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Chaos control of new Mathieu–van der Pol systems by fuzzy logic constant controllers

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ABSTRACT

In this paper, a new fuzzy logic controller—fuzzy logic constant controller (FLCC) is introduced to chaotic signals controlling. The main ideas of the FLCC are described as follows: (1) proving the two chaotic systems are going to achieve asymptotically stable via Lyapunov direct method; (2) via detecting the sign of the errors, the appropriate fuzzy logic control scheme is operated; (3) choosing the upper bound and the lower bound of the error derivatives of the chaotic signals to be the consequent parts (corresponding controllers).

Due to controllers in traditional method – derived by Lyapunov direct method, are always complicated, nonlinear form or the functions of errors, a new simplest controller—FLCC is presented in this paper to synchronize two chaotic signals. Through the FLCC, there are three main contributions can be obtained: (1) the mathematical models of the nonlinear chaotic systems can be unknown, all we have to do is capturing the signals of the unknown systems; (2) through the fuzzy logic rules, the strength of controllers can be adjusted via the corresponding membership functions (which are decided by the values of error derivatives); (3) by the FLCC, the chaotic system can be much more exactly and efficiently controlled to the trajectory of our goal than traditional ones. Three cases, original point, regular function and chaotic Qi system (with large values of initial conditions), are given to illustrate the effectiveness of our new FLC.

1. Introduction

Since Ott et al. [1] gave the famous OGY control method in 1990, the applications of the various methods to control a chaotic behavior in natural sciences and engineering are well known. For example, the adaptive control [2–5], the method of chaos control based on sampled data [6], the method of pulse feedback of systematic variable [7], the active control [8,9] and linear error feedback control [10,11]. However, when Lyapunov stability of zero solution of states is studied, the stability of solutions on the whole neighborhood region of the origin is demanded.

In recent years, some chaos control based on fuzzy systems has been proposed since the fuzzy set theory was initiated by Zadeh [12], such as fuzzy sliding mode controlling technique [13,14], LMI-based synchronization [15] and extended backstepping sliding mode controlling technique [16]. The fuzzy logic control (FLC) scheme has been widely developed for almost 40 years and has been successfully applied to many applications [17]. Many researchers have worked to improve the performance of the FLCs and ensure their stability. Li and Gatland in Refs. [18,19] proposed a more systematic design method for PD- and PI-type FLCs [20]. Choi et al. [21] presents a single input FLC ensuring stability. Ying [22] presents a practical design method for nonlinear fuzzy controllers, and many other researchers have results on the matter of the stability of FLCs, in Castillo et al. [23] and Cázarez et al. [24] was presented an extension of the Margaliot work [25] to built stable type-2 fuzzy logic controllers in Lyapunov sense.

Recently, Yau and Shieh [26] proposed an amazing new idea to design fuzzy logic controllers—constructing fuzzy rules subject to a common Lyapunov function such that the master-slave chaos systems satisfy stability in the Lyapunov sense. In Ref. [26], there are two main controllers in their slave system. One is used in elimination of nonlinear terms and the other is built by fuzzy rules subject to a common Lyapunov function. Therefore, the resulting controllers are nonlinear form. Otherwise, in Ref. [26], the regular form is necessary. In order to carry out the new method, the original system must to be transformed into their regular form.

In this paper, we propose a new strategy which is also constructing fuzzy rules subject to a Lyapunov direct method. Error dynamics are used to be upper bound and lower bound. Through this new approach, a simplest controller, constant controller, can be obtained and the difficulty in realization of complicated controllers in chaos synchronization by Lyapunov direct method can be also coped. Unlike conventional approaches, the resulting

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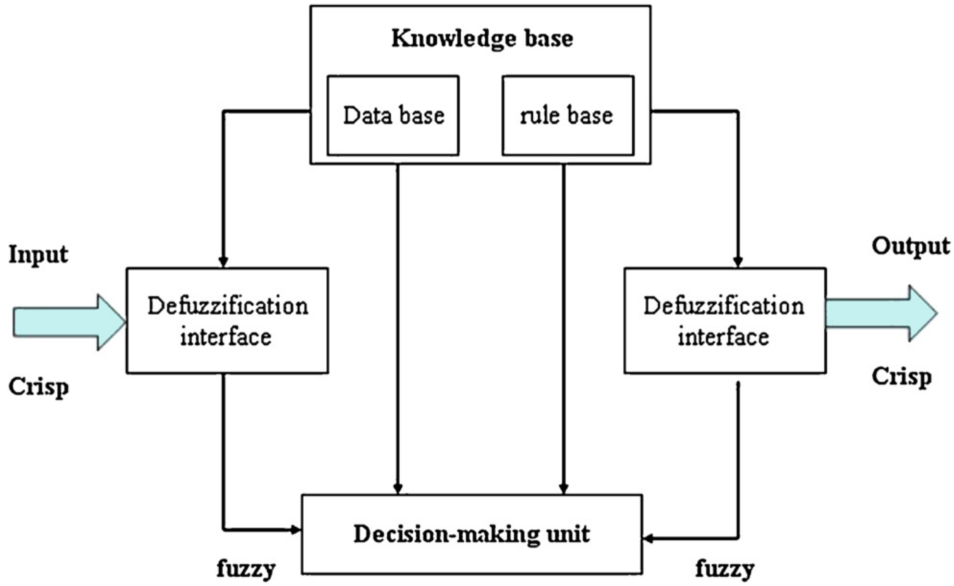

Fig. 1. The configuration of fuzzy logic controller.

control law has less maximum magnitude of the instantaneous control command and it can reduce the actuator saturation phenomenon in real physic system.

The rest of the paper is organized as follows: in Section 2, chaos control by FLCC scheme is presented. In Section 3, new chaotic Mathieu–van der Pol system is introduced. In Section 4, simulation results are shown. In Section 5 conclusions are given.

2. Chaos control by using the scheme of FLCC

2.1. Chaos control scheme

Consider the following chaotic system

$$\dot{\mathbf{x}} = \mathbf{f}(t, x) + u \tag{2-1}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T \in R^n$ is a the state vector, $\mathbf{f} : R_+ \times R^n \to R^n$ is a vector function and $u = [u_1, u_2, \ldots, u_n]^T \in R^n$ is the fuzzy logic controller needed to be designed.

The goal system which can be either chaotic or regular, is

$$\dot{\mathbf{y}} = g(t, y) \tag{2-2}$$

where $\mathbf{y} = [y_1, y_2, \ldots, y_n]^T \in R^n$ is a state vector, $g : R_+ \times R^n \to R^n$ is a vector function.

In order to make the chaos state $\mathbf{x}$ approaching the goal state $\mathbf{y}$, define $\mathbf{e} = \mathbf{x} - \mathbf{y} = [e_1, e_3, e_3, e_4]$ as the state error. The chaos control is accomplished in the sense that [13–22]:

$$\lim_{t\to\infty} \mathbf{e} = \lim_{t\to\infty} (\mathbf{x} - \mathbf{y}) = 0 \tag{2-3}$$

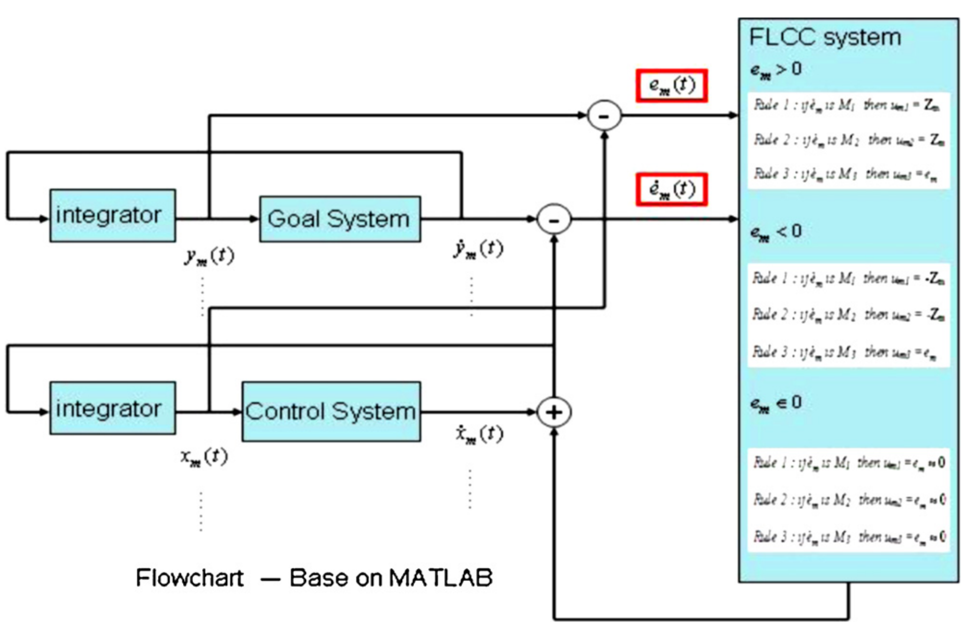

Fig. 2. The flowchart of FCLL designing based on MATLAB.

where

$$\mathbf{e} = \mathbf{x} - \mathbf{y} \tag{2-4}$$

From Eq. (2-4) we have the following error dynamics:

$$\dot{\mathbf{e}} = \dot{\mathbf{x}} - \dot{\mathbf{y}} = \mathbf{f}(t, x) - g(t, y) - u \tag{2-5}$$

According to Lyapunov direct method, we have the following Lyapunov function to derive the fuzzy logic controller for synchronization:

$$V = f(e_1, \dots e_m, \dots e_n) = \frac{1}{2}(e_1^2 + \cdots + e_m^2 + \cdots e_n^2) > 0 \tag{2-6}$$

The derivative of the Lyapunov function in Eq. (2-5) is:

$$\dot{V} = e_1\dot{e}_1 + \cdots + e_m\dot{e}_m + \cdots e_n\dot{e}_n \tag{2-7}$$

If the controllers included in $\dot{e}_1 \dots \dot{e}_m \dots \dot{e}_n$ can be suitably designed to achieve the target: $\dot{V} < 0$, then the two chaotic systems are asymptotically stable. The design process of FLCC is introduced in the following section.

2.2. Fuzzy logic constant controller design process

The basic configuration of the fuzzy logic system is shown in Fig. 1. It is composed of five function blocks [27]:

1. A rule base contains a number of fuzzy if–then rules.
2. A database defines the membership functions of the fuzzy sets used in fuzzy rules.
3. A decision-making unit performs the inference operations on the rules.
4. A fuzzification interface transforms the crisp inputs into degrees of match with linguistic value.
5. A defuzzification interface transforms the fuzzy results of the inference into a crisp output.

The fuzzy rule base consists of collection of fuzzy if–then rules expressed as the form if a is A then b is B, where a and b denote linguistic variables, A and B represent linguistic values which are characterized by membership functions. All of the fuzzy rules can be used to construct the fuzzy associated memory.

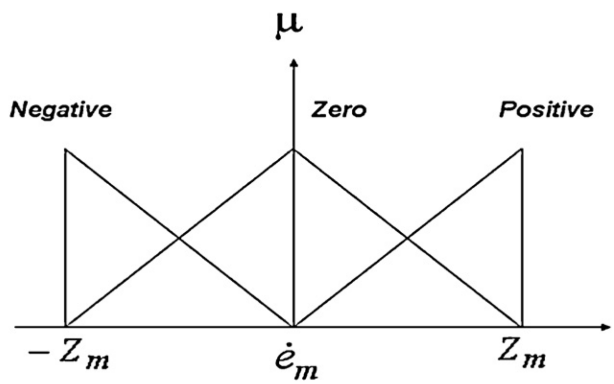

Fig. 3. Membership function.

We use one signal, error derivatives $\dot{e}(t) = [\dot{e}_1, \dot{e}_2, \dots \dot{e}_m, \dots \dot{e}_n]^T$, as the antecedent part of the proposed FLCC to design the control input u that will be used in the consequent part of the proposed FLCC as follows:

$$u = [u_1, u_2 \dots u_m, \dots u_n]^T \tag{2-8}$$

where u is a constant column vector and the FLCC accomplishes the objective to stabilize the error dynamics (2-5).

The strategy of the FLCC designing is proposed as follows and the configuration of the strategy is shown in Fig. 2.

Assume the upper bound and lower bound of $\dot{e}_m$ are Z_m and $-Z_m$, then the FLCC can be design step by step as follows:

(1) If e_m is detected as positive ($e_m > 0$), we have to design a controller for $\dot{e}_m < 0$, then $\dot{V} = e_m\dot{e}_m < 0$ can be achieved. Therefore we have the following ith if–then fuzzy rules as:

$$\text{Rule 1: if } \dot{e}_m \text{ is } M_{mi} \text{ then } u_{m1} = Z_m \tag{2-9}$$

$$\text{Rule 2: if } \dot{e}_m \text{ is } M_{m2} \text{ then } u_{m2} = Z_m \tag{2-10}$$

$$\text{Rule 3: if } \dot{e}_m \text{ is } M_{m3} \text{ then } u_{m3} = e_m \tag{2-11}$$

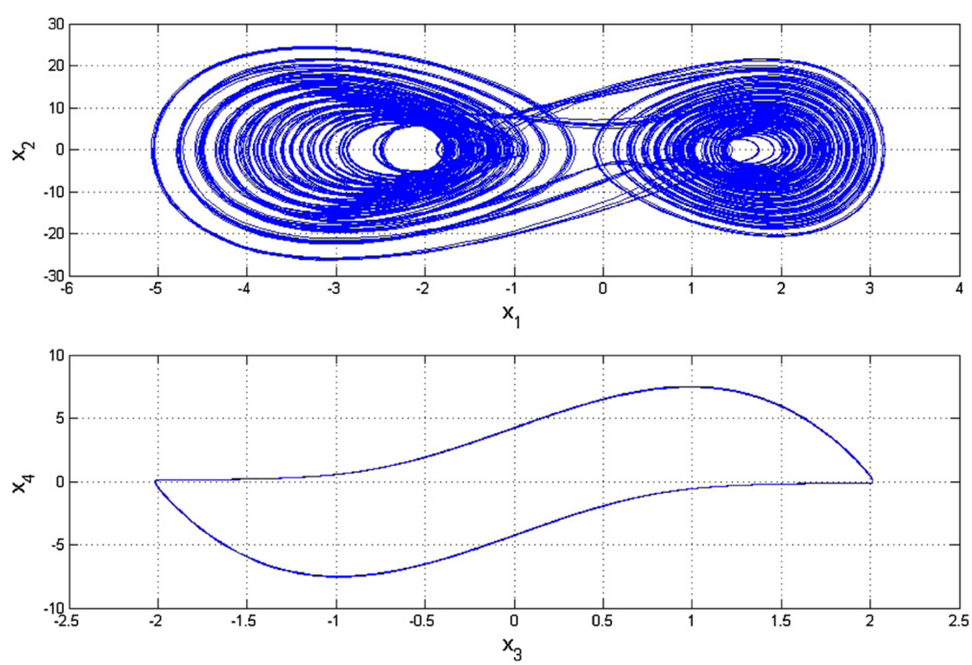

Fig. 4. Projections of phase portrait of new chaotic Mathieu–van der Pol system with $a = 10$, $b = 3$, $c = 0.4$, $d = 70$, $e = 1$, $f = 5$, and $g = 0.1$.

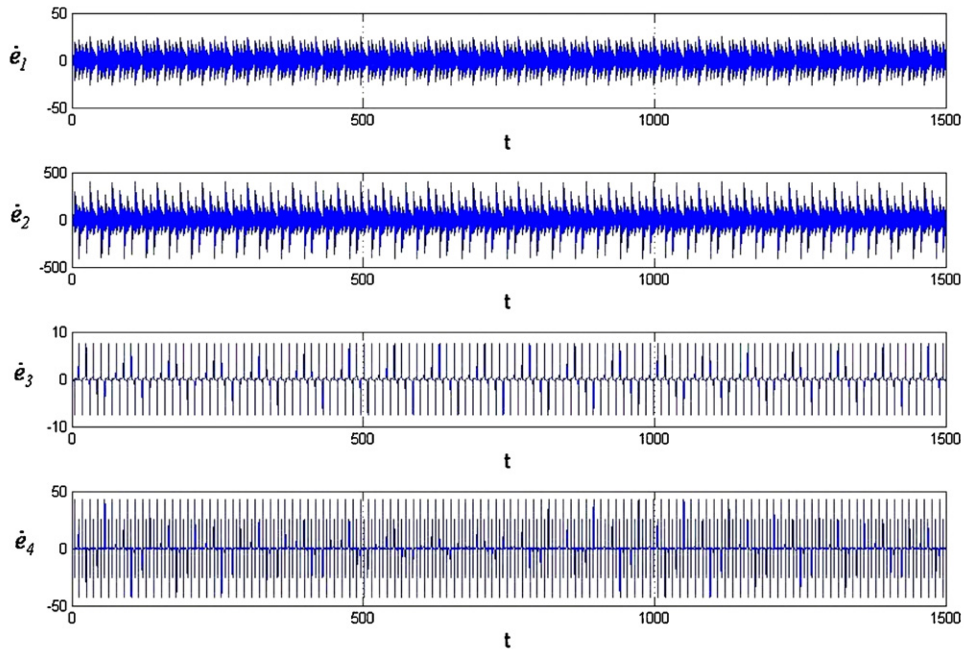

Fig. 5. Time histories of error derivatives for Case 1 (without controllers).

(2) If e_m is detected as negative ($e_m < 0$), we have to design a controller for $\dot{e}_m > 0$, then $\dot{V} = e_m \dot{e}_m < 0$ can be achieved. Therefore we have the following ith if–then fuzzy rules as:

$$\text{Rule 1}: \text{ if } \dot{e}_m \text{ is } M_{m1} \text{ then } u_{m1} = -Z_m \tag{2-12}$$

$$\text{Rule 2}: \text{ if } \dot{e}_m \text{ is } M_{m2} \text{ then } u_{m2} = -Z_m \tag{2-13}$$

$$\text{Rule 3}: \text{ if } \dot{e}_m \text{ is } M_{m3} \text{ then } u_{m3} = e_m \tag{2-14}$$

(3) If e_m approaches to zero, then the synchronization is nearly achieved. Therefore we have the following ith if–then fuzzy rules as:

$$\text{Rule 1}: \text{ if } \dot{e}_m \text{ is } M_{m1} \text{ then } u_{m1} = e_m \approx 0 \tag{2-15}$$

$$\text{Rule 2}: \text{ if } \dot{e}_m \text{ is } M_{m2} \text{ then } u_{m2} = e_m \approx 0 \tag{2-16}$$

$$\text{Rule 3}: \text{ if } \dot{e}_m \text{ is } M_{m3} \text{ then } u_{m3} = e_m \approx 0 \tag{2-17}$$

where $M_{m1} = \left|\dot{e}_m\right|/Z_m$, $M_{m2} = \left|\dot{e}_m\right|/Z_m$ and $M_{m3} = sgn((Z_m - \dot{e}_m)/Z_m) + sgn((\dot{e}_m - Z_m)/Z_m)$, M_{m1}, M_{m2} and M_{m3} refer to the membership functions of positive (P), negative (N) and zero (Z) separately which are presented in Fig. 3. For each case, u_{mi}, $i = 1$–3 is the i-rd output of $\dot{e}_m$ which is a constant controller. The centroid defuzzifier evaluates the output of all rules as follows:

$$u_m = \frac{\sum_{i=1}^{3} M_{mi} \times u_{mi}}{\sum_{i=1}^{3} M_{mi}} \tag{2-18}$$

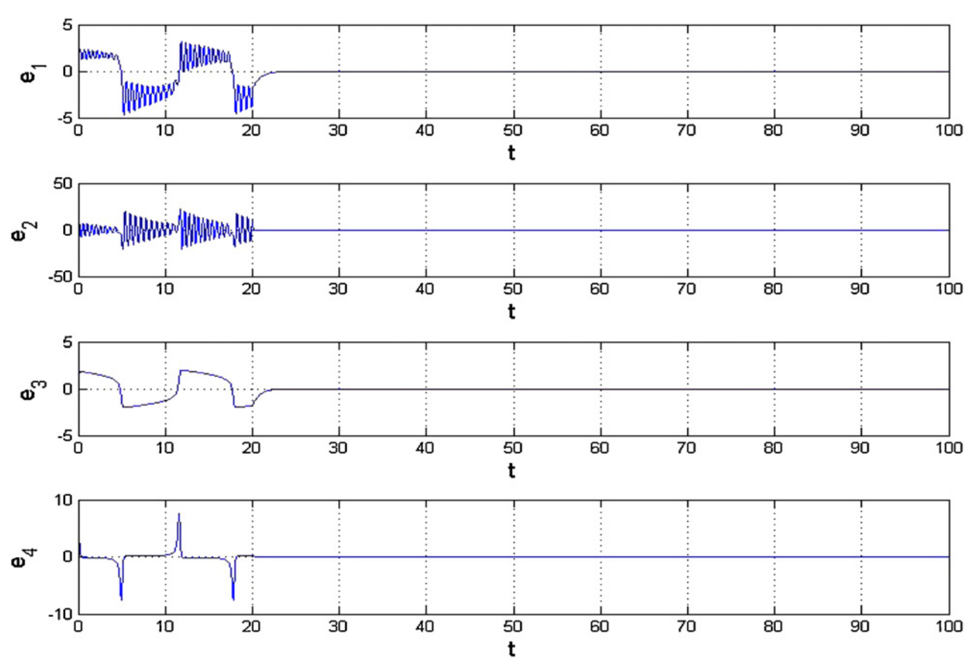

Fig. 6. Time histories of errors for Case 1—the FLCC is coming into after 20 s.

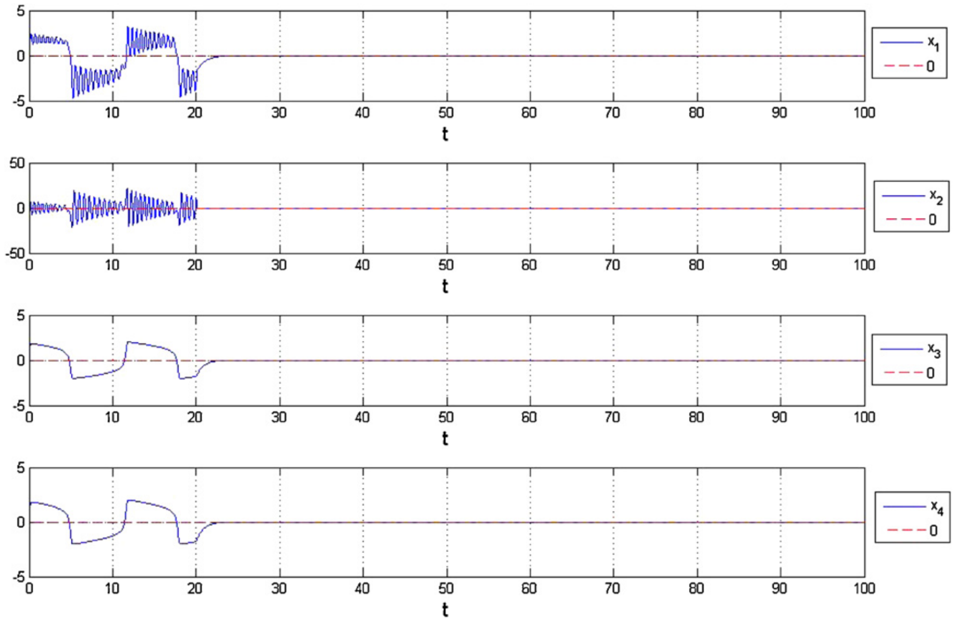

Fig. 7. Time histories of states for Case 1—the FLCC is coming into after 20 s.

Table 1
Rule-table of FLCC.

Rule	Antecedent $\dot{e}_m$	Consequent part u_{mi}
1	Negative (N)	u_{m1}
2	Positive (P)	u_{m2}
3	Zero (Z)	u_{m3}

The fuzzy rule base is listed in Table 1, in which the input variables in the antecedent part of the rules are $\dot{e}_m$ and the output variable in the consequent part is u_{mi}.

After designing appropriate fuzzy logic constant controllers and being substituted into Eq. (2-7), a negative definite of derivatives of Lyapunov function $\dot{V}$ can be obtained and the asymptotically stability of Lyapunov theorem can be achieved.

Consequently, the processes of FLCC designing to control a system following the trajectory of a goal system can be concluded as follows:

1. Construct a FLCC system in MATLAB (Simulink) following Fig. 2 and Eqs. (2-9)–(2-17).
2. Get the upper bound and lower bound of the error derivatives of the goal and control systems without any controller, i.e. $-Z_m \le \dot{e}_m \le Z_m$, which are used to be the constant controllers.
3. Design the membership functions of positive (P), negative (N) and zero (Z) $M_1 = \left|\dot{e}_m\right|/Z_m$, $M_2 = \left|\dot{e}_m\right|/Z_m$ and $M_3 = sgn((Z_m - \dot{e}_m)/Z_m) + sgn((\dot{e}_m - Z_m)/Z_m)$.

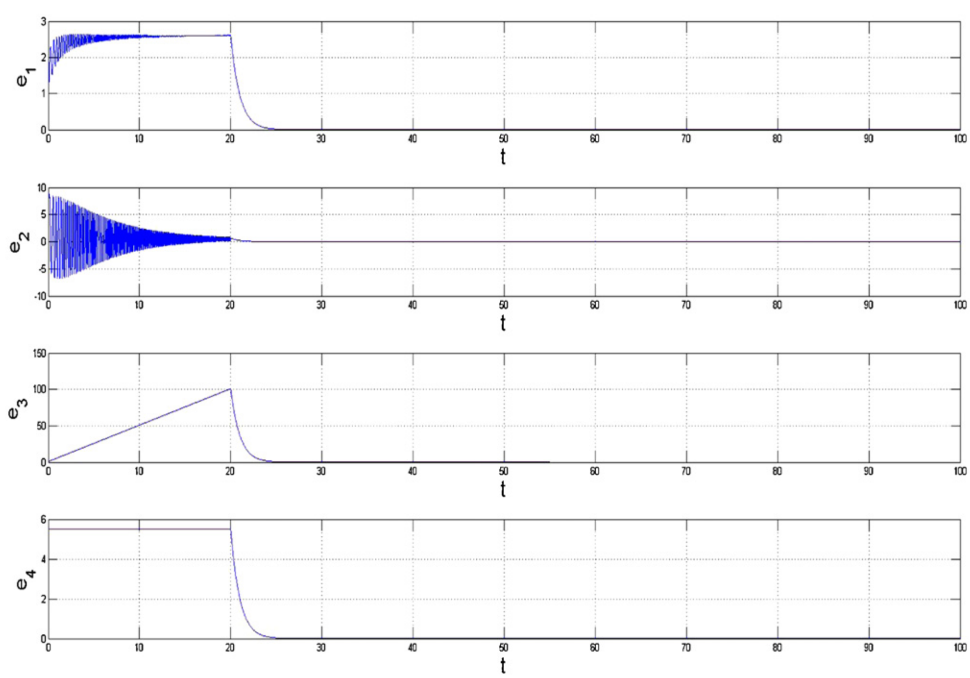

Fig. 8. Time histories of errors for Case 1—by nonlinear controllers (20 s).

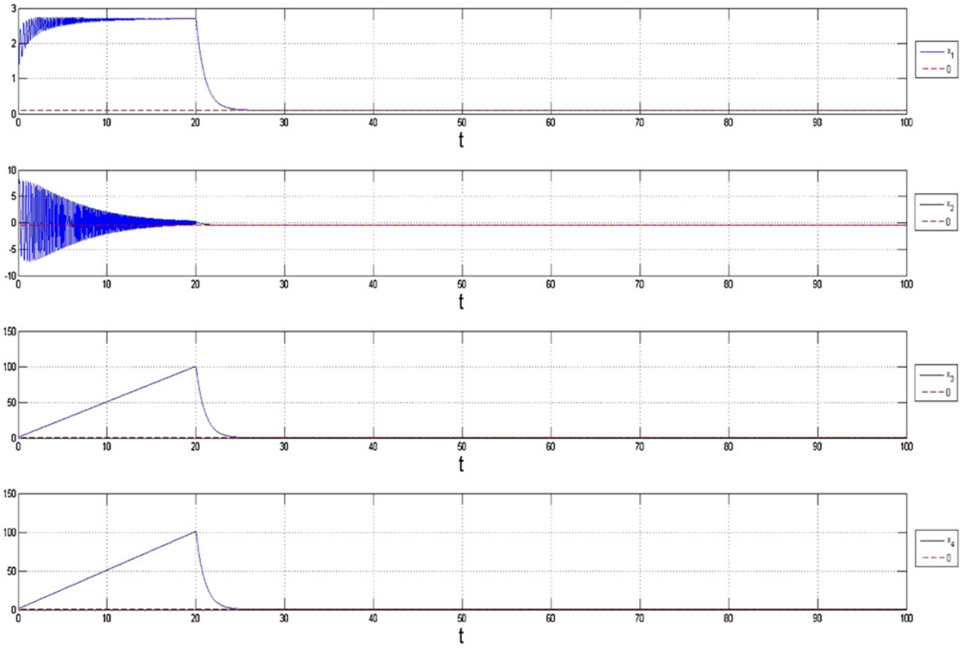

Fig. 9. Time histories of states for Case 1—by nonlinear controllers (20 s).

4. A negative definite of derivatives of Lyapunov function $\dot{V}$ can be obtained and the asymptotically stability of Lyapunov theorem can be achieved.

3. New chaotic Mathieu–van der Pol system

This section introduces new Mathieu–van der Pol system. Mathieu equation and van der Pol equation are two typical nonlinear non-autonomous systems:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -(a + b\ \sin\omega t)x_1 - (a + b\sin\omega t)x_1^3 - cx_2 + d\sin\omega t \end{cases} \tag{3-1}$$

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -ex_3 + f(1 - x_3^2)x_4 + g\ \sin\omega t \end{cases} \tag{3-2}$$

Exchanging $\sin\omega t$ in Eq. (3-1) with x_3 and $\sin\omega t$ in Eq. (3-2) with x_1, we obtain the autonomous new Mathieu–van der Pol system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -(a + bx_3)x_1 - (a + bx_3){x_1}^3 - cx_2 + dx_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -ex_3 + f(1 - x_3^2)x_4 + gx_1 \end{cases} \tag{3-3}$$

where a, b, c, d, e, f, g are uncertain parameters. This system exhibits chaos when the parameters of system are $a = 10$, $b = 3$, $c = 0.4$, $d = 70$,

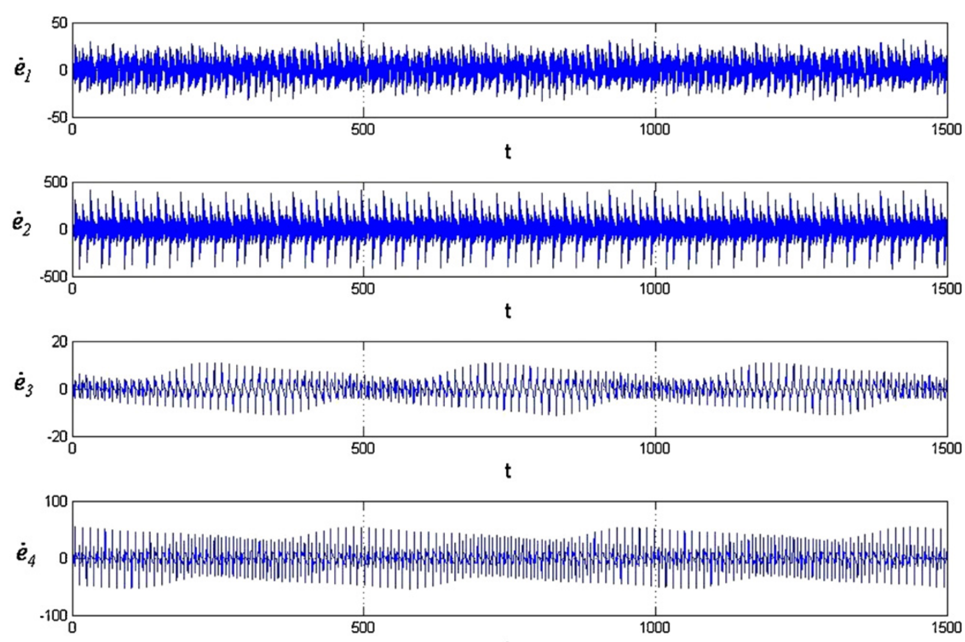

Fig. 10. Time histories of error derivatives for Case 2 (without controllers).

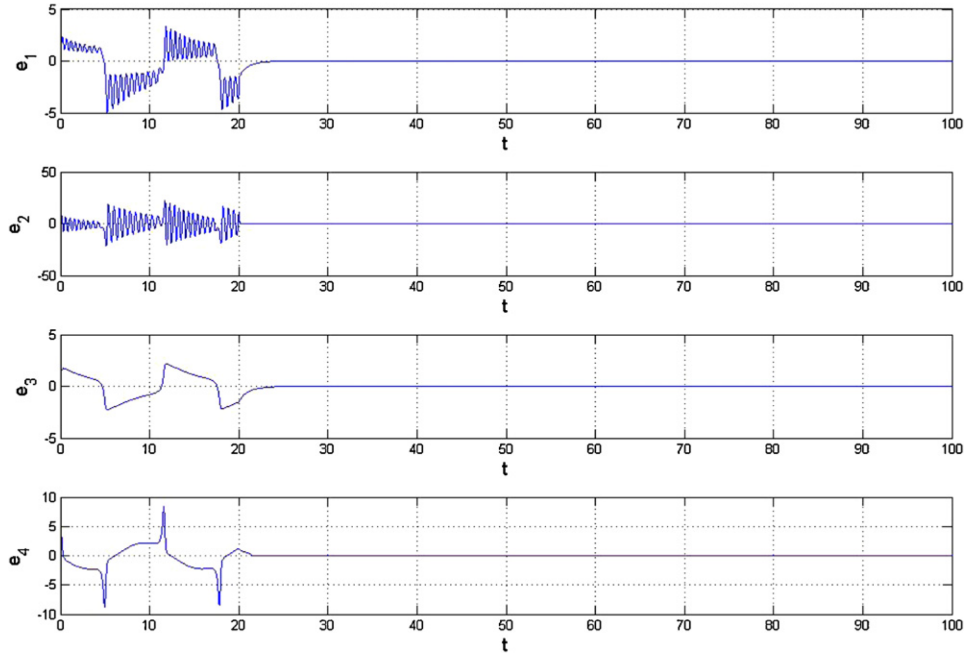

Fig. 11. Time histories of errors for Case 2—the FLCC is coming into after 20 s.

Table 2
Comparison between forms of controllers.

	FLCC (components)	Nonlinear controllers
u_1	$\pm Z_1 = \pm 50$ or $e_1 \approx 0$	$-x_2 - e_1$
u_2	$\pm Z_2 = \pm 500$ or $e_2 \approx 0$	$(a + bx_3)x_1 + (a + bx_3)x_1^3 + cx_2 - dx_3 - e_2$
u_3	$\pm Z_3 = \pm 10$ or $e_3 \approx 0$	$-x_4 - e_3$
u_4	$\pm Z_4 = \pm 10$ or $e_4 \approx 0$	$ex_3 - f(1 - x_3^2)x_4 - gx_1 - e_4$

$e = 1$, $f = 5$, $g = 0.1$ and the initial states of system are $(x_{10}, x_{20}, x_{30}, x_{40}) = (1, 5, 1, 5)$. The projections of phase portraits are shown in Fig. 4.

4. Simulation results

In order to lead (x_1, x_2, x_3, x_4) in Eq. (3-3) to the goal, we add control terms u_1, u_2, u_3 and u_4 to each equation of Eq. (3-3), respectively.

$$\begin{cases} \dot{x}_1 = x_2 + u_1 \\ \dot{x}_2 = -(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 + u_2 \\ \dot{x}_3 = x_4 + u_3 \\ \dot{x}_4 = -ex_3 + f(1 - x_3^2)x_4 + gx_1 + u_4 \end{cases} \tag{4-1}$$

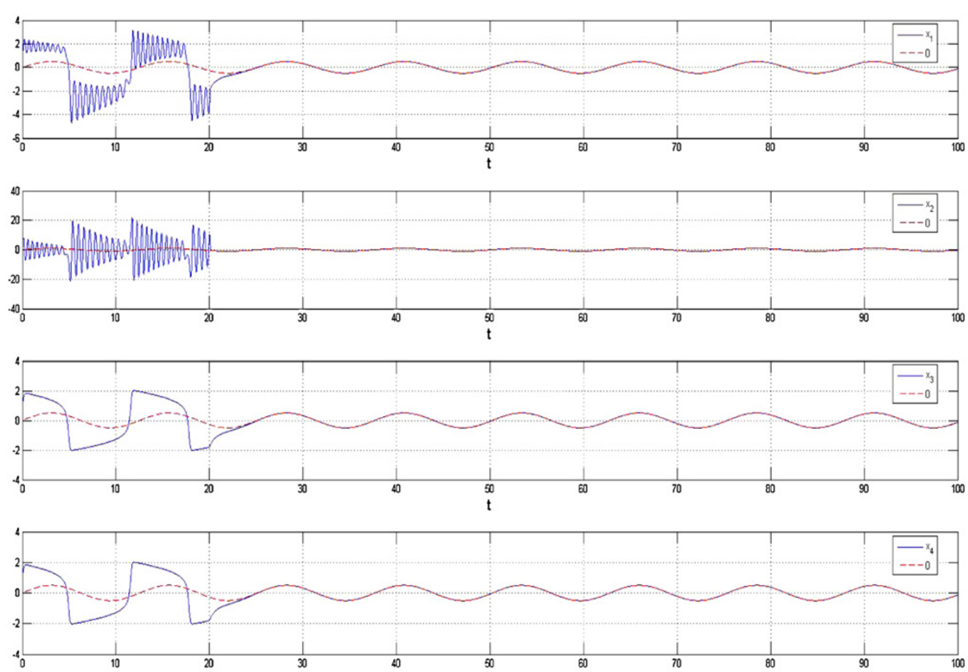

Fig. 12. Time histories of states for Case 2—the FLCC is coming into after 20 s.

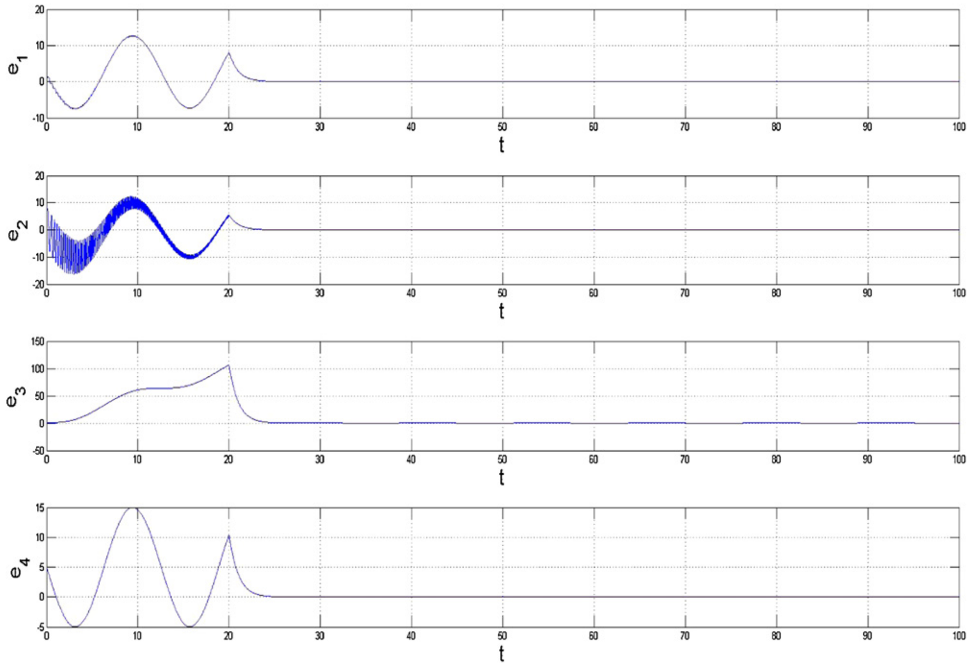

Fig. 13. Time histories of errors for Case 2—by nonlinear controllers (20 s).

Table 3
Comparison between error data at 99.96 s, 99.97 s, 99.98 s, 99.99 s and 100.00 s after the action of controllers.

Time	FLCC e_1	Nonlinear controllers e_1
99.96 s	−3.5694e−035	6.8001e−016
99.97 s	−3.5339e−035	6.8001e−016
99.98 s	−3.4987e−035	6.8001e−016
99.99 s	−3.4639e−035	6.8001e−016
100.00 s	−3.4294e−035	6.8001e−016
Time	FLCC e_2	Nonlinear controllers e_2
99.96 s	5.5438e−048	3.0531e−015
99.97 s	5.4885e−048	3.0531e−015
99.98 s	5.4340e−048	3.0531e−015
99.99 s	5.3799e−048	3.0531e−015
100.00 s	5.3264e−048	3.0531e−015
Time	FLCC e_3	Nonlinear controllers e_3
99.96 s	2.4019e−048	6.9389e−016
99.97 s	2.3780e−048	6.9389e−016
99.98 s	2.3543e−048	6.9389e−016
99.99 s	2.3309e−048	6.9389e−016
100.00 s	2.3077e−048	6.9389e−016
Time	FLCC e_4	Nonlinear controllers e_4
99.96 s	2.4710e−036	2.7756e−015
99.97 s	2.4464e−036	2.7756e−015
99.98 s	2.4221e−036	2.7756e−015
99.99 s	2.3980e−036	2.7756e−015
100.00 s	2.3741e−036	2.7756e−015

There are three cases to show the effectiveness and feasibility of the new approach—FLCC. *Case I*: Control the chaotic motion to zero, *Case II*: Control the chaotic motion to a regular function and *Case III*: Control the chaotic motion of the new Mathieu–van der Pol system to another chaotic motion of the Qi system. Furthermore, chaos control via traditional nonlinear controllers in *Cases I–III* is also given in tables and figures for comparison.

4.1. Case I: control the chaotic motion to zero by (1) FLCC and (2) traditional controllers

In this case, we will control the chaotic motion of the new Mathieu–van der Pol system (4-1) to zero. The goal is $y = 0$. The state error is $e_i = x_i - y_i = x_i$ ($i = 1, 2, 3, 4$) and error dynamics becomes

$$\begin{cases} \dot{e}_1 = \dot{x}_1 = x_2 + u_1 \\ \dot{e}_2 = \dot{x}_2 = -(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 + u_2 \\ \dot{e}_3 = \dot{x}_3 = x_4 + u_3 \\ \dot{e}_4 = \dot{x}_4 = -ex_3 + f(1 - x_3^2)x_4 + gx_1 + u_4 \end{cases} \quad (4\text{-}2)$$

Choosing Lyapunov function as:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2) \quad (4\text{-}3)$$

Its time derivative is:

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 = e_1(x_2 + u_1) \\ &\quad + e_2(-(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 + u_2) \\ &\quad + e_3(x_4 + u_3) + e_4(-ex_3 + f(1 - {x_3}^2)x_4 + gx_1 + u_4) \end{aligned} \quad (4\text{-}4)$$

(1) By FLCC

In order to design FLCC, we divide Eq. (4-3) into four parts:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2) = V_1 + V_2 + V_3 + V_4 \quad (4\text{-}5)$$

then the error derivative is

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 \quad (4\text{-}6)$$

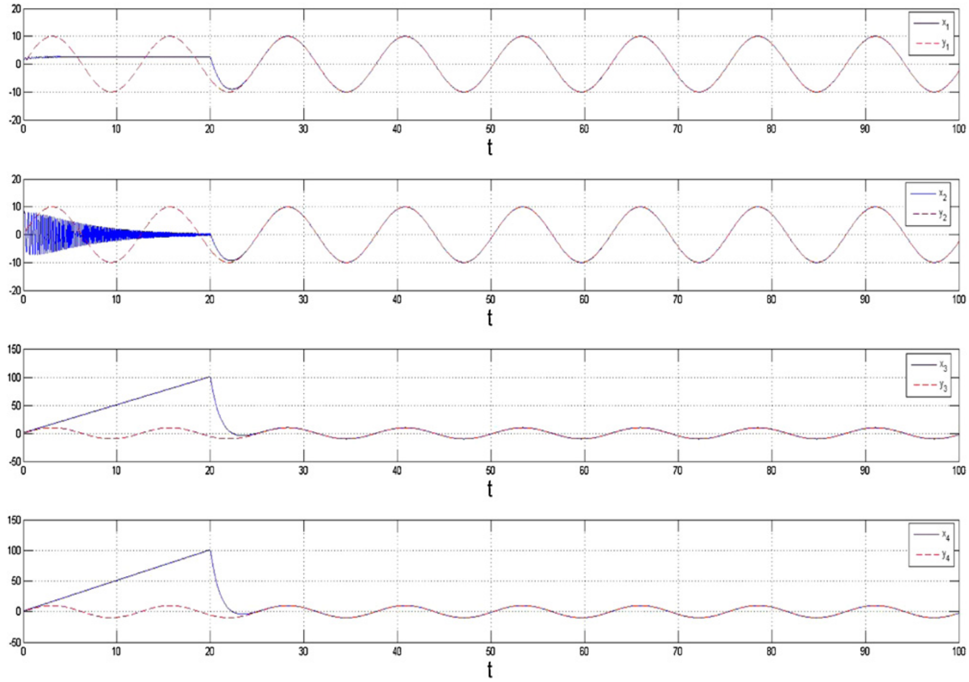

Fig. 14. Time histories of states for Case 2—by nonlinear controllers (20 s).

where

Part 1: $\dot{V}_1 = e_1\dot{e}_1 = e_1(x_2 + u_1)$
Part 2: $\dot{V}_2 = e_2\dot{e}_2 = e_2(-(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 + u_2)$
Part 3: $\dot{V}_3 = e_3\dot{e}_3 = e_3(x_4 + u_3)$
Part 4: $\dot{V}_4 = e_4\dot{e}_4 = e_4(-ex_3 + f(1 - x_3^2)x_4 + gx_1 + u_4)$

According to the process of the FLCC designing, the error derivatives of *Case I* without any controller are shown in Fig. 5, and the values of the upper bound and lower bound can be obtained as follows:

$$Z_1 = 50, \quad Z_2 = 500, \quad Z_3 = 10, \quad Z_4 = 10 \tag{4-7}$$

After getting $Z_1 \sim Z_4$, all the constant controllers, membership functions and the corresponding FLCC can be decided. Therefore, $\dot{V}_1 = e_1\dot{e}_1 < 0$, $\dot{V}_2 = e_2\dot{e}_2 < 0$, $\dot{V}_3 = e_3\dot{e}_3 < 0$ and $\dot{V}_4 = e_4\dot{e}_4 < 0$ can be achieved, then we have $\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 < 0$. It is clear that the derivative of Lyapunov function is negative definite and the

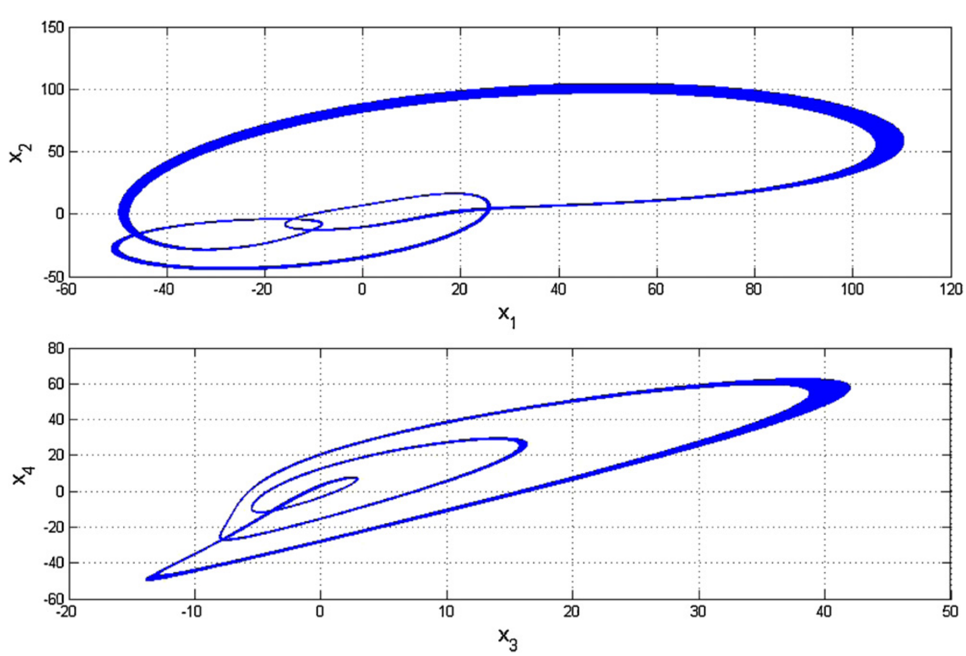

Fig. 15. Projections of phase portrait of chaotic Qi system with $a_1 = 30$, $b_1 = 10$, $c_1 = 1$, $d_1 = 10$.

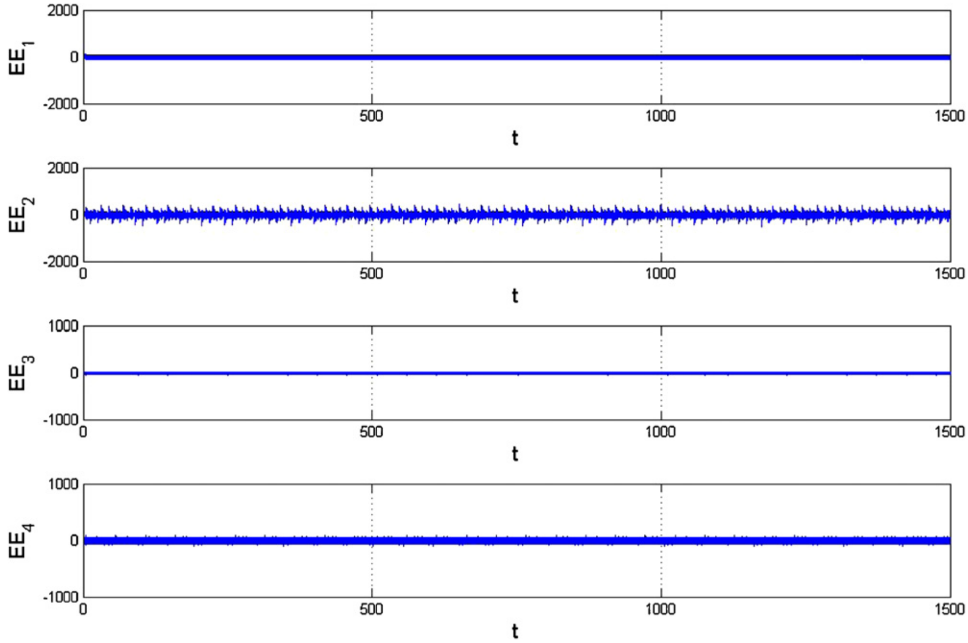

Fig. 16. Time histories of error derivatives for Case 3 (without controllers).

error dynamics system is going to achieve asymptotically stable. The simulation results are shown in Figs. 6 and 7.

(2) By traditional nonlinear controllers

In order to lead the error dynamics in Eq. (4-4) to achieve asymptotically stable, we have the following nonlinear controllers by traditional method as:

$$\begin{cases} u_1 = -x_2 - e_1 \\ u_2 = (a + bx_3)x_1 + (a + bx_3)x_1^3 + cx_2 - dx_3 - e_2 \\ u_3 = -x_4 - e_3 \\ u_4 = +ex_3 - f(1 - x_3^2)x_4 - gx_1 - e_4 \end{cases} \tag{4-8}$$

substituting all the nonlinear controllers in Eq. (4-8) into Eq. (4-4):

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 < 0 \tag{4-9}$$

which is a negative definite function and the error dynamics system is going to achieve asymptotically stable. The simulation results are shown in Figs. 8 and 9.

4.1.1. *Comparison with FLCC and traditional method*

In this section, numerical simulation results by FLCC and traditional controllers are listed in Tables 2 and 3 for comparison. Comparing the two simulation results in Tables 2 and 3, it is clear to find out that (1) the performance (accuracy and speed of convergence) of the states of errors converging to the original point by

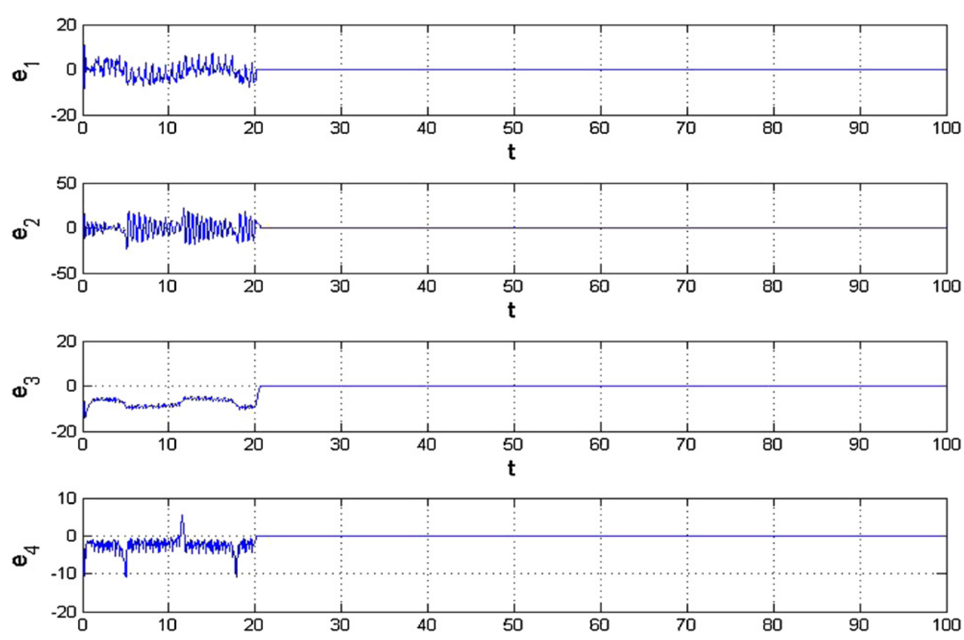

Fig. 17. Time histories of errors for Case 3—the FLCC is coming into after 20 s.

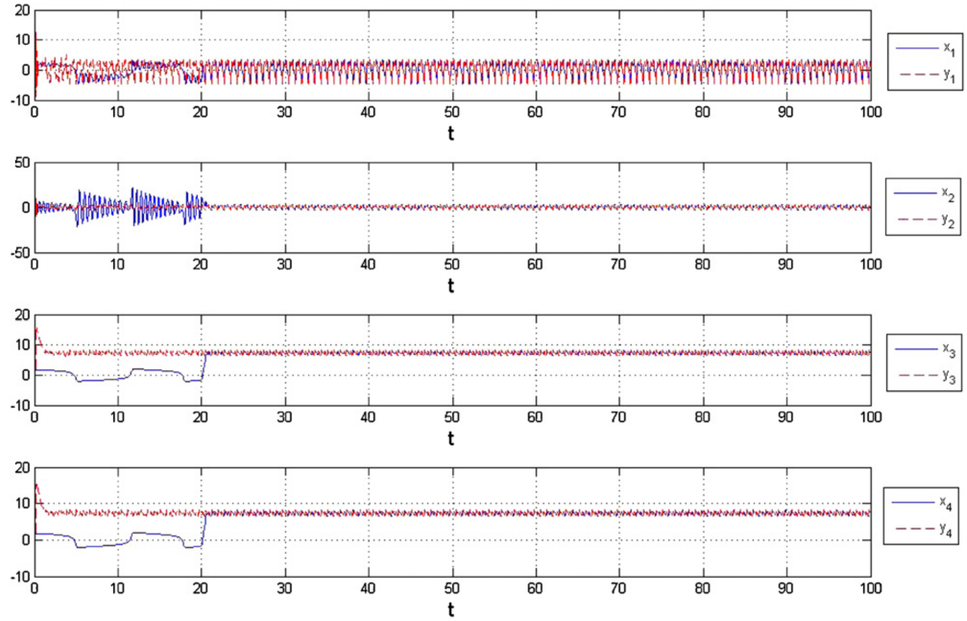

Fig. 18. Time histories of states for Case 3—the FLCC is coming into after 20 s.

Table 4
Comparison between forms of controllers.

	FLCC (components)	Nonlinear controllers
u_1	$\pm Z_1 = \pm 50$ or $e_1 \approx 0$	$-x_2 + \omega \times F_1 \cos \omega t - e_1$
u_2	$\pm Z_2 = \pm 500$ or $e_2 \approx 0$	$+(a + bx_3)x_1 + (a + bx_3)x_1^3 + cx_2 + dx_3 + \omega \times F_2 \cos \omega t - e_2$
u_3	$\pm Z_3 = \pm 20$ or $e_3 \approx 0$	$-x_4 + \omega \times F_3 \cos \omega t - e_3$
u_4	$\pm Z_4 = \pm 100$ or $e_4 \approx 0$	$ex_3 - f(1 - x_3^2)x_4 - gx_1 + \omega \times F_4 \cos \omega t - e_4$

Table 5
Comparison between error data at 29.96 s, 29.97 s, 29.98 s, 29.99 s and 30.00 s after the action of controllers.

Time	FLCC e_1	Nonlinear controllers e_1
29.96 s	0.00031450	0.0065468
29.97 s	0.00031388	0.0065308
29.98 s	0.00031325	0.0065146
29.99 s	0.00031262	0.0064984
30.00 s	0.00031197	0.0064820
Time	FLCC e_2	Nonlinear controllers e_2
29.96 s	0.00064831	0.0064258
29.97 s	0.00064702	0.0064110
29.98 s	0.00064572	0.0063961
29.99 s	0.00064440	0.0063810
30.00 s	0.00064306	0.0063658
Time	FLCC e_3	Nonlinear controllers e_3
29.96 s	0.00037285	0.0112380
29.97 s	0.00037216	0.0111750
29.98 s	0.00037146	0.0111130
29.99 s	0.00037075	0.0110510
30.00 s	0.00037003	0.0109890
Time	FLCC e_4	Nonlinear controllers e_4
29.96 s	0.00142410	0.0066563
29.97 s	0.00142150	0.0066392
29.98 s	0.00141900	0.0066220
29.99 s	0.00141630	0.0066047
30.00 s	0.00141370	0.0065873

FLCC is much better than the performance by traditional ones; (2) the controllers in FLCC designing are much simpler than traditional ones.

Consequently, even the system is so complicated and the controllers provided in FLCC are such a constant ones, the high performance and exact numerical simulation results can be still remained.

4.2. Case II: control the chaotic motion to a regular function by (1) FLCC and (2) traditional controllers

In this case we will control the chaotic motion of the new Mathieu–van der Pol system (4-1) to regular function of time. The goal is $y_i = F_i \sin \omega t$ $(i = 1, 2, 3, 4)$. The error equation

$$e_i = x_i - y_i = x_i - F_i \sin \omega t, \quad (i = 1, 2, 3, 4) \tag{4-10}$$

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty} (x_i - F_i \sin \omega t) = 0, \quad (i = 1, 2, 3, 4)$$

where $F_1 = 0.5$, $F_2 = 1$, $F_3 = 0.5$, $F_4 = 2$ and $\omega = 0.5$

The error dynamics is

$$\begin{cases} \dot{e}_1 = x_2 - \omega \times F_1 \cos \omega t + u_1 \\ \dot{e}_2 = -(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 - \omega \times F_2 \cos \omega t + u_2 \\ \dot{e}_3 = x_4 - \omega \times F_3 \cos \omega t + u_3 \\ \dot{e}_4 = -ex_3 + f(1 - x_3^2)x_4 + gx_1 - \omega \times F_4 \cos \omega t + u_4 \end{cases} \tag{4-11}$$

Choosing Lyapunov function as:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2) \tag{4-12}$$

Its time derivative is:

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 = e_1(x_2 - \omega \times F_1 \cos \omega t + u_1) \\ &\quad + e_2(-(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 - \omega \\ &\quad \times F_2 \cos \omega t + u_2) + e_3(x_4 - \omega \times F_3 \cos \omega t + u_3) \\ &\quad + e_4(-ex_3 + f(1 - x_3^2)x_4 + gx_1 - \omega \times F_4 \cos \omega t + u_4) \end{aligned} \tag{4-13}$$

(1) By FLCC

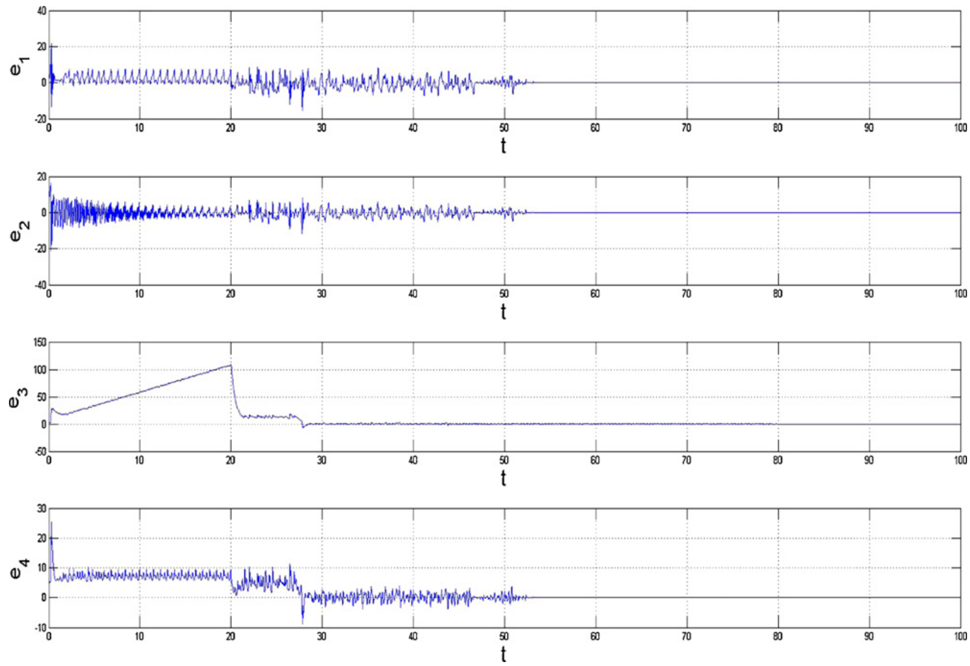

Fig. 19. Time histories of errors for Case 3—by nonlinear controllers (20 s).

In order to design FLCC, we divide Eq. (4-13) into four parts as follows:

Assume $V = (1/2)(e_1^2 + e_2^2 + e_3^2 + e_4^2) = V_1 + V_2 + V_3 + V_4$, then $\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4$, where $V_1 = (1/2)e_1^2$, $V_2 = (1/2)e_2^2$, $V_3 = (1/2)e_3^2$ and $V_4 = (1/2)e_4^2$.

Part 1: $\dot{V}_1 = e_1\dot{e}_1 = e_1(x_2 - \omega \times F_1 \cos\omega t + u_1)$
Part 2: $\dot{V}_2 = e_2\dot{e}_2 = e_2(-(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 - \omega \times F_2 \cos\omega t + u_2)$
Part 3: $\dot{V}_3 = e_3\dot{e}_3 = e_3(x_4 - \omega \times F_3 \cos\omega t + u_3)$
Part 4: $\dot{V}_4 = e_4\dot{e}_4 = e_4(-ex_3 + f(1 - x_3^2)x_4 + gx_1 - \omega \times F_4 \cos\omega t + u_4)$

According to the process of the FLCC designing, the error derivatives of *Case II* without any controller are shown in Fig. 10, and the values of the upper bound and lower bound can be obtained as follows:

$$Z_1 = 50, \quad Z_2 = 500, \quad Z_3 = 20, \quad Z_4 = 100 \tag{4-14}$$

After getting $Z_1 \sim Z_4$, all the constant controllers, membership functions and the corresponding FLCC can be decided. Therefore, $\dot{V}_1 = e_1\dot{e}_1 < 0$, $\dot{V}_2 = e_2\dot{e}_2 < 0$, $\dot{V}_3 = e_3\dot{e}_3 < 0$ and $\dot{V}_4 = e_4\dot{e}_4 < 0$ can be achieved, then we have $\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 < 0$. It is clear that the derivative of Lyapunov function is negative definite and the error dynamics system is going to achieve asymptotically stable. The simulation results are shown in Figs. 11 and 12.

(2) By traditional nonlinear controllers

In order to lead the error dynamics in Eq. (4-11) to achieve asymptotically stable, we have the following nonlinear controllers by traditional method as:

$$\begin{cases} u_1 = -x_2 + \omega \times F_1 \cos\omega t - e_1 \\ u_2 = +(a + bx_3)x_1 + (a + bx_3)x_1^3 + cx_2 + dx_3 + \omega \times F_2 \cos\omega t - e_2 \\ u_3 = -x_4 + \omega \times F_3 \cos\omega t - e_3 \\ u_4 = ex_3 - f(1 - x_3^2)x_4 - gx_1 + \omega \times F_4 \cos\omega t - e_4 \end{cases} \tag{4-15}$$

substituting all the nonlinear controllers in Eq. (4-15) into Eq. (4-13):

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 < 0 \tag{4-16}$$

which is a negative definite function and the error dynamics system is going to achieve asymptotically stable. The simulation results are shown in Figs. 13 and 14.

4.2.1. Comparison with FLCC and traditional method

Numerical simulation results by FLCC and traditional controllers are listed in Tables 4 and 5 for comparison. It can be found out that (1) the performance (accuracy and speed of convergence) of the states of errors converging to the original point by FLCC is much better than the performance by traditional ones; (2) the controllers in FLCC designing are much simpler than traditional ones.

Consequently, even the system is so complicated or regular form, and the controllers provided in FLCC are such a constant ones, the high performance and exact numerical simulation results can be still remained.

4.3. Case III: control the chaotic motion of the new Mathieu–van der Pol system to another chaotic motion of the Qi system via (1) FLCC and (2) traditional controllers

In Case III, we will control chaotic motion of the new Mathieu–van der Pol system (4-1) to that of the Qi system. The goal system for control is Qi system shown as follows:

$$\begin{cases} \dot{y}_1 = a_1(y_2 - y_1) + y_2y_3y_4 \\ \dot{y}_2 = b_1(y_1 + y_2) - y_1y_3y_4 \\ \dot{y}_3 = -c_1y_3 + y_1y_2y_4 \\ \dot{y}_4 = -d_1y_4 + y_1y_2y_3 \end{cases} \tag{4-17}$$

where y_1, y_2, y_3 and y_4 are the state variables of the system and a_1, b_1, c_1 and d_1 are all positive real parameters. This Qi system in Eq. (4-17) was recently introduced by Qi et al. [28] and it has been shown exhibit complex dynamical behavior including the familiar period-doubling route to chaos as well as hopf bifurcations. For the

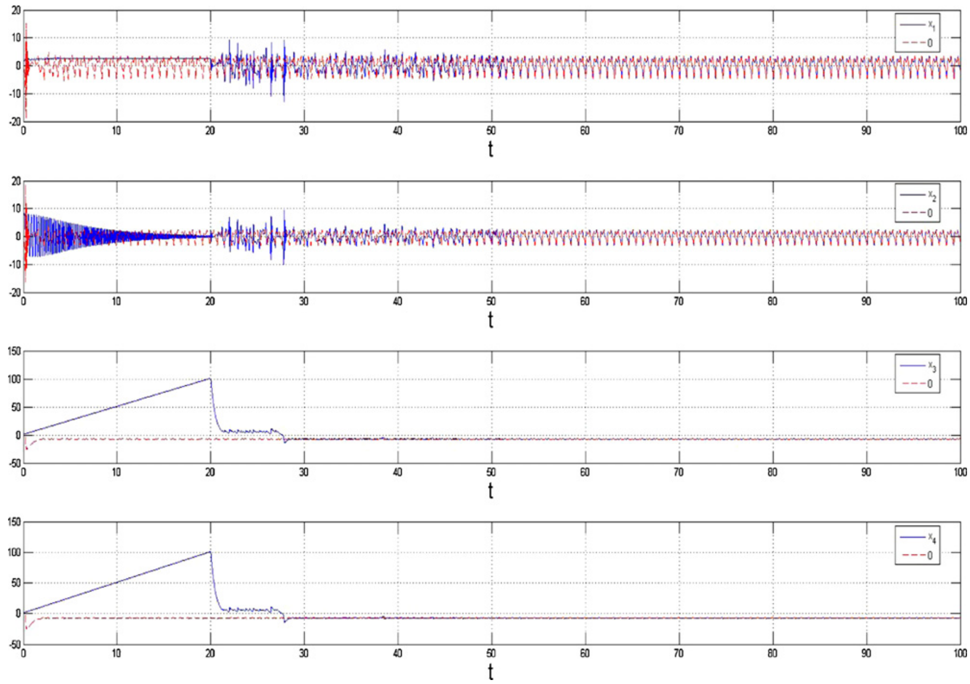

Fig. 20. Time histories of states for Case 3—by nonlinear controllers (20 s).

system parameters: $a_1 = 30$, $b_1 = 10$, $c_1 = 1$, $d_1 = 10$ and initial conditions $(y_{10}, y_{20}, y_{30}, y_{40}) = (20, 50, 20, 50)$, the Qi model exhibits chaotic motion which is shown in Fig. 15.

The error equation

$$e_i = x_i - y_i, \quad (i = 1, 2, 3, 4) \tag{4-18}$$

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty} (x_i - y_i) = 0, \quad (i = 1, 2, 3, 4)$$

The error dynamics is

$$\begin{cases} \dot{e}_1 = \dot{x}_1 = x_2 - (a_1(y_2 - y_1) + y_2y_3y_4) + u_1 \\ \dot{e}_2 = \dot{x}_2 = -(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 - (b_1(y_1 + y_2) - y_1y_3y_4) + u_2 \\ \dot{e}_3 = \dot{x}_3 = x_4 - (-c_1y_3 + y_1y_2y_4) + u_3 \\ \dot{e}_4 = \dot{x}_4 = -ex_3 + f(1 - x_3^2)x_4 + gx_1 - (-d_1y_4 + y_1y_2y_3) + u_4 \end{cases} \tag{4-19}$$

Choosing Lyapunov function as:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2) \tag{4-20}$$

Its time derivative is:

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 = e_1(x_2 - (a_1(y_2 - y_1) \\ &\quad + y_2y_3y_4) + u_1) + e_2(-(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 \\ &\quad - (b_1(y_1 + y_2) - y_1y_3y_4) + u_2) + e_3(x_4 - (-c_1y_3 + y_1y_2y_4) \\ &\quad + u_3) + e_4(-ex_3 + f(1 - x_3^2)x_4 + gx_1 - (-d_1y_4 + y_1y_2y_3) + u_4) \end{aligned} \tag{4-21}$$

(1) By FLCC

In order to design FLCC, we divide Eq. (4-12) into four parts as follows:

Assume $V = (1/2)(e_1^2 + e_2^2 + e_3^2 + e_4^2) = V_1 + V_2 + V_3 + V_4$, then $\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4$, where $V_1 = (1/2)e_1^2$, $V_2 = (1/2)e_2^2$, $V_3 = (1/2)e_3^2$ and $V_4 = (1/2)e_4^2$.

Part 1: $\dot{V}_1 = e_1\dot{e}_1 = e_1(x_2 - (a_1(y_2 - y_1) + y_2y_3y_4) + u_1)$
Part 2: $\dot{V}_2 = e_2\dot{e}_2 = e_2(-(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 - (b_1(y_1 + y_2) - y_1y_3y_4) + u_2)$
Part 3: $\dot{V}_3 = e_3\dot{e}_3 = e_3(x_4 - (-c_1y_3 + y_1y_2y_4) + u_3)$
Part 4: $\dot{V}_4 = e_4\dot{e}_4 = e_4(-ex_3 + f(1 - x_3^2)x_4 + gx_1 - (-d_1y_4 + y_1y_2y_3) + u_4)$

According to the process of the FLCC designing, the error derivatives of *Case III* without any controller are shown in Fig. 16, and the values of the upper bound and lower bound can be obtained as follows:

$$Z_1 = 2000, \quad Z_2 = 2000, \quad Z_3 = 1000, \quad Z_4 = 1000 \tag{4-22}$$

After getting $Z_1 \sim Z_4$, all the constant controllers, membership functions and the corresponding FLCC can be decided. Therefore, $\dot{V}_1 = e_1\dot{e}_1 < 0$, $\dot{V}_2 = e_2\dot{e}_2 < 0$, $\dot{V}_3 = e_3\dot{e}_3 < 0$ and $\dot{V}_4 = e_4\dot{e}_4 < 0$ can be achieved, then we have $\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 < 0$. It is clear that the derivative of Lyapunov function is negative definite and the error dynamics system is going to achieve asymptotically stable. The simulation results are shown in Figs. 17 and 18.

(2) By traditional nonlinear controllers

In order to lead the error dynamics in Eq. (4-19) to achieve asymptotically stable, we have the following nonlinear controllers by traditional method as:

$$\begin{cases} u_1 = -x_2 + (a_1(y_2 - y_1) + y_2y_3y_4) - e_1 \\ u_2 = (a + bx_3)x_1 + (a + bx_3)x_1^3 + cx_2 - dx_3 + (b_1(y_1 + y_2) - y_1y_3y_4) - e_2 \\ u_3 = -x_4 + (-c_1y_3 + y_1y_2y_4) - e_3 \\ u_4 = +ex_3 - f(1 - x_3^2)x_4 - gx_1 + (-d_1y_4 + y_1y_2y_3) - e_4 \end{cases} \tag{4-23}$$

Table 6
Comparison between forms of controllers.

	FLCC (components)	Nonlinear controllers
u_1	$\pm Z_1 = \pm 2000$ or $e_1 \approx 0$	$-x_2 + (a_1(y_2 - y_1) + y_2y_3y_4) - e_1$
u_2	$\pm Z_2 = \pm 2000$ or $e_2 \approx 0$	$(a + bx_3)x_1 + (a + bx_3)x_1^3 + cx_2 - dx_3 + (b_1(y_1 + y_2) - y_1y_3y_4) - e_2$
u_3	$\pm Z_3 = \pm 1000$ or $e_3 \approx 0$	$-x_4 + (-c_1y_3 + y_1y_2y_4) - e_3$
u_4	$\pm Z_4 = \pm 1000$ or $e_4 \approx 0$	$ex_3 - f(1 - x_3^2)x_4 - gx_1 + (-dy_4 + y_1y_2y_3) - e_4$

Table 7
Comparison between error data at 59.96 s, 59.97 s, 59.98 s, 59.99 s and 60.00 s after the action of controllers.

Time	FLCC e_1	Nonlinear controllers e_1
59.96 s	−7.0533e−011	−1.5015e−004
59.97 s	−6.9883e−011	−1.7386e−004
59.98 s	−6.9228e−011	−1.9902e−004
59.99 s	−6.8573e−011	−2.2194e−004
60.00 s	−6.7920e−011	−2.3330e−004
Time	**FLCC e_2**	**Nonlinear controllers e_2**
59.96 s	4.1470e−012	−1.3996e−004
59.97 s	4.1160e−012	−1.3715e−004
59.98 s	4.0852e−012	−1.2018e−004
59.99 s	4.0544e−012	−7.9564e−005
60.00 s	4.0239e−012	−2.2182e−006
Time	**FLCC e_3**	**Nonlinear controllers e_3**
59.96 s	0	9.1480e−007
59.97 s	0	−1.5572e−005
59.98 s	0	−4.0608e−005
59.99 s	0	−7.6908e−005
60.00 s	0	−1.2420e−004
Time	**FLCC e_4**	**Nonlinear controllers e_4**
59.96 s	−3.0018e−011	−9.9919e−005
59.97 s	−2.9837e−011	−1.3461e−004
59.98 s	−2.9644e−011	−1.7607e−004
59.99 s	−2.9441e−011	−2.2263e−004
60.00 s	−2.9231e−011	−2.6721e−004

substituting all the nonlinear controllers in Eq. (4-23) into Eq. (4-21):

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 < 0 \tag{4-24}$$

which is a negative definite function and the error dynamics system is going to achieve asymptotically stable. The simulation results are shown in Figs. 19 and 20.

4.3.1. Comparison with FLCC and traditional method

In this case, the new Mathieu–van der Pol system is controlled to another chaotic motion—the Qi system with large initial conditions $(y_{10}, y_{20}, y_{30}, y_{40}) = (20, 50, 20, 50)$. This case is illustrated to investigate the effectiveness and feasibility of the FLCC even if the two chaotic trajectories are far from each other. According to Figs. 17 and 19, the speed of controlling the error states to the original points via FLCC (about at 30 s) is much faster than the speed via nonlinear controller (about at 60 s). As a results, the numerical data in 59.96–60.00 s are further proposed for comparison.

Numerical simulation results by FLCC and traditional controllers are listed in Tables 6 and 7 for comparison. The two main superiorities still exist—(1) the performance (accuracy and speed of convergence) of the convergence of error states by FLCC is much better than by traditional method; (2) the controllers in FLCC designing are much simpler than traditional ones.

5. Conclusions

In this paper, a simplest controller—fuzzy logic constant controller (FLCC) is introduced to chaos control. The illustrations mentioned above demonstrate the better performance and accuracy of numerical simulation results of the synchronization via FLCC clearly even the fuzzy controllers are only a simple form of constant numbers.

As a result, three main contributions can be concluded—(1) high performance of the convergence of error states in synchronization; (2) the strength of the fuzzy controllers can be adjusted via membership functions; (3) fuzzy logic controllers are easy to produce. Furthermore, due to the characters of FLCC, the mathematical models of domain systems can be unknown, all we have to do is capturing the output signals, constructing the fuzzy logic system and calculating the area of the error derivatives, then we can control any output signal to another one. Hence, FLCC is such a potential tool and can be applied to various kinds of fields with lots of unknown functions—such as neuroscience, un-model bio-systems, and complicated brain network.

Acknowledgment

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