# Tunable liquid crystal lens for a holographic projection system

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### ABSTRACT

The tunable liquid crystal (LC) lens designed for a holographic projection system is demonstrated. By using a single patterned electrode LC lens, a solid lens and an encoded Fresnel lens on the LCoS panel, we can maintain the image size of the holographic projector with different wavelengths ( $\lambda$ :674nm, 532nm and 445nm). The zoom ratio of the holographic projection system depends on the lens power of the solid lens and the tunable lens power of the LC lens. The optical zoom function can help to solve the image size mismatching problem of the holographic projection system.

Keywords: Liquid-crystal lens; Holographic projection system,.

## 1. INTRODUCTION

The holographic projection system uses diffraction to generate an image.<sup>1-3</sup> Compared to a conventional projector generating images by amplitude modulated microdisplay, the holographic projector uses a spatial light modulator (SLM) which can modulate the phase of the incident light to generate the image by diffraction. Therefore, the holographic projection system has the possibility to generate a real 3D image.<sup>4</sup> However, the diffraction process is sensitive to the wavelength of the source, unlike the conventional projector. For a given phase-only Fourier hologram on the SLM, the image size will be proportional to the wavelength of the source, leading to image mismatch for different wavelengths<sup>5-7</sup>. In order to realize a color holographic projected image, the image size should be maintained with different light sources. In this paper, we will demonstrate that the image size can be maintained at different wavelengths ( $\lambda$ :674nm, 532nm and 445nm) by the use of a LC lens.

#### 2. OPERATING MECHANISM

In order to realize a holographic projection system with same image size at different wavelengths, we add a LC lens and a solid lens to a holographic projector. By correct choice of the lens power of the solid lens, we can maintain the image size in spite of the small tunable lens power of the LC lens (i.e. smaller than  $1.75m^{-1}$ ). The structure of the holographic system consists of 3 laser diodes ( $\lambda = 674nm$ , 532nm, and 445nm), 3 solid lenses, a mirror, a red light reflector, a beam splitter, a reflective liquid crystal on silicon (LCoS) panel and a LC lens combined with a solid lens, as shown in Fig. 1(a). The solid lenses in front of the laser diodes are used to calibrate the distance between the effective

Liquid Crystals XVII, edited by Iam Choon Khoo, Proc. of SPIE Vol. 8828, 88281B © 2013 SPIE · CCC code: 0277-786X/13/\$18 · doi: 10.1117/12.2024037 laser point sources and the LCoS panel. The mirror, red light reflector and beam splitter are used to guide the laser beams of the three different lasers on the same optical path. The laser beams are reflected by an LCoS panel and then pass through the LC lens and the solid lens to the observation plane. The polarization of the laser beams should be parallel to the rubbing direction of the LCoS panel and the LC lens. We put a screen in the observation plane to observe the diffraction pattern.



Fig. 1(a) The structure of the holographic projection system using a liquid crystal lens. (b) The schematic of effective optical system of (a)  $d_1$  is the effective radius of curvature of the incident light. The wavelengths of the laser diodes are 445nm, 532nm and 674nm.

The effective optical system of Fig. 1(a) is shown in Fig. 1(b), where we have omitted the solid lenses, the mirror, the red light reflector and the beam splitter. The distance  $d_1$  is an effective distance between the point source and the LCoS panel. We chose the positions and the focal lengths of the solid lenses to make  $d_1$  the same for the three different wavelengths. The distance between the LCoS panel and the LC lens is  $d_2$ , and the distance between the observation plane and the solid lens is  $d_3$ . We assume the distance between LC lens and the solid lens is zero. Following the Nazarathy and Shamir operator method for analyzing the coherent optical system, the transform operator T of the optical system can be expressed as<sup>8,9</sup>

$$T = R[d_3]Q[-(p_{\rm LC} + p_s)]R[d_2]f(x, y)Q[-p_{\rm F}]Q[\frac{1}{d_1}],$$
(1)

where R is the operator for free-space propagation, Q is the operator for the multiplication by a quadratic-phase exponential,  $p_{LC}$  and  $p_s$  are the lens power of the LC lens and that of the solid lens respectively, f(x,y) is the Fourier hologram displayed on the LCoS panel, and  $p_F$  is the lens power of the encoded Fresnel lens. By using the relations between the operators, we can expand Eq. (1) to the following equation

$$T = Q\left[\frac{p_{LC} + p_s}{d_3 \cdot (p_{LC} + p_s) - 1} + \left(\frac{1}{1 - d_3 \cdot (p_{LC} + p_s)}\right)^2 \cdot \frac{1}{d_2 + \frac{d_3}{1 - d_3 \cdot (p_{LC} + p_s)}}\right] \cdot Ff \cdot Q\left[\frac{1}{1 - d_3 \cdot (p_{LC} + p_s)} - p_F + \frac{1}{d_1}\right].$$
(2)
$$V\left[\frac{1}{1 - d_3 \cdot (p_{LC} + p_s)} \cdot \frac{1}{\lambda(d_2 + \frac{d_3}{1 - d_3 \cdot (p_{LC} + p_s)})}\right] \cdot Ff \cdot Q\left[\frac{1}{d_2 + \frac{d_3}{1 - d_3(p_{LC} + p_s)}} - p_F + \frac{1}{d_1}\right].$$

In Eq. (2), F and V are operators for Fourier transformation and scaling by a constant, respectively. Because the first Q operator in Eq. (2) will not modulate the amplitude of the observed image, we can ignore it. To obtain the Fraunhofer diffraction pattern, the second Q operator is then set to be unity, which means that

$$p_{LC} + p_s = \frac{(d_1 + d_2 + d_3) - d_1 \cdot d_3 \cdot p_F - d_1 \cdot d_2 \cdot p_F}{d_1 \cdot d_3 + d_2 \cdot d_3 - d_1 \cdot d_2 \cdot d_3 \cdot p_F}$$
(3)

Therefore, the transform operator T can be rewritten as

$$T = V \left[ \frac{1}{\lambda (d_2 - d_2 \cdot d_3 \cdot (p_{LC} + p_s) + d_3)} \right] \cdot Ff.$$
(4)

The Fourier transform of the hologram f(x,y) will be formed at the distance  $d_3$  with the magnification M, given by:

$$\mathbf{M} = \lambda \cdot \left[ d_2 + d_3 - d_2 \cdot d_3 \cdot (p_{LC} + p_s) \right].$$
(5)

For an electrically tunable system, all the optical components in the system are fixed, which means that  $d_1$ ,  $d_2$  and  $d_3$  are constants. From Eq. (5), we can control the magnification by tuning the lens power of LC lens,  $p_{LC}$ . From Eq. (3), it follows that the power of the encoded Fresnel lens,  $p_F$ , must be adjusted correspondingly in order to maintain the second Q operator unity and therefore the projected image at the observation plane. We assume that the minimum wavelength and the maximum wavelength of the light source in the system are  $\lambda_{min}$  and  $\lambda_{max}$  respectively. In order to maintain the magnification of the system with different wavelengths, the following equation should be satisfied.

$$\lambda_{\max}(d_2 + d_3 - d_2 d_3 \cdot (p_s + p_{LC\max}(\lambda_{\max}))) = \lambda_{\min}(d_2 + d_3 - d_2 d_3 \cdot p_s),$$
(6)

where  $p_{LCmax}(\lambda_{max})$  is the maximum required lens power of the LC lens at  $\lambda_{max}$ . By judicious choice of the lens power of the solid lens, a range of maximum lens powers of the LC lens can be accommodated:

$$p_{\rm LCmax}(\lambda_{\rm max}) = \frac{\left(d_2 + d_3 - p_s \cdot d_2 \cdot d_3\right) \cdot \left(\lambda_{\rm max} - \lambda_{\rm min}\right)}{d_2 \cdot d_3 \cdot \lambda_{\rm max}}.$$
(7)

Given the maximum lens power of the LC lens at  $\lambda_{max}$ ,  $p_{LCmax}(\lambda_{max})$ , the lens power of the solid lens should be

$$p_{s} = \frac{d_{2} + d_{3}}{d_{2}d_{3}} - \frac{p_{LC\max}(\lambda_{\max})\lambda_{\max}}{(\lambda_{\max} - \lambda_{\min})}.$$
(8)

By properly choosing the lens power of the solid lens  $p_s$  and electrically tuning the lens powers  $p_{LC}$  and  $p_F$ , we can maintain the magnification of the holographic projection system with different wavelengths.

## **3. EXPERIMENTAL RESULTS AND DISCUSSION**

To demonstrate a holographic projector with the same magnification for different wavelengths, we adopt the reflective LCoS panel and a LC lens with 2 mm aperture. The resolution of the LCoS panel is 1920 x 1200 with pixel pitch 8.1µm. The structure and the fabrication process of the LC lens was proposed.<sup>10</sup> The tunable lens power of the LC lens is important for designing the system. The measured voltage-dependent lens power of the LC lens using a laser diode with maximum wavelength of the system ( $\lambda_{max} = 674$ nm) is shown in Fig. 2. The measured lens power of the LC lens can be switched from 0 to 4.07 m<sup>-1</sup> when the applied voltage increases from 0 to 120 V<sub>rms</sub>.



Figure 2 : The lens power of the LC lens as a function of the applied voltages. The wavelength of the incident light is 674nm

In the experiments, the distances  $d_1$ ,  $d_2$  and  $d_3$  in Fig. 1(b) were set at 0.5m, 0.2m and 0.5m. We used a webcam (Logitech, PR9000) to capture the image on the screen. The lens powers of the solid lenses we used with the three lasers are 3 m<sup>-1</sup>, -3 m<sup>-1</sup> and -4.5m<sup>-1</sup>. We changed the lens power of the Fresnel lens which is encoded on the LCoS panel and the voltage of the LC lens. We recorded the lens powers  $p_F$  and  $p_{LC}$  when the contrast ratio of the captured image is maximum and the magnifications of the maximum wavelength ( $\lambda_{max} = 674$ nm) and the minimum wavelength ( $\lambda_{min} = 445$ nm) are the same. The required lens power of the LC lens  $p_{LCmax}(\lambda_{ma}x)$  as a function of the lens power of the solid lens is shown in Fig. 3.



Figure 3 : The relation between the lens power of the solid lens and the required lens power of the LC lens at the maximum wavelength.

When the lens power of the solid lens increased from  $-4.5\text{m}^{-1}$  to  $3\text{m}^{-1}$ , the required lens power of LC lens decreased from  $3.72\text{m}^{-1}$  to  $1.75\text{m}^{-1}$ . If the tunable lens power of LC lens is  $1\text{m}^{-1}$ , we can still achieve magnification maintaining by using a solid lens with lens power  $4.07\text{m}^{-1}$ . Smaller tunable lens power means the cell gap of the LC lens can be thinner and the response time can be shorter. From Eq. (5) and Eq. (7), when ps approaches  $(d_1 + d_2)/(d_1 \cdot d_2)$ , the required lens power of LC lens and the magnification will approach zero. Therefore, although we can maintain the magnification by using the LC lens with small tunable lens power combined with an additional solid lens having large lens power, we still prefer to use a LC lens with large lens power in order to have a reasonably large image. The images using different wavelengths captured using the same lens power  $p_{LC} = 0$ ,  $p_s=3\text{m}^{-1}$  and  $p_F = 0.75\text{m}^{-1}$  are shown in Fig. 4.



Figure 4 : The image of Bart Simpson produced by the holographic projector using the lens power  $p_{LC} = 0$ ,  $p_s=3m^{-1}$  and  $p_F = 0.75m^{-1}$  with the wavelength of the light source (a) 674nm, (b)532nm and (c)445nm.

The ratio of the magnification between Figs. 4(a), 4(b) and 4(c) are 1.516:1.197:1 which is the same as the ratio of the wavelengths 674:532:445. It shows that if we don't use a LC lens to control the magnification, the magnification will be proportional to the wavelength. The captured images which are tuned by LC lens and encoded Fresnel lens with the same magnification are shown in Fig. 5.



Figure 5 : The image of Bart Simpson produced by the holographic projector using the lens power  $p_s=3m^{-1}$  with (a)  $\lambda = 674$ nm,  $p_{LC} = 1.75m^{-1}$ ,  $p_F = -2.469 m^{-1}$ , (b)532nm,  $p_{LC} = 1.26m^{-1}$ ,  $p_F = -0.474 m^{-1}$ , and (c)445nm,  $p_{LC} = 0m^{-1}$ ,  $p_F = 0.75m^{-1}$ ,

The required lens power of the LC lens for the maximum wavelength of this system is  $1.75m^{-1}$ . From Eq. (5), the required lens power of other wavelengths smaller than  $\lambda_{max}$  will be smaller than  $1.75m^{-1}$ . Therefore, in the design of the tunable holographic projection system, we only need to check the required lens power of the LC lens for maximum wavelength and then we can maintain the magnification for all wavelengths between  $\lambda_{max}$  and  $\lambda_{min}$ . However, based on Fig. 5, the image quality needs to be improved. In order to improve the image quality, we can optimize the LC lens with less scattering and low aberration, and also enlarge the aperture of the LC lens and choose lasers with high uniformity.

# 4. CONCLUSION

We have demonstrated a holographic projector with the same magnification for different wavelengths by adding a LC lens and a solid lens. The required lens power of the LC lens for maximum wavelength to maintain the magnification continuously decreases from  $3.72m^{-1}$  to  $1.75m^{-1}$  as the lens power of the solid lens increases from  $-4.5m^{-1}$  to  $3m^{-1}$ . We can realize the system using a LC lens with small lens power (< $1.75m^{-1}$ ) by adding a solid lens with large lens power (> $3m^{-1}$ ). The operating principle is investigated and the calculated results agree with the experimental results. The advantages of such a holographic system are the simple structure design and simple imaging process. The magnification maintaining function enhances the feasibility to realize a color-sequential holographic projection system.

This research was supported by the National Science Council (NSC) in Taiwan under the contract no. 101-2112-M-009-011-MY3 and the Graduate Students Study Abroad Program. Neil Collings was supported by the EPSRC Platform Grant "Liquid crystal photonics" (EP/F00897X/1).

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