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Identification of Structural Stiffness Parameters via Wavelet Packet from Seismic Response

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Abstract

Vibration-based damage assessment approaches are conventionally and widely adopted to monitor the structural health of a building during the bases excitation process. This study explores the possibility of using structural stiffness parameters to locate the storeys of building that are damaged in an earthquake event. A damaged storeys usually exhibits stiffness parameters variations during a strong enough earthquake, and the corresponding stiffness of storey often change in the earthquake. The wavelet packet transform is applied to the measured acceleration responses of a structure to reconstruct the autoregressive with exogenous input (ARX) model in wavelet packet domain. The modal parameters of the structure are estimated directly through the identified coefficient matrixes of the ARX model. Next, the stiffness parameters of the structure that could be reconstructed form the identified natural frequencies and mode shapes would be used to detect the damage locating of a building. The accuracy of this procedure is numerically confirmed, the effects of the wavelet parameters and noise on the ability to accurately estimate the dynamic properties are also investigated.

Keywords: system identification; time series model; wavelet packet

1. Introduction

The identified structural stiffness matrix could be used to assess the health of structures directly. It is essential that obtained the modal parameters of an existing structure or bridge from the measured vibration in the field of the

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structural health monitoring system. System identification is a process to create mathematical model of structures using measurements of the dynamic responses and base excitation of structure. The modal parameters could be calculated from the determined mathematical model. Based on the properties of coefficient estimation approaches, the process of system identification are divided into two broad categories, online and offline estimation.

- Offline estimation algorithms estimate the coefficient matrix of time series model after collecting all the input and output data. These algorithms including: least square approaches [1]; maximum likelihood approaches [2]. The advantages of offline estimation algorithm claimed is well noise immunity by using long-range data. However, these kinds algorithm can't track the variation of system properties.
- Online estimation algorithms estimate the coefficient matrix of time series model when new responses is measureable during the vibration process. These algorithms including: recursive least square approaches [3]; recursive gradient approaches [4] and recursive maximum likelihood approaches [5]. The advantage of online estimation algorithms is that it can track the time-variant system properties. However, the disadvantage of these algorithms is sensitive to noise by using short-range data.

The vibration-based damage detection method, such as frequency-based method, stiffness-based method, flexibility-based and mode-shape-based method, would be use to locate the damaged storeys based on examining change in measured dynamic responses. The stiffness-based method, as the name suggests, the measured vibration would be used to constructing the stiffness matrix for detecting the health of structure. The stiffness-based process requires as follows: 1. identifying the modal parameters of structures from the measured vibration; 2. constructing the stiffness matrix of structures by identified modal parameters; 3. computing the variation of the identified stiffness matrix between the damaged and undamaged structure.

This study utilizes a simple and efficient modal identification method with wavelet packet decomposition to constructing the stiffness matrix from noisy dynamic responses. First, the time series model of a structure is established in Haar wavelet packet domain and the recursive least square approach is used to estimate the coefficient matrix of time series model. Next, the modal parameters of structure would be calculated by the estimated coefficient matrix of time series model. Then, the stiffness matrix of structure could be obtained from the identified modal parameters. However, it is difficult to identify the all structure's modal by the influence of noise. In here, the wavelet packet decomposition would be used to keep the interesting frequencies-band and reduce the effect of noise in uninteresting frequency-bands.

The proposed procedure is validated using the simulated earthquake acceleration responses of an eight-storey finite element model. The noisy simulated responses would be assumed and the Haar wavelet packet decomposition would be used in the process of the stiffness matrix construction.

2. Methodology

A building structure could be considered with viscous damping. The dynamic responses of building must satisfy the equation of motion, and could be described as follows

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f} \tag{1}$$

where **M**, **C** and **K** are mass, damping and stiffness matrices, respectively; $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$, respectively, are the acceleration, velocity and displacement responses vectors of the building, and **f** is the input force vectors. According to the central difference approach, equation of motion could be rewrite by time series model

$$\mathbf{y}_t = \sum_{i=1}^{I} \mathbf{\Phi}_i \mathbf{y}_{t-i\Delta t} + \sum_{j=0}^{J} \mathbf{\Theta}_j \mathbf{f}_{t-j\Delta t} \tag{2}$$

where, $\mathbf{y}_{t-i\Delta t}$ and $\mathbf{f}_{t-i\Delta t}$ are the vectors of measured responses and input forces at time $t\text{-}i\Delta t$, respectively; $1/\Delta t$ is the sampling rate of the measurement, $\mathbf{\Phi}_i$ and $\mathbf{\Theta}_j$ are matrices of coefficient functions to be determined in the model. Treating $\mathbf{y}_{t-i\Delta t}$ and $\mathbf{f}_{t-i\Delta t}$ as vector functions, and expand by discrete wavelet basis

$$\mathbf{y}_{t-i\Delta t}=\sum_{m=0}^{\bar{m}}\sum_{n=0}^{\bar{n}}\bar{\mathbf{y}}_i(m,n)\psi_{m,n}+\sum_{n=0}^{\bar{n}}\hat{\mathbf{y}}_i(\bar{m},n)\phi_{m,n};\quad \mathbf{f}_{t-j\Delta t}=\sum_{m=0}^{\bar{m}}\sum_{n=0}^{\bar{n}}\bar{\mathbf{f}}_j(m,n)\psi_{m,n}+\sum_{n=0}^{\bar{n}}\hat{\mathbf{f}}_j(\bar{m},n)\phi_{m,n} \tag{3}$$

where, $\phi(t)$ and $\psi(t)$are the scale function and mother function, respectively, and defined as follows

$$\psi_{m,n}(t)=2^{-m/2}\psi\left(2^{-m}t-n\right);\quad \phi_{m,n}(t)=2^{-m/2}\phi\left(2^{-m}t-n\right) \tag{4}$$

Equation (4) with $n\in\boldsymbol{Z}$ (the set of integers) can specify an orthonormal basis in Hebert space. The following are orthonormal properties

$$\left\langle\phi_{m,n}(t),\phi_{k,l}(t)\right\rangle=\begin{cases}1 & \text{if } m=k \ \& \ n=l\\ 0 & \text{else}\end{cases};\quad \left\langle\psi_{m,n}(t),\psi_{k,l}(t)\right\rangle=\begin{cases}1 & \text{if } m=k \ \& \ n=l\\ 0 & \text{else}\end{cases};\quad \left\langle\phi_{m,n}(t),\psi_{k,l}(t)\right\rangle=0 \tag{5}$$

Substituting Equations(3) into Equation(2), performing the inner product with respect to $\phi_{m,n}(t)$ and $\psi_{m,n}(t)$ on both sides of the resulting equation,respectively, and applying the orthonormal properties specified by Equation(5) yields

$$\hat{\mathbf{y}}_0(\bar{m},n)=\sum_{i=1}^{I}\mathbf{\Phi}_i\hat{\mathbf{y}}_i(\bar{m},n)+\sum_{j=0}^{J}\mathbf{\Theta}_j\hat{\mathbf{f}}_j(\bar{m},n),\quad \bar{\mathbf{y}}_0(m,n)=\sum_{i=1}^{I}\mathbf{\Phi}_i\bar{\mathbf{y}}_i(m,n)+\sum_{j=0}^{J}\mathbf{\Theta}_j\bar{\mathbf{f}}_j(m,n) \tag{6}$$

Rearranging Equation(6) for different values of m and n yields

$$\widehat{\boldsymbol{y}}_t=\overline{\boldsymbol{\varphi}}_t^T\boldsymbol{\theta}_t \tag{7}$$

where,

$$\overline{\boldsymbol{\varphi}}_t=\left[\widehat{\boldsymbol{y}}_{t-\Delta t}\ \cdots\ \widehat{\boldsymbol{y}}_{t-I\Delta t}\ \ \widehat{\boldsymbol{f}}_t\ \cdots\ \widehat{\boldsymbol{f}}_{t-J\Delta t}\right]^T;\ \boldsymbol{\theta}_t=\left[\boldsymbol{\Phi}_{t,1}\ \cdots\ \boldsymbol{\Phi}_{t,I}\ \ \boldsymbol{\Theta}_{t,0}\ \cdots\ \boldsymbol{\Theta}_{t,J}\right] \tag{8}$$

A recursive least square algorithm can be used to estimate and track the time-variant coefficient matrix of time series model as follows

$$\widehat{\boldsymbol{\theta}}_{t+1}=\widehat{\boldsymbol{\theta}}_t+\boldsymbol{L}_t\boldsymbol{\varepsilon}_t;\ \boldsymbol{\varepsilon}_t=\widehat{\boldsymbol{y}}_t-\overline{\boldsymbol{\varphi}}_t^T\widehat{\boldsymbol{\theta}}_t;\ \boldsymbol{L}_t=\boldsymbol{P}_t\overline{\boldsymbol{\varphi}}_t=\frac{\boldsymbol{P}_{t-1}\overline{\boldsymbol{\varphi}}_t}{\boldsymbol{\lambda}+\overline{\boldsymbol{\varphi}}_t^T\boldsymbol{P}_{t-1}\overline{\boldsymbol{\varphi}}_t};\ \boldsymbol{P}_t=\frac{1}{\boldsymbol{\lambda}}\left[\boldsymbol{P}_{t-1}-\boldsymbol{L}_t\overline{\boldsymbol{\varphi}}_t^T\boldsymbol{P}_{t-1}\right] \tag{9}$$

where, $\boldsymbol{L}_t$ is the filtering gain; $\boldsymbol{\varepsilon}_t$ is the a priori prediction error; $\boldsymbol{P}_t$ is the estimation covariance matrix and $\boldsymbol{\lambda}$ is the forgetting factor.

Eq. (2) reveals that the modal parameters of a structure are determined from $\mathbf{\Phi}_i$ with i=1,2,…,I. A matrix G is constructed from $\mathbf{\Phi}_i$ as

$$\mathbf{G}=\begin{bmatrix}\mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots\\ \mathbf{\Phi}_I & \mathbf{\Phi}_{I-1} & \mathbf{\Phi}_{I-2} & \mathbf{\Phi}_{I-3} & \cdots & \mathbf{\Phi}_1\end{bmatrix}_{\bar{N}\times\bar{N}} \tag{10}$$

where $\mathbf{I}$ is a lxl unit matrix, l is the dimension of $\mathbf{y}(t)$, and $\bar{N}=I\times l$; then the modal parameters of the structure under consideration can be directly determined from the eigenvalues and eigenvectors of $\mathbf{G}$ [6].

Consider the response of a structure described by equation of motions such as Eq. (1). Assuming properties damping, the orthogonality property of the mode shapes with respect to the mass and stiffness matrix [7], the stiffness matrix can be reconstructed by the follows formula

$$\mathbf{K} = \boldsymbol{\varphi}^{-\mathbf{T}} \mathbf{M}_D^{\mathbf{T}/2} \boldsymbol{\Lambda} \mathbf{M}_D^{1/2} \boldsymbol{\varphi}^{-1} \tag{11}$$

in which $\mathbf{M}_D$ and $\mathbf{K}_D$ are the diagonal modal mass matrix and diagonal modal stiffness matrix, respectively, $\boldsymbol{\Lambda} = \mathbf{M}_D^{-\mathbf{T}/2} \mathbf{K}_D \mathbf{M}_D^{-1/2}$.

3. Numerical Verification

The finite element of eight-storey frame was built to demonstrate the feasibility of the proposed procedure, and this frame was subjected to a simulated 1999 Chi-chi earthquake at its base in X direction (see Fig. 1). The geometric parameters of this six-storey-frame were 1.5m long, 1.1m wide and 8.48m high. The 5% of the modal damping ratio would be assumed during the simulation process. Plates were fixed on each floor, such that the total mass of frame at each floor was approximately 0.95ton. The columns have rectangular section of 15X2.5, with units is cm. Table 1 shows the theoretical modal parameters. This frame was subjected to base excitations of the 1999 Chi-chi earthquake. The acceleration responses of the base and all floors at t=4-10 seconds were used in evaluating modal parameters for the frame. Fig. 1 displays the time history of base excitation and acceleration responses in X direction of this numerical model subject the Chi-chi earthquake.

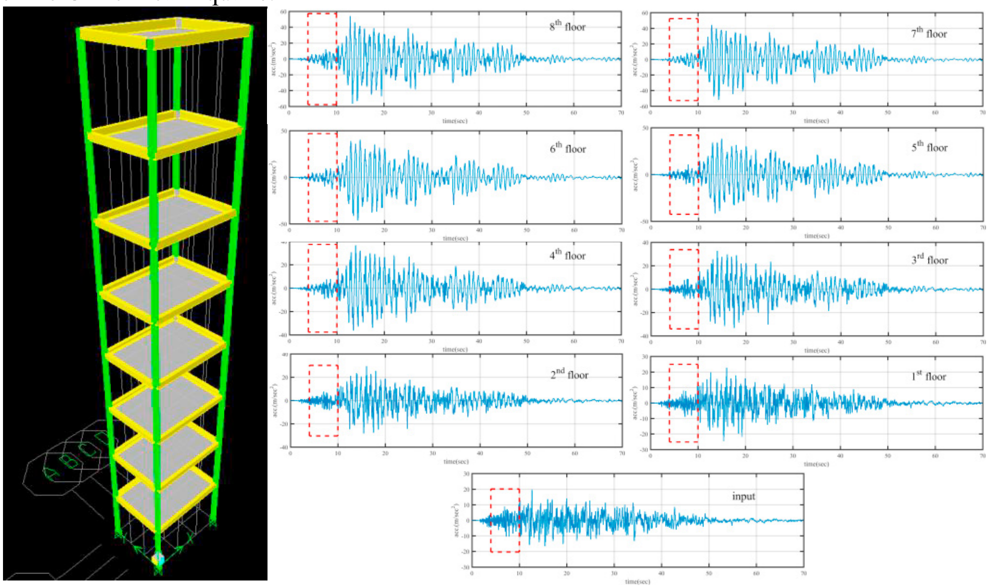


Fig. 1. Schematic diagram of an eight-storey finite element model and its simulation responses.

Table 1. Theoretical modal parameters of numerical model.

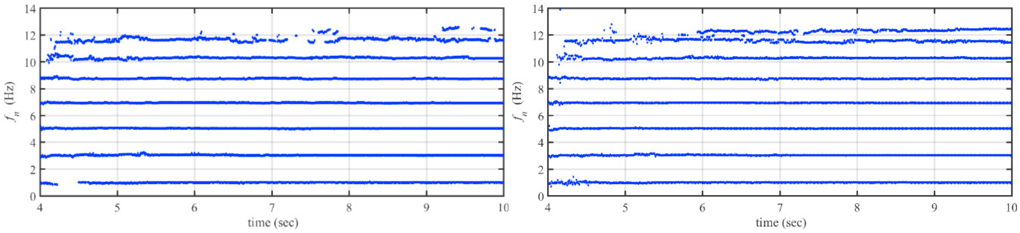
modal	1st	2nd	3rd	4th	5th	6th	7th	8th
f_n (Hz)	1.015	3.037	5.033	6.961	8.758	10.329	11.565	12.359
ξ (%)	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00

3.1. Effects of noise

Measured responses always contain some level of corrupted noise. To somewhat simulated this situation, noise with 2% variance of the signal-to-noise ratio (NSR) was randomly added to the computed responses and input.

When the simulated responses without noise, the accurate modal parameters could be yielded in (I,J)=2. Theoretically, raising (I,J) still yielded the results with very high accuracy. Therefore, the more and higher (I,J) would be used for processing of noisy responses. In the numerical test, the order of time series model (I,J) would be set form 10 to 20.

Figure 2(a) illustrates the stabilization diagrams of the natural frequencies obtained from the responses with NSR=2% in time domain. The 8th modal is difficult to find out, this modal only appeared briefly in t=9.2~9.4sec and t=9.6~9.8sec. It is potentially could be ignored.



(a) (b)

Fig. 2. Stabilization diagrams of the identified natural frequencies obtained (a) in time domain; (b) in wavelet packet domain.

In Haar wavelet packet domain, some uninterested frequency-bands would be removed. It is indicates that the wavelet packet decomposition exhibits a frequency filtering effect. Figure 2(b) plots the results identified from processing the noisy input and acceleration responses of all eight stories by using the vibration at t=4-10sec. The 8th modal could be accurately identified after t=6sec. Figure 4 illustrates the stabilization diagrams of the natural frequencies obtained from the responses with NSR=2% in wavelet packet domain.

Figure 3 once again describe the identified frequencies and mode shapes of eight modes in time domain and Haar wavelet packet domain with different colors. It is not obviously that the difference of the identified mode shapes from 1st to 5th in time domain or Haar wavelet packet domain. The relative error of the each identified natural frequencies are less than 2%.

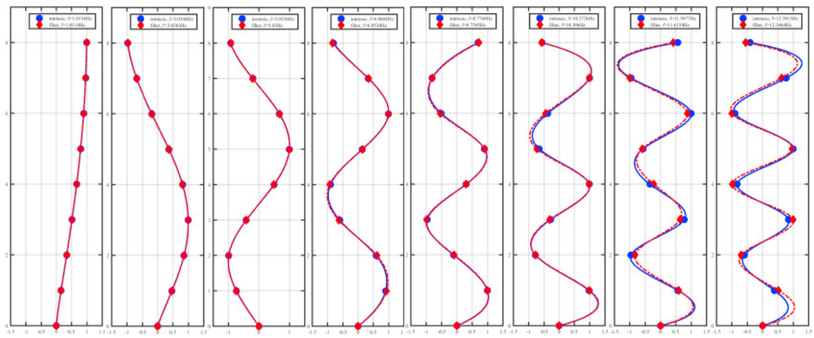


Fig. 4. Identified natural frequency and mode shapes of eight-storey finite element model.

3.2. Constructing stiffness matrix

In the stiffness matrix construction process, the mass properties of all degree of freedom is necessary. Under the assumption of given mass matrix, combined with the identified natural frequencies and mode shapes , the stiffness matrix of structure can be determined by Eq.(14). Table 2 shows the theoretical stiffness matrix of the eight-storey frame extracted from finite element model. Due to the values of off-tridiagonal elements less than 1% of the tridiagonal elements, in the follow numerical study, only the relative errors of the tridiagonal element would be discussed.

Table 3 shows the relative errors of the calculated stiffness using Eq. (14). The modal parameters that were identified from the simulation vibration of all observed degree of freedom in Eq. (14). For the identified results from the time series model in time domain, the maximum relative is 9.765%. It is evidently that the variant of the local stiffness less than 10% is difficult to detect from the identified modal parameters in time domain.

Table 2. The theoretical stiffness matrix of eight-storey frame. (unit: kN/m)

1742.10	-1851.10	115.20	-5.40	1.20	0.90	1.40	1.20
	2656.60	-1941.50	122.90	-6.40	0.10	0.10	-0.20
		3658.90	-1943.00	125.10	-6.00	0.10	-0.20
			3653.10	-1943.70	125.90	-5.60	0.10
				3651.50	-1944.10	125.40	-5.60
					3650.60	-1944.10	125.70
	sym.					3650.40	-1949.20
							3775.90

Next, for the identified results from the time series model in Haar wavelet packet domain, the wavelet packet decomposition could be used to keep certain interesting frequencies-band for the measured vibration. The maximum relative error is 3.85%. It is evidently that the variant of the local stiffness more than 5% could be easy detected form the identified modal parameters in Haar wavelet packet domain.

Table 3. The relative error of stiffness matrix on tridiagonal. (unit: %)

	K(8,8)	K(8,7)	K(7,7)	K(7,6)	K(6,6)	K(6,5)	K(5,5)	K(5,4)	K(4,4)	K(4,3)	K(3,3)	K(3,2)	K(2,2)	K(2,1)	K(1,1)
Haar wavelet packet domain	3.221	3.018	2.65	0.95	3.145	2.074	2.36	2.065	1.509	0.292	0.765	2.266	3.851	0.629	1.962
Time domain	5.733	9.098	9.32	6.528	9.765	7.127	7.89	5.064	6.882	1.718	6.765	8.924	9.415	3.275	2.446

4. Conclusions

This study extend the wavelet packet decomposition for creating the stiffness matrix of building from structural dynamic responses. The procedure of stiffness matrix creation from the measured vibration including three stages, 1. Determining the coefficient matrix of time series model in wavelet packet domain; 2.The modal parameters of a structure would be identified from measured seismic responses; 3.Constructing the stiffness from the identified natural frequencies and mode shapes.

In coefficient matrix determination stage, the time series model were constructed in Haar wavelet packet domain. The time-varying coefficient matrix in time series model were determined by a recursive least square approach. Then, the modal parameters of the structure were calculated from these coefficients. Finally, the stiffness matrix of the structure would be constructed by the identified natural frequencies and mode shapes.

The numerically simulated responses of an eight-storey finite element model subjected earthquake input were used to validate the proposed procedure. The effects of noise on the ability to identify the modal parameters were also investigated. The wavelet packet decomposition apply on the measured data did provide more accurately stiffness than the process intrinsic measurement data.

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