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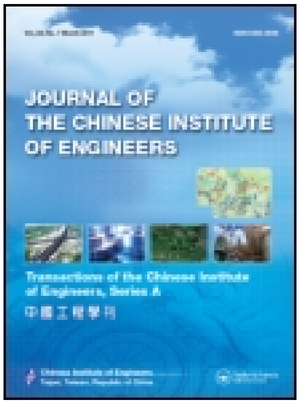

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Further investigation of vibration induced by harmonic loadings applied at circular rigid plate on half-space medium

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This article is the further investigation of vibration induced by harmonic loadings applied at a circular rigid plate on a half-space medium as reported by Liou (Liou, G.S., 2009. Vibrations induced by harmonic loadings applied at circular rigid plate on half-space medium. *Journal of Sound and Vibration*, 323, 257–269). The main purposes of this article are (1) to give the semi-analytic solution for vibration at any specific location in half-space medium; (2) to calculate the total average energy flux over one period of vibration time from the surfaces of a cylindrical cavity. The reason to evaluate the total average energy flux is that the fluctuation phenomenon of vibration amplitude along the distance from the vibration source (circular rigid plate) has been observed by Liou (2009) and can be seen from the numerical results in this article. To calculate the total average energy flux from the surfaces, piecewise linear distribution of energy flux intensity along z-direction for cylindrical surface and r-direction for bottom surface of the cylindrical cavity is assumed. The results show that the total average energy flux from the surfaces attenuates as distance from vibration source increases. This is due to the material damping in the half-space medium. The attenuation of total average energy flux is faster as the damping in the half-space medium is higher.

Keywords: ground vibration; wave propagation; energy flux; half-space

1. Introduction

Ambient vibration is an important design factor for structures housing precision equipment. Therefore, investigating the vibrations induced by a vibrational loading source has been of interest to many researchers in recent decades. To calculate the vibrations, Liou (2009) has employed the analytical solutions of wave propagation in half-space medium to obtain the vibration on any location of free surface of half-space medium. Sheng *et al*. (1999) and Krylov (1996) applied Euler beam Theory to model a whole train including rails, sleepers and ballast to evaluate the ground vibration induced by passing trains. Apsel and Luco (1983) have calculated the vibration in half-space medium due to point source.

Liou (2009) has shown that the amplitudes of some components of vibration on a free surface of half-space medium could fluctuate along distance from a vibration source (harmonic loadings applied at a rigid circular plate). This article will give the semi-analytic formulation to calculate the vibration in a half-space medium, and some numerical results are shown. The same fluctuation phenomenon is also observed for some vibration components at certain depths in the half-space medium.

In order to show that the total average energy flux over one period of vibration time passing through cylindrical cavity surfaces is attenuating as the cavity surfaces are getting farther from the vibration source, the total average energy flux passing through cylindrical cavity surfaces is calculated by assuming that the variation of intensity of energy flux is piecewise linear in r-direction for the bottom surface of the cavity and in z-direction for the cylindrical surface of the cavity Some numerical results of total average energy flux passing through different cavity surfaces will be shown.

2. Formulations

According to Liou's (2009) work, the vibrations at any specific location on a half-space medium (depth $z=0$) can be calculated by a semi-infinite integral. For the vibration at an arbitrary location in half-space medium ($z \geq 0$), the formulation can be obtained using the

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procedure described by Liou (1991, 1994). The expression can be stated as follows:

$$u_0 = -\int_0^\infty J\overline{Q}DP\,\mathrm{d}k \tag{1}$$

where Bessel function matrix J, Hankel transform matrix of piecewise linear tractions D and vector of traction intensities at nodal rings P are defined in the appendix; the displacement vector $u_0 = (u_r(r,z), u_z(r,z), u_\theta(r,z))^T$; and matrix $\overline{Q}$ can be expressed as follows:

$$\overline{Q} = \begin{bmatrix} B_1e^{-vz} + B_2e^{-v'z} & B_3e^{-vz} + B_4e^{-v'z} & 0 \\ B_4e^{-vz} + B_3e^{-v'z} & B_5e^{-vz} + B_6e^{-v'z} & 0 \\ 0 & 0 & B_7e^{-v'z} \end{bmatrix}, \tag{2}$$

where

$$B_1 = \frac{-2k^2Gv'}{\Lambda}, \quad B_2 = \frac{v'G(2k^2 - k_\beta^2)}{\Lambda}$$

$$B_3 = \frac{-kG(2k^2 - k_\beta^2)}{\Lambda},$$

$$B_4 = \frac{2kGvv'}{\Lambda}, \quad B_5 = \frac{vG(2k^2 - k_\beta^2)}{\Lambda}, \quad B_6 = -\frac{2k^2Gv}{\Lambda}$$

$$\text{and} \quad B_7 = \frac{1}{Gv'},$$

where k is wave number in r-direction; G is shear modulus;

$$v = \sqrt{k^2 - \left(\omega^2/c_p^2\right)}; \quad v' = \sqrt{k^2 - \left(\omega^2/c_p^2\right)};$$

$$k_\beta = \omega/c_s; \quad \Lambda = G\left[4k^2vv' - \left(2k^2 - k_\beta^2\right)^2\right];$$

c_p and c_s are compressional and shear wave velocities, respectively.

Also, the total average energy flux through the surfaces of cylindrical cavity shown in Figure 1 due to unit normalized loading can be calculated as follows:

$$E = \frac{\rho\omega^2}{4}\left(\frac{2\pi}{\pi}\right)\left(\int_0^{\bar{r}} \left(|u_r|^2 + |u_z|^2 + |u_\theta|^2\right)\big|_{z=\bar{z}} r\,\mathrm{d}r + \int_0^{\bar{z}} \left(|u_r|^2 + |u_z|^2 + |u_\theta|^2\right)\big|_{r=\bar{r}} \bar{r}\,\mathrm{d}z\right). \tag{3}$$

where ρ is mass density of half-space medium. In the equation, the total energy flux (E) through the surfaces has been averaged by the period $\frac{2\pi}{\omega}$ of time. In Equation (3), 2π is for Fourier component $n = 0$ and π is for Fourier component $n = 1$. Also, the integrations with respect to θ and time t have been done implicitly for E in Equation (3).

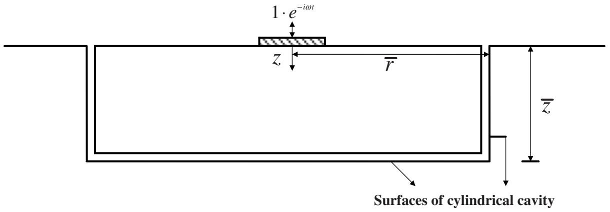

Figure 1. Half-space Medium with assumed surfaces of cylindrical cavity.

3. Numerical results

According to extensive investigations of numerical results, one can also observe that at some depth ($z \neq 0$) the fluctuation phenomenon along the distance from vibration source for some components of vibration as stated by Liou (2009) still exists. Figure 2 shows the numerical results for $|u_r|$ component due to unit normalized rocking loading applied to a rigid circular plate on half space. In Figure 2 the nondimensionalized frequency ϖ is 0.1; damping ratio is 0.001; Poisson ratio is 0.33; and all the quantities are nondimensionalized or normalized as in Liou's (2009) work. From this figure, one can see that the fluctuation is more severe as the nondimensionalized depth $\bar{z}$ increases.The depth $\bar{z}$ and the distance $\bar{r}$ have been nondimensionalized by shear wave length of 1 Hz frequency.

The above mentioned fluctuation phenomenon leads to curiosity as to the total average energy flux passing through the surfaces of different cylindrical cavities as shown in Figure 1 .To calculate the total average energy flux, Equation (3) is employed with the assumption that the variation of energy flux intensity is piecewise linear with respect to z direction and r direction in cylindrical coordinates.

In order to obtain accurate result, the sizes of the intervals $\Delta\bar{r}$ and $\Delta\bar{z}$ for piecewise linear distribution model have to be tested first. Table 1 shows the results for the total average energy flux for the case with $\bar{r} = 6.0$, $\bar{z} = 0.6$ and nondimensionalized frequency $\varpi = 1.0$. Damping ratio $\xi = 0.001$ and Poisson ratio $v = 0.33$ are selected for the results in the table. In this table and Table 2, the total average energy fluxes are calculated by Equation (3) with the terms $\frac{\rho\omega^2}{4}$ and $\left(\frac{2\pi}{\pi}\right)$ dropped, and the normalized applied loadings are equal to 1 by the ways of Liou's (2009) work. From Table 1, one can conclude that the numerical results are converging for each type of exciting loading and

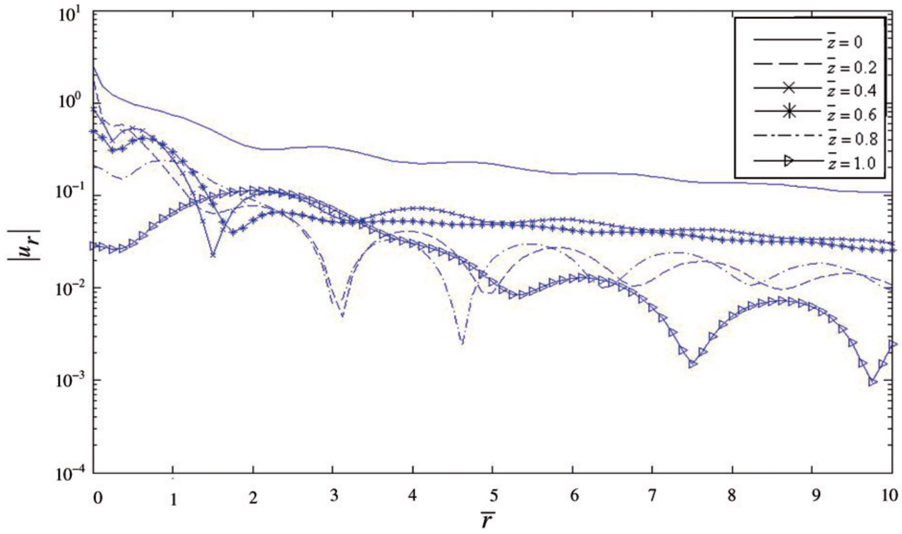

Fig. 2. Vibration amplitude of $\boldsymbol{u}_r$ due to unit normalized rocking loading ($\bar{\omega}=0.1$).

Table 1. Convergence test of total average energy flux for piecewise linear model by the case $\bar{r}=6.0$ and $\bar{z}=0.6$.

Excitation types		$\Delta\bar{r}=0.075$	$\Delta\bar{r}=0.100$	$\Delta\bar{r}=0.125$
Vertical	$\Delta\bar{z}=0.100$	0.066846	0.066846	0.066846
Rocking		84.11761	84.15501	84.20291
Horizontal		0.26684	0.26704	0.26724
Torsional		34.0742	34.0742	34.0744
Vertical	$\Delta\bar{z}=0.150$	0.066877	0.066877	0.066877
Rocking		84.17563	84.21303	84.26093
Horizontal		0.266858	0.267058	0.267258
Torsional		34.07419	34.07419	34.07439
Vertical	$\Delta\bar{z}=0.200$	0.066911	0.066911	0.066911
Rocking		84.24073	84.27813	84.32603
Horizontal		0.266878	0.267078	0.267278
Torsional		34.07418	34.07418	34.07438

Table 2. Total average energy flux through surfaces of cavity of $\bar{r}=0.5$ and $\bar{z}=0.1$.

$\xi=0.001$	1.9408
$\xi=0.01$	1.8907
$\xi=0.02$	1.8356
$\xi=0.03$	1.7811
$\xi=0.04$	1.7274
$\xi=0.05$	1.6746
$\xi=0.07$	1.5719
$\xi=0.10$	1.4261

the intervals $\Delta\bar{r}=0.125$ and $\Delta\bar{z}=0.100$ for piecewise linear model are good enough with precision up to three significant figures for the results of total average energy flux.

Therefore, $\Delta\bar{r}=0.125$ and $\Delta\bar{z}=0.100$ are employed in the piecewise linear model to calculate the total average energy flux passing through the surfaces of different cylindrical cavities for different damping ratios. The results are shown in Figure 3. In the figure, there are 20 results for each damping ratio. The 20 results are the total average energy fluxes passing through the surfaces (cylindrical surface and bottom circular surface) of the cavities of $\bar{r}=0.5$ with $\bar{z}=0.1$, $\bar{r}=1.0$ with $\bar{z}=0.2$, $\bar{r}=1.5$ with $\bar{z}=0.3,\ldots,$ up to $\bar{r}=10.0$ with $\bar{z}=2.0$, respectively. In the figure, one should note that the total average energy fluxes have been normalized by the total average energy fluxes of the case $\bar{r}=0.5$ and $\bar{z}=0.1$. The total average energy fluxes of the case $\bar{r}=0.5$ and $\bar{z}=0.1$ for different

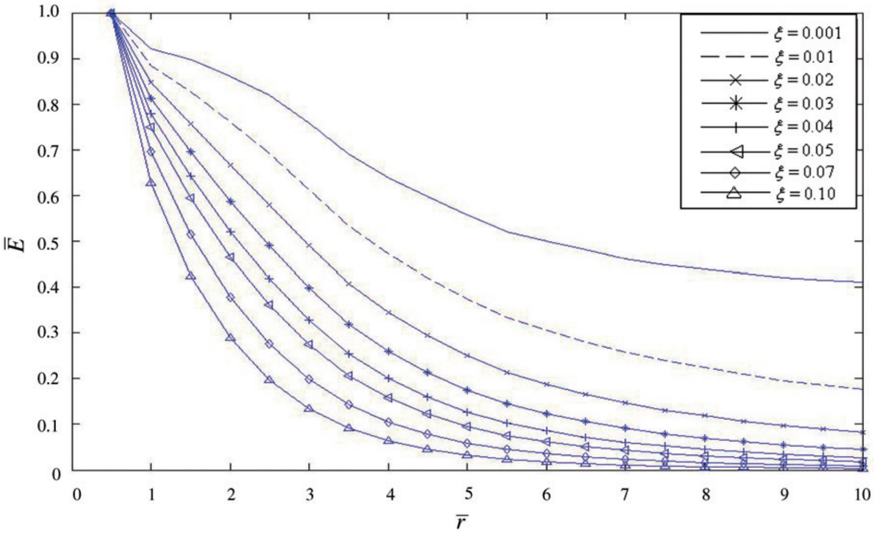

Fig. 3. Total average energy flux through cylindrical cavities due to unit normalized rocking loading ($\bar{\omega} = 0.1$).

damping ratios $\xi = 0.01 \sim 0.10$ are shown in Table 2. Also, Figure 3 shows that higher damping ratio will make the total average energy flux attenuate faster.

4. Conclusions

This article has derived the equation for calculating the vibration in half space medium due to an excitation at a circular rigid surface foundation. And a simple numerical scheme to calculate the energy flux through the surfaces of a cylindrical cavity is employed. After the numerical investigation has been done, the following numerical conclusions can be made.

(1) Although the fluctuation of vibration amplitude along the distance from the vibration source as shown in Figure 2 exists, the total average energy flux through a surface decreases as distance from the vibration source increases if damping is present in the medium.

(2) The energy flux is attenuating faster as the damping in the medium is greater.

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Nomenclatures

k	horizontal wave number in r direction.
ω	excitation frequency.
G	shear modulus.
c_p	compressional wave velocity.
c_s	shear wave velocity.
D	Hankel transform matrix.
J	Bessel function matrix.
P	vector of traction intensities at nodal rings.

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Appendix

The matrix J in Equation (1) can be expressed as

$$J = \begin{bmatrix} J_n'(kr) & 0 & (n/r)J_n(kr) \\ 0 & kJ_n(kr) & 0 \\ (n/r)J_n(kr) & 0 & J_n'(kr) \end{bmatrix}, \tag{A1}$$

where $J_n(kr)$ = first kind of Bessel function of order n; and $J'_n(kr) = [\mathrm{d}J_n(kr)/\mathrm{d}r]$

The matrix D in Equation (1) can be expressed as

$$D = \begin{bmatrix} -D^T_{n+1} + D^T_{n-1} & 0 & D^T_{n+1} + D^T_{n-1} \\ 0 & D^T_n & 0 \\ D^T_{n+1} + D^T_{n-1} & 0 & -D^T_{n+1} + D^T_{n-1} \end{bmatrix} \tag{A2}$$

where

$$\begin{aligned} D^T_{n+1} &= \int_0^{a_0} \frac{r}{2} J_{n+1}(kr) h^T \mathrm{d}r, \\ D^T_n &= \int_0^{a_0} r J_{n+1}(kr) h^T \mathrm{d}r, \end{aligned} \tag{A3}$$

and

$$D^T_{n-1} = \int_0^{a_0} \frac{r}{2} J_{n-1}(kr) h^T \mathrm{d}r,$$

and vector h^{T} is piecewise linear distribution model of interaction tractions (τ_{rz}, σ_{zz}, and $\tau_{\theta z}$). Vector P in Equation (1) is the intensity of interaction traction at the nodal rings. For detailed derivations of these equations, one can refer to Liou's (2009) work.