mathrm{II}}_{\mathrm{i}} < (\mathrm{i}-1)(n^L_r t^r_l - t^r_L)$, then, from Appendix A, the p.d.f. of $\mathrm{X}^{\mathrm{II}}_{\mathrm{i}}$, $\mathrm{f_i}(\mathrm{X}^{\mathrm{II}})$, can be written as:

$$\mathrm{f_i}\left(\mathrm{X}^{\mathrm{II}}\right) = \mathrm{f}_\gamma\left(\frac{-(\mathrm{i}-1)\mathrm{C}_2}{\mathrm{X}^{\mathrm{II}} - (\mathrm{i}-1)\mathrm{C}_1}\right) \cdot \frac{(\mathrm{i}-1)\mathrm{C}_2}{\left(\mathrm{X}^{\mathrm{II}} - (\mathrm{i}-1)\mathrm{C}_1\right)^2} \tag{12}$$

where $\mathrm{C}_1 = n^L_r t^r_l - t^h_L$, and $\mathrm{C}_2 = L(\delta_h - \delta_r)$. Consequently, in situations where only two routings, i.e. traveling via CR alone and traveling via HSR and progressive CR, are chosen, the proportion of Type II passengers who alight at station j of zone i, $i = 2, \ldots, n$, and choose to travel via CR alone, $\mathrm{C}^{\mathrm{II_i}}$, can be written as:

$$\mathrm{C}^{\mathrm{II}_\mathrm{i}} = \int_{-\infty}^{t_d + t^r_{\mathrm{w(i-1)}} - t_\mathrm{s}} \mathrm{f_i}\,(\mathrm{X}^{\mathrm{II}})d\mathrm{X}^{\mathrm{II}} \quad \text{for} \quad \mathrm{i} = 2, \ldots, n \tag{13}$$

and the proportion of Type II passengers who choose to travel via HSR and progressive CR is $1 - \mathrm{C}^{\mathrm{II}_\mathrm{i}}$. In zone 1, it is noted here that passengers are unable to travel via HSR and progressive CR.

4.2. CR alone vs. HSR and bracktracking CR

Similarly, let $T_{\mathrm{H2}} - T'_{\mathrm{R}} = 0$, then, from (4) and (5) in Sect. 2, we obtain

$$\begin{aligned} T_{\mathrm{H2}} - T'_{\mathrm{R}} = t_d + (t^h_{\mathrm{p2}} + t^r_{\mathrm{p2}} - t^r_{\mathrm{p}}) + t^r_{\mathrm{wi}} + \mathrm{i}\, t^h_L - [((\mathrm{i}-1)n^L_r + \mathrm{j}) \\ - (n^L_r - \mathrm{j})]t^r_l - t_\mathrm{s} = 0 \end{aligned} \tag{14}$$

Since $t^h_{\mathrm{p2}} + t^r_{\mathrm{p2}} - t^r_{\mathrm{p}} = [\mathrm{i}\, L\delta_h + (n^L_r - \mathrm{j})l\,\delta_r - ((\mathrm{i}-1)n^L_r + \mathrm{j})l\,\delta_r]/\gamma = \mathrm{i\,K} + 2[(\mathrm{i}-1)n^L_r + \mathrm{j}]\delta_r/\gamma$, then (14) can be simplified to:

$$(t_d + t'^r_{\mathrm{wi}} - t_\mathrm{s}) + 2(n^L_r - \mathrm{j})\left(l\,\frac{\delta_r}{\gamma} + t^r_l\right) = \mathrm{i}\,(n^L_r t^r_l - t^h_L - \mathrm{K}) = \mathrm{i\,X} \tag{15}$$

Random variables γ and X are respectively in left- and right-hand sides of (15), so markets resulting from the comparison between travel via CR alone and via HSR and backtracking CR can't be analyzed using X distribution alone. Let $\mathrm{Y} = (l\,\frac{\delta_r}{\gamma} + t^r_l)$, and $t^r_l < \mathrm{Y} < \infty$, since $\mathrm{X} = n^L_r t^r_l - t^h_L - \mathrm{K}$, therefore X is an increasing function of γ and Y is a decreasing function of γ. Any passenger who has a specific γ value will also have specific X and Y values. The p.d.f. of Y, $f\,(Y)$, can be derived from the theorem shown in Appendix A. That is:

$$\mathrm{f}\,(\mathrm{Y}) = \mathrm{f}_\gamma\left(\frac{l\,\delta_r}{\mathrm{Y} - t^r_l}\right) \cdot \frac{l\,\delta_r}{(\mathrm{Y} - t^r_l)^2} \tag{16}$$

X and Y are both random variables transformed from γ, though distributed with different parameters. The proof in Appendix B shows that if $\gamma_1 > \gamma_2$, then it is true for $\mathrm{X}_1 > \mathrm{X}_2$, and $\mathrm{Y}_1 < \mathrm{Y}_2$. Let γ' stand for a γ value, and X' stand for its corresponding X value. The position of X' in the X curve is then the same as that of γ' in the γ curve as shown in Fig. 5 a and b. On the other hand, the position of Y', the corresponding Y value, is on the opposite side of the Y curve as shown in Fig. 5 c.

From (15), if the (X, Y) values of Type II passengers, who alight at station j in zone i, makes $(t_d + t'^r_{\mathrm{wi}} - t_\mathrm{s}) + 2(n^L_r - \mathrm{j})\mathrm{Y} \geqq \mathrm{i\,X}$ held, they will then choose a CR train directly to their destinations; otherwise, they will choose to travel via an HSR train and a backtracking CR train. The market boundary along the corridor for these two routing strategies can be esti-

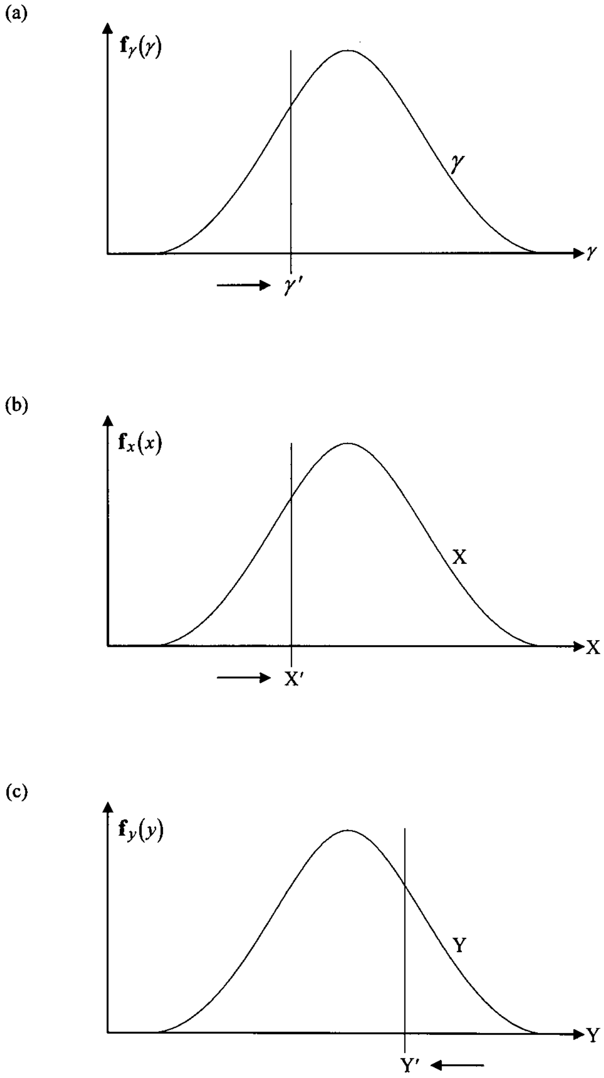

Fig. 5. The relationship among γ, X, and Y

mated by finding the (X, Y) values that satisfy the equality of (15). Let $(\bar{X}, \bar{Y})$ be (X, Y) values that satisfy the equality condition of (15). Then

$$(t_d + t'^{r}_{\mathrm{wi}} - t_{\mathrm{s}}) + 2\,(n_r^L - \mathrm{j})\bar{Y} = \mathrm{i}\,\bar{X} \tag{17}$$

From the definitions of X, and Y, we know

$$\bar{X} = (n_r^L t_l^r - t_L^h) - L(\delta_h - \delta_r) \cdot 1/\gamma \tag{18}$$

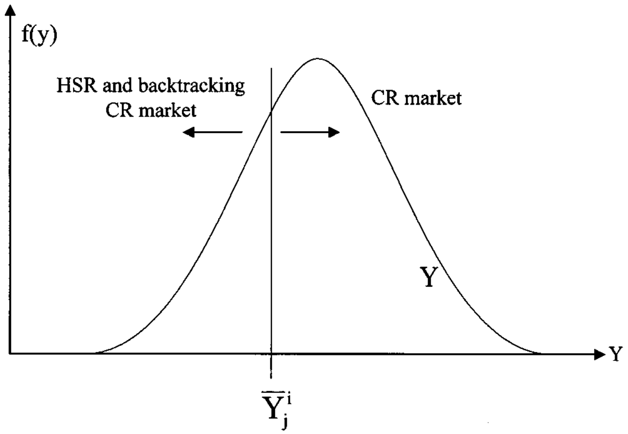

Fig. 6. Market segmentation of CR alone, and HSR and backtracking CR

$$\bar{Y} = l\delta_r \cdot 1/\gamma + t_l^r \tag{19}$$

Substitute $1/\gamma = (\bar{Y} - t_l^r)/l\delta_r$ derived from (19) into (18), rearrange, and solve the simultaneous equations for $\bar{X}$ and $\bar{Y}$, $\bar{Y}$ can then be solved for by:

$$\bar{Y} = \frac{i((n_r^L t_l^r - t_L^h) + \delta\, t_l^r) - (t_d + t'^r_{wi} - t_s)}{2\,(n_r^L - j) + i\delta} = \bar{Y}_j^i \tag{20}$$

where $\delta = L(\delta_h - \delta_r)/l\delta_r$. The value of $\bar{Y}$ in (20) will change as destination station j in zone i varies, so we use $\bar{Y}_j^i$ to represent $\bar{Y}$ value in (20). As shown in Fig. 6, for any Type II passengers who alight at station j in zone i, if their Y values are greater than $\bar{Y}_j^i$, then their X values are smaller than $\bar{X}$. That is

$$\begin{aligned}(t_d + t'^r_{wi} - t_s) + 2\,(n_r^L - j)Y > (t_d + t'^r_{wi} - t_s)\\ + 2\,(n_r^L - j)\bar{Y}_j^i = i\,\bar{X} > i\,X\end{aligned} \tag{21}$$

Therefore, these passengers will choose to travel in CR trains directly to their destination stations. On the other hand, passengers whose Y values are smaller than $\bar{Y}_j^i$ will have X values greater than $\bar{X}$. That is

$$\begin{aligned}(t_d + t'^r_{wi} - t_s) + 2\,(n_r^L - j)Y < (t_d + t'^r_{wi} - t_s)\\ + 2\,(n_r^L - j)\bar{Y}_j^i = i\,\bar{X} < i\,X\end{aligned} \tag{22}$$

therefore, these passengers will choose to travel via HSR trains and then transfer to CR trains that backtrack to their destination stations. In the same zone i, the value of $\bar{Y}_j^i$ decreases as j decrease as shown in (20). In other words, among all passengers whose destination stations are in zone i, those alighting at stations closer to the origin station 0, i.e. with smaller j values, are more likely to choose conventional trains, while those alighting at stations closer to joint station i, are more likely to travel in an HSR train to station i, then transfer to a CR train that backtracks to their destination stations. Similarly, in situations where only two routings, i.e. CR alone vs. HSR and backtracking CR, are compared, the proportion of Type II passengers who alight at any station j of zone i, $i=1,...,n$ and choose CR trains alone, $C_{1j}^{II_i}$, can be calculated by:

$$C_{1j}^{II_i} = \int_{\bar{Y}_j^i}^{\infty} f(Y)dY \quad \text{for} \quad i = 1, ..., n \tag{23}$$

Normally, the market for traveling beyond desired destinations in an HSR train and then backtracking via a CR train is competitive only when the total travel distance is long enough and transfer time is short. In reality, stations like this are usually located in large metropolitan areas with strong demand, thereby allowing small headway and waiting time. As noted before, passengers alighting at station j in zone 1 are unable to travel via HSR and progressive CR, so the proportion of these passengers, who choose to travel via HSR and backtracking CR can be calculated as $1 - C_{1j}^{II_1}$.

4.3. HSR and progressive CR vs. HSR and backtracking CR

Following the analyses described above, let $T_{H2} - T_{H1} = 0$, then

$$\begin{aligned} T_{H2} - T_{H1} = {} & (t_L^h + t_{wi}^{\prime r} + t_s - t_{w(i-1)}^r) + (t_{p2}^h + t_{p2}^r - t_{p1}^h - t_{p1}^r) \\ & - [(n_r^L - j) - j] l \, t_l^r = 0 \end{aligned} \tag{24}$$

Since $(t_{p2}^h + t_{p2}^r - t_{p1}^h - t_{p1}^r) = [L\delta_h + (n_r^L - j) l \delta_r - j \, l \delta_r]/\gamma = K + 2(n_r^L - j)\delta_r/\gamma$, therefore (24) can be simplified to:

$$(t_{wi}^{\prime r} + t_s - t_{w(i-1)}^r) + 2(n_r^L - j)\left(l \frac{\delta_r}{\gamma} + t_l^r\right) = (n_r^L t_l^r - t_L^h - K) = X \tag{25}$$

The form of (25) is similar to that of (15), so we may solve for $\bar{Y}_j^{\prime i}$ (similar to $\bar{Y}_j^i$ in the preceding subsection) by applying the same derivations. Thus,

$$\bar{Y}_j^{\prime i} = \frac{((n_r^L t_l^r - t_L^h) + \delta \, t_l^r) - (t_{wi}^{\prime r} - t_{w(i-1)}^r + t_s)}{2(n_r^L - j) + \delta} \tag{26}$$

Similarly, in situations where only two routing strategies, i.e. HSR and progressive CR vs. HSR and backtracking CR, are compared, the proportion of Type II passengers who alight at any station j of zone i, $i=2,...,n$, and

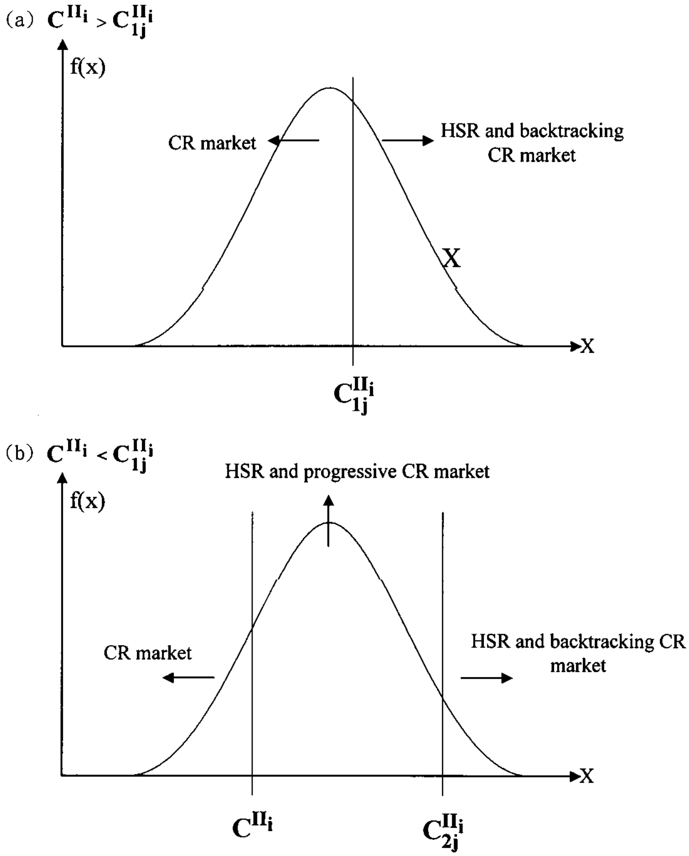

Fig. 7. Market segmentation of Type II passenger alighting at station *j* in zone *i*

choose to travel via HSR and progressive CR, $C_{2j}^{II_i}$, is an integral similar to (23)

$$C_{2j}^{II_i} = \int_{\bar{Y}_j^{\prime i}}^{\infty} f\,(Y)\,dY\,, \quad i = 2\,, \ldots, n \tag{27}$$

Consequently, in these situations, the proportion of Type II passenger who choose to travel via HSR and backtracking CR is $1 - C_{2j}^{II_i}$, $i = 2, \ldots, n$.

4.4. Synthesis

Type II passengers may have three routing alternatives. We have just analyzed their markets by comparing two of them each time as described in the preceding three subsections. C^{II_i}, $C_{1j}^{II_i}$, and $C_{2j}^{II_i}$ were derived in these analyses. Appendix C presents graphical proof and rules for integrating the results of these separate analyses. If $C^{II_i} > C^{II_i}_{1j}$, passengers alighting at station j of zone i are proven to travel via CR alone or via HSR and backtracking CR, and there is no possibility of traveling via HSR and progressive CR. The proportions of passengers chosing these two routes are, respectively, $C^{II_i}_{1j}$ and $1 - C^{II_i}_{1j}$ as shown in Fig. 7 a.

As shown in Appendix C, if $C^{II_i} < C^{II_i}_{1j}$, then it is true for $C^{II_i}_{2j} > C^{II_i}_{1j}$, i.e. $C^{II_i} < C^{II_i}_{1j} < C^{II_i}_{2j}$, then passengers alighting at station j of zone i have all three routing alternatives available. C^{II_i}, $C^{II_i}_{2j} - C^{II_i}$, and $1 - C^{II_i}_{2j}$ are, respectively, the proportions of passengers who choose to travel via CR alone, HSR and progressive CR, and HSR and backtracking CR, as shown in Fig. 7 b. Therefore, the markets for Type II passengers can be estimated by calculating C^{II_i}, $C^{II_i}_{1j}$, and $C^{II_i}_{2j}$, and applying the rules in Appendix C. Consequently, an overall market analysis for passengers originating at joint station 0 and alighting at different stations along the rail corridor will be completed by combining results of analyses for both Type I and Type II passengers.

Table 1. Values of parameters

Parameter	Symbol	Value
Number of zones of CR line divided by all HSR stations	n	9
Number of CR stations in a zone	m^i	7
Average spacing between two consecutive HSR stations	L	40 km
Average spacing between two consecutive CR stations	l	5 km
Cruise speed of HSR trains	V_h	250 km/h
Cruise speed of CR trains	V_r	110 km/h
Rate of acceleration of HSR trains	α	1.736 km/min^2
Rate of acceleration of HSR trains	β	1.44 km/min^2
Riding time between two consecutive HSR stations (including train stopping time)	t^h_L	14 min
Riding time between two consecutive CR stations (including train stopping time)	t^r_l	6 min
Unit distance fare for HSR trains	δ_h	0.1 $/km
Unit distance fare for CR trains	δ_r	0.05 $/km
Average stop time for a station	t_s	2 min
Waiting time for a progressive CR train at the joint station	t^r_w	15 min
Waiting time for a backtracking CR train at the joint station	t'^r_w	15 min
The mean of passenger’s time value (assuming normal distribution)	$\bar{\gamma}$	0.09 $/min
The standard deviation of passenger’s time value (assuming normal distribution)	σ	0.01 $/min

Table 2. Market shares for Type I passengers alighting at station *i*

Station *i*	t_d=−10 min (CR, HSR)	t_d=+10 min (CR, HSR)
1	(0.000021, 0.999979)	(0.476267, 0.523733)
2	(0.000259, 0.999741)	(0.056067, 0.943933)
3	(0.000625, 0.999375)	(0.023429, 0.976571)
4	(0.000978, 0.999022)	(0.014958, 0.985042)
5	(0.001281, 0.998719)	(0.011400, 0.988600)
6	(0.001535, 0.998465)	(0.009504, 0.990496)
7	(0.001747, 0.998253)	(0.008345, 0.991655)
8	(0.001926, 0.998074)	(0.007568, 0.992432)
9	(0.002077, 0.997923)	(0.007014, 0.992986)

Table 3. Market shares for Type II passengers alighting at station *j* of zone *i*

Zone *i*	Station *j*	t_d=−10 min (CR, HSR+CR–F[a], HSR+CR–B[b])	t_d=+10 min (CR, HSR+CR–F, HSR+CR–B)
1	1∼5	(1, 0, 0)	(1, 0, 0)
	6	(0.970300, 0, 0.029700)	(1, 0, 0)
	7	(0.998300. 0, 0.001700)	(1, 0, 0)
2	1∼6	(0.000021, 0.999979, 0)	(0.476400, 0.523600, 0)
	7	(0.000021, 0.999944, 0.000035)	(0.476400, 0.523565, 0.000035)
3	1∼6	(0.000260, 0.999740, 0)	(0.056060, 0.943940, 0)
	7	(0.000260, 0.999705, 0.000035)	(0.056060, 0.943905, 0.000035)
4	1∼6	(0.000626, 0.999375, 0)	(0.023430, 0.976570, 0)
	7	(0.000626, 0.999339, 0.000035)	(0.023430, 0.976535, 0.000035)
5	1∼6	(0.000978, 0.999022, 0)	(0.014960, 0.985040, 0)
	7	(0.000978, 0.998987, 0.000035)	(0.014960, 0.985005, 0.000035)
6	1∼6	(0.001281, 0.998719, 0)	(0.011400, 0.988600, 0)
	7	(0.001281, 0.998684, 0.000035)	(0.000060, 0.999905, 0.000035)
7	1∼6	(0.001535, 0.998465, 0)	(0.009504, 0.990496, 0)
	7	(0.001535, 0.998430, 0.000035)	(0.009504, 0.990461, 0.000035)
8	1∼6	(0.001747, 0.998253, 0)	(0.008345, 0.991655, 0)
	7	(0.001747, 0.998218, 0.000035)	(0.008345, 0.991620, 0.000035)
9	1∼6	(0.001926, 0.998074, 0)	(0.007568, 0.992432, 0)
	7	(0.001926, 0.998039, 0.000035)	(0.007568, 0.992397, 0.000035)

[a] Represents via HSR and progressive CR
[b] Represents via HSR and backtracking CR

5. Example and sensitivity analysis

Along the west corridor of Taiwan, there is an existing CR line. A new HSR project has been proposed in the same corridor to connect the terminal stations of Taipei and Kaohsiung. In between, there will be eight intermediate stations. The models developed in this research were applied to

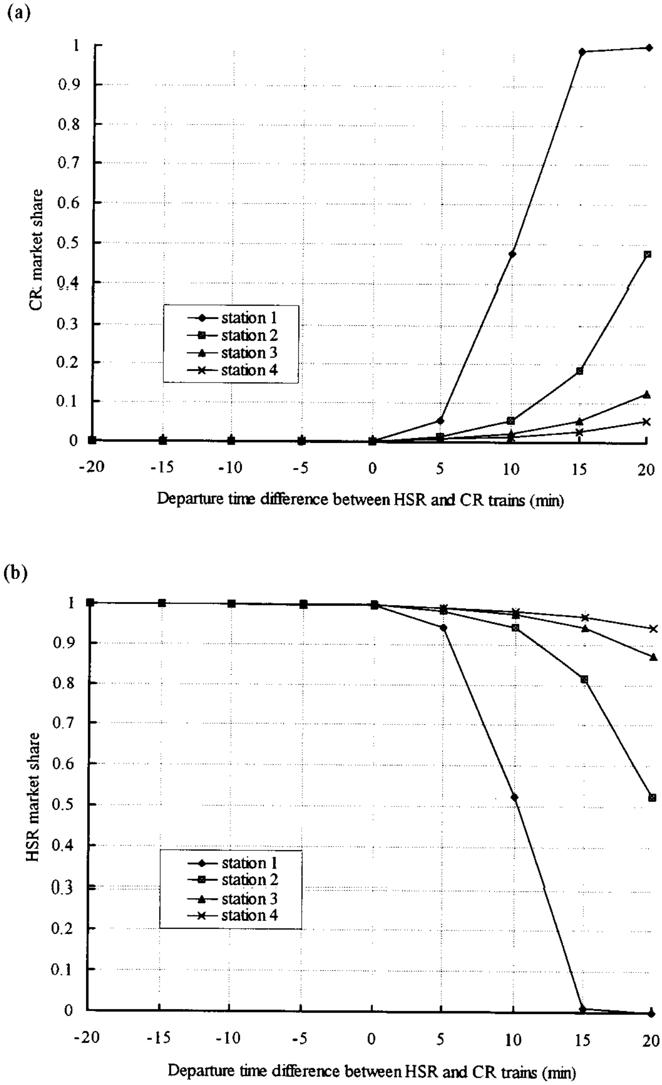

Fig. 8. The market share vs. departure time difference between HSR and CR trains

this corridor by using the data shown in Table 1. Most of the data are based on service characteristics of the existing CR line and planned HSR line (but also slightly modified to fit the model assumptions). The value of time is based on a socioeconomic survey of Taiwan and assumed to be distributed with a normal distribution with a mean of $\bar{\gamma} = 0.09$ \$/min and a standard deviation of $\sigma = 0.01$ \$/min. The model was programmed using Mathematica, and the results are summarized in Tables 2 and 3.

As shown in Table 2, when Type I passengers are confronted with $t_d = 10$ min, i.e., the latest CR train is 10 min earlier than the latest HSR

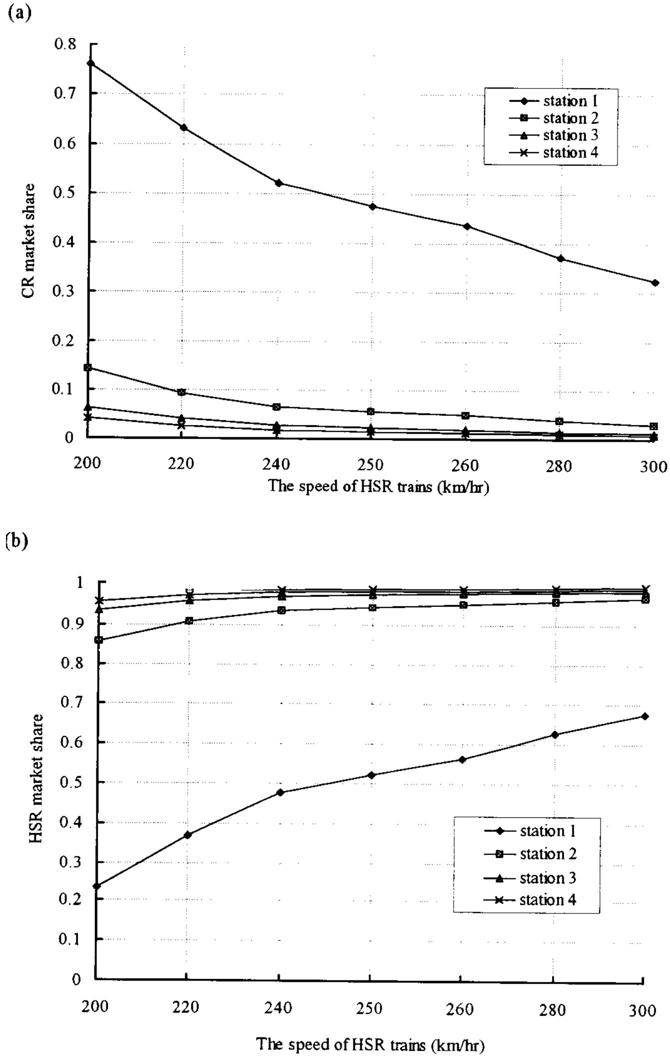

Fig. 9. The market share vs. the speed of HSR trains

train, except those alighting at joint station 1, most other Type I passengers who alight at stations other than station 1 will choose the HSR train. In other words, CR competes with HSR only in markets with trip lengths of less than 40 km, or approximately the range of commuting distance. Similarly, as shown in Table 3, when Type II passengers are confronted with $t_d = 10$ min, except for those alighting at CR stations in zones 1 and 2, most will choose to travel via HSR and progressive CR. Only a few passengers, who alight at the station closest to the next joint station, will choose to travel via HSR and backtracking CR. For $t_d = -10$ min, CR will attract only Type II passengers in zone 1.

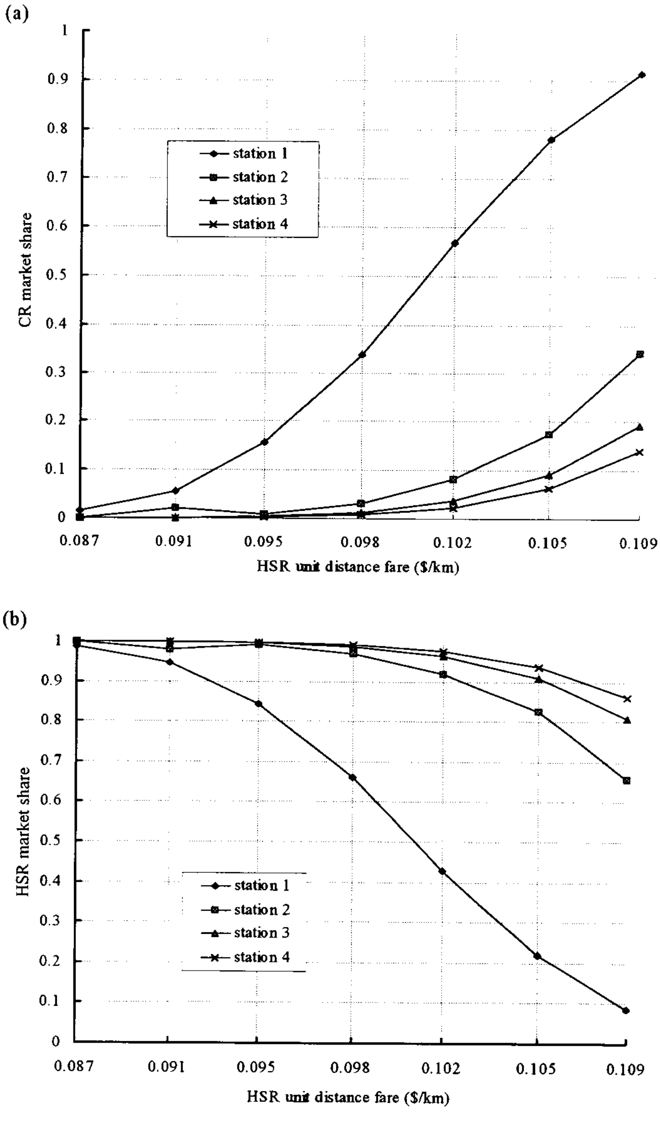

Fig. 10. The market share vs. HSR unit distance fare

Type I and Type II passengers have similar market share patterns along the corridor, i.e., the market for joint station i is similar to that for zone i, so we will perform sensitivity analysis only for Type I passengers. Figs. 8–12 show how market shares for HSR and CR are affected by changes in waiting time difference, t_d, HSR cruise speed, V_h, HSR unit distance fare, δ_h, the mean value of time of passengers, $\bar{\gamma}$, and the standard deviation of γ, σ. CR markets exist only for short travel distances when t_d is positive as shown in Fig. 8. The improved speed and reduced fares of HSR have the

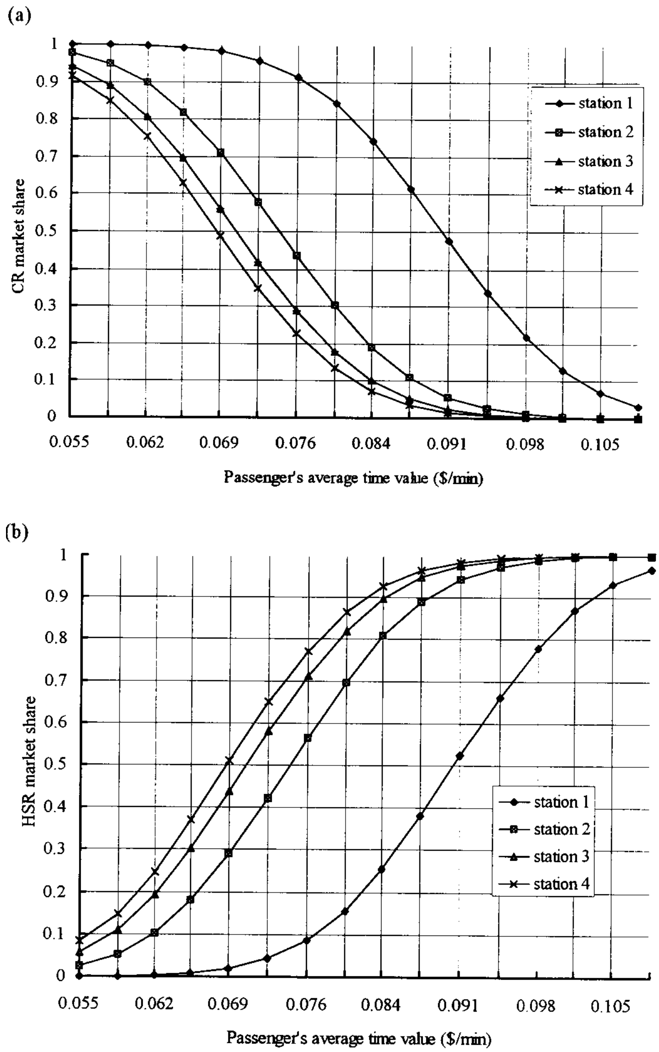

Fig. 11. The market share vs. passenger's average time value

effect of expanding its market share, and to a lesser extent for longer trips, as shown in Figs. 9 and 10. Fig. 11 shows that CR still can attract a moderate to high market share for short to medium travel distances when the average passenger's value of time is low. Finally, Fig. 12 shows that changes in the standard deviation of distribution of value of time also affect the market shares of CR and HSR but at a slower rate.

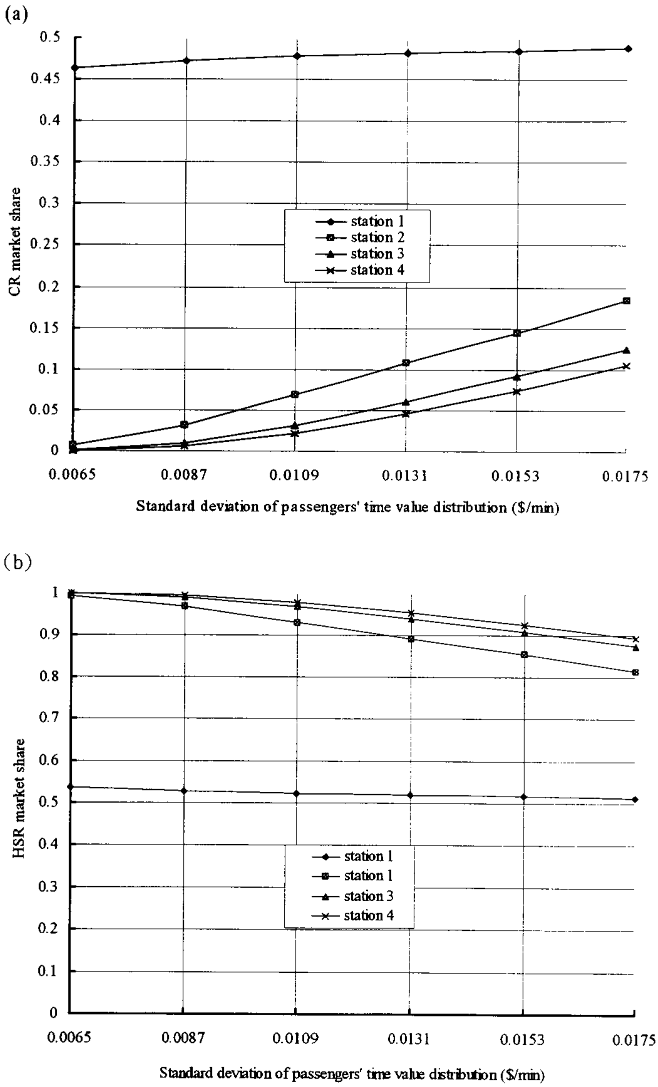

Fig. 12. The market share vs. the standard deviation of passengers' time value distribution

6. Extensions and algorithm

The market shares for HSR and CR along a transportation corridor can be analyzed by calculating and comparing C^{I_i}, C^{II_i}, $C^{II_i}_{1j}$, and $C^{II_i}_{2j}$ as shown in the preceding sections. On the basis of these analyses, we will discuss and describe consequences of relaxing some important assumptions.

6.1. Relaxing equivalent spacing assumption

Suppose there are n+1 HSR and CR joint stations (including the origin station 0) spaced with L_i, $i = 1, ..., n$ along the HSR line, and there are m_i CR stations (not including joint stations i–1 and i) in zone i of the CR lines, spaced with l_j^i, $j = 1, ..., m_i + 1$, then the riding time in an HSR train between two consecutive joint stations, t_{hi}, is $L_i/V_h + V_h/\alpha$, $i = 1, ..., n$ and the riding time in a CR train between two consecutive CR stations of zone i, t_{rj}^i, is $l_j^i/V_r + V_r/\beta$, $i = 1, ..., n$, $j = 1, ...\ m_i + 1$. The values for t_{hi} and t_{rj}^i can be obtained from timetables for existing HSR and CR lines. For planned HSR and CR lines, these values can be calculated by utilizing given V_h, V_r, α and β values.

Though it looks complex in the analysis shown in Sects. 3 and 4, there are several critical equations which play key roles in this paper. The critical equations that define market boundaries between HSR and CR are (7), (11), (15), and (25). All four of these equations show a similar physical meaning. That is (waiting and transfer time differences) + (fare conversion time difference) = (Riding time difference) between two routings. In Appendix D, we modify these four equations so as to obtain revised C^{I_i}, C^{II_i}, $C_{1j}^{II_i}$, and $C_{2j}^{II_i}$ of relaxing equivalent spacing assumption. Consequently, following the rules in Appendix C and synthesis description of Sect. 4, we can estimate market shares for different routes along the corridor by revised C^{I_i}, C^{II_i}, $C_{1j}^{II_i}$ and $C_{2j}^{II_i}$.

6.2. Relaxing value of time-related assumptions

Traditional methods used in route choice and zonal design problems for transit or rail corridors usually consider an "average" value of time per person to estimate the generalized travel cost or the generalized travel time (e.g., Wirasinghe and Seneviratne 1986; Furth 1986; Ghoneim and Wirasinghe 1987; Jansson and Ridderstolpe 1992, etc.). Instead of using an average value of time assumption, this paper treats an individual's value of time, γ, as a random variable, and derives its relationship with trip length, and the resulting route and mode choice of the individual. The p.d.f. of γ and other γ transformed variables are further used to accumulate individual route and mode choices so as to estimate markets for different HSR and CR routes. In this paper, we treat γ as a random variable in order to recognize passengers have differences in the value of time due to variations in income, age, trip purpose, etc. However, passengers may weigh differently between waiting/transfer time and in-vehicle riding time, and between traveling in an HSR train and traveling in a CR train. The theoretical framework developed in this paper won't change due to the incorporation of placing different weights on various time components. So, we may multiply the waiting/transfer time components by a relative ratio as compared with the value of time for in-vehicle riding time, and obtained from other empirical literature. The algorithm presented in Appendix E will show how to incorporate this consideration into the computation process.

6.3. Relaxing assumptions that trains make all stops and passengers board at station 0

In reality, HSR and CR trains may not stop at all stations. In these cases, we may just adjust joint station and zonal label n, and CR station label m_i to represent the station at which HSR and CR trains actually stop. Accordingly, joint station spacing, L_i, $i=1,...,n$, and CR station spacing l_j^i, $j=1,...,m_i+1$, should also be adjusted. All other procedures may follow those presented in Sect. 6.1, which describe the revisions for relaxing the equivalent spacing assumption.

The same procedure can also be applied in the analysis of markets for the two rail systems in cases where passengers board at other joint stations. The labels n, L_i, and l_j^i, are adjusted again, and all other procedures can follow those presented in Sect. 6.1. Furthermore, the model can be applied in analyzing the market of the two rail systems in cases where passengers and trains travel in the opposite direction by using station labels from n+1 to 1.

Finally, we present a simplified revision of an algorithm which shows how to operationalize the theoretical model presented in this paper in Appendix E.

7. Conclusions

This paper develops a new analytical model for exploring how passengers make use of HSR and CR serving the same rail corridor. Passengers are divided into two types according to whether they can take an HSR train directly to their destination stations or not. The route choices for each type of passenger are formulated to depend on the passenger's departure time, value of time, trip distance, fare and the service characteristics of HSR and CR. Instead of assuming an average value of time for all passengers, this paper treats an individual passenger's value of time as a random variable, and derives its relationship with the resulting rail route choice and market boundaries. The probability density functions of γ transformed variables are used to estimate market shares for different rail routes along various zones of the rail corridor. Theoretical modeling is operationalized and illustrated through an example. HSR is shown to serve most medium- to long-trip markets and CR is shown to serve commuter trip markets and collection/distribution markets for HSR. Extension for relaxing some model assumptions are discussed and a simplified algorithm is presented.

The new model may have several advantages over other simpler modal choice or route choice models. First, the model integrates rail passengers' mode choice and route choice into a framework, thus should help to better describe the actual phenomenon. Second, the model introduces new concepts such as time zones with the same waiting time differences, and the probability density functions of γ transformation variables and apply these concepts as a way to aggregate individual choices of passengers with different departure times and values of time. Though, the theoretical derivation of the model is technical and long, the application of the model is simple as shown in parameters listed in Table 1 and algorithm in Appendix E. Only estimation for statistical distribution of value of time requires additional efforts, while other parameter values are easy to collect. Therefore, this approach has the advantage of application simplicity as compared to the calibration process and aggregation problem in conventional logit model. Third, the model has potential application beyond the HSR and CR examined here and could be worth examining further. The results of the example illustrated in the paper could be valid only for the west corridor in Taiwan, and the findings are exploratory. Estimates for statistical distribution of value of time and other input values should be based on actual data in future application of the model in other corridors of interest.

Appendix A

Theorem. Let X be a random variable of the continuous type having p.d.f. f(x). Let $\mathcal{A}$ be the one-dimension space where f(x)>0. Consider the random variable Y=u(X), where y=u(x) defines a one-to-one transformation that maps the set $\mathcal{A}$ onto the set $\mathcal{B}$. Let the inverse of y=u(x) be denoted by x=w(y), and let the derivative dx/dy=w′(y) be continuous and not vanish for all points y in $\mathcal{B}$. Then the p.d.f. of the random variable Y=u(X) is given by

$$\begin{aligned} \mathrm{g}\,(\mathrm{y}) &= \mathrm{f}\,[\mathrm{w}(\mathrm{y})]\,|\mathrm{w}'(\mathrm{y})| \qquad \mathrm{y} \in \mathcal{B}\,, \\ &= \mathrm{o}\,, \qquad\qquad\qquad\quad elsewhere\,. \end{aligned}$$

It is given that $\mathrm{X} = (n_r^L t_l^r - t_L^h) - L\,(\delta_h - \delta_r)/\gamma$, where γ is a random variable and all other variables are exogenous in the definition of X. Let $\mathrm{C}_1 = n_r^L t_l^r - t_L^h$ and $\mathrm{C}_2 = L\,(\delta_h - \delta_r)$, then the p.d.f. of X, $\mathrm{f_X}\,(\mathrm{x})$, can be calculated as:

$$\because\ \gamma = -\mathrm{C}_2/\mathrm{X}^{\mathrm{I}} - \mathrm{C}_1\,, \quad d\gamma = \frac{\mathrm{C}_2}{(\mathrm{X}^{\mathrm{I}} - \mathrm{C}_1)^2}\,d\mathrm{x} \tag{A-1}$$

$$\therefore\ \mathrm{f_X}\,(\mathrm{X}) = \mathrm{f}_\gamma \left(-\frac{\mathrm{C}_2}{\mathrm{X} - \mathrm{C}_1}\right) \cdot \frac{\mathrm{C}_2}{(\mathrm{X} - \mathrm{C}_1)^2} \tag{A-2}$$

Let $\mathrm{X}^{\mathrm{I_i}} = \mathrm{iX} = \mathrm{iC}_1 - \mathrm{iC}_2/\gamma$, then the p.d.f. of $\mathrm{X}^{\mathrm{I_i}}$, $\mathrm{f}_{\mathrm{X}^{\mathrm{I_i}}}\,(\mathrm{X}^{\mathrm{I_i}})$, can be obtained directly from (A-2):

$$\mathrm{f}_{\mathrm{X}^{\mathrm{I_i}}}\,(\mathrm{X}^{\mathrm{I_i}}) = \mathrm{f}_\gamma \left(-\frac{\mathrm{i}\,\mathrm{C}_2}{\mathrm{X}^{\mathrm{I_i}} - \mathrm{i}\,\mathrm{C}_1}\right) \cdot \frac{\mathrm{i}\,\mathrm{C}_2}{(\mathrm{X}^{\mathrm{I_i}} - \mathrm{i}\,\mathrm{C}_1)^2} \tag{A-3}$$

Substituting $f_i(X^I)$ for $f_{X^{I_i}}(X^{I_i})$ and X^I for X^{I_i} in (A-3) will yield the simplified mathematical expression shown in (8). The probability density functions of other random variables such as X^{II_i}, Y, g^{I_i}, g^{II_i}, $g_{1j}^{II_i}$, and $g_{2j}^{II_i}$ can be obtained in the same way as shown above.

Appendix B

Suppose γ_1 and γ_2 represent the specific time values of two individuals and their respective (X, Y) values are (X_1, Y_1) and (X_2, Y_2). If $\gamma_1 > \gamma_2$, then the following conditions must hold by the definitions of X and Y:

$$X_1 = n_r^L t_l^r - t_L^h - L(\delta_h - \delta_r)/\gamma_1 > n_r^L t_l^r - t_L^h - L(\delta_h - \delta_r)/\gamma_2 = X_2 \quad \text{(B-1)}$$

$$Y_1 = l\delta_r/\gamma_1 + t_l^r < l\delta_r/\gamma_2 + t_l^r = Y_2 \quad \text{(B-2)}$$

Appendix C

We give here graphical proofs of the following statements:

1. If $C^{II_i} > C_{1j}^{II_i}$, passengers alighting at station j of zone i travel only via CR alone or via HSR and backtracking CR with no possibility of traveling via HSR and progressive CR. The proportions of passengers choosing these two routes are, respectively, $C_{1j}^{I_i}$ and $1 - C_{1j}^{II_i}$.

2. If $C^{II_i} < C_{1j}^{II_i}$, then it is true for $C_{2j}^{II_i} > C_{1j}^{II_i}$, i.e. $C^{II_i} < C_{1j}^{II_i} < C_{2j}^{II_i}$, then passengers alighting at station j of zone i have all three routing alternatives available. C^{II_i}, $C_{2j}^{II_i} - C^{II_i}$, and $1 - C_{2j}^{II_i}$ are, respectively, the proportions of passengers who choose to travel via CR alone, HSR and progressive CR, and HSR and backtracking CR.

From Appendix B, we know that when $\gamma_1 > \gamma_2$, it is true that $X_1 > X_2$, $Y_1 < Y_2$. Thus, for a specific γ value, γ', with its corresponding (X, Y) as (X', Y'), the following equality must hold:

$$\int_0^{\gamma'} f_y(\gamma) d\gamma = \int_0^{X'} f_X(X) dX = \int_{Y'}^{\infty} f_Y(Y) dY \quad \text{(C-1)}$$

(C-1) implies that the position of γ' in γ's distribution curve is the same as that of X' in X's distribution curve from the left, and the same as that of Y' in Y's distribution curve from the right. This implies that as long as we get the market boundary of station j in zone i derived by random variable Y, we get the market boundary of station j in zone i for X distribution graph simultaneously. That means, the positions where one is located in both dis-

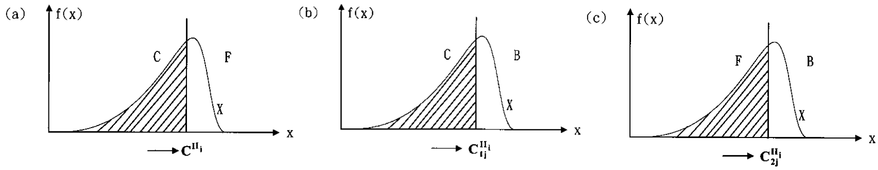

Fig. C.1

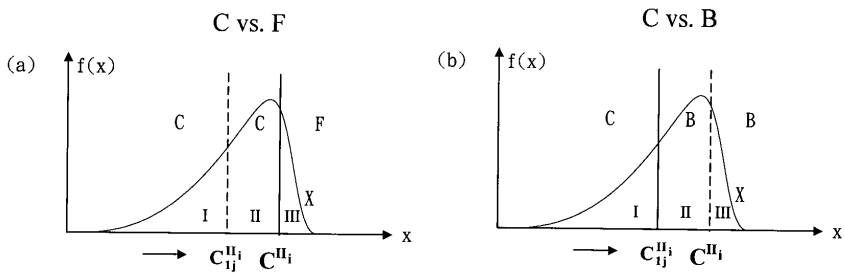

Fig. C.2

tribution curves of *X* and *Y* could be found, and the areas defined by a market boundary in *X′* and *Y′* distribution curves stand for the same group of passengers. Therefore, we can transform all the distribution graphs originally expressed in the phase of *Y*'s distribution into their respective *X* distribution graphs. The following illustrations will further be used to prove the two statements mentioned above.

All figures below are only for illustration purposes. Letters *C*, *F*, and *B* are used to stand for CR alone, HSR and progressive CR, and HSR and backtracking CR, respectively. Since we have got C^{II_i}, $C_{1j}^{II_i}$, $C_{2j}^{II_i}$ by means of the derivations shown in Sects. 3 and 4, therefore their market boundaries can be all expressed in the *X*-based distribution curve as shown in Fig. C.1.

1. If $C^{II_i} > C_{1j}^{II_i}$, we can divide the area below *X*'s distribution curve into three sections, I, II, and III, by using C^{II_i} and $C_{1j}^{II_i}$ as shown in Fig. C.2.

From Fig. C.2 (a), the attributed market of these three sections can be expressed respectively by "I: $C>F$", which stands for passengers with *X* value located in area I of *X* distribution curve in Fig. C.2 (a) will choose CR rather than HSR and progressive CR, and "II: $C>F$" and "III: $F>C$" represent in the same way as that of "I: $C>F$". Similarly, in Fig. C.2 (b) we have "I: $C>B$", "II: $B>C$", and "III: $B>C$". Therefore, we can make an inference as follows:

$$\text{"I: } C>\text{F and } B\text{"; "II: } B>C>F\text{"; "III: } F \text{ and } B>C\text{"} \quad \text{(C-2)}$$

(C-2) shows that Sect I and II under the *X* distribution belong to the markets for CR alone, and HSR and backtracking CR when $C^{II_i} > C_{1j}^{II_i}$. Sec-

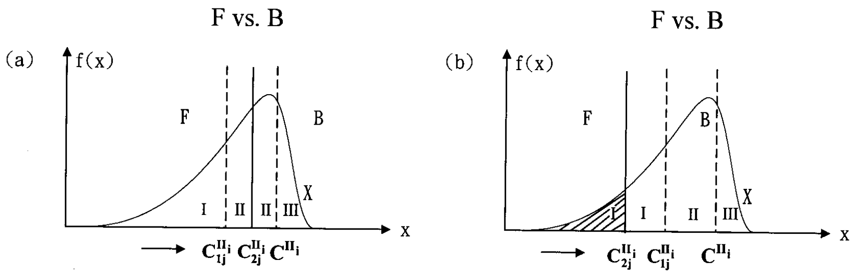

Fig. C.3

tion III can not be attributed to the *F* or *B* markets with certainty until $C^{II_i}_{2j}$ is jointly compared under $C^{II_i} > C^{II_i}_{1j}$.

(i). If $C^{II_i}_{2j} > C^{II_i}_{1j}$, the inference "Section between $C^{II_i}_{1j}$ and $C^{II_i}_{2j}$: *F*>*B*" could be made from Fig. C.3(a), and this is in contradiction with the inference "II: *B*>*C*>*F*" in (C-2).
(ii). If $C^{II_i}_{2j} < C^{II_i}_{1j}$, then we have market segments from Fig. C.3(b) as follows:

"The shade of I: *F*>*B*"; "The non-shade of I: *B*>*F*";
"II: *B*>*F*"; "III: *B*>*F*" (C-3)

From synthesizing the inferences of (C-2) and (C-3), we have "I: *C*>*F*>*B*"; "II: *B*>*C*>*F*"; and "III: *B*>*F*>*C*", that is, area I is attributed to the CR alone market, and area II and III are attributed to the HSR and backtracking CR markets. Thus we have proven that if $C^{II_i} > C^{II_i}_{1j}$, passengers alighting at station *j* of zone *i* travel only via CR alone or via HSR and backtracking CR, and have no possibility of traveling via HSR and progressive CR. The proportions of passengers choosing these two routes are, respectively, $C^{II_i}_{1j}$ and $1 - C^{II_i}_{1j}$ (see Fig. 7).

2. If $C^{II_i} < C^{II_i}_{1j}$, we divide the area below *X*'s distribution curve into three Sects. I, II, and III, with C^{II_i} and $C^{II_i}_{1j}$ as shown in Fig. C.4.

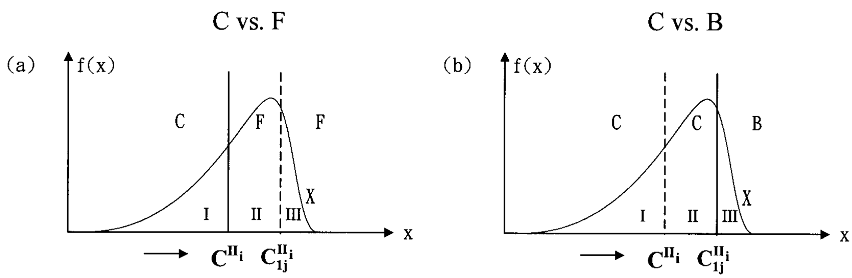

Fig. C.4

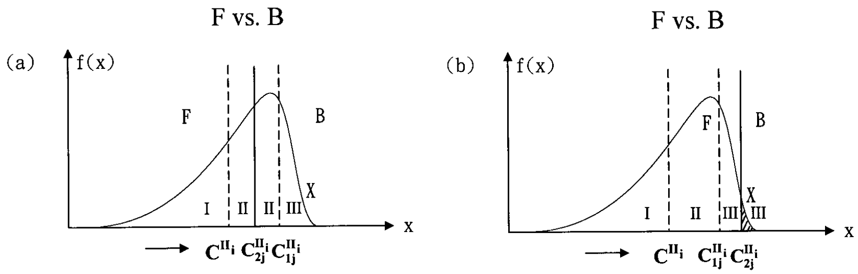

Fig. C.5

From Fig. C.4(a), the attributed markets of the three sections are expressed as "I: $C>F$"; "II: $F>C$"; "III: $B>F$"; and those from Fig. C.4(b) are "I: $C>B$"; "II: $C>B$"; "III: $B>C$". Thus, we can make an inference as follows:

$$\text{"I: } C>\text{F and } B\text{"; "II: F} > C > \text{B"; "III: } F \text{ and } B>C\text{"} \quad \text{(C-4)}$$

Thus, the areas I and II under the X distribution curve when $C^{II_i} < C_{1j}^{II_i}$ are inferred to belong to the CR alone market, and the HSR and backtracking CR market, respectively. Area III can not be attributed to the F or B market with certainty until $C_{2j}^{II_i}$ is jointly compared under $C^{II_i} > C_{1j}^{II_i}$.

(i). If $C_{2j}^{II_i} < C_{1j}^{II_i}$, the inference "area between $C_{2j}^{II_i}$ and $C_{1j}^{II_i}$ of II: $B>F$" can be made from Fig. C.5(a), and this is in contradiction with the inference "III: $F>C>B$" in (C-4).

(ii). If $C_{2j}^{II_i} > C_{1j}^{II_i}$, then we have market segments from Fig. C.5(b) as follows:

$$\text{"I: } F>B\text{"; "II: } F>B\text{"; "The non-shade of III: } F>B\text{";} \\ \text{"shade of III: } B>F\text{"} \quad \text{(C-5)}$$

From synthesizing the inferences of (C-4) and (C-5), we have "I: $C>F>B$"; "II: $F>C>B$"; "The shade of III: $F>C>B$", and "The non-shade of III: $B>F>C$". This implies that if $C^{II_i} < C_{1j}^{II_i}$, passengers alighting at station j of zone i will travel via CR alone, via HSR and progressive CR, and via HSR and backtracking CR. The proportions of passengers choosing these three routes are, respectively, C^{II_i}, $C_{2j}^{II_i} - C^{II_i}$, and $1 - C_{2j}^{II_i}$.

Appendix D

1. Revised C^{I_i} *for Type I passengers.* Suppose a Type I passenger alights at joint station i, then, for a given t_d, the fare conversion time difference between traveling via HSR and via CR can be expressed as:

$$t_{\mathrm{p}}^{h} - t_{\mathrm{p}}^{r} = (L_1 + L_2 + ... + L_{\mathrm{i}})(\delta_h - \delta_r)/\gamma = [(\delta_h - \delta_r)/\gamma] \sum_{\mathrm{k}=1}^{\mathrm{i}} L_{\mathrm{k}} \quad \text{(D-1)}$$

and the riding time difference between HSR and CR is

$$\Delta t_1 + \Delta t_2 + ... + \Delta t_{\mathrm{i}} = \sum_{\mathrm{k}=1}^{\mathrm{i}} \Delta t_{\mathrm{k}} \quad \text{(D-2)}$$

where Δt_{k}, $k = 1, ..., i$ represents the riding time difference at spacing k between HSR and CR. Δt_{k}, which includes both riding and stop time differences, can be estimated from timetables for both types of trains. (7) thus becomes

$$t_d = \sum_{\mathrm{k}=1}^{\mathrm{i}} \Delta t_{\mathrm{k}} - [(\delta_h - \delta_r)/\gamma] \sum_{\mathrm{k}=1}^{\mathrm{i}} L_{\mathrm{k}} \quad \text{(D-3)}$$

Considering the market of interest as a whole, then, similarly, the right hand side of (D-3) is a random variable transformed from γ. Denote this variables as $\mathrm{g}^{\mathrm{I}_{\mathrm{i}}}$, that is

$$\mathrm{g}^{\mathrm{I}_{\mathrm{i}}} = \sum_{\mathrm{k}=1}^{\mathrm{i}} \Delta t_{\mathrm{k}} - [(\delta_h - \delta_r)/\gamma] \sum_{\mathrm{k}=1}^{\mathrm{i}} L_{\mathrm{k}}, \quad -\infty < \mathrm{g}^{\mathrm{I}} < \sum_{\mathrm{k}=1}^{\mathrm{i}} \Delta t_{\mathrm{k}} \quad \text{(D-4)}$$

Then, from Appendix B, the p.d.f. of $\mathrm{g}^{\mathrm{I}_{\mathrm{i}}}$, $\mathrm{f}_{\mathrm{i}}\,(\mathrm{g}^{\mathrm{I}})$, is

$$\mathrm{f}_{\mathrm{i}}\,(\mathrm{g}^{\mathrm{I}}) = \mathrm{f}_{\gamma}\left(\frac{-\mathrm{C}_2}{\mathrm{g}^{\mathrm{I}} - \mathrm{C}_1}\right) \cdot \frac{\mathrm{C}_2}{(\mathrm{g}^2 - \mathrm{C}_1)^2}\ , \quad \mathrm{C}_1 = \sum_{\mathrm{k}=1}^{\mathrm{i}} \Delta t_{\mathrm{k}}\ ,$$
$$\mathrm{C}_2 = (\delta_h - \delta_r) \sum_{\mathrm{k}=1}^{\mathrm{i}} L_{\mathrm{k}} \quad \text{(D-5)}$$

and $\mathrm{C}^{\mathrm{I}_{\mathrm{i}}}$ can be revised as:

$$\mathrm{C}^{\mathrm{I}_{\mathrm{i}}} = \int_{-\infty}^{t_d} \mathrm{f}_{\mathrm{i}}\,(\mathrm{g}^{\mathrm{I}})\, d\mathrm{g}^{\mathrm{I}} \quad \text{(D-6)}$$

2. Revised $\mathrm{C}^{\mathrm{II}_{\mathrm{i}}}$ *for HSR and progressive CR vs. CR alone.* Suppose a Type II passenger who alights at station j of zone i, for a given t_d, the fare conversion time difference between HSR and progressive CR and CR alone can be expressed as:

$$t^h_{\mathrm{p1}} + t^r_{\mathrm{p1}} - t^r_{\mathrm{p}} = (L_1 + L_2 + \ldots + L_{\mathrm{i}-1})(\delta_h - \delta_r)/\gamma$$
$$= [(\delta_h - \delta_r)/\gamma] \sum_{\mathrm{k}=1}^{\mathrm{i}-1} L_{\mathrm{k}} \tag{D-7}$$

and the riding time difference between HSR and progressive CR and CR alone is

$$\Delta t_1 + \Delta t_2 + \ldots + \Delta t_{\mathrm{i}-1} = \sum_{\mathrm{k}=1}^{\mathrm{i}-1} \Delta t_{\mathrm{k}} \tag{D-8}$$

where Δt_{k}, $k=1,\ldots,i$–1 represents the riding time difference at spacing k between HSR and progressive CR and CR alone. (11) then can be rewritten as:

$$t_d + t^r_{\mathrm{w(i-1)}} - t_{\mathrm{s}} = \sum_{\mathrm{k}=1}^{\mathrm{i}-1} \Delta t_{\mathrm{k}} - [(\delta_h - \delta_r)/\gamma] \sum_{\mathrm{k}=1}^{\mathrm{i}-1} L_{\mathrm{k}} \tag{D-9}$$

Similarly, the right hand side of the equality in (D-9) is a random variable. Assume $\mathrm{g}^{\mathrm{II_i}}$ stands for this variable, and $-\infty < \mathrm{g}^{\mathrm{II_i}} < \sum_{\mathrm{k}=1}^{\mathrm{i}-1} \Delta t_{\mathrm{k}}$, then the p.d.f. of $\mathrm{g}^{\mathrm{II_i}}$ can be derived as:

$$\mathrm{f_i}\left(\mathrm{g^{II}}\right) = \mathrm{f}_\gamma \left(\frac{-\mathrm{C}_2}{\mathrm{g^{II}} - \mathrm{C}_1}\right) \cdot \frac{\mathrm{C}_2}{\left(\mathrm{g^{II}} - \mathrm{C}_1\right)^2} \tag{D-10}$$

where $\mathrm{C}_1 = \sum_{\mathrm{k}=1}^{\mathrm{i}-1} \Delta t_{\mathrm{k}}$, $\mathrm{C}_2 = (\delta_h - \delta_r) \sum_{\mathrm{k}=1}^{\mathrm{i}-1} L_{\mathrm{k}}$, and $\mathrm{C}^{\mathrm{II_i}}$ can be revised as:

$$\mathrm{C}^{\mathrm{II_i}} = \int_{-\infty}^{t_d + t^t_{\mathrm{w(i-1)}} - t_{\mathrm{s}}} \mathrm{f}_{(\mathrm{i}-1)}\left(\mathrm{g^{II}}\right) d\mathrm{g^{II}} \tag{D-11}$$

3. Revised $\mathrm{C}^{\mathrm{II_i}}_{\mathrm{1j}}$ *for CR alone vs. HSR and backtracking CR.* Suppose a Type II passenger who alights at station j of zone i, then, for a given t_d, the fare conversion time difference between CR alone and HSR and backtracking CR, $t^h_{\mathrm{p2}} + t^r_{\mathrm{p2}} - t^r_{\mathrm{p}}$, can be calculated as:

$$(\delta_h - \delta_r)/\gamma \cdot \sum_{\mathrm{k}=1}^{\mathrm{i}} L_{\mathrm{k}} + 2\,\delta_r/\gamma \cdot \sum_{\mathrm{k}=1}^{\mathrm{m_i}} l^{\mathrm{i}}_{(\mathrm{k}+1)}$$
$$= \left[(\delta_h - \delta_r) \cdot \sum_{\mathrm{k}=1}^{\mathrm{i}} L_{\mathrm{k}} + 2\,\delta_r \cdot \sum_{\mathrm{k}=1}^{\mathrm{m_i}} l^{\mathrm{i}}_{(\mathrm{k}+1)}\right]/\gamma \tag{D-12}$$

and the riding time difference between the two routings is

$$\sum_{\mathrm{k}=1}^{\mathrm{i}} \Delta t_{\mathrm{k}} - 2 \sum_{\mathrm{k}=\mathrm{j}}^{\mathrm{m_i}} t^{\mathrm{i}}_{r(\mathrm{k}+1)} \tag{D-13}$$

Therefore, (15) can be rewritten as:

$$(t_d + t'^{r}_{\mathrm{wi}} - t_{\mathrm{s}}) = \sum_{\mathrm{k=1}}^{\mathrm{i}} \Delta t_{\mathrm{k}} - 2\sum_{\mathrm{k=j}}^{\mathrm{m_i}} t^{\mathrm{i}}_{r(\mathrm{k+1})} - \left[(\delta_h - \delta_r)\sum_{\mathrm{k=1}}^{\mathrm{i}} L_{\mathrm{k}} + 2\,\delta_r \cdot \sum_{\mathrm{k=1}}^{\mathrm{m_i}} l^{\mathrm{i}}_{(\mathrm{k+1})}\right]/\gamma \qquad \text{(D-14)}$$

Denote the right hand side of (D-14) as $\mathrm{g}^{\mathrm{II_i}}_{\mathrm{1j}}$, and $-\infty < \mathrm{g}^{\mathrm{II_i}}_{\mathrm{1j}} < \sum_{\mathrm{k=1}}^{\mathrm{i}} \Delta t_{\mathrm{k}}$ $-2\sum_{\mathrm{k=j}}^{\mathrm{m_i}} t^{\mathrm{i}}_{r(\mathrm{k+1})}$, then the p.d.f. of $\mathrm{g}^{\mathrm{II_i}}_{\mathrm{1j}}$, $\mathrm{f_{ij}}\,(\mathrm{g}^{\mathrm{II}}_{1})$, is

$$\mathrm{f_{ij}}\,(\mathrm{g}^{\mathrm{II}}_{1}) = \mathrm{f}_{\gamma}\left(\frac{-\mathrm{C}_2}{\mathrm{g}^{\mathrm{II}}_{1} - \mathrm{C}_1}\right) \cdot \frac{\mathrm{C}_2}{(\mathrm{g}^{\mathrm{II}}_{1} - \mathrm{C}_1)^2} \qquad \text{(D-15)}$$

where $\mathrm{C}_1 = \sum_{\mathrm{k=1}}^{\mathrm{i}} \Delta t_{\mathrm{k}} - 2\sum_{\mathrm{k=j}}^{\mathrm{m_i}} t^{\mathrm{i}}_{r(\mathrm{k+1})}$, $\mathrm{C}_2 = \left[(\delta_h - \delta_r) \cdot \sum_{\mathrm{k=1}}^{\mathrm{i}} L_{\mathrm{k}} + 2\delta_r \cdot \sum_{\mathrm{k=j}}^{\mathrm{m_i}} l^{\mathrm{i}}_{(\mathrm{k+1})}\right]$, and the revised $\mathrm{C}^{\mathrm{II_i}}_{\mathrm{1j}}$ can be expressed as:

$$\mathrm{C}^{\mathrm{II_i}}_{\mathrm{1j}} = \int_{-\infty}^{t_d + t'^{r}_{\mathrm{wi}} - t_{\mathrm{s}}} \mathrm{f_{ij}}\,(\mathrm{g}^{\mathrm{II}}_{1})\, d\mathrm{g}^{\mathrm{II}}_{1} \qquad \text{(D-16)}$$

4. Revised $\mathrm{C}^{\mathrm{II_i}}_{\mathrm{2j}}$ *for HSR and progressive CR vs. HSR and backtracking CR.* The fare conversion time difference in this situation, $t^{h}_{\mathrm{p2}} + t^{r}_{\mathrm{p2}} - t^{h}_{\mathrm{p1}} - t^{r}_{\mathrm{p1}}$, can be expressed as:

$$L_{\mathrm{i}}\,(\delta_h - \delta_r)/\gamma + [2\,\delta_r/\gamma]\sum_{\mathrm{k=1}}^{\mathrm{m_i}} l^{\mathrm{i}}_{(\mathrm{k+1})} = \left[L_{\mathrm{i}}\,(\delta_h - \delta_r) + 2\,\delta_r \cdot \sum_{\mathrm{k=1}}^{\mathrm{m_i}} l^{\mathrm{i}}_{(\mathrm{k+1})}\right]/\delta \qquad \text{(D-17)}$$

and the riding time difference is

$$\Delta t_{\mathrm{i}} - 2\sum_{\mathrm{k=j}}^{\mathrm{m_i}} t^{\mathrm{i}}_{r(\mathrm{k+1})} \qquad \text{(D-18)}$$

Therefore, (25) can be rewritten as:

$$(t'^{r}_{\mathrm{wi}} - t^{r}_{\mathrm{w(i-1)}} + t_{\mathrm{s}}) = \Delta t_{\mathrm{i}} - 2\sum_{\mathrm{k=j}}^{\mathrm{m_i}} t^{\mathrm{i}}_{r(+1)} - \left[L_{\mathrm{i}}\,(\delta_h - \delta_r) + 2\,\delta_r \cdot \sum_{\mathrm{k=1}}^{\mathrm{m_i}} l^{\mathrm{i}}_{(\mathrm{k+1})}\right]/\gamma \qquad \text{(D-19)}$$

Denote the right hand side of (D-19) as $g_{2j}^{II_i}$, and $-\infty < g_{1j}^{II_i} < \Delta t_i - 2\sum_{k=j}^{m_i} t_{r(k+1)}^{i}$, then the p.d.f. of $g_{2j}^{II_i}$, $f_{ij}\left(g_2^{II}\right)$, is

$$f_{ij}\left(g_2^{II}\right) = f_\gamma\left(\frac{-C_2}{g_2^{II} - C_1}\right) \cdot \frac{C_2}{\left(g_2^{II} - C_1\right)^2} \tag{D-20}$$

where $C_1 = \Delta t_i - 2\sum_{k=j}^{m_i} t_{r(k+1)}^{i}$, $C_2 = \left[L_k\left(\delta_h - \delta_r\right) + 2\delta_r \cdot \sum_{k=j}^{m_i} l_{(k+1)}^{i}\right]$, and $C_{2j}^{II_i}$ can be revised as:

$$C_{2j}^{II_i} = \int_{-\infty}^{t'^{r}_{wi} - t^{r}_{w(i-1)} + t_s} f_{ij}\left(g_2^{II}\right) dg_2^{II} \tag{D-21}$$

Appendix E

Algorithm:

1. Collect data including $t^{r}_{w(i-1)}$, t'^{r}_{wi}, t_{ds}, Δt_k, t_{rj}^{i}), n, m_i, V_h, V_r, α, β, δ_h, δ_r, t_s, and $f_\gamma(\gamma)$. The first five variables may be estimated from timetables for HSR and CR trains.

2. Transform probability density functions and calculate C^{I_i} (i=1 to n), C^{II_i} (i=2 to n), $C_{1j}^{I_i}$ (i=1 to n, j=1 to m_i), $C_{2j}^{I_i}$ (i=2 to n, j=1 to m_i) by (D-2) to (D-13). The waiting/transfer time components in the upper bounds of the integrals (D-6), (D-11), (D-16), and (D-21), i.e. t_{ds}, $t_{ds} + t^{r}_{w(i-1)} - t_s$, $t_{ds} + t'^{r}_{wi} - t_s$, $t'^{r}_{wi} - t^{r}_{w(i-1)} + t_s$, may be multiplied by p, the ratio of waiting/transfer time value to the riding time value, so as to account for different weights placed by passengers on various time components. For instance,

$$C^{II_i} = \int_{-\infty}^{p\cdot\left(t_{ds} + t^{r}_{w(i-1)}\right) - t_s} f_{(i-1)}\left(g^{II}\right) dg^{II}\ . \tag{E-1}$$

3. Estimate market shares

Type I passengers: passengers departing from joint station 0 and alighting at joint station i, i=1,..., n. There are two markets, the market share for traveling via CR is C^{I_i}, and the market share for traveling via HSR is $1 - C^{I_i}$.

Type II passengers: passengers departing from joint station 0 and alighting at CR station j of zone i.

(1) Zone 1: there are only two competitive markets, the market share for traveling via CR alone is $C_{1j}^{II_i}$, and the market share for traveling via HSR and backtracking CR is $1 - C_{1j}^{II_i}$.
(2) Zone i, i=2,..., n: for i=2 to n, and for j=1 to $n_r^L - 1$,

if $C^{II_i} > C_{1j}^{II_i}$, then there are only two markets, the market share for traveling via CR alone is $C_{1j}^{II_i}$, and the market share for traveling via HSR and backtracking CR is $1 - C_{1j}^{II_i}$,
if $C^{II_i} < C_{1j}^{II_i}$, then there are three markets, the market share for traveling via CR alone is C^{II_i}, the market share for traveling via HSR and progressive CR is $C_{2j}^{II_i} - C^{II_i}$, and the market share for traveling via HSR and backtracking CR is $1 - C_{2j}^{II_i}$.

References

Brand D, Parody TE, Hsu PS, Tierney KF (1992) Forecasting high-speed rail ridership. Trans Res Rec 1341:12–18

Campbell JF (1992) Selecting routes to minimize urban travel time. Trans Res B 26(4):261–274

Cook TM, Russell RA (1980) Estimating urban travel times: A comparative study. Trans Res A 14:173–175

Furth PF (1986) Zonal route design for transit corridors. Trans Sci 20(1):1–12

Ghoneim NSA, Wirasinghe SC (1987) Optimum zone configuration for planned urban commuter rail lines. Trans Sci 21(2):106–114

Jansson K, Ridderstolpe B (1992) A method for the route choice problem in public transport systems. Trans Sci 26(3):246–251

Mandel B, Gaudry M, Rothengatter W (1994) Linear or nonlinear utility functions in Logit models? The impact on German high-speed rail demand forecasts. Trans Res B 28(2):91–101

Spiess H (1983) On optimal choice strategies in transit networks. Centre de Recherche sur les Transports, Université de Montreal

Wirasinghe SC, Seneviratne PN (1986) Rail line length in an urban transportation corridor. Trans Sci 20(4):237–245