

Simulation of precise orbit determination of COSMIC from onboard GPS zero-difference phase data with kinematic method

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ABSTRACT

COSMIC is a constellation mission to study the climate, ionosphere and geodesy. The main geodetic mission of COSMIC is to determine the global gravity field model and its temporal variations, which need the precise geometric orbit of COSMIC. GPS observations onboard COSMIC are simulated using the GPS precise final orbit and high-rate clock of CODE, the designed orbit of COSMIC and the GPS antennae for precise orbit determination (POD). The precise geometric orbits of COSMIC are determined with the kinematic method from the space-borne simulated observations to test the POD capability of GPS antennae. There are two POD GPS antennae onboard COSMIC, named as POD +X and -X. The orbit from POD -X antenna has the approximately same precision as that from POD +X antenna, and the errors of both are greater than the given random error while simulating GPS data. The main reason is that the designed positions of POD antennae are not good. There are the different angels between the boresight vector and zenith direction of two antennae. Another reason is that POD +X antenna is in the flying direction and POD -X antenna is in the inverse direction. In order to get the high precision of POD, a virtual antenna is constructed from POD +X and -X, whose center is the center of mass of COSMIC. Observations from POD +X and -X then are reduced to the virtual antenna. Comparing with the referenced orbit and the kinematic orbit from the virtual antenna, the precision of orbit is consistent to the given random error when simulating GPS data, up to centimeter level.

Keywords: COSMIC, precise kinematic orbit determination, GPS zero-difference phase data, virtual antenna

1 INTRODUCTION

COSMIC (Constellation Observing System for Meteorology, Ionosphere and Climate) is a mission to study the climate, ionosphere and geodesy, which is sponsored by the national space organization (NSPO), Taiwan, and the university corporation for atmospheric research (UCAR), USA. COSMIC will collect a large number of high-low satellite-to-satellite tracking observations to the whole earth. There are lots of GPS data onboard COSMIC, which have less mixed errors. Therefore an earth gravity field model to 50 degrees and temporal variations to 7 degrees can be recovered from COSMIC [Chao et al., 2000; Hwang, 2001]. A space-borne GPS receiver with two antennae is loaded in each satellite to determine the precise orbit. Two antennae are called POD +X and -X, respectively, and POD +X in the flying direction and POD -X in the inverse direction. There are also two GOX antennae for GPS occultation and a TBB is used to detect the ionosphere, which are used to study on the global ionospheric model and applications.

For the geodetic mission, COSMIC is mainly used to recover the earth gravity field model, which needs the precise geometric orbit. Now there are three methods commonly used to determine the orbit, which are the dynamic, reduced dynamic, and kinematic methods [Montenbruck and Gill, 2000; Tapley et al. 2004]. The dynamic method needs the complicated force models and there are many force parameters to be solved [Seeber, 1993 ; Guo et al., 2006]. The reduced dynamic method utilizes the known force models and lots of continuous tracking observations, and less force parameters are solved with the method. When there are the large number of continuous tracking observations, based on the geometric relations between observations and orbit, the satellite orbit can be determined with the kinematic method according to the least squares theory. But there appear the discontinuous phenomenon for the kinematic orbit because of the data gap, bad observations, and bad GPS satellite configuration. In the reduced dynamic method, the solution of a small quantity of force parameters has the equivalence of the optimization of time interval τ and the a prior standard

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deviation σ_a of force parameters [Wu et al. 1990 ; Visser and van den IJssel, 2003]. Therefore the reduced dynamic orbit is the balance between the a prior force models and the tracking data. In practice, the dynamic orbit is the ultimate condition for the reduced dynamic orbit, that is, $\tau = \infty$ and $\sigma_a = \mathbf{0}$. The kinematic orbit does not need the force models, which is another limiting condition of the reduced dynamic orbit, that is, $\tau = \mathbf{0}$ and $\sigma_a = \infty$.

The orbit altitudes of low-earth-orbit (LEO) satellites to detect the earth gravity field are about 200-800km, which need to be tracked continuously with GPS technique to get the precise orbits at the decimeter, or centimeter level [NRC, 1997]. There are two antennae for COSMIC to determine the precise orbit. First, the observations of COSMIC-borne GPS are simulated. Then the practical capability of POD for COSMIC with the kinematic method is checked in the paper.

2 SIMULATION OF GPS OBSERVATIONS ONBOARD COSMIC

A LEO satellite can be continuously tracked under all weather with GPS technique and the orbit can be determined with the dynamic or reduce-dynamic methods at the precision of decimeters or centimeters level [Bock, 2003; Schutz et al., 1994]. The basic GPS observations are the phase L [Leick, 2004]. The observation equations for GPS data onboard LEO satellite are

$$L_i = \rho + c(\delta_s - \delta^G) + c(\Delta r_s - \Delta r^G) - I_i + \lambda_i N_i + \varepsilon_L \quad (1)$$

where, $\rho = \left[(x_s - x^G)^2 + (y_s - y^G)^2 + (z_s - z^G)^2 \right]^{\frac{1}{2}}$ is the geometric distance between the LEO and GPS satellites. c is the light speed. δ_s is the receiver clock error. δ^G is the clock error for the GPS satellite. $\Delta r_s = -2\mathbf{r}_s \bullet \dot{\mathbf{r}}_s / c^2$ is the relative effect on the receiver clock. $\mathbf{r}_s = [x_s \ y_s \ z_s]^T$. $\dot{\mathbf{r}}_s = d\mathbf{r}_s / dt$. $\Delta r^G = -2\mathbf{r}^G \bullet \dot{\mathbf{r}}^G / c^2$ is the relative effect on the GPS satellite clock. $\mathbf{r}^G = [x^G \ y^G \ z^G]^T$. $\dot{\mathbf{r}}^G = d\mathbf{r}^G / dt$. I_i is the ionospheric delay. λ_i is the wavelength. N_i is the ambiguity. ε_L is the noise of phase data.

The GPS final orbit can be gotten with the delay of about 2 weeks, whose precision is better than 5 centimeters. The osculation orbit of COSMIC and the final GPS orbit of CODE are used to simulate GPS data. Selecting the designed orbit as the initial values, considering the tidal perturbation, solar and lunar gravitational forces, and the earth gravity field model EIGEN2 up to 70 degrees, the COSMIC orbit can be integrated with the numerical integrator based on the satellite dynamics. The study needs the clock correction at the interval of 30 seconds. CODE also provides the high rate clock data, whose precision is up to 0.1ns. The high rate clock data of CODE corresponding to the GPS final orbit is used in the paper. When simulating the GPS data, the receiver clock is thought to synchronize with the GPS clock, and the multi-path effect is neglected. In the body-fixed fame (X to the flying direction, Z to TBB, and XYZ is a right-hand coordinate system, whose origin is the mass center of satellite), the positions of POD +X and -X are shown in table 1.

The noise of pseudorange is up to 30 cm [Leick, 2004]. So when simulating GPS data, the random noise for the pseudorange is set to $\varepsilon_p = 0.3\text{m}$ and that for the phase data is $\varepsilon_L = 0.01\text{m}$. The area-to-mass ratio of COSMIC is 0.01, the altitude is 800km, and the boresight and azimuth vectors are shown in table 1. One day (Apr. 11, 2006) of GPS data are simulated for one COSMIC satellite.

Table 1 Physical parameters of antennae POD+X and -X

Antenna	X(m)	Y(m)	Z(m)	Angle ^a	Boresight(m)			Azimuth(m)		
					X	Y	Z	X	Y	Z
POD+X	0.4721	-0.0006	-0.2697	14.81	0.9667	0.0000	-0.2556	-0.9643	0.0000	-0.2647
POD-X	-0.4702	-0.0008	-0.2745	15.35	-0.2556	0.0000	-0.9667	0.2647	0.0000	-0.9643

^a This is an angle between zenith and x axis in degrees.

Figure 1 shows the number of GPS satellites visible for the POD +X and -X. The number of GPS satellites for each antenna is through 1 to 9. There are 20 percent of data less than 4 GPS satellites. The number of GPS satellites for POD -X outgo that for POD +X. The two antennae for orbit determination are located on the sides of satellite, not on the top.

The sight vector of antenna does not point to the zenith. The angle between the sight vector and the zenith for POD -X is 74.65° , which is less than that (75.19°) for POD +X. So the maximum number of GPS satellites is only up to 9.

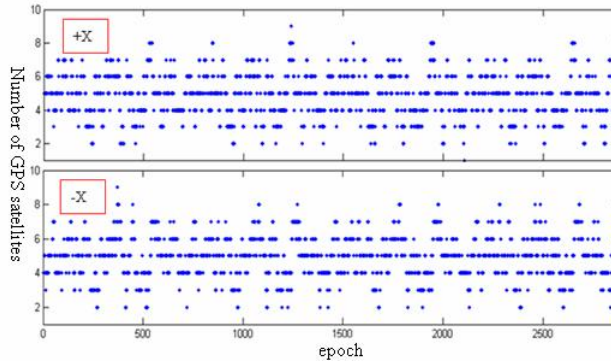


Figure 1 Number of GPS satellites visible for POD +X and -X

3 POD OF COSMIC WITH KINEMATIC METHOD

Because the ionospheric delay is the inverse ratio to the frequency square, the non-ionospheric-effect linear combination of observations LC can remove the effect of ionosphere, that is

$$LC = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2 = \rho + c(\delta_S - \delta^G) + c(\Delta r_S - \Delta r^G) + B_3 + \varepsilon_{LC} \quad (2)$$

in which, $B_3 = N_1(\lambda_1 + \lambda_2) + \frac{cf_2}{f_1^2 - f_2^2}(N_1 - N_2)$, and ε_{LC} is the noise. Suppose L_1 and L_2 with the same precision to be independent, whose precision is σ_1 . So the precision of LC is

$$\sigma_{LC} = \frac{\sigma_1 \sqrt{f_1^4 + f_2^4}}{f_1^2 - f_2^2} \approx 3\sigma_1 \quad (3)$$

(2) is the observing equation. Based on the least squares theory, the parameters including the positions and ambiguities can be solved with the precise point positioning method. Therefore at least 4 GPS satellites can be visible for COSMIC at any epoch to solve the unknown parameters.

The osculation orbit of COSMIC is known as the true orbit for the comparison with the kinematic orbit from the observing data of POD +X and -X. Figure 2 shows the orbit differences for POD +X and -X data. Table 2 shows the statistical results for the orbit differences of POD +X and -X.

The orbit precision from POD +X or -X alone is up to meter level, which is much greater than the random error of phase data. The number of GPS satellites is less than 4 at many epochs shown in figure 1. If the number of GPS satellites is less than 4, the unknown parameters cannot be solved, which causes the discontinuous orbit. If only 4 GPS satellites are visible for POD +X or -X, the reliability for the kinematic orbit is not high. There appear many peaks in figure 2, which are caused from the observing quantity and quality. So the kinematic orbit of COSMIC from only POD +X or -X has the low precision.

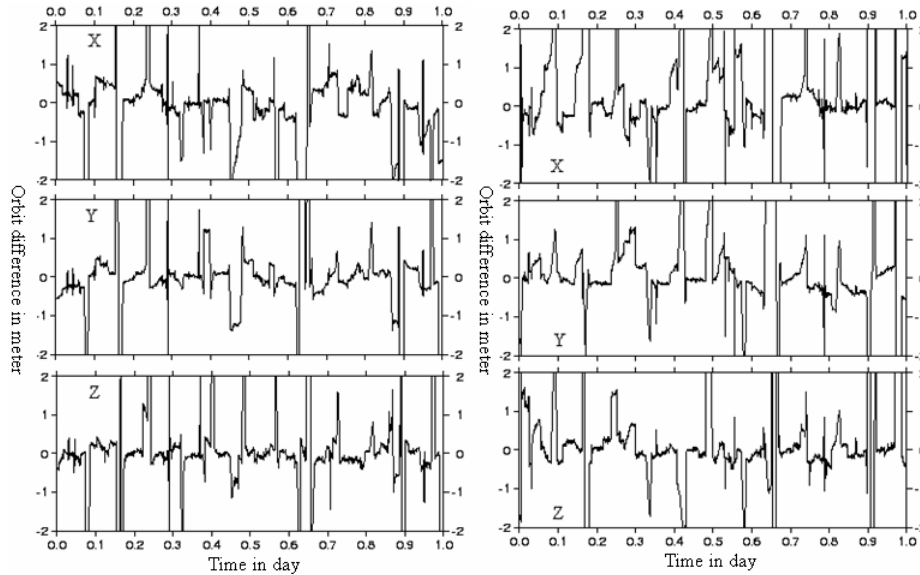


Figure 2 Comparisons between the kinematic and true orbits for POD +X (left) and -X (right)

To improve the orbit precision, the differences greater than 1m between the kinematic and true orbits are considered as the gross errors which should be deleted. The kinematic orbits at these epochs are replaced by the reduced-dynamic orbits, because the precision of the reduced-dynamic orbit is up to decimeter or centimeter level [Guo, 2006]. Then the orbit is filtered and smoothed with the Gauss filter to get the smooth and continuous orbit. Figures 3 show the differences between the smooth kinematic and referenced orbits for POD +X and -X, respectively. Table 2 shows the statistical results for the orbit comparison. So the precision of smooth orbit can be up to 0.1 meter through the Gaussian filtering.

The distance between POD +X and -X is fixed. To validate the characteristics and design for POD antennae, the positions of POD +X and -X are calculated, respectively, then the distance is solved. Comparing the solving distance with the known distance, the mean difference is 9 millimeters and the root mean squares (RMS) is $\pm 28\text{mm}$.

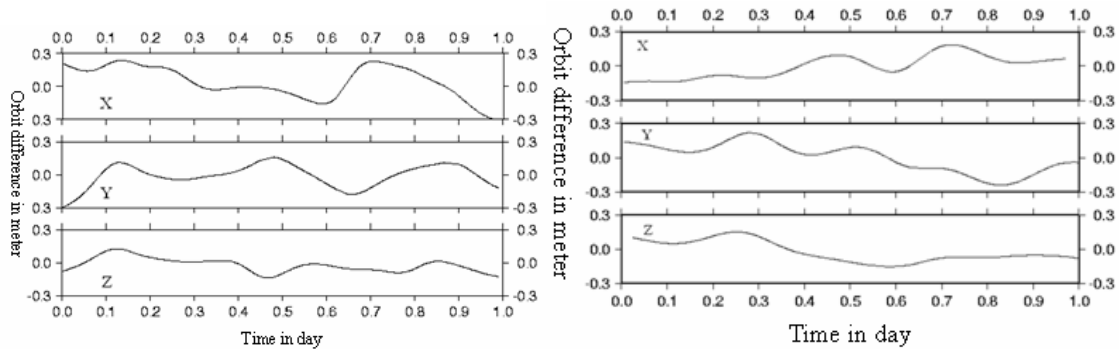


Figure 3 Comparisons between the smooth and true orbits from POD +X (left) and -X (right)

Table 2 Statistical results for comparison between kinematic and true orbits in meters before and after the Gaussian filtering

	POD +X		POD -X		POD +X ^a		POD -X ^a	
	Mean difference	STD	Mean difference	STD	Mean difference	STD	Mean difference	STD
X	-0.493	4.078	-0.258	8.058	0.065	0.083	-0.019	0.052
Y	-0.116	3.166	-0.054	2.417	-0.012	0.036	0.010	0.073
Z	0.122	11.064	1.386	14.136	-0.024	0.039	-0.024	0.035

^a These two rows give results through the Gaussian filtering

4 ORBIT IMPROVEMENT

The main reason for the bad orbit from only POD +X or -X is the shortage of observations for one antenna. To increase the observing data, the observations from POD +X and -X are combined to one referenced antenna, which is called the virtual antenna whose phase center is the COSMIC center of mass. So there are more GPS satellites visible and the observing data are increased for the antenna.

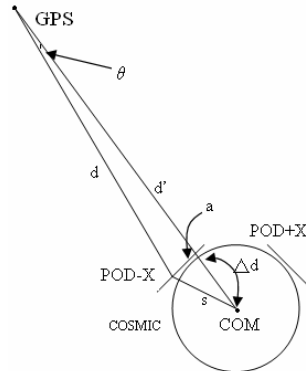


Figure 4 Reduction of POD antennae

Figure 4 shows how to make the virtual antenna from POD +X and -X, in which d is the distance between the GPS satellite and antenna center, d' is the distance between the GPS satellite and the center of mass of COSMIC, s is the distance between the antenna center and the center of mass, a is the distance between the antenna center and GPS satellite, and Δd is the distance correction for the reduction of antenna center to the center of mass. Then

$$\Delta d = \sqrt{s^2 - 2d^2 + \frac{d}{d'}(d^2 + d'^2 - s^2)} \quad (4)$$

For the GPS observation, the phase correction is

$$\Delta L_i = L_i + \Delta d / \lambda_i \quad (5)$$

The observations from POD +X and -X can be combined to be observations of the virtual antenna with the antenna reduction technique. The GPS satellite condition visible for the virtual antenna is shown in figure 5. There are at least 5 GPS satellites visible for the virtual antenna, and maximum number is up to 12. 40 percent of observations are from more than 8 GPS satellites. So the observing condition is very better than that of only POD +X or -X, which can avoid the discontinuous observing phenomenon and improve the precision and reliability for the precise orbit determination. Then the orbits are determined from the observations of virtual antenna with the kinematic method, and smoothed with the Gauss filter. Figure 6 shows the comparison between the improved orbit and the true orbit, and the statistical results are shown in table 3. From figure 5 and table 3, we can find the precision of orbit is up to 55 millimeters, which is consistent to the random error for the GPS observation. This indicates that the technique of virtual antenna can improve the kinematic orbit which can satisfy the recovery of earth gravity field and temporal variation study.

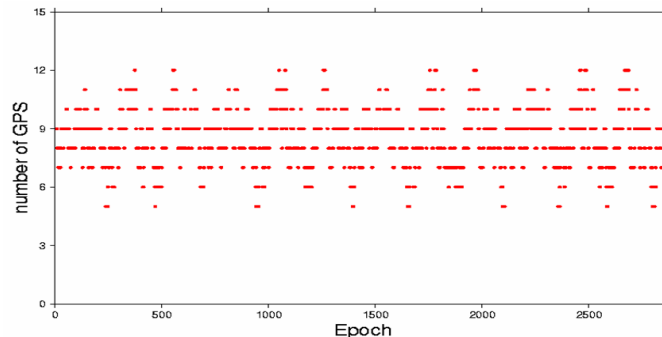


Figure 5 Number of GPS satellites visible for the virtual antenna

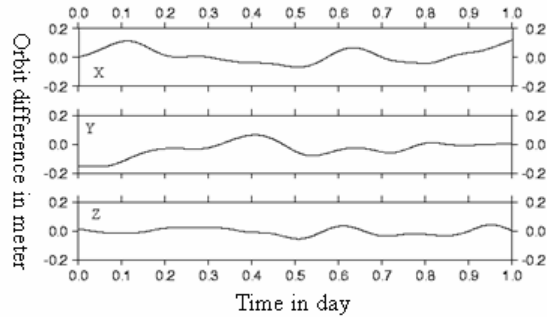


Figure 6 Comparison between the true orbit and the orbit from virtual antenna

Table 3 Statistical results for the improved kinematic orbit in meters

Coordinate	Mean difference	Standard deviation
X	0.004	0.036
Y	-0.011	0.026
Z	-0.002	0.029

5 CONCLUSIONS

To insure the mission of COSMIC, validate the capability of kinematic orbit determination, and ensure the orbit precision for the geodetic study, the simulation of POD of COSMIC using satellite-borne GPS zero-difference phase data with the kinematic method is made in the paper. The COSMIC-borne GPS data for POD +X and -X are simulated given the designed orbit, GPS final orbit and clock data of CODE, and a certain random error. The precise orbit is determined using the zero-difference non-ionosphere linear combination observation of phase data with the kinematic method. The precisions of orbit determination from only POD +X or -X are consistent, but greater than the given random error, which is made from the designed positions of POD antennae. The observations from POD +X and -X are combined to be those from a virtual antenna to improve the orbit determination with Gauss filter. The precision of orbit determination from the observations of virtual antenna is consistent to the given random error. The simulation test indicates that the kinematic method for the satellite-borne GPS phase data can satisfy the geodetic mission of COSMIC.

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