



# The venture capital entry model on game options with jump-diffusion process

Chuan-Chuan Ko<sup>a,\*</sup>, Tyrone T. Lin<sup>b</sup>, Chyan Yang<sup>a</sup>

<sup>a</sup> Institute of Business & Management, National Chiao Tung University, 118, Sec. 1, Chung-hsiao W. Road, Taipei 100 Taiwan

<sup>b</sup> Department of International Business, National Dong Hwa University, 1, Sec. 2, Da-hsueh Road, Shou-feng, Hualien 974 Taiwan

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## ABSTRACT

This paper aims to apply game options to construct the optimal decision-making and management tool for venture capital (VC) firms. This model emphasizes the inferences with game options on the market structures formed by different competition and investment strategies of the two VC firms to reflect the investment returns. These market structures are classified into an entry-deterred game (specific monopoly), a leader's dominated strategies (duopoly), and simultaneous investment. It is considered how to select investment timing to avoid any potential competitive threats in order to provide the optimal expected threshold values for the investment decisions of VC firms.

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## 1. Introduction

Venture capital (VC) firms pursue the highly risky and profitable investment models that anticipate the appreciation of capital in the long term. They invest in the companies with the potential for rapid growth. As the status of the fledgling companies in which venture capitalists invest is highly uncertain, their earnings may drop as a result of the existence of potential competitors that grab the market share.

The paper applies the real options approach (ROA) to evaluate the feasibility of the projects taken by these companies. Assume that a start-up company's unit contribution margin (that is, sales price minus variable cost of production) follows both the geometric Brownian motion (continuous process) and Poisson process (discrete process) in their jump-diffusion processes,<sup>1</sup> and the paper applies the game theory to examine the investment behavior of two VC firms. The different competitive strategies adopted by these two VC firms reflect the additional sales to obtain extra (potential investment returns), which are assumed to be a hyperbolic function. The paper further constructs an investment strategy model based on the market structures of an entry-deterred game (specific monopoly), a leader's dominating strategies (duopoly), and simultaneous investment entries formed

by the competitive behavior of the two VC firms in order to derive the optimal threshold value for investment decisions.

The paper will discuss that when the start-up company enters the market, it will attempt to make use of the established VC investment companies. When the unit contribution margin under uncertain profitability is the decision variable, the paper considers the previous investment and deferred investment behavior in the start-up company by VC firms and conducts a numerical analysis of entry decision threshold, relevant sensitivity analysis, and numerical example explanation.

## 2. Literature review

VC firms put funds into the start-up company with potential growth, as bearing high risks. To identify an appropriate level of risk treatment has become a key strategy to make profits in today's economy. Many researches regarding corporate optimum risk management have been done. Wu and Olson (2008) studied a variety of risk evaluation models within supply chains: chance constrained programming, data envelopment analysis, and multi-objective programming models. Wu et al. (2010) considered a three-dimensional early warning approach for product development risk management, which was proposed by integrating graphical evaluation and review technique with failure modes and effect analysis. Wu and Olson (2009a) discussed various risks modeling to optimize risk management. Risk management has become a key point to corporate development. Several risk evaluation methods even focus on measuring the risk value. The research has shown that the synthetic approaches to manage the risks facing an organization and the most effective ways to

\* Corresponding author. Tel.: +886 2 82189871; fax: +886 2 82122149.

E-mail addresses: [kochuanchuan@just.edu.tw](mailto:kochuanchuan@just.edu.tw) (C.-C. Ko), [tjlin@mail.ndhu.edu.tw](mailto:tjlin@mail.ndhu.edu.tw) (T.T. Lin), [professor.yang@gmail.com](mailto:professor.yang@gmail.com) (C. Yang).

<sup>1</sup> A number of papers have addressed the stochastic differential equations includes Shimko (1992), Dixit and Pindyck (1994), and Smit and Trigeorgis (2004).

take risk include new business philosophies such as corporate risk management (Wu et al., 2006; Wu and Olson, 2009b, 2010a,b). Moreover, real options analysis is one of the most appropriate methods for assessing the investments in VC firms involving uncertainty. When assessing the value of an investment project, apart from the expected future net cash inflow, the evaluation should include the management flexibility value implied by the uncertainty of investment environments. This includes the probability that managers will receive new information, high room for managerial flexibility, and the ability of decision-makers to respond to new information, etc. (Copeland and Antikarov, 2001).

Several authors have evaluated VC investment strategies. Lin and Huang (2004) noted that a start-up company raises funds from different types of VC firms and the most suitable VC entry mode is established with the reflection of investment profitability under the special effectiveness function. Under risk aversion, the proposed model conducts the most suitable VC establishment aimed at the project investment support standard generated by VC firms. Lin et al. (2007) applied the ROA in which the entire model assumed that the expected discounted factor and the jump-diffusion process were incorporated into the ROA to assess the value of a start-up company and determine the threshold of the exit timing of liquidation or convertibility when establishing the optimal disinvestment pricing model for VC firms. Kannianen and Keuschnigg (2003) mentioned that a VC firm not only had to provide capital support to the start-up company, but also had to increase the firm value. In a VC investment project, the most appropriate amount of investment in the start-up company should be determined based on the stringent management problem. Concerning the volume of investment combination and the transaction condition between management consultants, it is especially important to reduce the management consultant fee of the combined investment company and the management cost of the VC investment expert. Rosenberg (2003) explained that the VC investment expert not only invests capital in the start-up company, but also invests professional knowledge, management technique, time, and business negotiation so as to help nurture the start-up company into becoming an enterprise of high profitability. Takezawa et al. (2007) applied an option framework to quantify the underlying risk and proposed an optimization problem to select the optimal ownership structure and supply contract for maximizing the total shareholders' value of the parent.

The traditional approach is to make investment decisions based on net present value. This assumes the existence of a static investment environment and takes only net cash flows into consideration. However, it is very important to analyze a dynamic investment environment in order to devise a flexible investment strategy to cope with future uncertainties in the investment environment. For that reason, the ROA has rapidly gained popularity as an investment decision method. The investment decisions based on the ROA emphasize the value of flexible management and options (Myers, 1987; Dixit and Pindyck, 1994). In the recent years, scholars have stressed that the influence of decisions from competitors is also an important factor affecting the value of flexible management. Smit and Trigeorgis (2006) pointed out that strategic investment projects should be based on an expanded (or strategic) net present value (NPV) criterion that incorporated not only the passive (or direct) NPV of expected cash flows from investing immediately and the flexibility value from active management (real options), but also the strategic (game theory) value from competitive interactions. Smit and Ankum (1993) applied the game options principle as an analytical tool to evaluate a project's value and support the overall operating and investment strategy. Smit (2003) showed that the game options approach could make a

more complete assessment of a strategic option value in an interactive competitive setting. Miller and Waller (2003) pointed out that project planning was an important decision management tool and encouraged managers to utilize real options to process investment evaluation under future uncertain conditions and to explore how to use the opportunity to evade potential threats. Yeo and Qiu (2003) suggested utilizing the ROA to allow for a more feasible judgment in making investment decisions.

Aloysius (2002) introduced the concept that the most suitable investment decision for investors involved cooperation via symmetrical information in the duopoly market. The advantage for competitor is that cooperation will not be the most appropriate approach. Kong and Kwok (2007) applied real options and game theory to analyze the oligopoly market. Assuming that there are two competing firms and they have incurred asymmetric sunk costs, there will be a leading investor in the market. The firm with a competitive edge will make the first investment, or the two firms will invest at the same time. If a firm is more competitive, it will enter the market by setting up an optimal investment threshold value for the market leader and follower. When the preemptive thresholds of both firms happen to coincide, the two firms will enter the market simultaneously. Smit and Trigeorgis (2006) applied real options and game theory to the investment planning of strategic alliances. Pawlina and Kort (2006) noted that in an oligopoly, the investment costs are asymmetric and there is an optimal investment strategy. The study's result shows that a marginal increase in the investment cost of the firm with a cost disadvantage can enhance that firm's own value within a certain range of the asymmetry level. Jin et al. (2009) used a financial tool "option-based" mathematical model for the joint production and the maintenance system provided useful maintenance decisions in the environment of uncertain demand.

De Giovanni et al. (2008) analyzed the dynamic structure of a return process using subordinated laws and showed how subordinated models can be used to price contingent claims. The subordinated asset price models will consider the hyperbolic model. Kalashnikov et al. (2009) justified the concept of conjectural variations equilibrium applied to the mixed duopoly model by demonstrating the concavity of the expected profit function. Huang and Hsu (2008) enhanced the capability of explaining intemporal decision-making behavior and proposed an anticipative hyperbolic discounted utility model that revised the conventional hyperbolic discounted utility model by introducing anticipative parameters under the consideration of the anticipation of future gains or losses. Therefore, the paper assumes that the investment additional sales to obtain extra (the investment returns) from the competition between two competing VC firms form a hyperbolic function.

### 3. Proposed model

The paper adopts real options combining game theory, it evaluates theoretical models to figure out the threshold for unit contribution margin based on following the geometric Brownian motion (GBM) involved in a Poisson jump-process. Assuming that there are only two VC firms in the newly created market, when they are interested in investing in the start-up company, the investment scale is equal, which means the same investment input. Based on the investment of two VC firms, the start-up company can gain more additional sales to obtain extra and added values. Furthermore, eternal factors impact the market. Different strategies verify the additional sales to obtain extra. The two VC firms have different competitive advantages; although their investment inputs are the same, contributions and sharing are different.

The project value of the start-up company is affected by the external market environment and its operating condition. Assuming that the VC firm investing in the start-up company follows the stochastic unit contribution margin,  $P$ , the value variation growth over time is described by the jump-diffusion process which includes the GBM (continuous process)  $P_G$  and Poisson process (discrete process)  $P_D$  as follows:

$$dP \equiv dP_G + dP_D = \alpha P dt + \sigma P dW + \theta P dq. \tag{1}$$

Among the above factors,  $\alpha$  is the drift over time,  $\sigma$  is the volatility over time,  $dW$  is the increment of a standard Wiener process  $W$  of zero mean and unit standard deviation  $\sqrt{dt}$ , and  $\theta$  is the deterministic amplitude specifying the jump size (fall) in the jump-process  $0 \leq \theta \leq -1$ . The jump-process follows the Poisson process with an arrival rate of  $\lambda$  and then  $dq$  and  $dW$  are independent (so that  $E(dW \times dq) = 0$ ):

$$dq = \begin{cases} 1, & \text{with prob. } \lambda dt, \\ 0, & \text{with prob. } 1 - \lambda dt. \end{cases} \tag{2}$$

The stochastic  $P$  follows the jump-diffusion process because the start-up company is also affected by the competition from other firms. When the start-up company applies for new product development,  $P$  will decrease if competitors also apply for new patents.

Then, the different competitive and investment strategies of the two VC firms will be distinguished by the condition of a leader's dominating strategies (duopoly), an entry-deterred game (specific monopoly), and simultaneous investment based on the market condition and competitor strategy of the project. Assume the start-up company and the two VC firms negotiate the additional sales to obtain extra and  $D(x_1, x_2)$  belongs to the two VC firms. That is the investments of the two VC firms' different competition and investment strategies reflect the additional sales to obtain extra investment behavior function as  $D(x_1, x_2)$ .  $D_1(x_1, x_2)$  belongs to VC firm 1.  $D_2(x_1, x_2)$  belongs to VC firm 2. Moreover,

$$D(x_1, x_2) = D_1(x_1, x_2) + D_2(x_1, x_2), \text{ among which}$$

$$\begin{cases} x_1: & \text{VC firm 1 project decision} \\ & (0: \text{if VC firm 1 has not invested; } 1: \text{if VC firm 1 has invested).} \\ x_2: & \text{VC firm 2 project decision} \\ & (0: \text{if VC firm 2 has not invested; } 1: \text{if VC firm 2 has invested).} \end{cases}$$

The paper examines the market acceptance of the products of new ventures against the background of the market uncertainties of the two VC firms. It estimates the responses of the opponents under a competitive landscape in order to determine whether investments are viable. Table 1 illustrates possible scenarios. The paper assumes that VC firm 1 is the leading competitor.

In order to conduct a further study on the effect generated by the additional sales to obtain extra  $D(x_1, x_2)$  behavior of both parties under a duopoly market, the correlation of variables  $x_1$  and  $x_2$  is assumed as follows:

$$D(x_1, x_2) = (x_1 + h)(x_2 + k), \tag{3}$$

where  $h, k \in R$  and  $k > h > 1$ .  $k$  and  $h$  separately represent the technology, finance, market, and business know-how in VC firm 1 and VC firm 2. The paper assume VC firm 1 as the market leader. Its business know-how has a comparative advantage, which is hard for VC firm 2 to compete. When dealing with the start-up company, VC firm 1 can have a better bargain. According to the assumption in the paper, VC firm 1 can have larger shares of the pies in the start-up company. As the two VC firms intend to invest in the start-up company, their investment strategies are a function of mutual speculation and influence.

The additional sales to obtain extra of the two VC firms is expressed by Eq. (3). Thus, it is assumed that  $D_i(x_1, x_2)$ ,  $i = 1, 2$  are the additional sales to obtain extra for VC firm 1 and VC firm 2 upon different investment strategies, respectively. When the two VC firms evaluate the benefits of investing in a start-up company, the four expected additional sales to obtain extra investment scenarios by different opponents' reactions and their investment strategies are shown in Table 2.

The four expected additional sales to obtain extra scenarios from the various opponents' reactions and their investment strategies are as follows: (1)  $[D_1(0,0), D_2(0,0)]$  represents the condition that both VC firms are taking a "waiting" strategy. The expected additional sales to obtain extra are  $D(0,0) = h \times k$ . VC firm 1 shares  $D_1(0,0) = Z_{0,0} \times h \times k$  and VC firm 2 shares  $D_2(0,0) = (1 - Z_{0,0}) \times h \times k$ ,  $0 \leq Z_{0,0} \leq 1$ ; (2)  $[D_1(1,0), D_2(1,0)]$ : when VC firm 1 invests first while VC firm 2 adopts the waiting strategy, the expected additional sales to obtain extra are  $D(1,0) = h \times k + k$ . VC firm 1 shares  $D_1(1,0) = Z_{1,0} \times (h \times k + k)$  and VC firm 2 shares  $D_2(1,0) = (1 - Z_{1,0}) \times (h \times k + k)$ ,  $0 \leq Z_{1,0} \leq 1$ ; (3)  $[D_1(0,1),$

**Table 1**  
The function of investment behavior.

VC firm's strategy	Explanation
$D(0,0)$	Neither VC firm has invested in this project, $x_1=0, x_2=0$
$D(1,0)$	VC firm 1 is a leader; VC firm 2 is a follower, $x_1=1, x_2=0$
$D(0,1)$	VC firm 1 has not invested; VC firm 2 has invested, $x_1=0, x_2=1$
$D(1,1)$	Both VC firms have invested in this project, $x_1=1, x_2=1$

**Table 2**  
Share of the additional sales to obtain extra matrix for VC firm 1 and VC firm 2 upon different competition strategy.

		VC firm 2	
		Wait	Invest
VC firm 1	Wait	$[D_1(0,0), D_2(0,0)]$ Note: VC firm 1: $D_1(0,0) = Z_{0,0} \times h \times k$ VC firm 2: $D_2(0,0) = (1 - Z_{0,0}) \times h \times k$	$[D_1(0,1), D_2(0,1)]$ Note: VC firm 1: $D_1(0,1) = Z_{0,1} \times (h \times k + h)$ VC firm 2: $D_2(0,1) = (1 - Z_{0,1}) \times (h \times k + h)$
	Invest	$[D_1(1,0), D_2(1,0)]$ Note: VC firm 1: $D_1(1,0) = Z_{1,0} \times (h \times k + k)$ VC firm 2: $D_2(1,0) = (1 - Z_{1,0}) \times (h \times k + k)$	$[D_1(1,1), D_2(1,1)]$ Note: VC firm 1: $D_1(1,1) = Z_{1,1} \times (h \times k + k + h)$ VC firm 2: $D_2(1,1) = (1 - Z_{1,1}) \times (h \times k + k + h)$

Note: 1. Distributing the additional sales to obtain extra in each cell for [VC firm 1, VC firm 2]. 2. Eq. (3) is derived by the two VC firms' additional sales to obtain extra sharing under different strategies:  $D(x_1, x_2) = (x_1 + h)(x_2 + k) = x_1x_2 + hx_2 + kx_1 + hk = D_1(x_1, x_2) + D_2(x_1, x_2)$ .  $D_1(x_1, x_2) = Z_{x_1, x_2} \times D(x_1, x_2)$ ,  $D_1(x_1, x_2)$  is the additional sales to obtain extra for VC firm 1.  $D_2(x_1, x_2) = (1 - Z_{x_1, x_2}) \times D(x_1, x_2)$ ,  $D_2(x_1, x_2)$  is the additional sales to obtain extra for VC firm 2. 3.  $0 \leq Z_{x_1, x_2} \leq 1$ ,  $Z_{x_1, x_2}$  is the distribution ratio for VC firm 1 under  $D(x_1, x_2)$ .  $(1 - Z_{x_1, x_2})$  is the distribution ratio for VC firm 2.

$D_2(0,1)$ : VC firm 1 delays investments by forgoing investment opportunities, while VC firm 2 remains interested in making investments. The expected additional sales to obtain extra are  $D(0,1)=h \times k+h$ . VC firm 1 shares  $D_1(0,1)=Z_{0,1} \times (h \times k+h)$  and VC firm 2 shares  $D_2(0,1)=(1-Z_{0,1}) \times (h \times k+h)$ ,  $0 \leq Z_{0,1} \leq 1$ ; (4)  $[D_1(1,1), D_2(1,1)]$ : VC firm 1 and VC firm 2 adopt a cooperation strategy by investing at the same time. Here, the expected additional sales to obtain extra are  $D(1,1)=1+h \times k+h+k$ . VC firm 1 shares  $D_1(1,1)=Z_{1,1} \times (1+h \times k+k+h)$  and VC firm 2 shares  $D_2(1,1)=(1-Z_{1,1}) \times (1+h \times k+k+h)$ ,  $0 \leq Z_{1,1} \leq 1$ .

Therefore, the four expected response investment additional sales to obtain extra are  $D(1,1) > D(1,0) > D(0,1) > D(0,0)$ . Since VC firm 1 and VC firm 2 implement the investments simultaneously, they adopt the cooperation strategy with synthetic effects viewing the market optimistically. The expected investment additional sales to obtain extra  $D(1,1)$  are the highest. The second is  $D(1,0)$ . VC firm 1 dominates the investment opportunity and sets a high threshold, which then makes VC firm 2 give up the investment opportunity. The third is  $D(0,1)$ . VC firm 2 views the market optimistically, while VC firm 1 views it pessimistically. VC firm 2 implements the investment and then becomes special monopoly as illustrated  $D(1,0) > D(0,1)$ . The last is  $D(0,0)$ . Neither of the two VC firms enter the market although they are interested in investing and are the start-up company's consultants. The market will reflect that and increase the returns. The start-up company will pay the consultation fees to the two VC firms. The descending of ranking is  $D(1,1) > D(1,0) > D(0,1) > D(0,0)$ . Below, the paper attempts to conduct the market entry threshold under different market environments formed by different VC competition strategies.

### 3.1. Specific monopoly model

First, VC firm 1 delays or even opts out of investments due to its negative view of the market. However, VC firm 2 holds a different perspective. Believing that early investments will create a niche, it is determined to go ahead with investments. As a result, the market becomes a specific monopoly. The potential profitability value of the entry-deterred game (specific monopoly) investment is  $M(P)$  and according to Itô's Lemma (Itô, 1951), the increment can be calculated as follows:

$$dM(P) = M_P(P)dP_C + \frac{1}{2}M_{PP}(P)dP_C^2 + [M(P(1+\theta)) - M(P)]dq \quad (4)$$

By incorporating Eq. (1) and (2) into Eq. (4), we can derive the following expected value:

$$E[dM(P)] = \{ \alpha PM_P(P) + \frac{1}{2}\sigma^2 P^2 M_{PP}(P) + \lambda M(P(1+\theta)) - \lambda M(P) \} dt \quad (5)$$

Here, the change of the potential profitability value is formed by the change of capital gains. Following the Bellman equation, the continuation region is given by

$$\begin{aligned} \gamma M(P)dt &= E[dM(P)] + PD_2(0,1)dt \\ &= \left\{ \alpha PM_P(P) + \frac{1}{2}\sigma^2 P^2 M_{PP}(P) + \lambda M(P(1+\theta)) - \lambda M(P) + PD_2(0,1) \right\} dt, \end{aligned} \quad (6)$$

where  $\gamma$  is a discount rate, and the above formula explains that the expected potential profitability value of the unit time is equivalent to unit contribution margin and can satisfy the assumption that the unit contribution margin is quite reasonable (equivalent to the condition of satisfying the risk premium). Furthermore,  $[\alpha PM_P(P) + \frac{1}{2}\sigma^2 P^2 M_{PP}(P) + \lambda M(P(1+\theta)) - \lambda M(P)] dt$  is formed by the change of capital gains in unit time  $dt$ , and unit contribution margin multiplied by the additional sales to obtain extra  $PD_2(0,1) dt$  is formed by the cash flow change in unit time  $dt$ .

The above profitability value is formed by cash flows (see Appendix 1 for the particular solution part) and capital gains (see Appendix 2 for the general solution part). Thus it can be seen that the profitability value function of sole investment in the project is

$$M(P) = \begin{cases} A_1 P^{\beta_1} + \frac{PD_2(0,0)}{\gamma - \alpha - \lambda \theta}, & P < P_S^* \\ \frac{PD_2(0,1)}{\gamma - \alpha - \lambda \theta} - I, & P \geq P_S^* \end{cases} \quad (7)$$

where

$$\beta_1 = \frac{-(\alpha - (1/2)\sigma^2) + \sqrt{(\alpha - (1/2)\sigma^2)^2 + 2\sigma^2(\lambda + \gamma - \lambda(1+\theta))^{\beta_1}}}{\sigma^2} > 1$$

and it is assumed that  $\gamma - \alpha - \lambda \theta > 0$ . Therefore,  $\gamma - \alpha$  is the expected return,  $\lambda$  is the jump-diffusion process following the Poisson process with an uncertain arrival rate, and  $\theta$  is the magnitude of influence for the jump size in the jump-diffusion process. Here,  $I$  denotes the sunk cost of investment equity shares by VC firms,  $P_S^*$  represents the threshold of a specific monopoly investment that VC firms invest in the start-up company, and  $A_1$  is the undetermined parameter.

However, the solutions in Eq. (7) have straightforward economic interpretations. In the region  $P < P_S^*$ , VC firm 2 chooses waiting to invest in the new start-up company. Its value includes the expected potential investment returns, capital gains  $A_1 P^{\beta_1}$ , and the consultation fees  $PD_2(0,0)$ . Moreover, there is positive probability that  $P$  process will move into the region  $P \geq P_S^*$  at the certain future time when the investment will resume and profits  $PD_2(0,1)/(\gamma - \alpha - \lambda \theta) - I$  will accrue. The value  $M(P)$  when  $P < P_S^*$  is just the expected present value of such future flow. Next consider the region  $P \geq P_S^*$ . Suppose for a moment that the firm is forced to continue operation of the project forever (Dixit and Pindyck, 1994).

The value-matching condition (VMC) is utilized as follows: before the investment in the project, the profitability value function of the threshold is equivalent to the beneficial value function of the threshold after the investment in the project (which means the satisfactory value is the only condition). The smooth-pasting condition (SPC) is also utilized as follows: the marginal value of the project should be equivalent during the first order of differential function (meaning the equivalent condition of satisfying the marginal value). According to the VMC and SPC (Dixit and Pindyck, 1994), the threshold  $P_S^*$  and parameter  $A_1$  under the special monopoly of Eq. (7) are indicated in the following equation:

$$\begin{cases} \text{VMC} : A_1 (P_S^*)^{\beta_1} + \frac{P_S^* D_2(0,0)}{\gamma - \alpha - \lambda \theta} = \frac{P_S^* D_2(0,1)}{\gamma - \alpha - \lambda \theta} - I, \\ \text{SPC} : A_1 \beta_1 (P_S^*)^{\beta_1 - 1} + \frac{D_2(0,0)}{\gamma - \alpha - \lambda \theta} = \frac{D_2(0,1)}{\gamma - \alpha - \lambda \theta}. \end{cases} \quad (8)$$

After sorting Eq. (8), the threshold  $P_S^*$  and undetermined parameter  $A_1$  of the unit contribution margin of the investment under the special monopoly are as follows:

$$P_S^* = \frac{\beta_1}{\beta_1 - 1} \frac{I(\gamma - \alpha - \lambda \theta)}{D_2(0,1) - D_2(0,0)}, \quad (9)$$

$$A_1 = \frac{(P_S^*)^{1 - \beta_1}}{\beta_1} \frac{D_2(0,1) - D_2(0,0)}{\gamma - \alpha - \lambda \theta}. \quad (10)$$

### 3.2. Leader's dominating strategy (duopoly) model

Assuming that there are two VC firms in the newly created market, the leader invests in this project and the strategy for the follower is to wait for more opportune timing.

3.2.1. Follower's value function and threshold

The first step is to come up with the solution of the market entry threshold value for the follower. When the leader has invested in this project, the potential profitability value of the follower's investment is  $F(P)$ , and according to Itô's Lemma (Itô, 1951), the increment can be calculated as follows (derived from  $M(P)$ ):

$$F(P) = \begin{cases} N_1 P^{\beta_1} + \frac{PD_2(0,0)}{\gamma-\alpha-\lambda\theta}, & P < P_F^* \\ \frac{PD_2(1,1)}{\gamma-\alpha-\lambda\theta} - I, & P \geq P_F^* \end{cases} \quad (11)$$

Here,  $D_2(1,1)$  denotes the additional sales to obtain extra for VC firm 2 when the two VC firms enter the market at the same time,  $P_F^*$  represents the threshold of the duopoly of the follower's investment, and  $N_1$  is the undetermined parameter. According to the VMC and SPC (Dixit and Pindyck, 1994), the threshold  $P_F^*$  and undetermined parameter  $N_1$  under the duopoly of the follower are indicated as follows:

$$\begin{cases} \text{VMC} : N_1 (P_F^*)^{\beta_1} + \frac{P_F^* D_2(0,0)}{\gamma-\alpha-\lambda\theta} = \frac{P_F^* D_2(1,1)}{\gamma-\alpha-\lambda\theta} - I, \\ \text{SPC} : N_1 \beta_1 (P_F^*)^{\beta_1-1} + \frac{D_2(1,0)}{\gamma-\alpha-\lambda\theta} = \frac{D_2(1,1)}{\gamma-\alpha-\lambda\theta}. \end{cases} \quad (12)$$

After sorting Eq. (12), the threshold  $P_F^*$  and undetermined parameter  $N_1$  of the unit contribution margin of the investment under the duopoly of the follower are as follows:

$$P_F^* = \frac{\beta_1}{\beta_1-1} \frac{I(\gamma-\alpha-\lambda\theta)}{D_2(1,1)-D_2(0,0)}, \quad (13)$$

$$N_1 = \frac{(P_F^*)^{1-\beta_1}}{\beta_1} \frac{D_2(1,1)-D_2(1,0)}{\gamma-\alpha-\lambda\theta}. \quad (14)$$

3.2.2. Leader's value function and threshold

Before the follower invests in this project, the leader can obtain  $PD_1(1,0)$  return. However, when the follower invests in the market, the return will change to  $PD_1(1,1)$ , where  $L(P)$  is the potential profitability value of the leader. According to Itô's Lemma (Itô, 1951), the increment is described as follows (derived from  $M(P)$ ):

$$E[dL(P)] = \{ \alpha PL_P(P) + \frac{1}{2} \sigma^2 P^2 L_{PP}(P) + \lambda L(P(1+\theta)) - \lambda L(P) \} dt. \quad (15)$$

Here, the change of the potential profitability value is formed by the change of capital gains.

Following the Bellman equation, the continuation region is then given by

$$\begin{aligned} \gamma L(P) dt &= E[dL(P)] + PD_1(1, x_2) dt, x_2 = 0, 1 \\ &= \alpha PL_P(P) dt + \frac{1}{2} \sigma^2 P^2 L_{PP}(P) dt + \lambda [L(P(1+\theta)) - L(P)] dt \\ &\quad + PD_1(1, x_2) dt, x_2 = 0, 1. \end{aligned} \quad (16)$$

As mentioned above, the change of the potential profitability value is formed by the changes of capital gains and cash flows.

Moreover,  $\{ \alpha PL_P(P) + \frac{1}{2} \sigma^2 P^2 L_{PP}(P) + \lambda L(P(1+\theta)) - \lambda L(P) \} dt$  is formed by the change of capital gains in unit time  $dt$ , and unit contribution margin multiplied by the additional sales to obtain extra  $PD_1(1, x_2) dt$ ,  $x_2 = 0, 1$  is formed by the change of cash flow in unit time  $dt$ . When  $P < P_F^*$ , the potential profitability value function of the leader's investment in the project is

$$L(P) = E_1 P^{\beta_1} + E_2 P^{\beta_2} + \frac{PD_1(1,0)}{\gamma-\alpha-\lambda\theta}, \quad (17)$$

where  $E_1, E_2$  are the parameters of the pending decision. To satisfy the boundary condition, when  $P=0$ , the potential profitability value function  $L(P)$  is equal to 0; when  $P_F^*$  is discounted to  $P_L^*$ , the potential profitability value function is equal to the profitability

value function of  $P \geq P_F^*$  as follows:

$$\begin{cases} L(0) = 0, \\ L(P_F^*) = \frac{P_F^* D_1(1,1)}{\gamma-\alpha-\lambda\theta}. \end{cases} \quad (18)$$

After sorting Eqs. (17) and (18), the undetermined parameters  $E_1, E_2$  and potential profitability value function of the leader's investment  $L(P)$  are as follows:

$$\begin{cases} E_1 = (P_F^*)^{1-\beta_1} \frac{D_1(1,1)-D_1(1,0)}{\gamma-\alpha-\lambda\theta}, \\ E_2 = 0, \end{cases} \quad (19)$$

$$L(P) = \begin{cases} \frac{PD_1(1,0)}{\gamma-\alpha-\lambda\theta} + \left(\frac{P}{P_F^*}\right)^{\beta_1} \frac{P_F^* [D_1(1,1)-D_1(1,0)]}{\gamma-\alpha-\lambda\theta}, & P < P_F^* \\ \frac{PD_1(1,1)}{\gamma-\alpha-\lambda\theta}, & P \geq P_F^* \end{cases} \quad (20)$$

The potential project value  $G(P)$  before the leader's investment is  $K_1 P^{\beta_1} + PD_1(0,0)/(\gamma-\alpha-\lambda\theta)$  (the inference is the same as shown in Appendices 1 and 2) and  $K_1$  is the undetermined parameter for the value of capital gains of the project before the leader's investment. For the VMC, before investing in the project, the profitability value function of the leader threshold is equivalent to the beneficiary value function of the leader threshold. The profitability value  $L(P_L^*)$  of the project invested by the leader under the threshold  $P_L^*$  is equivalent to the potential investment profitability value of the leader  $G(P_L^*)$  plus the sunk cost of investment  $I$ . That is

$$G(P_L^*) + I = L(P_L^*). \quad (21)$$

For the SPC, the marginal value of the project and under the equivalent first order of differentiation, the undetermined parameter  $K_1$  and the threshold  $P_L^*$  can be found and after arrangement the result is as follows:

$$\begin{cases} \text{VMC} : K_1 (P_L^*)^{\beta_1} + \frac{P_L^* D_1(0,0)}{\gamma-\alpha-\lambda\theta} = E_1 (P_L^*)^{\beta_1} + \frac{P_L^* D_1(1,0)}{\gamma-\alpha-\lambda\theta} - I, \\ \text{SPC} : K_1 \beta_1 (P_L^*)^{\beta_1-1} + \frac{D_1(0,0)}{\gamma-\alpha-\lambda\theta} = \beta_1 E_1 (P_L^*)^{\beta_1-1} + \frac{D_1(1,0)}{\gamma-\alpha-\lambda\theta}. \end{cases} \quad (22)$$

After sorting Eq. (22), the threshold  $P_L^*$  and undetermined parameter  $K_1$  of the unit contribution margin of the investment under the duopoly of the leader are as follows:

$$P_L^* = \frac{\beta_1}{\beta_1-1} \frac{I(\gamma-\alpha-\lambda\theta)}{D_1(1,0)-D_1(0,0)}, \quad (23)$$

$$K_1 = \frac{(P_L^*)^{1-\beta_1}}{\beta_1} \frac{D_1(1,0)-D_1(0,0)}{\gamma-\alpha-\lambda\theta} + E_1. \quad (24)$$

3.3. Simultaneous investment model

Assuming that two VC firms enter into investment simultaneously, as well, VC firm 1 and VC firm 2 implement strategies of cooperation. According to Itô's Lemma (Itô, 1951), the increment is described as follows (derived from  $M(P)$ ):

$$J(P) = \begin{cases} H_1 P^{\beta_1} + \frac{PD(0,0)}{\gamma-\alpha-\lambda\theta}, & P < P_J^* \\ \frac{PD(1,1)}{\gamma-\alpha-\lambda\theta} - I, & P \geq P_J^* \end{cases} \quad (25)$$

According to the VMC and SPC (Dixit and Pindyck, 1994), the unit contribution margin threshold  $P_J^*$  and parameter  $H_1$  under the simultaneous investment are:

$$\begin{cases} \text{VMC} : H_1 (P_J^*)^{\beta_1} + \frac{P_J^* D(0,0)}{\gamma-\alpha-\lambda\theta} = \frac{P_J^* D(1,1)}{\gamma-\alpha-\lambda\theta} - I, \\ \text{SPC} : H_1 \beta_1 (P_J^*)^{\beta_1-1} + \frac{D(0,0)}{\gamma-\alpha-\lambda\theta} = \frac{D(1,1)}{\gamma-\alpha-\lambda\theta}. \end{cases} \quad (26)$$

After sorting Eq. (26), the threshold  $P_J^*$  and undetermined parameter  $H_1$  of the unit contribution margin of the investment

under the simultaneous investment are as follows:

$$P_j^* = \frac{\beta_1}{\beta_1 - 1} \frac{I(\gamma - \alpha - \lambda\theta)}{D(1,1) - D(0,0)}, \tag{27}$$

$$H_1 = \frac{(P_j^*)^{1-\beta_1} D(1,1) - D(0,0)}{\beta_1 \gamma - \alpha - \lambda\theta}. \tag{28}$$

#### 4. Numerical analysis

The numerical analysis is based on the 2008 Taiwan Venture Capital Yearbook, published by the Taiwan Venture Capital Association (TVCA) (2008) and the Taiwan Economic Journal to determine relevant variables. The assumed simulation parameter value of this section is as follows: the drift over time  $\alpha=0.22$ , the volatility over time  $\sigma=0.56$ , the discount rate  $\gamma=0.39$ , the deterministic amplitude specifying the jump size in the jump-process  $\theta=-0.28$ , the arrival rate  $\lambda=0.18$ , and the sunk cost  $I=18$  million. The additional sales to obtain extra is  $D(x_1, x_2) = (x_1 + h)(x_2 + k)$  and then  $h=1$  million and  $k=3$  million. The two VC firms generate the four expected additional sales to obtain extra under different strategies. The distribution ratios are  $Z_{0,0}=0.6$ ,  $Z_{1,0}=1$ ,  $Z_{0,1}=0$ ,  $Z_{1,1}=0.6$ . The additional sales to obtain extra for the two VC firms adopting different investment strategies are shown in Table 3.

Different market structures, including the specific monopoly, the leader's dominating strategies (duopoly), and simultaneous entry are formed into the model according to different investment strategies of the two VC firms. The numerical analysis is based on the assumptions of the aforesaid parameter values and the model developed in Section 2 so as to derive the optimal market entry unit contribution margin threshold values under the optimal investment strategy. By combining with the previous simulated parameter value, the results are collectively arranged as in Table 4.

The calculation result of the numerical example is as follows. It is assumed that VC firm 1 is the market leader. If the market leader forgoes investment opportunities and VC firm 2 continues to invest, the market will become an entry-deterred game (specific monopoly). In this instance, the unit contribution margin threshold value under the optimal investment strategy is  $P_S^* = 4.117$  (dollar). If the investment environment is upbeat, VC firm 1, as the market leader, will invest as soon as possible. In this instance, the unit contribution margin threshold value of the market leader under the optimal investment strategy is  $P_L^* = 2.744$

**Table 3**  
Share of the additional sales to obtain extra matrix for VC firm 1 and VC firm 2 upon different competition strategy (unit: million).

		VC firm 2	
		Wait	Invest
VC firm 1	Wait	(1.8, 1.2)	(0.0, 4.0)
	Invest	(6.0, 0.0)	(4.8, 3.2)

**Table 4**  
The optimal investment threshold.

Market condition	Threshold value
Special monopoly	$P_S^* = 4.117$ (dollar)
Duopoly	$P_L^* = 2.744$ (dollar)
	$P_F^* = 5.763$ (dollar)
Simultaneous investment	$P_j^* = 2.305$ (dollar)

(dollar). Because VC firm 2 is the market follower, the unit contribution margin threshold value of the market follower under the optimal investment strategy is  $P_F^* = 5.763$  (dollar). Another possible scenario is that both VC firms cooperate and invest at the same time in order to create synergy and a win-win result. Accordingly, the unit contribution margin threshold value under the optimal investment strategy is  $P_j^* = 2.305$  (dollar).

By summarizing the above-mentioned results and based on the collective sorting of the value functions of various stages and relevant thresholds, the correlation can be described as shown in Table 4, which explains when the market becomes a duopoly, the threshold for the market leader  $P_L^* = 2.744$  (dollar) is lower than that for the follower  $P_F^* = 5.763$  (dollar) because the market leader boasts competitive advantages and is the first to make investments. The investment strategy of the market follower in a duopoly is more conservative because the market leader is already in the market. Unless new companies demonstrate obvious advantages, the market follower will simply wait for future investment opportunities. Therefore, the unit contribution margin threshold value of the investment strategy is higher at  $P_F^* = 5.763$  (dollar). However, if the market leader chooses to drop investment opportunities, but the market follower goes ahead with investments, the investment market will become an entry-deterred game (specific monopoly). Because the market leader forgoes investment opportunities, the market follower will become more cautious in evaluating the investment environment. The decision maker requires a higher unit contribution margin to enter the market. The unit contribution margin threshold value of the investment strategy is  $P_S^* = 4.117$  (dollar). Under the scenario that the two VC firms collaborate to generate synergy and achieve a win-win result, their investment strategies will become more competitive between VC firms. Therefore, the unit contribution margin threshold value under the optimal investment strategy is  $P_j^* = 2.305$  (dollar). In the overall market, both VC firms enter the market. VC firm 1 is the earliest entrant in the duopoly market. The second entrant to the market is the entry-deterred game (specific monopoly) and then the simultaneous entries of the two VC firms working together. The latest entrant to the market is the market follower in a duopoly market. Its investment strategy is also the most conservative.

#### 5. Sensitivity analysis

This section describes the sensitivity analysis on the relevant variables presented in the paper: the volatility over time  $\sigma$ , the discount rate  $\gamma$ , the achievement rate  $\lambda$ , the deterministic amplitude specified for the jump size (fall) in the jump-process  $\theta$ , etc. It is assumed that other parameters are constant when exploring the influence of the change of a single parameter on the optimal threshold value.

First, the paper focus on the volatility over time  $\sigma$ . Table 5 lists the influence of its change on the optimal threshold value under the optimal investment decisions.

**Table 5**  
Threshold values for different values of  $\sigma$ .

$\sigma$	Special monopoly	Duopoly		Simultaneous investment $P_j^*$
	$P_S^*$	$P_L^*$	$P_F^*$	
0.36	3.316	2.211	4.642	1.857
0.46	3.688	2.459	5.163	2.065
0.56	4.117	2.744	5.763	2.305
0.66	4.600	3.067	6.440	2.576
0.76	5.137	3.425	7.192	2.877

As shown in Table 5, when  $\sigma$  increases, the levels of uncertainties and risks associated with the investments by VC firms in the start-up company also increase. The optimal threshold value rises as a result. When the risk increases, decision makers will adopt a waiting strategy and hope for better opportunities as they are pessimistic about the investment environment.

Table 6 shows the influence of the change of the discount rate  $\gamma$  on the optimal threshold value under the optimal investment decisions.

The rise of the discount rate  $\gamma$  reflects the increased rate of return required by decision makers. At this point, investment strategies become conservative. No investments will be made until the unit contribution margin grows to a higher level. The optimal threshold value under the optimal investment decisions will also increase.

Tables 7 and 8, respectively, show the influences of the changes of the achievement rate  $\lambda$  and the magnitude of influence for the jump size (fall) in the Jump-process  $\theta$  on the optimal threshold value under the optimal investment decisions.

The achievement rate  $\lambda$  refers to the probability of sudden events such as a financial tsunami or financial crisis upon the domestic and international economies. The deterministic amplitude specified for the jump size in the jump-process  $\theta$  refers to the intensity level of the influence of the sudden events on the investment environment. When an adverse sudden event occurs, the economic cycle will fall into a trough and investments will stop until the economic recovery is anticipated. Therefore, when  $\lambda$  and  $\theta$  expand, the optimal threshold value under the optimal investment decisions will also increase.

**Table 6**  
Threshold values for different values of  $\gamma$ .

$\gamma$	Special monopoly	Duopoly		Simultaneous investment
	$P_S^*$	$P_L^*$	$P_F^*$	
0.29	3.251	2.167	4.552	1.821
0.34	3.688	2.459	5.163	2.065
0.39	4.117	2.744	5.763	2.305
0.44	4.539	3.026	6.354	2.542
0.49	4.955	3.303	6.937	2.775

**Table 7**  
Threshold values for different values of  $\lambda$ .

$\lambda$	Special monopoly	Duopoly		Simultaneous investment
	$P_S^*$	$P_L^*$	$P_F^*$	
0.12	4.046	2.696	5.662	2.265
0.15	4.080	2.720	5.713	2.285
0.18	4.117	2.744	5.763	2.305
0.21	4.154	2.769	5.815	2.326
0.24	4.191	2.794	5.867	2.347

**Table 8**  
Threshold values for different values of  $\theta$ .

$\theta$	Special monopoly	Duopoly		Simultaneous investment
	$P_S^*$	$P_L^*$	$P_F^*$	
-0.18	4.025	2.683	5.635	2.254
-0.23	4.069	2.712	5.696	2.278
-0.28	4.117	2.744	5.763	2.305
-0.33	4.170	2.780	5.838	2.335
-0.38	4.228	2.818	5.919	2.368

## 6. Conclusion

This model emphasizes the following: (1) the inferences with game options on the market structures formed by different competition and investment strategies of two VC firms in order to reflect the investment returns. These market structures are classified into an entry-deterred game (specific monopoly), a leader's dominating strategies (duopoly), and simultaneous investment; (2) how to select investment timing to avoid the potential competitive threats in order to provide the optimal expected threshold values for the investment decisions of VC firms. The purpose is to derive the threshold values for the optimal market entries in different market structures and to anticipate the time required for obtaining the threshold values and potential revenues in order to provide a reference for investment decision makers.

## Appendix 1

Let the profitability value  $M(P)$  of the extraordinary solution equation be written as follows:  $M(P) = CPD(1,0)$  (the part of cash flows). By applying it to the following equation we obtain

$$\gamma M(P) = \alpha P M_P(P) + \frac{1}{2} \sigma^2 P^2 M_{PP}(P) + \lambda M(P(1+\theta)) - \lambda M(P) + PD(0,1). \tag{A1}$$

According to Itô's Lemma (Itô, 1951) and following the Bellman equation [see Eqs. (4)–(6)], the solution obtained from the above equation is

$$C = \frac{1}{\gamma - \alpha - \lambda \theta} \tag{A2}$$

and

$$M(P) = \frac{PD(0,1)}{\gamma - \alpha - \lambda \theta}. \tag{A3}$$

## Appendix 2

Let the profitability value  $M(P)$  of the general solution equation be  $M(P) = A_1 P^{\beta_1} + A_2 P^{\beta_2}$ . It makes sense to require that  $P=0$ . It will remain  $M(P)=0$ . To make the profitability value go to zero, we must set the corresponding coefficient  $A_2=0$ . The general solution equation is  $M(P)=AP^\beta$ . By applying it to Eq. (5), we obtain

$$\gamma M(P) = \{ \alpha M(P)\beta + \frac{1}{2} \sigma^2 \beta(\beta-1)M(P) + \lambda M(P(1+\theta)) - \lambda M(P) \}, \tag{A4}$$

$$\gamma AP^\beta = \alpha \beta AP^\beta + \frac{1}{2} \sigma^2 \beta(\beta-1)AP^\beta + \lambda AP^\beta (1+\theta)^\beta - \lambda AP^\beta, \tag{A5}$$

$$\gamma = \alpha \beta + \frac{1}{2} \sigma^2 \beta(\beta-1) + \lambda(1+\theta)^\beta - \lambda, \tag{A6}$$

$$\frac{1}{2} \sigma^2 \beta^2 + (\alpha - \frac{1}{2} \sigma^2) \beta - (\lambda + \gamma - \lambda(1+\theta)^\beta) = 0. \tag{A7}$$

The solution of  $\beta$  is

$$\beta = \frac{-(\alpha - \frac{1}{2} \sigma^2) \pm \sqrt{(\alpha - \frac{1}{2} \sigma^2)^2 + 2\sigma^2(\lambda + \gamma - \lambda(1+\theta)^\beta)}}{\sigma^2}. \tag{A8}$$

We will generally denote the variable in the equation by  $\beta$  and the whole quadratic expression by  $f(\beta)$ :

$$f(\beta) = \frac{1}{2} \sigma^2 \beta^2 + \left( \alpha - \frac{1}{2} \sigma^2 \right) \beta - (\lambda + \gamma - \lambda(1+\theta)^\beta). \tag{A9}$$

The coefficient of  $\beta^2$  in  $f(\beta)$  is positive, so the graph is an upward-pointing parabola that goes to  $\infty$  as  $\beta$  goes to  $\pm \infty$ . Also,  $f(1) = \alpha - \gamma + \lambda \theta < 0$ , ( $\lambda > 0, \gamma > \alpha, 0 > \theta > -1$ ) and  $f(0) = -\gamma < 0$  (see Dixit and Pindyck, 1994).

Therefore,

$$\beta_1 = \frac{-(\alpha - \frac{1}{2}\sigma^2) + \sqrt{(\alpha - \frac{1}{2}\sigma^2)^2 + 2\sigma^2(\lambda + \gamma - \lambda(1 + \theta)^\beta)}}{\sigma^2} > 1, \quad (\text{A10})$$

$$\beta_2 = \frac{-(\alpha - \frac{1}{2}\sigma^2) - \sqrt{(\alpha - \frac{1}{2}\sigma^2)^2 + 2\sigma^2(\lambda + \gamma - \lambda(1 + \theta)^\beta)}}{\sigma^2} < 0. \quad (\text{A11})$$

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