PHYSICAL REVIEW D **96,** 023529 (2017)

Bianchi type I anisotropic power-law solutions for the Galileon models

Tuan Q. Do[1,*] and W. F. Kao[2,†]

[1]*Faculty of Physics, VNU University of Science, Vietnam National University, Hanoi 120000, Vietnam*
[2]*Institute of Physics, Chiao Tung University, Hsin Chu 30010, Taiwan*
(Received 31 May 2017; published 25 July 2017)

We will study the cosmological solutions of a class of Galileon models. A special class of the Bianchi type I power-law solutions will be presented in this paper. We will show that these Bianchi type I power-law solutions are stable during the inflationary phase. In addition, we will also show that the presence of a phantom field induces at least an unstable mode for the perturbation equation and destabilizes the power-law solutions.

DOI: 10.1103/PhysRevD.96.023529

I. INTRODUCTION

Cosmic inflation [1] has served as a successful paradigm of modern cosmology in resolving several cosmological problems including the *monopole*, *horizon*, and *flatness* problems [1]. In addition, it also offers a precise framework to accommodate the observations of the Wilkinson Microwave Anisotropy Probe (WMAP) [2] and Planck satellites [3] designed for the survey of the cosmic microwave background (CMB) radiation. Indeed, many observations have been shown to be consistent with the theoretical predictions of the standard inflationary models [2,3]. There are, however, some large scale CMB anomalies observed by the WMAP [2] and the Planck [3]. For example, the hemispherical asymmetry and the cold spot are beyond the predictions of standard inflationary models. As a result, these exotic features of CMB imply that the early universe should be slightly anisotropic rather than isotropic. Hence, isotropic inflationary models, based on the homogeneous and isotropic Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime [4], should be modified by the introduction of the anisotropic inflation models, e.g., the anisotropic Bianchi spacetimes [5,6].

The cosmic no-hair conjecture proposed by Hawking and his colleagues [7,8] claims that all classical hairs (or equivalently spatial anisotropic directions) of the early universe will disappear once the vacuum energy dominates the late-time universe. More precisely, Hawking claimed [7] that all cosmological models with a positive cosmological constant Λ will approach a late-time homogeneous and isotropic de Sitter spacetime. This conjecture was partially proven by Wald [9]. It was shown that all non-type-IX Bianchi spacetimes will evolve toward the late-time de Sitter spacetime if the dominant energy condition and the strong energy condition are both satisfied. For the type-IX Bianchi spacetime, it will behave similarly if the cosmological constant Λ is sufficiently large [9]. Recently, there have been a number of discussions concerning the validity of the cosmic no-hair conjecture in various cosmological models including the higher curvature models [10,11], the Lorentz Chern-Simons theory [12], the massive vector theories [13], the supergravity-motivated models [14–20], the nonlinear massive gravity models [21], and the massive bigravity [22]. Moreover, the cosmic no-hair conjecture has also been extensively discussed in the case of inhomogeneous and anisotropic cosmology [23]. It is worth noting that a "holographic" proof for this conjecture using the idea that the entropy of our expanding universe will reach a maximum-entropy value has been done recently in Ref. [24]. Observationally, improved constraints on the isotropy of the universe have been obtained in a general test using Planck's data on the CMB temperature and polarization [25].

Among the models mentioned above [10–22], there is a famous counterexample to the cosmic no-hair conjecture derived from a supergravity-motivated model proposed by Kanno, Soda, and Watanabe (KSW) [14]. An interesting review on this model can be found in Ref. [15]. As a result, a spatial anisotropy of the Bianchi type I (BI) metric, interpreted as a spatial hair, has been shown to be non-vanishing in the postinflationary phase derived from a special coupling term between scalar and electromagnetic fields, i.e., $f^2(\phi)F_{\mu\nu}F^{\mu\nu}$ [14].

In addition, the BI metric has been shown to be an attractor solution to the KSW model for both canonical scalar field [14] and noncanonical scalar fields models including the Dirac-Born-Infeld (DBI) and supersymmetric Dirac-Born-Infeld (SDBI) fields [17,18]. Moreover, the cosmological implications of the KSW model such as the effect of anisotropic expansion on the correlations between T, E, and B modes of CMB and on the primordial gravitational waves have also been carried out in detail in Refs. [19,20]. Among the generalizations of the KSW model, there is a scenario [16,17] introduced by the inclusion of an additional phantom field with negative kinetic energy. The phantom field is expected to be responsible for the dark energy associated with the

*tuanqdo@vnu.edu.vn
†gore@mail.nctu.edu.tw

expanding of our current universe [26]. As a result, the inclusion of the phantom field turns the corresponding anisotropic cosmological solutions unstable during the inflationary phase [16,17]. As a result, the final state of our universe will tend toward an isotropic state as predicted by the cosmic no-hair conjecture.

In addition to the (S)DBI models, there is another interesting noncanonical type of scalar field model known as the Galileon scalar field model with an action that is invariant under the Galilean symmetry $\partial_\mu \pi \to \partial_\mu \pi + b_\mu$, with b_μ a constant vector [27]. Developed from the Dvali-Gabadadze-Porrati braneworld model [28] by Nicolis, Rattazzi, and Trincherini [27], the Galileon model introduces higher derivative terms of the Galileon (scalar) field but admits only second order field equations, which ensure that it will be free from the Ostrogradsky instability of the Galileon models [29]. The Galilean symmetry is, however, broken in curved space [30]. A covariant version of the Galileon model was therefore introduced in Ref. [30] as a more general extension [27]. It has thus been discussed extensively in Refs. [31–35]. More interestingly, a generalized version of the covariant Galileon model was also proposed in Ref. [31]. This model has also been shown in Ref. [32] to be equivalent to the higher derivatives model proposed by Horndeski a long time ago [36]. Note also that the generalized Galileon model can be reduced to the DBI model [17] in a proper limit. Moreover, the generalized Horndeski model, with the generalized Galileon model as its subclass, has also been proposed in Ref. [37].

In this paper, we would like to find both isotropic and anisotropic inflationary solutions in the context of the covariant Galileon field coupled to the electromagnetic field. We will also examine whether the cosmic no-hair conjecture holds in the Galileon-vector model in the absence (or presence) of the phantom field [26]. As a result will be shown in a moment, the BI inflationary power-law solutions found in the Galileon-vector model turn out to be stable. The presence of a phantom field does lead to the collapse of the corresponding BI power-law solutions as expected [14,16,17].

This paper will be organized as follows: (i) A brief review and the motivation of this paper have been summarized in Sec. I. (ii) A set of isotropic power-law solutions of the Galileon model will be presented in Sec. II. (iii) In Sec. III, a set of BI power-law solutions of the Galileon-vector model will be presented along with the stability analysis of this set of solutions. (iv) In addition, we will show the power-law solutions are a set of attractor solutions in Sec. IV. (v) The BI power-law solutions of the Galileon-vector-phantom model along with their stability analysis will be shown in Sec. V. (vi) In addition, a general stability analysis for a more general Galileo-vector-phantom model will be presented in Sec. VI. (vii) Finally, concluding remarks will be given in Sec. VII.

II. GALILEON MODELS AND THE ISOTROPIC POWER-LAW SOLUTIONS

In this paper, we will focus on a simple scenario of the covariant Galileon model, also known as the G-inflation model [34], with the action given by

$$S = \int d^4x\sqrt{g}\left\{\frac{M_p^2}{2}R + K(\phi,X) - G(\phi,X)\Box\phi\right\}$$
$$= \int d^4x\sqrt{g}\left\{\frac{M_p^2}{2}R + k_0\exp\left[\frac{\tau\phi}{M_p}\right]X - g_0\exp\left[\frac{\lambda\phi}{M_p}\right]X\Box\phi\right\}, \tag{2.1}$$

focusing on exponential interactions given by

$$K(\phi,X) = k_0\exp\left[\frac{\tau\phi}{M_p}\right]X; \qquad G(\phi,X) = g_0\exp\left[\frac{\lambda\phi}{M_p}\right]X. \tag{2.2}$$

$K(\phi,X)$ and $G(\phi,X)$ can in general be any arbitrary functional of the scalar field ϕ with its kinetic term defined as $X \equiv -\partial_\mu\phi\partial^\mu\phi/2$ [30]. It will be shown later that this choice will lead us to power-law solutions. Note that M_p is the Planck mass, while τ, λ, k_0, and g_0 are field parameters.

As a result, the variational equation of action (2.1) with respect to $g_{\mu\nu}$ leads to the corresponding Einstein field equations [34],

$$M_p^2 G_{\mu\nu} \equiv M_p^2\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) = \mathcal{T}_{\mu\nu} = \sum_{i=2}^{3}\mathcal{T}_{\mu\nu}^{(i)}, \tag{2.3}$$

with

$$\mathcal{T}_{\mu\nu}^{(2)} = k_0(Xg_{\mu\nu} + D_\mu\phi D_\nu\phi)\exp\left[\frac{\tau\phi}{M_p}\right], \tag{2.4}$$

$$\mathcal{T}_{\mu\nu}^{(3)} = -g_0\left[\left(-D_\sigma XD^\sigma\phi + 2\frac{\lambda}{M_p}X^2\right)g_{\mu\nu} + (D_\mu\phi D_\nu X + D_\nu\phi D_\mu X) + \left(\Box\phi + 2\frac{\lambda}{M_p}X\right)D_\mu\phi D_\nu\phi\right]\exp\left[\frac{\lambda\phi}{M_p}\right], \tag{2.5}$$

and D_μ the covariant derivative. In addition, the variational equation of the Galileon field $\phi \equiv \phi(t)$ can be shown to be [34]

$$\sum_{i=2}^{3} \mathcal{E}^{(i)} = 0, \tag{2.6}$$

with

$$\mathcal{E}^{(2)} = k_0\left(\Box\phi - \frac{\tau}{M_p}X\right)\exp\left[\frac{\tau\phi}{M_p}\right], \tag{2.7}$$

$$\mathcal{E}^{(3)} = g_0\left\{2\frac{\lambda^2}{M_p^2}X^2 - [(\Box\phi)^2 - (D_\mu D_\nu\phi)^2 - R_{\mu\nu}D^\mu\phi D^\nu\phi] - 2\frac{\lambda}{M_p}D_\mu X D^\mu\phi\right\}\exp\left[\frac{\lambda\phi}{M_p}\right]. \tag{2.8}$$

In order to obtain (an)isotropic cosmological solutions, we will consider the BI metric space with a metric given by [14–17]

$$\begin{aligned} ds^2 &= -dt^2 + a_1^2dx^2 + a_2^2dy^2 + a_3^2dz^2 \\ &= -dt^2 + \exp[2\alpha(t) - 4\sigma(t)]dx^2 \\ &\quad + \exp[2\alpha(t) + 2\sigma(t)](dy^2 + dz^2) \end{aligned} \tag{2.9}$$

with a_i, α, and σ the scale factors, the isotropy parameter, and the anisotropy parameter, respectively. The relation among a_i, α, and σ can be read off directly from the above equations. Note that σ should be much smaller than α in order to accommodate the observational data obtained from WMAP [2] and Planck [3]. In addition, the scalar field will also be assumed to be a function of time only, $\phi = \phi(t)$, in accord with the BI metric space.

Note that Eq. (2.8) reduces to

$$\mathcal{E}^{(3)} = g_0\left\{\frac{\lambda^2}{M_p^2}\frac{\dot\phi^2}{2} - \left[6H\frac{\ddot\phi}{\dot\phi} + 9H^2 - \left(\sum_{i=1}^{3}H_i^2\right) - R_{00}\right] + \frac{2\lambda}{M_p}\ddot\phi\right\}\dot\phi^2\exp\left[\frac{\lambda\phi}{M_p}\right] \tag{2.10}$$

with the help of the identity

$$(\Box\phi)^2 - (D_\mu D_\nu\phi)^2 = (\Box\phi)^2 - [(\partial_\mu\partial_\nu - \Gamma^\sigma_{\mu\nu}\partial_\sigma)\phi][(\partial^\mu\partial^\nu + g^{\mu\kappa}\Gamma^\nu_{\kappa\sigma}\partial^\sigma)\phi] = 6H\dot\phi\,\ddot\phi + 9H^2\dot\phi^2 - \left(\sum_{i=1}^{3}H_i^2\right)\dot\phi^2. \tag{2.11}$$

Here $H \equiv (\sum_{i=1}^3 H_i)/3$ and $H_i \equiv \dot a_i/a_i$ ($i = 1-3$) are the mean Hubble parameter and its components, respectively. Note again that the most general field equations for arbitrary functionals $K(\phi, X)$ and $G(\phi, X)$ can be found in Ref. [34].

In addition, the component equations of the Einstein equation (2.3) can be shown to be

$$-3M_p^2(\dot\alpha^2 - \dot\sigma^2) + \frac{k_0}{2}\exp\left[\frac{\tau\phi}{M_p}\right]\dot\phi^2 + 3g_0\exp\left[\frac{\lambda\phi}{M_p}\right]\dot\alpha\dot\phi^3 - \frac{g_0\lambda}{2M_p}\exp\left[\frac{\lambda\phi}{M_p}\right]\dot\phi^4 = 0, \tag{2.12}$$

$$-M_p^2\ddot\alpha - 3M_p^2\dot\alpha^2 + \frac{g_0}{2}\exp\left[\frac{\lambda\phi}{M_p}\right](\ddot\phi + 3\dot\alpha\dot\phi)\dot\phi^2 = 0, \tag{2.13}$$

$$\ddot\sigma + 3\dot\alpha\dot\sigma = 0. \tag{2.14}$$

Note that one of the Eqs. (2.12)–(2.14) is redundant following the constraint $D_\mu K^{\mu\nu} = D_\mu(M_p^2 G^{\mu\nu} - \mathcal{T}^{\mu\nu}) = 0$ derived from the Bianchi identity and energy conservation law. As a result, we have a nonredundant set of three equations for the field variables ϕ, α, and σ. Following Refs. [14,16,17], we will try to find a set of power-law solutions of the following forms:

$$\alpha = \zeta\log t; \qquad \sigma = \eta\log t; \qquad \frac{\phi}{M_p} = \xi\log t + \phi_0. \tag{2.15}$$

The power law solution for a system with scalar-vector coupling is known to violate Hawking's conjecture. Hence it is important to examine all possible extensions of these scalar-vector coupling systems. In particular, the Galileon model is a very important generalization along this approach [14]. It is straightforward to show that the power-law ansatz shown above is probably the only known and consistent way to induce a consistent set of power-law solutions with all terms in the ordinary differential equations (ODEs) varying as $1/t^2$. In particular, the kinetic terms $\dot\alpha^2$, $\dot\sigma^2$, and $g^{00}\dot\phi^2$ are all required to evolve as $1/t^2$. On the other hand, the power-law solution puts a strong constraint on the possible interactions that can be introduced consistently in the system. This is the main reason that we are focusing on the model shown in action (2.1). Indeed, we will show in the next section that a consistent set of power-law solutions will require that $\tau = 0$ even if we introduce a nontrivial interaction term in the first place. In addition, our main interest is to study the effect of some of the Galileon terms step by step. The result indicates that the system ends up with a set of field equations that still endorses a set of stable anisotropic power-law solutions in a fairly complicated way.

A. Isotropic limit

First of all, Eq. (2.14) reduces to the algebraic equation

$$\eta(3\zeta - 1) = 0. \tag{2.16}$$

Inflationary (or equivalently strongly expanding) solutions requires that $\zeta \gg 1$. As a result, the solution

$\zeta = 1/3$ is not an interesting solution. We will focus on the solution

$$\eta = 0 \tag{2.17}$$

that represents an isotropic solution with $\sigma = 0$. Note again the σ-related equation (2.14) is absent from the field equations if we started out with the isotropic FLRW metric in the first place. Therefore, the solution $\zeta = 1/3$ to the Eq. (2.14) is not a solution for the isotropic FLRW metric space. Finally, it can be shown that Eqs. (2.6), (2.12), and (2.13) reduce immediately to the following set of algebraic equations:

$$-k_0(3\zeta - 1)\xi - 9\zeta(\zeta - 1)\xi^2 v - 2\lambda\xi^3 v + \frac{\lambda^2}{2}\xi^4 v = 0, \tag{2.18}$$

$$-3\zeta^2 + k_0\frac{\xi^2}{2} + 3\zeta\xi^3 v - \frac{\lambda}{2}\xi^4 v = 0, \tag{2.19}$$

$$\zeta - 3\zeta^2 + \frac{3}{2}\zeta\xi^3 v - \frac{\xi^3}{2} v = 0. \tag{2.20}$$

Here, we have introduced a new variable,

$$v = g_0 M_p \exp[\lambda\phi_0], \tag{2.21}$$

for convenience. We note that there are two additional constraints, $\tau = 0$ and $\xi = 2/\lambda$, which will make sure that each term in the field equations is proportional to $1/t^2$. It is clear that the solution $\tau = 0$ leads to

$$K(\phi, X) = k_0 X = k_0\frac{\dot{\phi}^2}{2}. \tag{2.22}$$

As a result, we obtain, from Eqs. (2.19) and (2.20), that

$$k_0 = -\frac{\lambda^2}{2}(3\zeta^2 - 2\zeta), \tag{2.23}$$

$$v = \frac{\lambda^3}{4}\zeta. \tag{2.24}$$

We can insert these relations back into Eq. (2.18) for a consistent check. Equation (2.23) appears as a simple degree-two algebraic equation of ζ. If we treat k_0 as a field parameter, the solution to ζ can be shown to be

$$\zeta_\pm = \frac{1}{3} \pm \frac{\sqrt{\lambda^2 - 6k_0}}{3\lambda}. \tag{2.25}$$

In order to figure out whether these solutions are inflationary solutions, we will focus on two critical cases below.

Case 1: $k_0 = 1$ such that $K(\phi, X)$ represents the kinetic term of a canonical scalar field. As a result, the corresponding ζ is given by

$$\zeta = \frac{1}{3} + \frac{\sqrt{\lambda^2 - 6}}{3\lambda}, \tag{2.26}$$

with $\lambda \geq \sqrt{6}$. It is then easy to show that $1/3 < \zeta < 2/3$. Hence, the Galileon model does not admit any isotropic inflationary power-law solutions that require $\zeta \gg 1$.

Case 2: $k_0 = -1$ such that $K(\phi, X)$ represents the kinetic term of a phantom field. As a result, the corresponding ζ is given by

$$\zeta = \frac{1}{3} + \frac{\sqrt{\lambda^2 + 6}}{3\lambda}. \tag{2.27}$$

It is straightforward to show that $\zeta \simeq 2/3$ for $\lambda \gg 1$. On the other hand, we have $\zeta \simeq \sqrt{6}/(3\lambda) \gg 1$ if $\lambda \ll 1$. Hence, the isotropic inflationary power-law solutions to the Galileon model exist for all $\lambda \ll 1$.

III. GALILEON-VECTOR MODEL

A. The model and its anisotropic power-law solutions

Note that the Galileon model (2.1) can only admit isotropic power-law solutions ($\eta = 0$) according to the equation for anisotropic parameter σ as shown in Eq. (2.14). Hence, if we would like to obtain anisotropic power-law solutions to the Galileon with $\eta \neq 0$, a non-minimally coupling Galileon model should be considered. For example, an electromagnetic field can do the job [14–17]. In this section, we would like to try if we can find anisotropic solutions by introducing a gauge field (A_μ) coupling to the Galileon field (ϕ) via the interacting term $f^2(\phi)F_{\mu\nu}F^{\mu\nu}$ [14–17] motivated by the supergravity theory [14,15]. Here $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of gauge field A_μ and $f(\phi)$ is a functional of the scalar field ϕ. To be more specific, we will focus on the Galileon-vector model with an action given by

$$S = \int d^4x\sqrt{g}\left\{\frac{M_p^2}{2}R + k_0\exp\left[\frac{\tau\phi}{M_p}\right]X - g_0\exp\left[\frac{\lambda\phi}{M_p}\right]X\Box\phi - \frac{f_0^2}{4}\exp\left[-\frac{2\rho\phi}{M_p}\right]F_{\mu\nu}F^{\mu\nu}\right\}. \tag{3.1}$$

Note that, following Refs. [14,16,17], the functional $f(\phi)$ is also chosen as an exponential function

$$f(\phi) = f_0\exp\left[-\frac{\rho\phi}{M_p}\right] \tag{3.2}$$

with f_0 and $\rho > 0$ two different coupling constants. Note that the negative sign in $\exp[-\rho\phi/M_p]$ is chosen to accommodate the inflationary power-law solutions as will be shown explicitly in a moment.

As a result, the field equations of this model can be shown to be

$$\partial_\mu\{\sqrt{g}[f^2(\phi)F^{\mu\nu}]\} = 0, \tag{3.3}$$

$$M_p^2\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) - \mathcal{T}^{(2)}_{\mu\nu} - \mathcal{T}^{(3)}_{\mu\nu} + g_{\mu\nu}\frac{f^2(\phi)}{4}F^{\lambda\sigma}F_{\lambda\sigma} - f^2(\phi)F_{\mu\gamma}F_\nu^\gamma = 0, \tag{3.4}$$

$$\mathcal{E}^{(2)} + \mathcal{E}^{(3)} - \frac{f_\phi(\phi)f(\phi)}{2}F_{\mu\nu}F^{\mu\nu} = 0, \tag{3.5}$$

respectively. In addition, we also write $f_\phi \equiv df/d\phi$ for convenience. Note that $\mathcal{T}^{(i)}_{\mu\nu}$ are defined in Eqs. (2.4) and (2.5). Moreover, $\mathcal{E}^{(i)}$ are defined in Eqs. (2.7) and (2.8).

A consistent configuration of the gauge field A_μ is given by [14,16,17]

$$A_\mu = (0, A_x(t), 0, 0). \tag{3.6}$$

As a result, Eq. (3.3) reduces to

$$\dot{A}_x(t) = f^{-2}(\phi)\exp[-\alpha - 4\sigma]p_A, \tag{3.7}$$

with p_A the constant of integration [14]. Hence, Eq. (3.5) can be written as

$$\mathcal{E}^{(2)} + \mathcal{E}^{(3)} + f^{-3}f_\phi\exp[-4\alpha - 4\sigma]p_A^2 = 0. \tag{3.8}$$

Furthermore, the Einstein equation (3.4) reduces to the following component equations:

$$-3M_p^2(\dot\alpha^2 - \dot\sigma^2) + \frac{k_0}{2}\exp\left[\frac{\tau\phi}{M_p}\right]\dot\phi^2 + 3g_0\exp\left[\frac{\lambda\phi}{M_p}\right]\dot\alpha\dot\phi^3 - \frac{g_0\lambda}{2M_p}\exp\left[\frac{\lambda\phi}{M_p}\right]\dot\phi^4 + \frac{f^{-2}}{2}\exp[-4\alpha - 4\sigma]p_A^2 = 0, \tag{3.9}$$

$$-M_p^2\ddot\alpha - 3M_p^2\dot\alpha^2 + \frac{g_0}{2}\exp\left[\frac{\lambda\phi}{M_p}\right](\ddot\phi + 3\dot\alpha\dot\phi)\dot\phi^2 + \frac{f^{-2}}{6}\exp[-4\alpha - 4\sigma]p_A^2 = 0, \tag{3.10}$$

$$\ddot\sigma + 3\dot\alpha\dot\sigma - \frac{f^{-2}}{3M_p^2}\exp[-4\alpha - 4\sigma]p_A^2 = 0. \tag{3.11}$$

By introducing a new variable [14,16,17],

$$l = \frac{f_0^{-2}p_A^2}{M_p^2}\exp[2\rho\phi_0], \tag{3.12}$$

we can derive a set of algebraic equations from Eqs. (3.8)–(3.11) given below,

$$-k_0\xi(3\zeta - 1) - 9\xi^2\zeta(\zeta - 1)v - 2\lambda\xi^3 v + \frac{\lambda^2}{2}\xi^4 v - \rho l = 0, \tag{3.13}$$

$$-3\zeta^2 + 3\eta^2 + k_0\frac{\xi^2}{2} + 3\xi^3\zeta v - \frac{\lambda}{2}\xi^4 v + \frac{l}{2} = 0, \tag{3.14}$$

$$\zeta - 3\zeta^2 + \frac{1}{2}\xi^3(3\zeta - 1)v + \frac{l}{6} = 0, \tag{3.15}$$

$$(3\zeta - 1)\eta - \frac{l}{3} = 0. \tag{3.16}$$

Note that the following constraints are required in order that each term in these equations evolves as $1/t^2$:

$$\tau = 0, \tag{3.17}$$

$$\xi = \frac{2}{\lambda}, \tag{3.18}$$

$$-\rho\xi + 2\zeta + 2\eta = 1. \tag{3.19}$$

These relations also demonstrate the power of the exponential form interactions adopted in this paper. The exponential interactions are in fact the most important feature to the existence of a set of consistent power-law solutions. These equations put forward a set of constraints to the field parameters (τ, λ, ρ). This is in fact the key to the existence of a set of consistent power-law solutions. We are dealing with a system with three variables (ζ, η, ξ). In addition, we can show that

$$l = 3\eta(3\zeta - 1), \tag{3.20}$$

$$\zeta + \eta = z + \frac{1}{2}, \tag{3.21}$$

from Eqs. (3.16) and (3.19), respectively with the help of Eq. (3.18), i.e., $\xi = 2/\lambda$. Here we have introduced a new parameter $z \equiv \rho/\lambda$ for convenience.

Note that, if we choose $f(\phi) = f_0\exp[\rho\phi/M_p]$ with $\rho > 0$, we will obtain the following solution: $\zeta = -\eta - \rho/\lambda - 1/2 < 0$ for positive λ [14,16,17]. This means that the inflationary constraint $\zeta \gg 1$ is inconsistent with this choice. This is the reason why we chose the negative sign in the definition of exponential coupling $f(\phi) = f_0\exp[-\rho\phi/M_p]$.

Moreover, Eqs. (3.14) and (3.15) can be rewritten as

$$k_0 = -\frac{\lambda^2}{8}(18\zeta^2 - 18z\zeta - 15\zeta + 12z^2 + 10z + 2) = -\frac{\lambda^2}{8}[(2z + 1)(6z - 1) - 18z\eta + 3\eta(6\eta - 1)], \tag{3.22}$$

$$v = \frac{\lambda^3}{16}(6\zeta - 2z - 1) = \frac{\lambda^3}{8}(2z + 1 - 3\eta), \quad (3.23)$$

respectively, with the help of Eqs. (3.20) and (3.21).

As a result, Eq. (3.13) is automatically satisfied with the solutions given in Eqs. (3.20)–(3.23) due to the Bianchi identity mentioned earlier. Note that the algebraic Eq. (3.22) can be solved immediately by treating k_0 as a field parameter. The result is

$$\zeta = \zeta_\pm = \frac{5}{12} + \frac{z}{2} \pm \frac{\sqrt{\Delta}}{12}. \quad (3.24)$$

Here, we have defined a new variable Δ as

$$\Delta \equiv 9 - \frac{64k_0}{\lambda^2} - 60z^2 - 20z. \quad (3.25)$$

In fact, we can show that Δ is always positive,

$$\Delta = (6z+1)^2 - 24\eta(6z + 1 - 6\eta) = (6z + 1 - 12\eta)^2 \quad (3.26)$$

with the help of Eq. (3.22). Hence, it is straightforward to show that

$$\sqrt{\Delta} = |6z + 1 - 12\eta|. \quad (3.27)$$

Moreover, the positivity of Δ implies that we always have the following inequality for k_0 such as

$$k_0 \le -\frac{\lambda^2}{64}(60z^2 + 20z - 9). \quad (3.28)$$

In addition, the constraint $v \ge 0$ leads to the following constraints:

$$3\eta \le 2z + 1. \quad (3.29)$$

Note that the constraints $\zeta + \eta > 0$ and $\zeta - 2\eta > 0$ must be obeyed if the solution ζ represents an expanding solution. With the help of Eq. (3.21), we can show that the constraint $\zeta + \eta > 0$ is obeyed for all positive λ and ρ. On the other hand, the constraint $\zeta - 2\eta > 0$ implies that

$$3\eta < z + \frac{1}{2}. \quad (3.30)$$

In addition, $l > 0$ implies that $\eta > 0$ and $\zeta > 1/3$. Note that we have excluded the alternative solution $\zeta < 1/3$ and $\eta < 0$. The alternative choice will lead to the result $z < -1/6$. Hence, the alternative choice does not represent an expanding solution. As a result, $\zeta > 1/3$ implies that $\eta < z + 1/6$. All constraints on η show that

$$0 < \eta < \frac{z}{3} + \frac{1}{6} \quad (3.31)$$

is the final constraint required for η. Therefore, η is expected to be considerately smaller than z and is the only constraint for the existence of the expanding solutions. Hence, the corresponding constraint on k_0 is that

$$k_0 \lesssim -\frac{\lambda^2}{8}[(2z+1)(3z-1) + 3\eta(6\eta - 1)]. \quad (3.32)$$

This constraint can easily be achieved for expanding solutions obeying the constraint $\eta < z/3 + 1/6$. In particular, $k_0 \le -3\lambda^2 z^2/4$ is a direct constraint for the existence of the inflationary solution derived from the requirement that $z \gg 1$.

B. Inflationary solutions

Next, we would like to examine whether these expanding solutions represent inflationary solutions with the constraints $\zeta + \eta \gg 1$ and $\zeta - 2\eta \gg 1$. Note that the solution $\zeta = \zeta_- = 1/3 + \eta$ cannot represent an inflationary solution for $\eta \ll \zeta$. Therefore, the only possible inflationary solution is $\zeta = \zeta_+$. First of all, the first constraint

$$\zeta + \eta = z + \frac{1}{2} \gg 1 \quad (3.33)$$

implies that

$$z \gg 1, \quad (3.34)$$

or equivalently $\rho \gg \lambda$. It is clear that if $\lambda \ll 1$, then $\rho \simeq O(1)$ would be enough for the solution to drive the inflation.

In addition, the second constraint, $\zeta - 2\eta \gg 1$, can be rewritten as

$$\zeta - 2\eta = z + \frac{1}{2} - 3\eta \gg 1 \quad (3.35)$$

implies that

$$3\eta \ll z - \frac{1}{2}. \quad (3.36)$$

Both constraints can easily be achieved if $z \gg 1$ and $\eta \ll z$. Note again that the constraint $k_0 \le -3\lambda^2 z^2/4$ is required, according to Eq. (3.32), for the existence of the inflationary solution.

Note that $\Delta \simeq 36z^2$ if $\eta \simeq 1/12$ as indicated by Eq. (3.27). Therefore, if $\eta \simeq 1/12 \ll \zeta$, we should have the following constraints:

$$k_0 \simeq -\frac{3\lambda^2}{2} z^2; \; v \simeq \frac{\lambda^3}{4} z. \tag{3.37}$$

This assures that the power-law solutions obtained here can represent inflationary solutions with the anisotropy parameter η negligibly small in comparison with the isotropy parameter ζ. As a result, the constraint $\eta \ll \zeta$ is consistent with the observations of WMAP and Planck.

C. Stability analysis of anisotropic inflationary power-law solutions

We will try to find out whether the anisotropic power-law solutions obtained are stable during the inflationary phase. This can be done by considering a set of compatible power-law perturbations of the following form: $\delta\alpha = At^n$, $\delta\sigma = Bt^n$, and $\delta\phi = M_p C t^n$ [16,17]. Here A, B, and C are all constants. As a result, perturbing Eqs. (3.8), (3.10), and (3.11) around the anisotropic power-law solutions leads us to a set of algebraic equations, which can be rewritten as a matrix equation,

$$\mathcal{D}\begin{pmatrix} A \\ B \\ C \end{pmatrix} \equiv \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = 0, \tag{3.38}$$

where the explicit definitions of A_{ij} are given by

$$\begin{aligned} A_{11} &= \frac{12}{\lambda^2} v n^2 + \frac{6}{\lambda}\left[\frac{6}{\lambda}(2\zeta - 1)v + k_0\right] n - 4\rho l; \\ A_{12} &= -4\rho l; \\ A_{13} &= \left[\frac{4}{\lambda}(3\zeta - 2)v + k_0\right] n^2 \\ &\quad + \left[\frac{36}{\lambda}\zeta(\zeta - 1)v + \frac{8}{\lambda} v + (3\zeta - 1)k_0\right] n \\ &\quad + \frac{36}{\lambda}\zeta(\zeta - 1)v + \frac{8}{\lambda} v + 2\rho^2 l, \end{aligned} \tag{3.39}$$

$$\begin{aligned} A_{21} &= -n^2 - \left(6\zeta - \frac{12}{\lambda^3} v - 1\right) n - \frac{2l}{3}; \qquad A_{22} = -\frac{2l}{3}; \\ A_{23} &= \frac{2}{\lambda^2} v n^2 + \frac{6}{\lambda^2}(3\zeta - 1) v n + \frac{4}{\lambda^2}(3\zeta - 1) v + \frac{\rho}{3} l, \end{aligned} \tag{3.40}$$

$$\begin{aligned} A_{31} &= 3\eta n + \frac{4l}{3}; \qquad A_{32} = n^2 + (3\zeta - 1)n + \frac{4l}{3}; \\ A_{33} &= -\frac{2\rho}{3} l. \end{aligned} \tag{3.41}$$

It is noted that the stability of the corresponding anisotropic inflationary power-law solutions will be determined by the value of n, which can be obtained by solving Eq. (3.38). In particular, we will have stable or unstable anisotropic power-law solutions during the inflationary phase for negative or positive n, respectively. Mathematically, nontrivial solutions to Eq. (3.38) exist only when

$$\det \mathcal{D} = 0. \tag{3.42}$$

It is straightforward to show that this determinant equation gives a degree 6 polynomial equation of n,

$$n(a_6 n^5 + \cdots + a_1) = 0, \tag{3.43}$$

with

$$a_1 = \frac{8(3\zeta - 1) v l}{\lambda^4} E, \tag{3.44}$$

$$a_6 = \frac{24}{\lambda^4} v^2 + \frac{12}{\lambda}\zeta v - \frac{8}{\lambda} v + k_0, \tag{3.45}$$

where

$$\begin{aligned} E = [&\lambda^3(15\zeta^2 - 3\zeta\eta - 13\zeta + 2\eta + 2) \\ &- 2\lambda^2\rho(3\zeta - 3\eta - 1) + 4\lambda k_0 + 24\zeta v - 8v]. \end{aligned} \tag{3.46}$$

As a result, nontrivial solutions to Eq. (3.43) must obey the following equation:

$$f(n) \equiv a_6 n^5 + \cdots + a_1 = 0. \tag{3.47}$$

It is known that this equation will have at least one positive root corresponding to an unstable mode of the anisotropic power-law solutions if $a_1 a_6 < 0$ as discussed in Refs. [16,17]. We will therefore discuss the sign of the coefficients a_1 and a_6 during the inflationary phase corresponding to two following constraints, $z \gg 1$ and $\eta \ll z$, to find out whether the cosmic no-hair conjecture is violated. As a result, we are able to show that

$$a_6 \sim \lambda^2 z(3z - 2) > 0, \tag{3.48}$$

according to Eqs. (3.36), (3.37), and the definition of a_6 given by Eq. (3.45). Next, our goal is the sign of a_1. It is clear that the sign of E will be that of a_1 since $\zeta \gg 1$, $v > 0$, and $l > 0$ as shown above. In particular, $E > 0$ will indicate $a_1 > 0$, and vice versa. During the inflationary phase, in which $\zeta \sim z \gg 1$ and $\zeta \gg \eta$, we can approximately revalue E as follows:

$$E \simeq 9\lambda^3 z^2 > 0. \tag{3.49}$$

It is now clear that $a_1 > 0$. The result, in which $a_6 > 0$ and $a_1 > 0$, means that the unstable mode may not be able to survive in this model. This is consistent with investigations done in the previous papers for both canonical and noncanonical (DBI and SDBI) scalar fields [17]. Hence, we need to provide more proof to conclude whether the anisotropic inflationary solutions of the Galileon-vector model are indeed stable against field perturbations. To confirm this claim, we are going to list the other coefficients a_i's ($i = 2$–5) of the polynomial $f(n)$ in Eq. (3.47) and show that they are all positive definite. If this is true, $f(n)$ will remain positive for all $n \geq 0$. To do this, we need first to simplify the components A_{ij}'s of matrix $\mathcal{D}$ by keeping only the leading terms. The results are

$$A_{11} \simeq \frac{12}{\lambda^2} v n^2 + \frac{6}{\lambda}\left(\frac{12}{\lambda}\zeta + k_0\right) n - 4\rho l;$$
$$A_{13} \simeq \left(\frac{12}{\lambda}\zeta v + k_0\right) n^2 + \left(\frac{36}{\lambda}\zeta^2 v + 3\zeta k_0\right) n + \frac{36}{\lambda}\zeta^2 v + 2\rho^2 l, \tag{3.50}$$

$$A_{21} \simeq -n^2 - \left(6\zeta - \frac{12}{\lambda^3} v\right) n - \frac{2l}{3};$$
$$A_{23} \simeq \frac{2}{\lambda^2} v n^2 + \frac{18}{\lambda^2}\zeta v n + \frac{12}{\lambda^2}\zeta v + \frac{\rho}{3} l, \tag{3.51}$$

$$A_{32} \simeq n^2 + 3\zeta n + \frac{4l}{3}. \tag{3.52}$$

Note that we have listed A_{ij}'s that contain only leading terms. Thanks to these simplifications along with the approximated values of ζ, k_0, and v as defined in Eqs. (3.21) and (3.37), the approximated values of the coefficients a_i's ($i = 2$–5) can be evaluated as follows:

$$a_5 \simeq 36\rho^2 z, \tag{3.53}$$

$$a_4 \simeq \rho^2(135z^2 + 18z + 6l), \tag{3.54}$$

$$a_3 \simeq \rho^2(162z^3 + 108z^2 + 36zl + 3\eta l), \tag{3.55}$$

$$a_2 \simeq \rho^2(162z^3 + 54z^2 l + 45z\eta l + 18zl). \tag{3.56}$$

It is apparent that all coefficient a_i's ($i = 2$–5) are positive if η, z, and l are all positive. This result proves our claim that the anisotropic power-law solutions of the Galileon-vector model with gauge field are indeed stable during the inflationary phase. Hence, the cosmic no-hair conjecture seems to be generally violated in the context of the KSW model for both canonical and noncanonical scalar fields ϕ due to the existence of the coupling term between scalar and gauge fields, $f^2(\phi)F^2$ [14–17].

IV. AUTONOMOUS EQUATIONS AND ATTRACTOR SOLUTION

The stability of the new set of anisotropic inflationary solutions has been shown with a power-law perturbation approach [16,17]. We can also show that these solutions are attractor solutions with the help of a set of autonomous equations [14,16,17]. Indeed, the field equations (3.8)–(3.11) can be written as

$$\frac{dX}{d\alpha} = -\frac{\ddot{\alpha}}{\dot{\alpha}^2} X - 3X + \frac{Z^2}{3}, \tag{4.1}$$

$$\frac{dY}{d\alpha} = -\frac{\ddot{\alpha}}{\dot{\alpha}^2} Y + \frac{\ddot{\phi}}{M_p \dot{\alpha}^2}, \tag{4.2}$$

$$\frac{dZ}{d\alpha} = -\frac{\ddot{\alpha}}{\dot{\alpha}^2} Z - 2(X+1)Z + \rho Y Z, \tag{4.3}$$

with

$$X = \frac{\dot{\sigma}}{\dot{\alpha}}; \quad Y = \frac{\dot{\phi}}{M_p \dot{\alpha}}; \quad Z = \frac{f_0^{-1} p_A}{M_p \dot{\alpha}} \exp\left[\frac{\rho\phi}{M_p} - 2\alpha - 2\sigma\right], \tag{4.4}$$

and the auxiliary variables

$$W_\tau = \exp\left[\frac{\tau\phi}{2M_p}\right]; \qquad W_\lambda = \sqrt{g_0 M_p} \dot{\alpha} \exp\left[\frac{\lambda\phi}{2M_p}\right] \tag{4.5}$$

obeying the equation

$$\frac{dW_\lambda}{d\alpha} = \frac{\ddot{\alpha}}{\dot{\alpha}^2} W_\lambda + \frac{\lambda Y}{2} W_\lambda. \tag{4.6}$$

Here we have parametrized the field equations by α instead of t with $d\alpha = \dot{\alpha} dt$. We will show in a moment that [14,16,17] the anisotropic fixed points of autonomous equations are identical to the new set of power-law solutions we obtained earlier. As a result, by showing that the fixed points are attractor solutions is equivalent to the proof that the corresponding power-law solutions are stable solutions. First of all, it is straightforward to show that the equation $dW_\tau/d\alpha = 0$ leads to a trivial solution $\tau = 0$ ($W_\tau = 1$) for $\dot{\phi} \neq 0$. This is consistent with the power-law solutions found in Sec. III. Note that the auxiliary variable W_λ^2 can be defined from the Hamiltonian equation (3.9),

$$W_\lambda^2 = \frac{6(X^2 - 1) + k_0 Y^2 + Z^2}{\lambda Y^4 - 6Y^3}. \tag{4.7}$$

The fixed points of the autonomous equations are defined as the solutions to the equations $dX/d\alpha = 0$, $dY/d\alpha = 0$, $dZ/d\alpha = 0$, and $dW_\lambda/d\alpha = 0$. As a result, we can obtain

$$Z^2 = -3X(2X - \rho Y - 1) \tag{4.8}$$

from the equations $dX/d\alpha = 0$ and $dZ/d\alpha = 0$. In addition, we can show that

$$Y = \frac{4(X+1)}{\lambda+2\rho}, \tag{4.9}$$

$$\frac{\ddot{\alpha}}{\dot{\alpha}^2} = -\frac{\lambda Y}{2} \tag{4.10}$$

from the equations $dZ/d\alpha = 0$ and $dW_\lambda/d\alpha = 0$. Note that we will ignore the trivial solutions $Z = 0$ and $W_\lambda = 0$ to the equations $dZ/d\alpha = 0$ and $dW_\lambda/d\alpha = 0$.

In order to write the autonomous equations closely as equations of dynamical variables X, Y, Z, and W_λ, we need to solve Eqs. (3.8) and (3.10) and rewrite all irrelevant variables as autonomous variables. Indeed, we can write Eq. (3.8) as

$$k_0\left(\frac{\ddot{\phi}}{M_p\dot{\alpha}^2}+3Y\right)+\left[\frac{3\ddot{\alpha}}{\dot{\alpha}^2}+\frac{2\ddot{\phi}}{M_p\dot{\alpha}^2}\left(\frac{3}{Y}-\lambda\right)-\frac{\lambda^2Y^2}{2}+9\right]\times Y^2W_\lambda^2+\rho Z^2 = 0. \tag{4.11}$$

In addition, we can also write Eq. (3.10) as

$$-\frac{\ddot{\alpha}}{\dot{\alpha}^2}+\frac{1}{2}\left(\frac{\ddot{\phi}}{M_p\dot{\alpha}^2}+3Y\right)Y^2W_\lambda^2+\frac{Z^2}{6}-3 = 0. \tag{4.12}$$

As a result, Eqs. (4.11) and (4.12) can be solved to give

$$\frac{\ddot{\phi}}{M_p\dot{\alpha}^2} = -\frac{9Y^5W_\lambda^4-Y^2W_\lambda^2(\lambda^2Y^2-Z^2)+6k_0Y+2\rho Z^2}{YW_\lambda^2(3Y^3W_\lambda^2-4\lambda Y+12)+2k_0}, \tag{4.13}$$

$$\frac{\ddot{\alpha}}{\dot{\alpha}^2} = -\frac{1}{6}\left\{\frac{3Y^2W_\lambda^2[9Y^5W_\lambda^4-Y^2W_\lambda^2(\lambda^2Y^2-Z^2)+6k_0Y+2\rho Z^2]}{YW_\lambda^2(3Y^3W_\lambda^2-4\lambda Y+12)+2k_0}-9Y^3W_\lambda^2-Z^2+18\right\}. \tag{4.14}$$

As a result, we can write the autonomous equations as

$$\frac{dX}{d\alpha} = \frac{X}{6}\left\{\frac{3Y^2W_\lambda^2[9Y^5W_\lambda^4-Y^2W_\lambda^2(\lambda^2Y^2-Z^2)+6k_0Y+2\rho Z^2]}{YW_\lambda^2(3Y^3W_\lambda^2-4\lambda Y+12)+2k_0}-9Y^3W_\lambda^2-Z^2+18\right\}-3X+\frac{Z^2}{3}, \tag{4.15}$$

$$\frac{dY}{d\alpha} = \frac{Y}{6}\left\{\frac{3Y^2W_\lambda^2[9Y^5W_\lambda^4-Y^2W_\lambda^2(\lambda^2Y^2-Z^2)+6k_0Y+2\rho Z^2]}{YW_\lambda^2(3Y^3W_\lambda^2-4\lambda Y+12)+2k_0}-9Y^3W_\lambda^2-Z^2+18\right\} - \frac{9Y^5W_\lambda^4-Y^2W_\lambda^2(\lambda^2Y^2-Z^2)+6k_0Y+2\rho Z^2}{YW_\lambda^2(3Y^3W_\lambda^2-4\lambda Y+12)+2k_0}, \tag{4.16}$$

$$\frac{dZ}{d\alpha} = \frac{Z}{6}\left\{\frac{3Y^2W_\lambda^2[9Y^5W_\lambda^4-Y^2W_\lambda^2(\lambda^2Y^2-Z^2)+6k_0Y+2\rho Z^2]}{YW_\lambda^2(3Y^3W_\lambda^2-4\lambda Y+12)+2k_0}-9Y^3W_\lambda^2-Z^2+18\right\}-2(X+1)Z+\rho YZ. \tag{4.17}$$

The fixed points can be found by observing that Eq. (4.12) implies that

$$\frac{\ddot{\phi}}{M_p\dot{\alpha}^2} = \frac{-9W_\lambda^2Y^3-3\lambda Y-Z^2+18}{3W_\lambda^2Y^2}, \tag{4.18}$$

with the help of Eq. (4.10). As a result, the equation $dY/d\alpha = 0$ reduces to

$$(X+1)(2X-6z-1)(2S_1X^2+S_2X+2S_3) = 0 \tag{4.19}$$

with

$$S_1 = 2\left(6z^2+5z+1+4\frac{k_0}{\lambda^2}\right), \tag{4.20}$$

$$S_2 = 12z^2-8z-7+32\frac{k_0}{\lambda^2}, \tag{4.21}$$

$$S_3 = 12z^2+4z+1+8\frac{k_0}{\lambda^2}, \tag{4.22}$$

and $z \equiv \rho/\lambda$ with the help of Eqs. (4.7), (4.8), (4.9), (4.10), and (4.18). There is a set of trivial solutions $X = -1$ and $X = 1/2+3z$ (corresponding to $\zeta = 1/3$). In addition, the nontrivial solutions of Eq. (4.19) read

$$X_\pm = \frac{-12z^2+8z+7-32\frac{k_0}{\lambda^2}\pm 3(2z+1)\sqrt{\Delta}}{8(6z^2+5z+1+4\frac{k_0}{\lambda^2})}, \tag{4.23}$$

with Δ defined in Eq. (3.25). We can show that these solutions are identical to the anisotropic power-law solutions, i.e., Eqs. (3.24) and (3.21), we obtained in Sec. III. Indeed, it is straightforward to show that

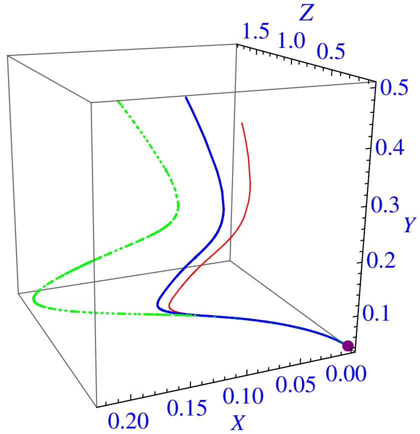

FIG. 1. The anisotropic fixed point with the field parameters chosen as $\lambda = 0.1$, $\rho = 50$, and $k_0 = -3\rho^2/2$ is shown as attractor solutions in this figure. All trajectories in the phase space of X, Y, and Z, with different initial conditions converge to the anisotropic fixed point (plotted as the purple point). The initial conditions $[X(t=0), Y(t=0), Z(t=0)]$ are set as (0.1, 0.45, 0.2) for the thin solid red curve, (0.15, 0.5, 0.15) for the thick solid blue curve, and (0.2, 0.5, 0.2) for the green dotted curve, respectively.

$$X_\pm = \frac{2z+1}{2\zeta_\mp} - 1 \tag{4.24}$$

with the help of Eq. (3.21). Note also that only the solution X_- corresponds to inflationary solutions ζ_+. In addition, the dynamical variables Y and Z can also be determined in terms of X_- as shown in Eqs. (4.8) and (4.9).

Note that Y is always positive similar to X and Z. Besides, $X = \eta/\zeta \ll 1$, $Y = \xi/\zeta \sim 2/\rho \ll 1$, and $Z^2 = l/\zeta^2 \sim 9\eta/\zeta \ll 1$ during the inflationary phase with $\rho \gg \lambda$, $\zeta \simeq z$, and $k_0 \simeq -3\lambda^2 z^2/2$. As a result, we can show the attractor behavior of the anisotropic fixed point (X_-, Y, Z) by plotting the numerical solutions of the autonomous equations with various initial conditions. This result shown in Fig. 1 indicates that the anisotropic (power-law) inflationary solution of the Galileon-vector model is indeed a stable attractor solution [14,16,17].

V. GALILEON-VECTOR-PHANTOM MODEL WITH A GAUGE FIELD AND PHANTOM FIELD

A. The model and its anisotropic power-law solutions

Besides the dark energy problem, the role of the phantom field has long been a focus of string theory and the evolution of the early universe [38]. It is also known to be critical in the study of black hole solutions [39]. The presence of the phantom field may play different roles in various areas of interest. In particular, the presence of a phantom field is known to turn the stable power-law solutions into unstable in a number of models [16,17]. Hence we would like to study the effect of the phantom field in this model. Following Refs. [16,17], we will include the kinetic and potential terms of an extra scalar field, $\psi \equiv \psi(t)$, in the action

$$S = \int d^4x\sqrt{g}\left\{\frac{M_p^2}{2}R + k_0 \exp\left[\frac{\tau\phi}{M_p}\right]X - g_0 \exp\left[\frac{\lambda\phi}{M_p}\right]X\Box\phi - \frac{\omega}{2}\partial_\mu\psi\partial^\mu\psi - V(\psi) - \frac{1}{4}f^2(\phi,\psi)F_{\mu\nu}F^{\mu\nu}\right\}, \tag{5.1}$$

with ω a positive or negative constant for our choice. Note that $\omega < 0$ and $\omega > 0$ represent canonical and phantom scalar fields, respectively. Note also that the scalar field coupling $f(\phi)$ has been changed to $f(\phi,\psi)$ in the action (5.1). In particular, the introduction of the phantom field is expected to turn the anisotropic inflationary solutions unstable similar to the results shown in [16,17]. We will show in a moment that this is also true for this Galileon-vector model.

As a result, the following field equations of the action (5.1) can be shown to be

$$\partial_\mu\{\sqrt{g}[f^2(\phi,\psi)F^{\mu\nu}]\} = 0, \tag{5.2}$$

$$\mathcal{E}^{(2)} + \mathcal{E}^{(3)} - \frac{f_\phi(\phi,\psi)f(\phi,\psi)}{2}F_{\mu\nu}F^{\mu\nu} = 0, \tag{5.3}$$

$$\Box\psi - \frac{V_\psi(\psi)}{\omega} - \frac{f_\psi(\phi,\psi)f(\phi,\psi)}{2\omega}F_{\mu\nu}F^{\mu\nu} = 0, \tag{5.4}$$

$$M_p^2\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) - \mathcal{T}^{(2)}_{\mu\nu} - \mathcal{T}^{(3)}_{\mu\nu} - \omega\partial_\mu\psi\partial_\nu\psi + g_{\mu\nu}\left[\frac{\omega}{2}\partial^\sigma\psi\partial_\sigma\psi + V(\psi) + \frac{f^2(\phi,\psi)}{4}F^{\lambda\sigma}F_{\lambda\sigma}\right] - f^2(\phi,\psi)F_{\mu\gamma}F^\gamma_\nu = 0. \tag{5.5}$$

Here $\mathcal{T}^{(2,3)}_{\mu\nu}$ and $\mathcal{E}^{(2,3)}$ are defined in Eqs. (2.4) and (2.5) and Eqs. (2.7) and (2.8), respectively. Furthermore, Eqs. (5.3) and (5.4) can be reduced to

$$\mathcal{E}^{(2)} + \mathcal{E}^{(3)} + f^{-3}f_\phi \exp[-4\alpha - 4\sigma]p_A^2 = 0, \tag{5.6}$$

$$-\ddot{\psi} - 3\dot{\alpha}\dot{\psi} - \frac{V_\psi}{\omega} + \frac{f^{-3}f_\psi}{\omega}\exp[-4\alpha - 4\sigma]p_A^2 = 0, \tag{5.7}$$

with the help of the field equation (5.2) for the gauge field. In addition, the Einstein field equations (5.5) can be shown to be:

$$-3M_p^2(\dot{\alpha}^2 - \dot{\sigma}^2) + \frac{k_0}{2}\exp\left[\frac{\tau\phi}{M_p}\right]\dot{\phi}^2 + 3g_0\exp\left[\frac{\lambda\phi}{M_p}\right]\dot{\alpha}\dot{\phi}^3 - \frac{g_0\lambda}{2M_p}\exp\left[\frac{\lambda\phi}{M_p}\right]\dot{\phi}^4 + \frac{\omega\dot{\psi}^2}{2} + V + \frac{f^{-2}}{2}\exp[-4\alpha - 4\sigma]p_A^2 = 0, \tag{5.8}$$

$$-M_p^2\ddot{\alpha}-3M_p^2\dot{\alpha}^2+\frac{g_0}{2}\exp\left[\frac{\lambda\phi}{M_p}\right](\ddot{\phi}+3\dot{\alpha}\dot{\phi})\dot{\phi}^2+V+\frac{f^{-2}}{6}\exp[-4\alpha-4\sigma]p_A^2=0, \tag{5.9}$$

$$\ddot{\sigma}+3\dot{\alpha}\dot{\sigma}-\frac{f^{-2}}{3M_p^2}\exp[-4\alpha-4\sigma]p_A^2=0. \tag{5.10}$$

Similar to the approach shown earlier in Sec. III, we will assume that the power-law solutions take the following forms [14,16,17]:

$$\alpha=\zeta\log t;\qquad \sigma=\eta\log t;\qquad \frac{\phi}{M_p}=\xi_1\log t+\phi_0;\qquad \frac{\psi}{M_p}=\xi_2\log t+\psi_0. \tag{5.11}$$

We will also propose the following exponential potential $V(\psi)$ and gauge kinetic coupling function $f(\phi,\psi)$:

$$V(\psi)=V_0\exp\left[\frac{\hat{\lambda}\psi}{M_p}\right], \tag{5.12}$$

$$f(\phi,\psi)=f_0\exp\left[-\frac{\rho\phi}{M_p}\right]\exp\left[\frac{\hat{\rho}\psi}{M_p}\right], \tag{5.13}$$

which are compatible with the power-law solutions. By introducing the variables

$$\hat{w}=V_0\exp[\hat{\lambda}\psi_0], \tag{5.14}$$

$$\hat{l}=\frac{f_0^{-2}p_A^2}{M_p^2}\exp[2\rho\phi_0-2\hat{\rho}\psi_0], \tag{5.15}$$

with $\hat{\lambda}$ and $\hat{\rho}$ some positive constants similar to λ and ρ, the field equations (5.6)–(5.10) reduce to the following set of algebraic equations:

$$-k_0\xi_1(3\zeta-1)-9\xi_1^2\zeta(\zeta-1)v-2\lambda\xi_1^3v+\frac{\lambda^2}{2}\xi_1^4v-\rho\hat{l}=0, \tag{5.16}$$

$$-\xi_2(3\zeta-1)-\frac{\hat{\lambda}}{\omega}\hat{w}+\frac{\hat{\rho}}{\omega}\hat{l}=0, \tag{5.17}$$

$$-3\zeta^2+3\eta^2+k_0\frac{\xi_1^2}{2}+3\xi_1^3\zeta v-\frac{\lambda}{2}\xi_1^4v+\frac{\omega\xi_2^2}{2}+\hat{w}+\frac{\hat{l}}{2}=0, \tag{5.18}$$

$$\zeta-3\zeta^2+\frac{1}{2}\xi_1^3(3\zeta-1)v+\hat{w}+\frac{\hat{l}}{6}=0, \tag{5.19}$$

$$(3\zeta-1)\eta-\frac{\hat{l}}{3}=0. \tag{5.20}$$

Note that the following constraints are required such that each term in the field equations (5.6)–(5.10) evolves as t^{-2}:

$$\tau=0, \tag{5.21}$$

$$\xi_1=\frac{2}{\lambda}, \tag{5.22}$$

$$\xi_2=\frac{-2}{\hat{\lambda}}, \tag{5.23}$$

$$-\rho\xi_1+\hat{\rho}\xi_2+2\zeta+2\eta=1. \tag{5.24}$$

As a result, Eqs. (5.20) and (5.24) imply that

$$\hat{l}=3\eta(3\zeta-1), \tag{5.25}$$

$$\zeta+\eta=z+\hat{z}+\frac{1}{2}. \tag{5.26}$$

Here we have defined new variables $z\equiv\rho/\lambda$ and $\hat{z}\equiv\hat{\rho}/\hat{\lambda}$ for convenience. In addition, k_0, v, and $\hat{\omega}$ can be shown to be

$$\begin{aligned}k_0&=-\frac{\lambda^2}{8}\left\{18(2\hat{z}+1)\zeta^2-3\left[(2\hat{z}+1)(6z+6\hat{z}+5)+\frac{8\omega}{\hat{\lambda}^2}\right]\zeta+2(3z+6\hat{z}+1)(2z+2\hat{z}+1)+\frac{16\omega}{\hat{\lambda}^2}\right\}\\&=-\frac{\lambda^2}{8}\left[(6z+6\hat{z}-1)\left(2z+2\hat{z}+1-\frac{4\omega}{\hat{\lambda}^2}\right)-18\eta(2\hat{z}+1)(z+\hat{z})+3\eta(2\hat{z}+1)(6\eta-1)+\frac{24\omega}{\hat{\lambda}^2}\eta\right],\end{aligned} \tag{5.27}$$

$$\begin{aligned}v&=\frac{\lambda^3}{16}\left[6(2\hat{z}+1)\zeta-(6\hat{z}+1)(2z+2\hat{z}+1)-\frac{8\omega}{\hat{\lambda}^2}\right]\\&=\frac{\lambda^3}{16}\left[2(2z+2\hat{z}+1)-6\eta(2\hat{z}+1)-\frac{8\omega}{\hat{\lambda}^2}\right],\end{aligned} \tag{5.28}$$

$$\begin{aligned}\hat{w}&=-\frac{1}{2}(3\zeta-1)\left[6\hat{z}\zeta-3\hat{z}(2z+2\hat{z}+1)-\frac{4\omega}{\hat{\lambda}^2}\right]\\&=\left(3z+3\hat{z}+\frac{1}{2}-3\eta\right)\left(3\hat{z}\eta+\frac{2\omega}{\hat{\lambda}^2}\right).\end{aligned} \tag{5.29}$$

Note that Eq. (5.16) is automatically satisfied due to the Bianchi identity. This equation also provides an independent check for the correctness of this set of power-law solutions. Similar to Sec. III, Eq. (5.27) can be solved by treating k_0 as a field parameter. The result is

$$\zeta_\pm=\frac{5}{12}+\frac{z+\hat{z}}{2}+\frac{2\omega}{3\hat{\lambda}^2(2\hat{z}+1)}\pm\frac{\sqrt{\hat{\Delta}}}{36\lambda^2\hat{\lambda}^2(2\hat{z}+1)}, \tag{5.30}$$

with $\hat{\Delta}$ defined by

$$\hat{\Delta} = 9\lambda^4[\hat{\lambda}^2(2\hat{z}+1)(6z+6\hat{z}+5)+8\omega]^2 - 144\lambda^2\hat{\lambda}^2(2\hat{z}+1)\{\lambda^2[\hat{\lambda}^2(3z+6\hat{z}+1)(2z+2\hat{z}+1) + 8\omega] + 4\hat{\lambda}^2 k_0\}. \quad (5.31)$$

Note that the parameter $\hat{\Delta}$ has to be positive for the existence of a real solution. Indeed, we can write $\hat{\Delta}$ as

$$\hat{\Delta} = 9\lambda^4\hat{\lambda}^4\left[(2\hat{z}+1)(6z+6\hat{z}-12\eta+1)-\frac{8\omega}{\hat{\lambda}^2}\right]^2. \quad (5.32)$$

Hence $\hat{\Delta}$ is indeed a positive parameter.

As a result, the positivity of $\hat{l}$ implies that $\eta > 0$ and $\zeta > 1/3$. In addition, the constraint $v > 0$ implies that

$$3\eta < \frac{1}{2\hat{z}+1}\left(2z+2\hat{z}+1-\frac{8\omega}{\hat{\lambda}^2}\right) \quad (5.33)$$

following the expression of v given in Eq. (5.28). On the other hand, the positivity of $\hat{w}$ leads to an inequality for η,

$$3\eta > -\frac{2\omega}{\hat{\lambda}^2\hat{z}}, \quad (5.34)$$

according to Eq. (5.29). In addition, we also need the following constraints for the scale factors to represent the expanding solution: $\zeta+\eta > 0$ and $\zeta - 2\eta > 0$. As a result, the constraint $\zeta+\eta>0$ can easily be achieved for all $z+\hat{z}>0$. On the other hand, the constraint $\zeta-2\eta>0$ implies that

$$3\eta < z+\hat{z}-\frac{1}{2}. \quad (5.35)$$

Note that we also have a number of constraints for η in this model due to the existence of the phantom field. We will discuss in a moment the case with η assuming the lowest and highest bounds in accord with a set of inflationary solutions.

B. Inflationary solutions

We will focus on the inflationary limit of the power-law solutions corresponding to the constraints for the scale factors $\zeta+\eta \gg 1$ and $\zeta-2\eta\gg 1$. As a result, the first constraint is equivalent to

$$z+\hat{z}\gg 1 \quad (5.36)$$

according to Eq. (5.26). Note that $\rho \gg \lambda \sim \mathcal{O}(1)$ and $\hat{\rho} \gg \hat{\lambda}\sim\mathcal{O}(1)$ will be enough to satisfy the constraint required. On the other hand, the second constraint $\zeta - 2\eta \gg 1$ is equivalent to

$$3\eta \ll z+\hat{z}-\frac{1}{2}. \quad (5.37)$$

Apparently, $z+\hat{z}\gg 1$ is also good enough for this constraint. In addition, Eq. (5.34) implies that

$$\eta > -\frac{2\omega}{3\hat{\lambda}^2\hat{z}} \quad (5.38)$$

for a phantom field ψ with $\omega = -1 < 0$. As a result, η obeys the following constraint:

$$-\frac{2\omega}{3\hat{\lambda}^2\hat{z}} < \eta \ll \frac{1}{3}\left(z+\hat{z}-\frac{1}{2}\right). \quad (5.39)$$

On the other hand, $\eta > 0$ will be enough for the constraint (5.34) if $\omega = 1$. Therefore the constraint for η will be

$$0<\eta\ll\frac{1}{3}\left(z+\hat{z}-\frac{1}{2}\right) \quad (5.40)$$

for a canonical field ψ with $\omega = 1$. Note also that Eq. (5.32) implies that

$$\hat{\Delta}\simeq 9\lambda^4\hat{\lambda}^4\left[(2\hat{z}+1)(6z+6\hat{z})-\frac{8\omega}{\hat{\lambda}^2}\right]^2, \quad (5.41)$$

if

$$\zeta \simeq z+\hat{z}\gg 1 \quad (5.42)$$

during the inflationary phase. Equivalently, we should have the following approximation:

$$\eta\simeq\frac{1}{12};\qquad k_0\simeq -\frac{3\lambda^2}{8}(z+\hat{z})(4z+3\hat{z});$$
$$v\simeq\frac{\lambda^3}{16}(4z+3\hat{z});\qquad w\simeq\frac{3}{4}\hat{z}\zeta\simeq\frac{3}{4}\hat{z}(z+\hat{z}) \quad (5.43)$$

during the inflationary phase. It is clear that this set of power-law solutions can be made consistent with the observational data of WMAP and Planck.

C. Stability analysis of anisotropic inflationary power-law solutions

Similar to Sec. III B [16,17], we will consider the power-law perturbations of the following forms: $\delta\alpha = At^n$, $\delta\sigma = Bt^n$, $\delta\phi = M_p C t^n$, and $\delta\psi = M_p D t^n$ with constants A, B, C, and D. The perturbations will blow up at time infinity if $n>0$, meaning that the set of power-law solutions is unstable. On the other hand, the corresponding solution is stable if all n is negative definite. Indeed, perturbing Eqs. (5.6), (5.7), (5.9), and (5.10) leads to the following matrix equation:

$$\hat{\mathcal{D}}\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \equiv \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} & \hat{A}_{13} & \hat{A}_{14} \\ \hat{A}_{21} & \hat{A}_{22} & \hat{A}_{23} & \hat{A}_{24} \\ \hat{A}_{31} & \hat{A}_{32} & \hat{A}_{33} & \hat{A}_{34} \\ \hat{A}_{41} & \hat{A}_{42} & \hat{A}_{43} & \hat{A}_{44} \end{bmatrix}\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = 0, \quad (5.44)$$

with

$$\hat{A}_{11} = A_{11}(l \to \hat{l}); \qquad \hat{A}_{12} = A_{12}(l \to \hat{l});$$
$$\hat{A}_{13} = A_{13}(l \to \hat{l}); \qquad \hat{A}_{14} = -2\rho\hat{\rho}\,\hat{l}, \quad (5.45)$$

$$\hat{A}_{21} = -\frac{6n}{\hat{\lambda}} + \frac{4}{\omega}\hat{\rho}\,\hat{l}; \qquad \hat{A}_{22} = \frac{4}{\omega}\hat{\rho}\,\hat{l}; \qquad \hat{A}_{23} = -\frac{2}{\omega}\rho\hat{\rho}\,\hat{l};$$
$$\hat{A}_{24} = n^2 + (3\zeta - 1)n + \frac{\hat{\lambda}^2}{\omega}\hat{w} + \frac{2}{\omega}\hat{\rho}^2\hat{l}, \quad (5.46)$$

$$\hat{A}_{31} = A_{21}(l \to \hat{l}); \qquad \hat{A}_{32} = A_{22}(l \to \hat{l});$$
$$\hat{A}_{33} = A_{23}(l \to \hat{l}); \qquad \hat{A}_{34} = \hat{\lambda}\,\hat{w} - \frac{\hat{\rho}}{3}\hat{l}, \quad (5.47)$$

$$\hat{A}_{41} = A_{31}(l \to \hat{l}); \qquad \hat{A}_{42} = A_{32}(l \to \hat{l});$$
$$\hat{A}_{43} = A_{33}(l \to \hat{l}); \qquad \hat{A}_{44} = \frac{2\hat{\rho}}{3}\hat{l}. \quad (5.48)$$

Note that the expressions of $A_{ij}(l)$ for i, $j = 1, 2, 3$ have already been defined in Eqs. (3.39)–(3.41). In addition, the notation $A_{ij}(l \to \hat{l})$ means that each l in A_{ij} is replaced by $\hat{l}$.

As a result, nontrivial solutions to Eq. (5.44) exist only when

$$\det \hat{\mathcal{D}} = 0. \quad (5.49)$$

This determinant equation can be reduced to a degree 8 polynomial equation of n as follows:

$$n(b_8 n^7 + \cdots + b_1) = 0, \quad (5.50)$$

with

$$b_8 = a_6, \quad (5.51)$$

$$b_1 \simeq \frac{24\hat{\lambda}^2\hat{l}v\hat{w}\zeta^2}{\omega\lambda}\left[18\hat{z}\zeta + 15\zeta - 6z - \frac{12\omega}{\hat{\lambda}^2}\right], \quad (5.52)$$

by keeping only the leading terms in the definition of b_1 for simplicity. Note that $a_6 > 0$ is defined earlier in Sec. III. Hence b_8 is also a positive coefficient. On the other hand, the sign of b_1 depends on the signature of ω. Indeed, $b_1 > 0$ if $\omega > 0$. Similarly, $b_1 < 0$ if $\omega < 0$. Note that there exists at least a positive root to the determinant equation $\det \hat{\mathcal{D}} = 0$ if $b_1 b_8 < 0$ [16,17]. It is apparent that $b_1 < 0$ for $\omega = -1$ if the parameters obey the approximations given by Eqs. (5.42) and (5.43) during the inflationary phase. Therefore, this set of power-law solutions is indeed unstable during the inflationary phase if ψ is a phantom field with $\omega = -1$ [16,17,26]. Therefore, the presence of a phantom field will destabilize the power-law solutions during the inflationary phase as expected.

VI. STABILITY ANALYSIS FOR A MORE GENERAL GALILEON-VECTOR MODEL

The action of a general Galileon-vector-phantom model is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + K(\phi, X) - G(\phi, X)\Box\phi + \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - V(\psi) - \frac{1}{4}f_1^2(\phi)f_2^2(\psi)F_{\mu\nu}F^{\mu\nu}\right]. \quad (6.1)$$

Note that the Planck unit has been set with $M_p = 1$ for convenience. As a result, the corresponding field equations can be shown to be [34]

$$\partial_\mu\{\sqrt{-g}[f_1^2(\phi)f_2^2(\psi)F^{\mu\nu}]\} = 0, \quad (6.2)$$

$$\mathcal{E}^{(2)} + \mathcal{E}^{(3)} - \frac{1}{2}f_1(\phi)f_2^2(\psi)f_{1\phi}(\phi)F_{\mu\nu}F^{\mu\nu} = 0, \quad (6.3)$$

$$\Box\psi + V_\psi(\psi) + \frac{1}{2}f_1^2(\phi)f_2(\psi)f_{2\psi}(\psi)F_{\mu\nu}F^{\mu\nu} = 0, \quad (6.4)$$

$$\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + \partial_\mu\psi\partial_\nu\psi - \mathcal{T}^{(2)}_{\mu\nu} - \mathcal{T}^{(3)}_{\mu\nu} + g_{\mu\nu}\left[-\frac{1}{2}\partial^\sigma\psi\partial_\sigma\psi + V(\psi) + \frac{f_1^2(\phi)f_2^2(\psi)}{4}F^{\lambda\sigma}F_{\lambda\sigma}\right] - f_1^2(\phi)f_2^2(\psi)F_{\mu\gamma}F_\nu^\gamma = 0, \quad (6.5)$$

with $\mathcal{E}^{(i)}$ and $\mathcal{T}^{(i)}_{\mu\nu}$ given by [34]

$$\mathcal{E}^{(2)} = \Box\phi K_X + K_{XX}D_\mu X D^\mu\phi - 2XK_{X\phi} + K_\phi, \quad (6.6)$$

$$\mathcal{E}^{(3)} = -2\Box\phi G_\phi + 2XG_{\phi\phi} - G_X[(\Box\phi)^2 - (D_\mu D_\nu\phi)^2 - R_{\mu\nu}D^\mu\phi D^\nu\phi] + 2G_{X\phi}(X\Box\phi - D_\mu X D^\mu\phi) - G_{XX}(D_\mu X D^\mu X + \Box\phi D_\mu X D^\mu\phi), \quad (6.7)$$

$$\mathcal{T}^{(2)}_{\mu\nu} = Kg_{\mu\nu} + K_X D_\mu\phi D_\nu\phi, \quad (6.8)$$

$$\mathcal{T}^{(3)}_{\mu\nu} = (G_X D_\sigma X D^\sigma\phi - 2XG_\phi)g_{\mu\nu} - (G_X\Box\phi + 2G_\phi)D_\mu\phi D_\nu\phi - G_X(D_\mu\phi D_\nu X + D_\nu\phi D_\mu X). \quad (6.9)$$

We wish to know the role of the phantom field in the stability of the inflationary universe for a more general

Galileon-vector model. We have shown in Sec. III that power-law solutions put a set of strong constraints on the interacting terms. We also show in Sec. V that the presence of the phantom field introduces at least an unstable mode to the stable solutions obtained in Sec. III. We expect the phantom field will also introduce at least an unstable mode for a more general Galileon-vector mode. Hence we will assume that a set of stable solutions exists for the Galileon-vector model when $\psi = 0$ in the action (6.1) in the first place. Then we will try to show that the presence of the phantom field does introduce an unstable mode for the perturbation equations. To be more specific, we will focus on a more general Galileon-vector-phantom model with the interaction terms specified as

$$K(\phi, X) = K(X); \qquad G(\phi, X) = g(\phi)X, \tag{6.10}$$

with arbitrary $K(X)$ and $g(\phi)$ [34]. Note also that we will adopt the same BI metric and vector field presented in Sec. III. As a result, Eqs. (6.3) and (6.4) reduce, respectively, to

$$\begin{aligned} &-(\ddot{\phi} + 3\dot{\alpha}\dot{\phi})K_X - \ddot{\phi}\dot{\phi}^2 K_{XX} - 3\dot{\phi}[\ddot{\alpha}\dot{\phi} + 3\dot{\alpha}^2\dot{\phi} + 2\dot{\alpha}\ddot{\phi}]g \\ &\quad + 2\ddot{\phi}\dot{\phi}^2 g_\phi + \frac{\dot{\phi}^4}{2} g_{\phi\phi} + f_1^{-3} f_2^{-2} f_{1\phi} \exp[-4\alpha - 4\sigma] p_A^2 = 0, \end{aligned} \tag{6.11}$$

$$-\ddot{\psi} - 3\dot{\alpha}\dot{\psi} + V_\psi - f_1^{-2} f_2^{-3} f_{2\psi} \exp[-4\alpha - 4\sigma] p_A^2 = 0. \tag{6.12}$$

Furthermore, the Einstein equation (6.5) can be shown to be

$$\begin{aligned} 3(\dot{\alpha}^2 - \dot{\sigma}^2) &= -K + \dot{\phi}^2 K_X + 3\dot{\alpha}\dot{\phi}^3 g - \frac{\dot{\phi}^4}{2} g_\phi - \frac{\dot{\psi}^2}{2} + V \\ &\quad + \frac{f_1^{-2} f_2^{-2}}{2} \exp[-4\alpha - 4\sigma] p_A^2, \end{aligned} \tag{6.13}$$

$$\begin{aligned} \ddot{\alpha} + 3\dot{\alpha}^2 &= -K + \frac{\dot{\phi}^2}{2} K_X + \frac{\dot{\phi}^2}{2} (\ddot{\phi} + 3\dot{\alpha}\dot{\phi}) g + V \\ &\quad + \frac{f_1^{-2} f_2^{-2}}{6} \exp[-4\alpha - 4\sigma] p_A^2, \end{aligned} \tag{6.14}$$

$$\ddot{\sigma} + 3\dot{\alpha}\dot{\sigma} = \frac{f_1^{-2} f_2^{-2}}{3} \exp[-4\alpha - 4\sigma] p_A^2. \tag{6.15}$$

A. Perturbation

Assuming that there exists a set of analytic solutions to field equations (6.11), (6.12), (6.14), and (6.15), we need to perturb the field equations in order to find out whether this set of solutions is stable. Note that there are a total of five equations [(6.11)–(6.15)] available for our analysis. We need, however, only four independent equations for four variables. It is known that there is a redundant equation that can be related to the other four equations due to the Bianchi identity $D_a(G^{ab} - T^{ab}) = 0$. As a result, we can focus on the field equations (6.11), (6.12), (6.14), and (6.15). Therefore, we can perturb these equations with the exponential perturbations of the following field variables:

$$\begin{aligned} \delta\alpha &= A \exp[\omega t], & \delta\sigma &= B \exp[\omega t], \\ \delta\phi &= C \exp[\omega t], & \delta\psi &= D \exp[\omega t]. \end{aligned} \tag{6.16}$$

The result can be written as a matrix equation,

$$\mathcal{D}_g \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \equiv \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = 0, \tag{6.17}$$

with

$$\begin{aligned} B_{11} &= -3\dot{\phi}^2 g\omega^2 - 3\dot{\phi}[K_X + 2(\ddot{\phi} + 3\dot{\alpha}\dot{\phi})g]\omega - 4\frac{f_{1\phi}}{f_1}\Sigma; \\ B_{12} &= -4\frac{f_{1\phi}}{f_1}\Sigma; \\ B_{13} &= -(K_X + \dot{\phi}^2 K_{XX} + 6\dot{\alpha}\dot{\phi}\, g - 2\dot{\phi}^2 g_\phi)\omega^2 \\ &\quad - \{3\dot{\alpha}K_X + 3\dot{\phi}(\ddot{\phi} + \dot{\alpha}\dot{\phi})K_{XX} + \ddot{\phi}\dot{\phi}^3 K_{XXX} \\ &\quad + 6[\ddot{\alpha}\dot{\phi} + \dot{\alpha}(\ddot{\phi} + 3\dot{\alpha}\dot{\phi})]g - 4\ddot{\phi}\dot{\phi}\, g_\phi - 2\dot{\phi}^3 g_{\phi\phi}\}\omega \\ &\quad - 3\dot{\phi}[\ddot{\alpha}\dot{\phi} + \dot{\alpha}(2\ddot{\phi} + 3\dot{\alpha}\dot{\phi})]g_\phi + 2\ddot{\phi}\dot{\phi}^2 g_{\phi\phi} + \frac{\dot{\phi}^4}{2} g_{\phi\phi\phi} \\ &\quad - \left[3\left(\frac{f_{1\phi}}{f_1}\right)^2 - \frac{f_{1\phi\phi}}{f_1}\right]\Sigma; \\ B_{14} &= -2\frac{f_{1\phi}}{f_1}\frac{f_{2\psi}}{f_2}\Sigma, \end{aligned} \tag{6.18}$$

$$\begin{aligned} B_{21} &= -3\dot{\psi}\omega + 4\frac{f_{2\psi}}{f_2}\Sigma; \quad B_{22} = 4\frac{f_{2\psi}}{f_2}\Sigma; \\ B_{23} &= 2\frac{f_{1\phi}}{f_1}\frac{f_{2\psi}}{f_2}\Sigma; \\ B_{24} &= -\omega^2 - 3\dot{\alpha}\omega + V_{\psi\psi} + \left[3\left(\frac{f_{2\psi}}{f_2}\right)^2 - \frac{f_{2\psi\psi}}{f_2}\right]\Sigma, \end{aligned} \tag{6.19}$$

$$\begin{aligned} B_{31} &= -\omega^2 - 3\left(2\dot{\alpha} - \frac{\dot{\phi}^3}{2}\right)\omega - \frac{2}{3}\Sigma; \qquad B_{32} = -\frac{2}{3}\Sigma; \\ B_{33} &= \frac{\dot{\phi}^2}{2} g\omega^2 + \left[\frac{\dot{\phi}^3}{2} K_{XX} + \dot{\phi}\left(\ddot{\phi} + \frac{9}{2}\dot{\alpha}\dot{\phi}\right)g\right]\omega \\ &\quad + \frac{\dot{\phi}^2}{2}(\ddot{\phi} + 3\dot{\alpha}\dot{\phi})g_\phi - \frac{1}{3}\frac{f_{1\phi}}{f_1}\Sigma; \\ B_{34} &= V_\psi - \frac{1}{3}\frac{f_{2\psi}}{f_2}\Sigma, \end{aligned} \tag{6.20}$$

$$B_{41} = -3\dot{\sigma}\omega - \frac{4}{3}\Sigma; \qquad B_{42} = -\omega^2 - 3\dot{\alpha}\omega - \frac{4}{3}\Sigma;$$

$$B_{43} = -\frac{2}{3}\frac{f_{1\phi}}{f_1}\Sigma; \qquad B_{44} = -\frac{2}{3}\frac{f_{2\psi}}{f_2}\Sigma. \tag{6.21}$$

Note that $\Sigma \equiv f_1^{-2}(\phi) f_2^{-2}(\psi)\exp[-4\alpha - 4\sigma]p_A^2 > 0$ acts as an additional variable. In addition, the following assumptions [16,17] will be adopted for the existence of an inflationary solutions:

$$\alpha \gg \sigma, \quad \dot{\alpha} > 0, \quad \dot{\alpha} \gg \dot{\sigma}, \quad \dot{\alpha} \gg \dot{\phi}, \quad \dot{\alpha} \gg \dot{\psi}. \tag{6.22}$$

Furthermore, we will also assume that $V(\psi)$ is a nearly flat potential with the following property:

$$V \gg V_\psi \gg V_{\psi\psi} > 0. \tag{6.23}$$

Accordingly, similar assumptions will also be imposed on the positive coupling functions $g(\phi)$ and f_i,

$$g_\phi \gg g_{\phi\phi} \gg g_{\phi\phi\phi} > 0, \tag{6.24}$$

$$\left(\frac{f_{1\phi}}{f_1}\right)^2 \geq \frac{f_{1\phi\phi}}{f_1}, \tag{6.25}$$

$$\left(\frac{f_{2\psi}}{f_2}\right)^2 \geq \frac{f_{2\psi\psi}}{f_2}, \tag{6.26}$$

such that f_i and g change slowly as ϕ evolves.

As mentioned above, the sign of ω is critical to the stability of the anisotropic solutions. Indeed, $\omega > 0$ or $\omega < 0$ corresponds to the existence of unstable or stable anisotropic solutions. Hence we need to solve the algebraic equation of ω derived from determinant equation, $\det \mathcal{D}_g = 0$.

First of all, the coefficients B_{ij}'s can be approximated as

$$B_{11} \simeq -3\dot{\phi}^2 g\omega^2 - 3\dot{\phi}(K_X + 6\dot{\alpha}\dot{\phi}g)\omega - 4\frac{f_{1\phi}}{f_1}\Sigma, \tag{6.27}$$

$$\begin{aligned} B_{13} \simeq & -(K_X + \dot{\phi}^2 K_{XX} + 6\dot{\alpha}\dot{\phi}g - 2\dot{\phi}^2 g_\phi)\omega^2 \\ & - [3\dot{\alpha}K_X + 3\dot{\alpha}\dot{\phi}^2 K_{XX} + \ddot{\phi}\dot{\phi}^3 K_{XXX} + 18\dot{\alpha}^2\dot{\phi}g \\ & - 4\ddot{\phi}\dot{\phi}g_\phi - 2\dot{\phi}^3 g_{\phi\phi}]\omega \\ & - 9\dot{\alpha}^2\dot{\phi}^2 g_\phi + 2\ddot{\phi}\dot{\phi}^2 g_{\phi\phi} + \frac{\dot{\phi}^4}{2}g_{\phi\phi\phi} \\ & - \left[3\left(\frac{f_{1\phi}}{f_1}\right)^2 - \frac{f_{1\phi\phi}}{f_1}\right]\Sigma, \end{aligned} \tag{6.28}$$

$$B_{33} \simeq \frac{\dot{\phi}^2}{2}g\omega^2 + \left[\frac{\dot{\phi}^3}{2}K_{XX} + \frac{9}{2}\dot{\alpha}\dot{\phi}^2 g\right]\omega + \frac{3}{2}\dot{\alpha}\dot{\phi}^3 g_\phi - \frac{1}{3}\frac{f_{1\phi}}{f_1}\Sigma, \tag{6.29}$$

during the inflationary phase under the assumptions (6.22). As a result, the determinant equation $\det \mathcal{D}_g = 0$ reduces to a degree 8 algebraic equation of ω,

$$\omega(c_8\omega^7 + \cdots + c_1) = 0, \tag{6.30}$$

with

$$c_8 = K_X + \dot{\phi}^2 K_{XX} + \frac{3}{2}\dot{\phi}^4 g^2 + 6\dot{\alpha}\dot{\phi}g - 2\dot{\phi}^2 g_\phi, \tag{6.31}$$

$$c_1 = M\Sigma^3 + N\Sigma^2 + P\Sigma, \tag{6.32}$$

$$M = -2\left[\left(\frac{f_{1\phi}}{f_1}\right)^2 - \frac{f_{1\phi\phi}}{f_1}\right]\left[\left(\frac{f_{2\psi}}{f_2}\right)^2 - \frac{f_{2\psi\psi}}{f_2}\right](5\dot{\alpha} - \dot{\sigma} - \dot{\phi}^3), \tag{6.33}$$

$$\begin{aligned} N = & -2\left[\left(\frac{f_{1\phi}}{f_1}\right)^2 - \frac{f_{1\phi\phi}}{f_1}\right]\left\{2\left[3\frac{f_{2\psi}}{f_2}(\dot{\alpha} - \dot{\sigma}) - \dot{\psi}\right]V_\psi + (5\dot{\alpha} - \dot{\sigma} - \dot{\phi}^3)V_{\psi\psi}\right\} \\ & - \dot{\phi}^2\left[\left(\frac{f_{2\psi}}{f_2}\right)^2 - \frac{f_{2\psi\psi}}{f_2}\right]\left[(5\dot{\alpha} - \dot{\sigma} - \dot{\phi}^3)(18\dot{\alpha}^2 g_\phi - 4\ddot{\phi}g_{\phi\phi} - \dot{\phi}^2 g_{\phi\phi\phi}) + 6\dot{\alpha}\dot{\phi}^2(K_X + 6\dot{\alpha}\dot{\phi}g)g_\phi \right. \\ & \left. + 18\frac{f_{1\phi}}{f_1}\dot{\alpha}\dot{\phi}(\dot{\alpha} - \dot{\sigma})g_\phi\right], \end{aligned} \tag{6.34}$$

$$\begin{aligned} P = & -\dot{\phi}^2\left\{2\left[3\frac{f_{2\psi}}{f_2}(\dot{\alpha} - \dot{\sigma}) - \dot{\psi}\right](18\dot{\alpha}^2 g_\phi - 4\ddot{\phi}g_{\phi\phi} - \dot{\phi}^2 g_{\phi\phi\phi})V_\psi + \left[(5\dot{\alpha} - \dot{\sigma} - \dot{\phi}^3)(18\dot{\alpha}^2 g_\phi - 4\ddot{\phi}g_{\phi\phi} - \dot{\phi}^2 g_{\phi\phi\phi}) \right.\right. \\ & \left.\left. + 6\dot{\alpha}\dot{\phi}^2(K_X + 6\dot{\alpha}\dot{\phi}g)g_\phi + 18\frac{f_{1\phi}}{f_1}\dot{\alpha}\dot{\phi}(\dot{\alpha} - \dot{\sigma})g_\phi\right]V_{\psi\psi}\right\}. \end{aligned} \tag{6.35}$$

Under the slowly changing potential prescribed by the constraints shown in Eqs. (6.22)–(6.26), it is clear that M, N, and P are all negative. As a result, we can show that $c_1 < 0$.

B. Galileon-vector model

We will try to show that $c_8 > 0$ if the solutions to the corresponding Galileon-vector model (in the absence of the ψ field) is a stable set of solutions. This then proves that the presence of the phantom field introduces at least an unstable mode from the result $c_1 c_8 < 0$. Indeed, the perturbation equations for the Galileon-vector model can be written as

$$\hat{\mathcal{D}}_g \begin{pmatrix} A \\ B \\ C \end{pmatrix} \equiv \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{31} & B_{32} & B_{33} \\ B_{41} & B_{42} & B_{43} \end{bmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = 0. \quad (6.36)$$

In addition, the determinant equation $\det \hat{\mathcal{D}}_g = 0$ reduces to a degree 6 algebraic equation of ω,

$$\omega(d_6\omega^5 + \cdots + d_1) = 0, \quad (6.37)$$

with

$$d_6 = c_8, \quad (6.38)$$

$$d_1 = 2\left[\left(\frac{f_{1\phi}}{f_1}\right)^2 - \frac{f_{1\phi\phi}}{f_1}\right](5\dot{\alpha} - \dot{\sigma} - \dot{\phi}^3)\hat{\Sigma}^2 + \dot{\phi}^2\left[(5\dot{\alpha} - \dot{\sigma} - \dot{\phi}^3)(18\dot{\alpha}^2 g_\phi - 4\ddot{\phi} g_{\phi\phi} - \dot{\phi}^2 g_{\phi\phi\phi}) + 6\dot{\alpha}\dot{\phi}^2(K_X + 6\dot{\alpha}\,\dot{\phi}\, g)g_\phi + 18\frac{f_{1\phi}}{f_1}\dot{\alpha}\,\dot{\phi}(\dot{\alpha} - \dot{\sigma})g_\phi\right]\hat{\Sigma}. \quad (6.39)$$

Here $\Sigma \to \hat{\Sigma} \equiv f_1^{-2}(\phi)\exp[-4\alpha - 4\sigma]p_A^2 > 0$ due to the vanishing of the phantom field ψ.

Note that $d_1 d_6 > 0$ is required in order to exclude an unstable mode from the perturbation equation $\det \hat{\mathcal{D}}_g = 0$. It is straightforward to show that $d_1 c_1 < 0$, under the same assumption that the potentials change slowly. As a result, we have proven that $c_1 c_8 < 0$, and hence there exists at least an unstable mode to the perturbation equation $\det \mathcal{D}_g = 0$. Therefore, the conclusion is that the presence of the phantom field does destabilize the inflationary solution as expected for the Galileon-vector-phantom model in the BI metric space. Hence the presence of the phantom field appears to support the cosmic no-hair conjecture in a more general context [16,17].

VII. CONCLUSION

Cosmic inflation [1] has served as a successful paradigm of modern cosmology in resolving several cosmological problems [1]. In addition, it also provides a useful framework to accommodate the observations of WMAP [2] and Planck [3]. Many observations have been shown to be consistent with the theoretical predictions of the standard inflationary models [2,3]. There are, however, some large scale CMB anomalies observed by the WMAP [2] and the Planck [3]. Hence, isotropic inflationary models, based on the homogeneous and isotropic FLRW)spacetime [4], should be modified by the introduction of the anisotropic inflation models, e.g., the anisotropic Bianchi spacetimes [5,6].

In addition to many successful scalar-vector models, with a special scalar-vector interaction $f(\phi)^2F^2$, the covariant Galileon-vector model appears to be a promising model for the anisotropically inflationary universe [17,27–37]. Hence we propose to study the dynamic feature of a special class of the Galileon model. As a result, we have found many useful results.

In summary, we have presented a set of isotropic power-law solutions for the Galileon-vector model in Sec. II. In Sec. III, a set of BI power-law solutions for the Galileon-vector model is presented along with the stability analysis of this set of solutions. We also show that the solutions we found are a set of attractor solutions in Sec. IV. The BI power-law solutions of the Galileon-vector-phantom model along with their stability analysis is shown in Sec. V. In addition, a general stability analysis for a more general Galileo-vector-phantom model is presented in Sec. VI. Our results show that the BI power-law solutions are indeed stable for the Galileon-vector model with a coupling term of the form $f^2(\phi)F_{\mu\nu}F^{\mu\nu}$. In addition, the presence of a phantom field introduces an unstable mode to the field equations. This shows that a stable solution for the Galileon-vector model will be destabilized by the introduction of a phantom field. Finally, we also present a solution-independent stability analysis for a more general Galileon-vector model. In particular, we also show the power-law solutions we obtained are a set of attractor solutions by solving a set of autonomous equations. The result shows that the presence of a phantom field does tend to destabilize the stable solution for the Galileon-vector model. These results indicate that Galileon-vector models deserve more attention. Hopefully, the results shown in this paper will be helpful to the search for a more realistic inflationary model.

ACKNOWLEDGMENTS

T. Q. D. is supported in part by the Vietnam National Foundation for Science and Technology Development (NAFOSTED) under Grant No. 103.01-2017.12. W. F. K. is supported in part by the Ministry of Science and Technology (MOST) of Taiwan under Contract No. MOST 104-2112-M-009-020-MY3. We thank the referee for useful comments and suggestions.

[1] A. H. Guth, The inflationary universe: A possible solution to the horizon and flatness problems, Phys. Rev. D **23**, 347 (1981); A. D. Linde, A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, Phys. Lett. **108B**, 389 (1982); A. D. Linde, Chaotic inflation, Phys. Lett. **129B**, 177 (1983).

[2] E. Komatsu *et al.* (WMAP Collaboration), Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Cosmological interpretation, Astrophys. J. Suppl. Ser. **192**, 18 (2011); G. Hinshaw *et al.* (WMAP Collaboration), Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Cosmological parameter results, Astrophys. J. Suppl. Ser. **208**, 19 (2013).

[3] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2015 results. XX. Constraints on inflation, Astron. Astrophys. **594**, A20 (2016); P. A. R. Ade *et al.* (Planck Collaboration), Planck 2015 results. XVI. Isotropy and statistics of the CMB, Astron. Astrophys. **594**, A16 (2016).

[4] T. Buchert, A. A. Coley, H. Kleinert, B. F. Roukema, and D. L. Wiltshire, Observational challenges for the standard FLRW model, Int. J. Mod. Phys. D **25**, 1630007 (2016).

[5] G. F. R. Ellis and M. A. H. MacCallum, A class of homogeneous cosmological models, Commun. Math. Phys. **12**, 108 (1969); G. F. R. Ellis, The Bianchi models: Then and now, Gen. Relativ. Gravit. **38**, 1003 (2006).

[6] C. Pitrou, T. S. Pereira, and J. P. Uzan, Predictions from an anisotropic inflationary era, J. Cosmol. Astropart. Phys. 04 (2008) 004; A. E. Gumrukcuoglu, C. R. Contaldi, and M. Peloso, Inflationary perturbations in anisotropic backgrounds and their imprint on the CMB, J. Cosmol. Astropart. Phys. 11 (2007) 005.

[7] G. W. Gibbons and S. W. Hawking, Cosmological event horizons, thermodynamics, and particle creation, Phys. Rev. D **15**, 2738 (1977); S. W. Hawking and I. G. Moss, Supercooled phase transitions in the very early universe, Phys. Lett. **110B**, 35 (1982).

[8] J. D. Barrow, Cosmic no hair theorems and inflation, Phys. Lett. B **187**, 12 (1987).

[9] R. M. Wald, Asymptotic behavior of homogeneous cosmological models in the presence of a positive cosmological constant, Phys. Rev. D **28**, 2118 (1983).

[10] J. D. Barrow and S. Hervik, Anisotropically inflating universes, Phys. Rev. D **73**, 023007 (2006); On the evolution of universes in quadratic theories of gravity, Phys. Rev. D **74**, 124017 (2006); Simple types of anisotropic inflation, Phys. Rev. D **81**, 023513 (2010); J. Middleton, On the existence of anisotropic cosmological models in higher order theories of gravity, Classical Quantum Gravity **27**, 225013 (2010).

[11] W. F. Kao and I. C. Lin, Stability conditions for the Bianchi type II anisotropically inflating universes, J. Cosmol. Astropart. Phys. 01 (2009) 022; Anisotropically inflating universes in a scalar-tensor theory, Phys. Rev. D **79**, 043001; Stability of the anisotropically inflating Bianchi type VI expanding solutions, Phys. Rev. D **83**, 063004 (2011).

[12] N. Kaloper, Lorentz Chern-Simons terms in Bianchi cosmologies and the cosmic no hair conjecture, Phys. Rev. D **44**, 2380 (1991); C. Chang, W. F. Kao, and I. C. Lin, Stability analysis of the Lorentz Chern-Simons expanding solutions, Phys. Rev. D **84**, 063014 (2011).

[13] S. Kanno, M. Kimura, J. Soda, and S. Yokoyama, Anisotropic inflation from vector impurity, J. Cosmol. Astropart. Phys. 08 (2008) 034; B. Himmetoglu, C. R. Contaldi, and M. Peloso, Instability of Anisotropic Cosmological Solutions Supported by Vector Fields, Phys. Rev. Lett. **102**, 111301 (2009); J. D. Barrow, M. Thorsrud, and K. Yamamoto, Cosmologies in Horndeski's second-order vector-tensor theory, J. High Energy Phys. 02 (2013) 146; L. Heisenberg, R. Kase, and S. Tsujikawa, Anisotropic cosmological solutions in massive vector theories, J. Cosmol. Astropart. Phys. 11 (2016) 008.

[14] S. Kanno, J. Soda, and M. a. Watanabe, Anisotropic power-law inflation, J. Cosmol. Astropart. Phys. 12 (2010) 024; M. a. Watanabe, S. Kanno, and J. Soda, Inflationary Universe with Anisotropic Hair, Phys. Rev. Lett. **102**, 191302 (2009).

[15] A. Maleknejad, M. M. Sheikh-Jabbari, and J. Soda, Gauge fields and inflation, Phys. Rep. **528**, 161 (2013); J. Soda, Statistical anisotropy from anisotropic inflation, Classical Quantum Gravity **29**, 083001 (2012).

[16] T. Q. Do, W. F. Kao, and I. C. Lin, Anisotropic power-law inflation for a two scalar fields model, Phys. Rev. D **83**, 123002 (2011); T. Q. Do and S. H. Q. Nguyen, Anisotropic power-law inflation in a two-scalar-field model with a mixed kinetic term, Int. J. Mod. Phys. D **26**, 1750072 (2017).

[17] T. Q. Do and W. F. Kao, Anisotropic power-law inflation for the Dirac-Born-Infeld theory, Phys. Rev. D **84**, 123009 (2011); T. Q. Do and W. F. Kao, Anisotropic power-law solutions for a supersymmetry Dirac-Born-Infeld theory, Classical Quantum Gravity **33**, 085009 (2016).

[18] R. Emami, H. Firouzjahi, S. M. Sadegh Movahed, and M. Zarei, Anisotropic inflation from charged scalar fields, J. Cosmol. Astropart. Phys. 02 (2011) 005; K. Murata and J. Soda, Anisotropic inflation with non-Abelian gauge kinetic function, J. Cosmol. Astropart. Phys. 06 (2011) 037; S. Hervik, D. F. Mota, and M. Thorsrud, Inflation with stable anisotropic hair: Is it cosmologically viable?, J. High Energy Phys. 11 (2011) 146; K. Yamamoto, M. a. Watanabe, and J. Soda, Inflation with multi-vector hair: The fate of anisotropy, Classical Quantum Gravity **29**, 145008 (2012); M. Thorsrud, D. F. Mota, and S. Hervik, Cosmology of a scalar field coupled to matter and an isotropy-violating Maxwell field, J. High Energy Phys. 10 (2012) 066; A. Maleknejad and M. M. Sheikh-Jabbari, Revisiting cosmic no-hair theorem for inflationary settings, Phys. Rev. D **85**, 123508 (2012); K. i. Maeda and K. Yamamoto, Inflationary dynamics with a non-Abelian gauge field, Phys. Rev. D **87**, 023528 (2013); J. Ohashi, J. Soda, and S. Tsujikawa, Anisotropic non-Gaussianity from a two-form field, Phys. Rev. D **87**, 083520 (2013); Anisotropic power-law *k*-inflation, Phys. Rev. D **88**, 103517 (2013); A. Ito and J. Soda, Designing anisotropic inflation with form fields, Phys. Rev. D **92**, 123533 (2015); A. A. Abolhasani, M. Akhshik, R. Emami, and H. Firouzjahi, Primordial statistical anisotropies: The effective field theory approach, J. Cosmol. Astropart. Phys. 03 (2016) 020; S. Lahiri, Anisotropic inflation in Gauss-Bonnet gravity,

J. Cosmol. Astropart. Phys. 09 (2016) 025; M. Karciauskas, Dynamical analysis of anisotropic inflation, Mod. Phys. Lett. A **31**, 1640002 (2016).

[19] M. a. Watanabe, S. Kanno, and J. Soda, The nature of primordial fluctuations from anisotropic inflation, Prog. Theor. Phys. **123**, 1041 (2010); A. E. Gumrukcuoglu, B. Himmetoglu, and M. Peloso, Scalar-scalar, scalar-tensor, and tensor-tensor correlators from anisotropic inflation, Phys. Rev. D **81**, 063528 (2010); M. a. Watanabe, S. Kanno, and J. Soda, Imprints of anisotropic inflation on the cosmic microwave background, Mon. Not. R. Astron. Soc. **412**, L83 (2011); J. Ohashi, J. Soda, and S. Tsujikawa, Observational signatures of anisotropic inflationary models, J. Cosmol. Astropart. Phys. 12 (2013) 009; N. Bartolo, S. Matarrese, M. Peloso, and A. Ricciardone, Anisotropic power spectrum and bispectrum in the $f(\phi)F^2$ mechanism, Phys. Rev. D **87**, 023504 (2013); X. Chen, R. Emami, H. Firouzjahi, and Y. Wang, The TT, TB, EB and BB correlations in anisotropic inflation, J. Cosmol. Astropart. Phys. 08 (2014) 027; R. Emami, H. Firouzjahi, and M. Zarei, Anisotropic inflation with the nonvacuum initial state, Phys. Rev. D **90**, 023504 (2014).

[20] A. Ito and J. Soda, MHz gravitational waves from short-term anisotropic inflation, J. Cosmol. Astropart. Phys. 04 (2016) 035; R. Emami and H. Firouzjahi, Clustering fossil from primordial gravitational waves in anisotropic inflation, J. Cosmol. Astropart. Phys. 10 (2015) 043; M. Fukushima, S. Mizuno, and K. i. Maeda, Gravitational baryogenesis after anisotropic inflation, Phys. Rev. D **93**, 103513 (2016).

[21] A. De Felice, A. E. Gumrukcuoglu, and S. Mukohyama, Massive Gravity: Nonlinear Instability of the Homogeneous and Isotropic Universe, Phys. Rev. Lett. **109**, 171101 (2012); A. E. Gumrukcuoglu, C. Lin, and S. Mukohyama, Anisotropic Friedmann-Robertson-Walker universe from nonlinear massive gravity, Phys. Lett. B **717**, 295 (2012); T. Q. Do and W. F. Kao, Anisotropically expanding universe in massive gravity, Phys. Rev. D **88**, 063006 (2013); W. F. Kao and I. C. Lin, Bianchi type I expanding universe in Weyl-invariant massive gravity, Phys. Rev. D **90**, 063003 (2014); T. Q. Do, Higher dimensional nonlinear massive gravity, Phys. Rev. D **93**, 104003 (2016).

[22] Y. Sakakihara, J. Soda, and T. Takahashi, On cosmic no-hair in bimetric gravity and the Higuchi bound, Prog. Theor. Exp. Phys. **2013**, 033E02 (2013); K. i. Maeda and M. S. Volkov, Anisotropic universes in the ghost-free bigravity, Phys. Rev. D **87**, 104009 (2013); T. Q. Do, Higher dimensional massive bigravity, Phys. Rev. D **94**, 044022 (2016).

[23] M. Kleban and L. Senatore, Inhomogeneous anisotropic cosmology, J. Cosmol. Astropart. Phys. 10 (2016) 022; W. E. East, M. Kleban, A. Linde, and L. Senatore, Beginning inflation in an inhomogeneous universe, J. Cosmol. Astropart. Phys. 09 (2016) 010.

[24] S. M. Carroll and A. Chatwin-Davies, Cosmic equilibration: A holographic no-hair theorem from the generalized second law, arXiv:1703.09241.

[25] D. Saadeh, S. M. Feeney, A. Pontzen, H. V. Peiris, and J. D. McEwen, How Isotropic Is the Universe?, Phys. Rev. Lett. **117**, 131302 (2016).

[26] R. R. Caldwell, A phantom menace?, Phys. Lett. B **545**, 23 (2002); Y. F. Cai, E. N. Saridakis, M. R. Setare, and J. Q. Xia, Quintom cosmology: Theoretical implications and observations, Phys. Rep. **493**, 1 (2010).

[27] A. Nicolis, R. Rattazzi, and E. Trincherini, The Galileon as a local modification of gravity, Phys. Rev. D **79**, 064036 (2009); P. Creminelli, A. Nicolis, and E. Trincherini, Galilean genesis: An alternative to inflation, J. Cosmol. Astropart. Phys. 11 (2010) 021.

[28] G. R. Dvali, G. Gabadadze, and M. Porrati, 4D gravity on a brane in 5D Minkowski space, Phys. Lett. B **485**, 208 (2000).

[29] R. P. Woodard, Ostrogradsky's theorem on Hamiltonian instability, Scholarpedia **10**, 32243 (2015); T. j. Chen, M. Fasiello, E. A. Lim, and A. J. Tolley, Higher derivative theories with constraints: Exorcising Ostrogradski's ghost, J. Cosmol. Astropart. Phys. 02 (2013) 042.

[30] C. Deffayet, G. Esposito-Farese, and A. Vikman, Covariant Galileon, Phys. Rev. D **79**, 084003 (2009); C. Deffayet, S. Deser, and G. Esposito-Farese, Generalized Galileons: All scalar models whose curved background extensions maintain second-order field equations and stress-tensors, Phys. Rev. D **80**, 064015 (2009).

[31] C. Deffayet, X. Gao, D. A. Steer, and G. Zahariade, From k-essence to generalized Galileons, Phys. Rev. D **84**, 064039 (2011); C. Deffayet and D. A. Steer, A formal introduction to Horndeski and Galileon theories and their generalizations, Classical Quantum Gravity **30**, 214006 (2013).

[32] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, Generalized G-inflation: Inflation with the most general second-order field equations, Prog. Theor. Phys. **126**, 511 (2011); C. Charmousis, E. J. Copeland, A. Padilla, and P. M. Saffin, General Second-Order Scalar-Tensor Theory and Self-Tuning, Phys. Rev. Lett. **108**, 051101 (2012); C. Charmousis, E. J. Copeland, A. Padilla, and P. M. Saffin, Self-tuning and the derivation of a class of scalar-tensor theories, Phys. Rev. D **85**, 104040 (2012).

[33] A. De Felice and S. Tsujikawa, Cosmology of a Covariant Galileon Field, Phys. Rev. Lett. **105**, 111301 (2010); A. De Felice and S. Tsujikawa, Generalized Galileon cosmology, Phys. Rev. D **84**, 124029 (2011); S. Nesseris, A. De Felice, and S. Tsujikawa, Observational constraints on Galileon cosmology, Phys. Rev. D **82**, 124054 (2010); C. Burrage, C. de Rham, D. Seery, and A. J. Tolley, Galileon inflation, J. Cosmol. Astropart. Phys. 01 (2011) 014; M. Trodden and K. Hinterbichler, Generalizing Galileons, Classical Quantum Gravity **28**, 204003 (2011); X. Gao, Conserved cosmological perturbation in Galileon models, J. Cosmol. Astropart. Phys. 10 (2011) 021; A. De Felice and S. Tsujikawa, Inflationary non-Gaussianities in the most general second-order scalar-tensor theories, Phys. Rev. D **84**, 083504 (2011); X. Gao, T. Kobayashi, M. Yamaguchi, and J. Yokoyama, Primordial Non-Gaussianities of Gravitational Waves in the Most General Single-Field Inflation Model, Phys. Rev. Lett. **107**, 211301 (2011); A. De Felice and S. Tsujikawa, Cosmological constraints on extended Galileon models, J. Cosmol. Astropart. Phys. 03 (2012) 025; J. Ohashi and S. Tsujikawa, Potential-driven Galileon inflation, J. Cosmol. Astropart. Phys. 10 (2012) 035; G. Leon and E. N. Saridakis, Dynamical analysis of generalized

Galileon cosmology, J. Cosmol. Astropart. Phys. 03 (2013) 025.

[34] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, Inflation Driven by the Galileon Field, Phys. Rev. Lett. **105**, 231302 (2010); C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, Imperfect dark energy from kinetic gravity braiding, J. Cosmol. Astropart. Phys. 10 (2010) 026; K. Kamada, T. Kobayashi, M. Yamaguchi, and J. Yokoyama, Higgs G-inflation, Phys. Rev. D **83**, 083515 (2011); T. Kobayashi, M. Yamaguchi, and J. Yokoyama, Primordial non-Gaussianity from G-inflation, Phys. Rev. D **83**, 103524 (2011); X. Gao and D. A. Steer, Inflation and primordial non-Gaussianities of "generalized Galileons", J. Cosmol. Astropart. Phys. 12 (2011) 019; T. Kobayashi, N. Tanahashi, and M. Yamaguchi, Multifield extension of G inflation, Phys. Rev. D **88**, 083504 (2013); S. Unnikrishnan and S. Shankaranarayanan, Consistency relation in power law G-inflation, J. Cosmol. Astropart. Phys. 07 (2014) 003; S. Hirano, T. Kobayashi, and S. Yokoyama, Ultra slow-roll G-inflation, Phys. Rev. D **94**, 103515 (2016).

[35] C. de Rham and A. J. Tolley, DBI and the Galileon reunited, J. Cosmol. Astropart. Phys. 05 (2010) 015; K. Van Acoleyen and J. Van Doorsselaere, Galileons from Lovelock actions, Phys. Rev. D **83**, 084025 (2011); C. de Rham and L. Heisenberg, Cosmology of the Galileon from massive gravity, Phys. Rev. D **84**, 043503 (2011); C. de Rham, Galileons in the Sky, C. R. Phys. **13**, 666 (2012).

[36] G. W. Horndeski, Second-order scalar-tensor field equations in a four-dimensional space, Int. J. Theor. Phys. **10**, 363 (1974).

[37] J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, New Class of Consistent Scalar-Tensor Theories, Phys. Rev. Lett. **114**, 211101 (2015); J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, Exploring gravitational theories beyond Horndeski, J. Cosmol. Astropart. Phys. 02 (2015) 018.

[38] T. Erler and C. Maccaferri, The phantom term in open string field theory, J. High Energy Phys. 06 (2012) 084.

[39] G. Clement, J. C. Fabris, and M. E. Rodrigues, Phantom black holes in Einstein-Maxwell-dilaton theory, Phys. Rev. D **79**, 064021 (2009).