International Journal of Control, Automation, and Systems (2011) 9(5):850-856 http://www.springer.com/12555
DOI 10.1007/s12555-011-0505-8

Proportional-Derivative Unknown Input Observer Design Using Descriptor System Approach for Non-Minimum Phase Systems

Huan-Chan Ting, Jeang-Lin Chang, and Yon-Ping Chen

Abstract: This paper considers the problem of estimating the state of an MIMO linear system with unknown inputs in the state and output. Through a series of linear transformations in the state and output equations, the original system can be transformed into a descriptor system form. The proposed proportional derivative observer can accurately estimate the system state and avoid the peaking phenomenon. Moreover, the approach developed in this paper does not require the derivatives of the output and can be applied to the system with unstable zeros (with respect to the relation between the output and the unknown input). Finally, our algorithm can prove the valid feasibility and the property of disturbance attenuation through demonstrating a simulation-base example.

Keywords: Descriptor system, non-minimum phase, state estimation, unknown input observer.

1. INTRODUCTION

External disturbances and coefficient variations of a plant bring difficulties designing the controller. These unknown perturbations are incorporated into the system model, as unknown inputs. When the system is subject to the unknown inputs, the standard Luenberger observer can not obtain the perfect estimation. As a result, the design of state observer for systems with unknown inputs, called the unknown input observer (UIO) design, is an important topic in several control applications [1-19]. However, these papers [1-4] only consider the unknown input in the state equation. In this regard, Darouach *et al.* [2] presented a full-order UIO, and Hou and Muller [3] established a reduced-order UIO. UIO can also apply to the fault detection and isolation problems [5-8]. When there exists unknown inputs in state and output equations of the system model, these papers [9-14] developed different design methods under some constraints of system matrices. Alwi *et al.* [13] proposed an effective sliding mode observer to estimate states and noises for LTI systems with disturbances and measurement noises. Sharma and Aldeen [15,16] decoupled the unknown inputs from the rest of the system through a series of coordinate transformations. There are two necessary and sufficient conditions to check the existence of a stable UIO. These two important conditions are that the transfer function matrix between the unknown input and the system output must be minimum phase and with relative degree one. If one of the conditions mentioned above is not satisfied, how to design a stable UIO is a difficult research problem. Releasing the minimum phase condition, this paper proposes a reduced-order UIO design method to effectively estimate the state in which there exists unknown inputs in the state and output equations of the system with unstable zeros. In contrast with [13], they presented a specific linear matrix inequality (LMI) to design parameters of the observer and guarantee the estimation convergence simultaneously. In this paper, the matrix stabilizing the estimated error dynamics is determined by a pole-placement method, and then another independent matrix is used to suppress the effect of unknown inputs.

On the other hand, Boutayeb *et al.* [17] involved the descriptor system approach to design a nonlinear observer for simultaneously estimating the system state and unknown inputs. Based on the same method, Fernando and Trinh [18] designed a reduced-order functional observer of the linear systems where both input and output disturbances are presented. Gao and Wang [20] developed the descriptor system observer to reduce the effect of measurement noises. They constructed a modified proportional derivative observer to asymptotically estimate the system state and output noise at the same time.

In this paper, we introduce the descriptor system approach, developed by Gao and Wang [20], into the observer design for linear systems with unknown inputs in the state and output. A series of similar transformations is first proposed to transform the original system into a descriptor system form. Then a proportional derivative observer is designed to estimate the system state in which the derivative gain and the proportional gain can adjust the unknown input amplification and ensure the robust stability of the estimation error dynamics, respectively. Although the perfect estimation

Manuscript received July 26, 2010; revised January 18, 2011; accepted June 17, 2011. Recommended by Editorial Board member Young Soo Suh under the direction of Editor Young Il Lee. This work was supported by the National Science Council under grant NSC-99-2221-E-161-607.

Huan-Chan Ting and Yon-Ping Chen are with the Institute of Electrical Control Engineering, National Chiao-Tung University, 1001 University Rd., Hsinchu 300, Taiwan (e-mails: hcting.ece92g@nctu.edu.tw, ypchen@mail.nctu.edu.tw).

Jeang-Lin Chang is with the Department of Electrical Engineering, Oriental Institute of Technology, 58, Sec. 2, Sichuan Rd., Banqiao Dist., New Taipei City 220, Taiwan (e-mail: jlchang@ee.oit.edu.tw).

is impossible, the proposed algorithm can be successfully implemented in the systems with unstable invariant zeros and the estimation error is finally bounded in a small region around zero. Moreover, the observer in this paper does not require the derivative of the output and is capable of avoiding the peaking phenomenon.

In the next section, a class of controlled systems is first introduced with some important assumptions in relation to the system matrices. Sections 3 and 4 describe some coordinate transformation and the unknown input observer design. To verify the developed observer, a numeric example is shown in section 5. Finally, section 6 gives concluding remarks.

2. PROBLEM FORMULATION

Consider the following MIMO linear system with the unknown input as

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \bar{\mathbf{A}}\mathbf{x}(t)+\bar{\mathbf{B}}\mathbf{u}(t)+\bar{\mathbf{D}}\mathbf{d}(t),\\ \mathbf{y}(t) &= \bar{\mathbf{C}}\mathbf{x}(t)+\bar{\mathbf{F}}\mathbf{d}(t),\end{aligned} \tag{1}$$

where $\mathbf{x}\in\mathbb{R}^n$, $\mathbf{u}\in\mathbb{R}^m$, $\mathbf{d}\in\mathbb{R}^l$, and $\mathbf{y}\in\mathbb{R}^p$ are the system state vector, the control input vector, the unknown input vector, and the system output vector, respectively. Suppose that system (1) is detectable and the system matrices with appropriate dimensions are known. Without loss of generality, we assume that $\text{rank}(\bar{\mathbf{F}}) = k$ and $\text{rank}(\bar{\mathbf{C}}) = p$ where $p \ge l$ and $0 < k \le l$. Although the input $\mathbf{d}(t)$ is unknown (cannot be measured), the target is to design a UIO which can accurately estimate the system state $\mathbf{x}(t)$ for any instant of time. Using the standard Luenberger observer to estimate the state of system (1) precisely is difficult due to the existence of unknown inputs. Busawon and Kabore [21] have shown that the conventional Luenberger observer is not adequate for handling measurement noises. Several authors [1-4,9-16] have solved this problem and proposed different UIO design methods. There exists two conditions checking the existence of a stable UIO [1-4,9-16]. The first is that the system with respect to the relation between the output and the unknown input must be minimum phase, i.e. the invariant zeros of the system are located on the left-half open plane. The second is that the relative degree of the transfer matrix function from the unknown input to the system output is one. For a linear system with the unknown inputs in the state and output equations and unstable invariant zeros, a UIO design using the descriptor system approach is proposed in this paper.

3. COORDINATE TRANSFORMATION AND SYSTEM ANALYSIS

Since $\text{rank}(\bar{\mathbf{F}}) = k$, applying the singular value decomposition [22] can obtain

$$\mathbf{U}\bar{\mathbf{F}}\mathbf{V} = \begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_k\end{bmatrix}, \tag{2}$$

where the matrices $\mathbf{U}\in\mathbb{R}^{p\times p}$ and $\mathbf{V}\in\mathbb{R}^{l\times l}$ are nonsingular. Define the following vectors

$$\mathbf{d}(t) = \mathbf{V}\begin{bmatrix}\mathbf{d}_1(t)\\ \mathbf{d}_2(t)\end{bmatrix} \text{ and } \bar{\mathbf{y}}(t) = \mathbf{U}\mathbf{y}(t), \tag{3}$$

where $\mathbf{d}_1\in\mathbb{R}^{l-k}$ and $\mathbf{d}_2\in\mathbb{R}^{k}$. As a result, system (1) can be rewritten as

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \bar{\mathbf{A}}\mathbf{x}(t)+\bar{\mathbf{B}}\mathbf{u}(t)+\bar{\mathbf{D}}_1\mathbf{d}_1(t)+\bar{\mathbf{D}}_2\mathbf{d}_2(t),\\ \bar{\mathbf{y}}(t) &= \mathbf{C}\mathbf{x}(t)+\begin{bmatrix}\mathbf{0}\\ \mathbf{I}_k\end{bmatrix}\mathbf{d}_2(t),\end{aligned} \tag{4}$$

where $\mathbf{C} = \mathbf{U}\bar{\mathbf{C}}$ and $\bar{\mathbf{D}}\mathbf{V} = [\bar{\mathbf{D}}_1\ \ \bar{\mathbf{D}}_2]$. Assuming rank $(\mathbf{C}\bar{\mathbf{D}}_1) = \text{rank}(\bar{\mathbf{D}}_1) = l-k$, there exists a transformation $\mathbf{T}_1\in\mathbb{R}^{n\times n}$ such that the matrix $\bar{\mathbf{D}}_1$ can be partitioned as

$$\mathbf{T}_1\bar{\mathbf{D}}_1 = \begin{bmatrix}\mathbf{D}_1\\ \mathbf{0}\end{bmatrix}, \tag{5}$$

where $\mathbf{D}_1\in\mathbb{R}^{(l-k)\times(l-k)}$ is invertible. Let $[\bar{\mathbf{C}}_1\ \ \bar{\mathbf{C}}_2] = \mathbf{C}\mathbf{T}_1^{-1}$ where $\bar{\mathbf{C}}_1\in\mathbb{R}^{p\times(l-k)}$ and $\bar{\mathbf{C}}_2\in\mathbb{R}^{p\times(n-l+k)}$, we can obtain that

$$\begin{aligned}\text{rank}(\mathbf{C}\bar{\mathbf{D}}_1) &= \text{rank}(\bar{\mathbf{C}}\mathbf{T}_1^{-1}\mathbf{T}_1\bar{\mathbf{D}}_1)\\ &= \text{rank}\left(\begin{bmatrix}\bar{\mathbf{C}}_1 & \bar{\mathbf{C}}_2\end{bmatrix}\begin{bmatrix}\mathbf{D}_1\\ \mathbf{0}\end{bmatrix}\right) = \text{rank}(\bar{\mathbf{C}}_1\mathbf{D}_1) = l-k.\end{aligned} \tag{6}$$

From $\text{rank}(\bar{\mathbf{D}}_1) = l-k$ and (6) we have $\text{rank}(\bar{\mathbf{C}}_1) = l-k$. Define the transformation $\mathbf{U}_1 = \begin{bmatrix}\bar{\mathbf{C}}_1^{+}\\ \Phi_1\end{bmatrix}\in\mathbb{R}^{p\times p}$ where $\Phi_1^T\in\mathbb{R}^{p\times(p-l+k)}$. The matrix $\bar{\mathbf{C}}_1^{+} = (\bar{\mathbf{C}}_1^T\bar{\mathbf{C}}_1)^{-1}\bar{\mathbf{C}}_1^T$ is a null space of $\bar{\mathbf{C}}_1^T$. Multiplying $\mathbf{U}_1$ and $\mathbf{C}\mathbf{T}_1^{-1}$ attains

$$\mathbf{U}_1\mathbf{C}\mathbf{T}_1^{-1} = \mathbf{U}_1\begin{bmatrix}\bar{\mathbf{C}}_1 & \bar{\mathbf{C}}_2\end{bmatrix} = \begin{bmatrix}\mathbf{I}_{l-k} & \mathbf{C}_{12}\\ \mathbf{0} & \mathbf{C}_{22}\end{bmatrix}, \tag{7}$$

where $\mathbf{C}_{12} = \bar{\mathbf{C}}_1^{+}\bar{\mathbf{C}}_2$ and $\mathbf{C}_{22} = \Phi_1\bar{\mathbf{C}}_2$. Furthermore, we introduce a transformation $\mathbf{T}_2 = \begin{bmatrix}\mathbf{I}_{l-k} & -\mathbf{C}_{12}\\ \mathbf{0} & \mathbf{I}_{n-l+k}\end{bmatrix}$ and define $\mathbf{z} = \begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{bmatrix} = \mathbf{T}_2^{-1}\mathbf{T}_1\mathbf{x}$ where $\mathbf{z}_1\in\mathbb{R}^{l-k}$ and $\mathbf{z}_2\in\mathbb{R}^{n-l+k}$. Notice that

$$\begin{aligned}&\mathbf{T}_2^{-1}\mathbf{T}_1\bar{\mathbf{A}}\mathbf{T}_1^{-1}\mathbf{T}_2 = \begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}\\ \mathbf{A}_{21} & \mathbf{A}_{22}\end{bmatrix},\quad \mathbf{T}_2^{-1}\mathbf{T}_1\bar{\mathbf{B}} = \begin{bmatrix}\mathbf{B}_1\\ \mathbf{B}_2\end{bmatrix},\\ &\mathbf{T}_2^{-1}\mathbf{T}_1\bar{\mathbf{D}}_2 = \begin{bmatrix}\mathbf{D}_{12}\\ \mathbf{D}_{22}\end{bmatrix},\ \text{and}\ \mathbf{U}_1\begin{bmatrix}\mathbf{0}\\ \mathbf{I}_k\end{bmatrix} = \begin{bmatrix}\mathbf{F}_{12}\\ \mathbf{F}_{22}\end{bmatrix}.\end{aligned} \tag{8}$$

Based on the mentioned transformations above, system (4) can be transformed into the following form:

$$\begin{aligned}\dot{\mathbf{z}}_1(t) &= \mathbf{A}_{11}\mathbf{z}_1(t)+\mathbf{A}_{12}\mathbf{z}_2(t)+\mathbf{B}_1\mathbf{u}(t)+\mathbf{D}_{12}\mathbf{d}_2(t)+\mathbf{D}_1\mathbf{d}_1(t),\\ \dot{\mathbf{z}}_2(t) &= \mathbf{A}_{21}\mathbf{z}_1(t)+\mathbf{A}_{22}\mathbf{z}_2(t)+\mathbf{B}_2\mathbf{u}(t)+\mathbf{D}_{22}\mathbf{d}_2(t),\\ \mathbf{y}_1(t) &= \mathbf{z}_1(t)+\mathbf{F}_{12}\mathbf{d}_2(t),\\ \mathbf{y}_2(t) &= \mathbf{C}_{22}\mathbf{z}_2(t)+\mathbf{F}_{22}\mathbf{d}_2(t),\end{aligned} \tag{9}$$

where $\mathbf{y}_1 \in \mathbb{R}^{l-k}$ and $\mathbf{y}_2 \in \mathbb{R}^{p-l+k}$. Observing (9) can obtain $\mathbf{z}_1 = \mathbf{y}_1 - \mathbf{F}_{12}\mathbf{d}_2$. Substitute $\mathbf{z}_1$ into (9) to obtain

$$\begin{aligned}\dot{\mathbf{z}}_2(t) &= \mathbf{A}_{22}\mathbf{z}_2(t)+\mathbf{B}_2\mathbf{u}(t)+\mathbf{A}_{21}\mathbf{y}_1(t)\\ &\quad+\left(\mathbf{D}_{22}-\mathbf{A}_{21}\mathbf{F}_{12}\right)\mathbf{d}_2(t)\\ &= \mathbf{A}_{22}\mathbf{z}_2(t)+\mathbf{B}_2\mathbf{u}(t)+\mathbf{A}_{21}\mathbf{y}_1(t)+\mathbf{D}_2\mathbf{d}_2(t),\\ \mathbf{y}_2(t) &= \mathbf{C}_{22}\mathbf{z}_2(t)+\mathbf{F}_{22}\mathbf{d}_2(t),\end{aligned} \tag{10}$$

where $\mathbf{D}_2 = \mathbf{D}_{22} - \mathbf{A}_{21}\mathbf{F}_{12}$. In the following, a reduced-order observer algorithm is proposed by the descriptor system form of estimating simultaneously the system state $\mathbf{z}_2$ and the unknown input $\mathbf{d}_2$. Define a vector $\mathbf{w} \in \mathbb{R}^{n-l+2k}$ as $\mathbf{w} = \begin{bmatrix}\mathbf{z}_2\\ \mathbf{d}_2\end{bmatrix}$ and

$$\begin{aligned}&\mathbf{E}=\begin{bmatrix}\mathbf{I}_{n-l+k} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix},\quad \mathbf{H}=\begin{bmatrix}\mathbf{A}_{22} & \mathbf{D}_2\\ \mathbf{0} & -\mathbf{I}_k\end{bmatrix},\\ &\mathbf{G}=\begin{bmatrix}\mathbf{B}_2 & \mathbf{A}_{21}\\ \mathbf{0} & \mathbf{0}\end{bmatrix},\quad \mathbf{v}=\begin{bmatrix}\mathbf{u}\\ \mathbf{y}_1\end{bmatrix},\quad \text{and}\quad \mathbf{M}=\begin{bmatrix}\mathbf{0}\\ \mathbf{I}_k\end{bmatrix}.\end{aligned} \tag{11}$$

Then the descriptor system form of system (10) is given by

$$\begin{aligned}\begin{bmatrix}\mathbf{I}_{n-l+k} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{z}}_2(t)\\ \dot{\mathbf{d}}_2(t)\end{bmatrix} &= \begin{bmatrix}\mathbf{A}_{22} & \mathbf{D}_2\\ \mathbf{0} & -\mathbf{I}_k\end{bmatrix}\begin{bmatrix}\mathbf{z}_2(t)\\ \mathbf{d}_2(t)\end{bmatrix}\\ &\quad+\begin{bmatrix}\mathbf{B}_2 & \mathbf{A}_{21}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{u}(t)\\ \mathbf{y}_1(t)\end{bmatrix}+\begin{bmatrix}\mathbf{0}\\ \mathbf{I}_k\end{bmatrix}\mathbf{d}_2(t)\\ &= \mathbf{H}\mathbf{w}(t)+\mathbf{G}\mathbf{v}(t)+\mathbf{M}\mathbf{d}_2(t),\\ \mathbf{y}_2(t) &= \begin{bmatrix}\mathbf{C}_{22} & \mathbf{F}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{z}_2(t)\\ \mathbf{d}_2(t)\end{bmatrix}=\mathbf{C}_1\mathbf{w}(t),\end{aligned} \tag{12}$$

where $\mathbf{C}_1$=[$\mathbf{C}_{22}$ $\mathbf{F}_{22}$]. The detectability of (12) is proven in the following lemmas.

Lemma 1: If the following conditions hold:

$$\operatorname{rank}\left(\begin{bmatrix}s\mathbf{I}_n-\bar{\mathbf{A}} & -\bar{\mathbf{D}}_1\\ \mathbf{C} & \mathbf{0}\end{bmatrix}\right)=n+l-k \quad \forall s\in\mathbb{C}^+ \quad \text{and}$$

$$\operatorname{rank}(\mathbf{C}\bar{\mathbf{D}}_1)=\operatorname{rank}(\bar{\mathbf{D}}_1)=l-k,$$

then the pair ($\mathbf{A}_{22}$, $\mathbf{C}_{22}$) is detectable.

Proof: According to $\operatorname{rank}(\mathbf{C}\bar{\mathbf{D}}_1)=\operatorname{rank}(\bar{\mathbf{D}}_1)=l-k$ and (8), we have

$$\begin{aligned}&\operatorname{rank}\left(\begin{bmatrix}s\mathbf{I}_n-\bar{\mathbf{A}} & -\bar{\mathbf{D}}_1\\ \mathbf{C} & \mathbf{0}\end{bmatrix}\right)\\ &=\operatorname{rank}\left(\begin{bmatrix}s\mathbf{I}_{l-k}-\mathbf{A}_{11} & -\mathbf{A}_{12} & -\mathbf{D}_1\\ -\mathbf{A}_{21} & s\mathbf{I}_{n-l+k}-\mathbf{A}_{22} & \mathbf{0}\\ \mathbf{I}_{l-k} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_{22} & \mathbf{0}\end{bmatrix}\right)\\ &=\operatorname{rank}\left(\begin{bmatrix}s\mathbf{I}_{n-l+k}-\mathbf{A}_{22}\\ \mathbf{C}_{22}\end{bmatrix}\right)+2\left(l-k\right)=n+l-k.\end{aligned}$$

Since $\operatorname{rank}\left(\begin{bmatrix}s\mathbf{I}_n-\bar{\mathbf{A}} & -\bar{\mathbf{D}}_1\\ \mathbf{C} & \mathbf{0}\end{bmatrix}\right)=n+l-k \quad \forall s\in\mathbb{C}^+$, we can ensure the following relationship,

$$\operatorname{rank}\left(\begin{bmatrix}s\mathbf{I}_{n-l+k}-\mathbf{A}_{22}\\ \mathbf{C}_{22}\end{bmatrix}\right)=n-l+k \quad \forall s\in\mathbb{C}^+$$

and complete the proof.

Lemma 2: The descriptor system (12) is completely detectable, i.e.,

(i) $\operatorname{rank}\left(\begin{bmatrix}s\mathbf{E}-\mathbf{H}\\ \mathbf{C}_1\end{bmatrix}\right)=n-l+2k, \quad \forall s\in\mathbb{C}^+$,

(ii) $\operatorname{rank}\left(\begin{bmatrix}\mathbf{E}\\ \mathbf{C}_1\end{bmatrix}\right)=n-l+2k.$

The completely detectable system represents that it has neither unstable finite nor infinite output decoupling zeros [22,23].

Proof: From rank($\mathbf{F}_{22}$)=k and Lemma 1, we have

$$\begin{aligned}\operatorname{rank}\left(\begin{bmatrix}s\mathbf{E}-\mathbf{H}\\ \mathbf{C}_1\end{bmatrix}\right)&=\operatorname{rank}\left(\begin{bmatrix}s\mathbf{I}_{n-k}-\mathbf{A}_{22} & -\mathbf{D}_2\\ \mathbf{0} & \mathbf{I}_k\\ \mathbf{C}_{22} & \mathbf{F}_{22}\end{bmatrix}\right)\\ &=\operatorname{rank}\left(\begin{bmatrix}s\mathbf{I}_{n-k}-\mathbf{A}_{22}\\ \mathbf{C}_{22}\end{bmatrix}\right)+k\\ &=n-l+2k \quad \forall s\in\mathbb{C}^+\end{aligned}$$

and

$$\begin{aligned}\operatorname{rank}\left(\begin{bmatrix}\mathbf{E}\\ \mathbf{C}_1\end{bmatrix}\right)&=\operatorname{rank}\left(\begin{bmatrix}\mathbf{I}_{n-l+k} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\\ \mathbf{C}_{22} & \mathbf{F}_{22}\end{bmatrix}\right)\\ &=n-l+k+\operatorname{rank}(\mathbf{F}_{22})=n-l+2k.\end{aligned}$$

Hence, system (12) is completely detectable. The proof of the lemma is finished.

Design $\mathbf{K}_1=\begin{bmatrix}\mathbf{0}\\ \mathbf{K}_{12}\mathbf{F}_{22}^+\end{bmatrix}\in\mathbb{R}^{(n-l+2k)\times(p-l+k)}$ where the gain matrix $\mathbf{K}_{12}\in\mathbb{R}^{k\times k}$ is invertible and $\mathbf{F}_{22}^+ = (\mathbf{F}_{22}^T\mathbf{F}_{22})^{-1}\mathbf{F}_{22}^T$. Note that

$$\begin{aligned}&\left(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1\right)^{-1}\mathbf{M}\\ &=\left(\begin{bmatrix}\mathbf{I}_{n-l+k} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{0}\\ \mathbf{K}_{12}\mathbf{F}_{22}^+\end{bmatrix}\begin{bmatrix}\mathbf{C}_{22} & \mathbf{F}_{22}\end{bmatrix}\right)^{-1}\mathbf{M}\\ &=\begin{bmatrix}\mathbf{I}_{n-k} & \mathbf{0}\\ -\mathbf{F}_{22}^+\mathbf{C}_{22} & \mathbf{K}_{12}^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{0}\\ \mathbf{I}_k\end{bmatrix}=\begin{bmatrix}\mathbf{0}\\ \mathbf{K}_{12}^{-1}\end{bmatrix}.\end{aligned} \tag{13}$$

4. OBSERVER FORMULATION AND DESIGN

The proportional derivative observer for system (12) is designed as

$$\begin{aligned}&(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)\dot{\eta}(t)=(\mathbf{H}+\mathbf{L}\mathbf{C}_1)\eta(t)\\&\quad+\left((\mathbf{H}+\mathbf{L}\mathbf{C}_1)(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\mathbf{K}_1-\mathbf{L}\right)\mathbf{y}_2(t)+\mathbf{G}\mathbf{v}(t),\\&\hat{\mathbf{w}}(t)=\eta(t)+(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\mathbf{K}_1\mathbf{y}_2(t),\end{aligned}\tag{14}$$

where $\eta\in\mathbb{R}^{n-l+2k}$ is an auxiliary state of the observer and $\hat{\mathbf{w}}=\begin{bmatrix}\hat{\mathbf{z}}_2\\ \hat{\mathbf{d}}_2\end{bmatrix}\in\mathbb{R}^{n-l+2k}$ denotes the estimation of $\boldsymbol{w}$. Adding $\mathbf{K}_1\dot{\mathbf{y}}_2$ into both sides of the first equation in (12) obtains

$$\begin{aligned}(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)\dot{\mathbf{w}}(t)&=(\mathbf{H}+\mathbf{L}\mathbf{C}_1)\mathbf{w}(t)+\mathbf{G}\mathbf{v}(t)\\&\quad+\mathbf{K}_1\dot{\mathbf{y}}_2(t)-\mathbf{L}\mathbf{y}_2(t)+\mathbf{M}\mathbf{d}_2(t).\end{aligned}\tag{15}$$

Substitute $\eta=\hat{\mathbf{w}}-\left(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1\right)^{-1}\mathbf{K}_1\mathbf{y}_2$ into (14) to obtain

$$\begin{aligned}(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)\dot{\hat{\mathbf{w}}}(t)&=(\mathbf{H}+\mathbf{L}\mathbf{C}_1)\hat{\mathbf{w}}(t)\\&\quad-\mathbf{L}\mathbf{y}_2(t)+\mathbf{K}_1\dot{\mathbf{y}}_2(t)+\mathbf{G}\mathbf{v}(t).\end{aligned}\tag{16}$$

Define $\tilde{\mathbf{w}}=\mathbf{w}-\hat{\mathbf{w}}=\begin{bmatrix}\tilde{\mathbf{z}}_2\\ \tilde{\mathbf{d}}_2\end{bmatrix}$ as the estimation error of $\boldsymbol{w}$. It follows from (15) and (16) that the dynamics of $\tilde{\mathbf{w}}$ can be expressed as

$$\begin{aligned}\dot{\tilde{\mathbf{w}}}(t)&=(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\left((\mathbf{H}+\mathbf{L}\mathbf{C}_1)\tilde{\mathbf{w}}(t)+\mathbf{M}\mathbf{d}_2(t)\right)\\&=((\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\mathbf{H}-\mathbf{N}\mathbf{C}_1)\tilde{\mathbf{w}}(t)\\&\quad+(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\mathbf{M}\mathbf{d}_2(t)\\&=((\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\mathbf{H}-\mathbf{N}\mathbf{C}_1)\tilde{\mathbf{w}}(t)+\mathbf{K}_{12}^{-1}\mathbf{d}_2(t),\end{aligned}\tag{17}$$

where $\mathbf{N}=-(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\mathbf{L}$. As a result, the design rule is to choose the gain $\mathbf{K}_{12}$ reducing the effect of $\mathbf{d}_2$, and the matrix N ensuring the stability of the error dynamics. Define the estimation state $\hat{\mathbf{x}}=\mathbf{T}_1^{-1}\mathbf{T}_2\begin{bmatrix}\mathbf{y}_1-\mathbf{F}_{12}\hat{\mathbf{d}}_2\\ \hat{\mathbf{z}}_2\end{bmatrix}$ and the estimation error

$$\tilde{\mathbf{x}}(t)=\mathbf{T}_1^{-1}\mathbf{T}_2\begin{bmatrix}\mathbf{0}&-\mathbf{F}_{12}\\ \mathbf{I}_{n-l+k}&\mathbf{0}\end{bmatrix}\tilde{\mathbf{w}}(t),\tag{18}$$

where the dynamics of $\tilde{\mathbf{w}}$ is given by

$$\dot{\tilde{\mathbf{w}}}(t)=((\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\mathbf{H}-\mathbf{N}\mathbf{C}_1)\tilde{\mathbf{w}}(t)+\mathbf{K}_{12}^{-1}\mathbf{d}_2(t).$$

The estimation performance of the developed observer is shown in the following theorem.

Theorem 1: Consider system (4) which satisfies the following conditions

$$\operatorname{rank}\left(\begin{bmatrix}s\mathbf{I}_n-\bar{\mathbf{A}}&-\bar{\mathbf{D}}_1\\ \mathbf{C}&\mathbf{0}\end{bmatrix}\right)=n+l-k\quad\forall s\in\mathbb{C}^+\ \text{and}$$

$$\operatorname{rank}(\mathbf{C}\bar{\mathbf{D}}_1)=\operatorname{rank}(\bar{\mathbf{D}}_1)=l-k,$$

and the observer (14). If the unknown input $\mathbf{d}_2$ is bounded and the matrix $((\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\mathbf{H}-\mathbf{N}\mathbf{C}_1)$ is stabilizing by N, then the estimation error is finally bounded in a small region.

Proof: Since $\mathbf{x}=\mathbf{T}_1^{-1}\mathbf{T}_2\begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{bmatrix}=\mathbf{T}_1^{-1}\mathbf{T}_2\begin{bmatrix}\mathbf{y}_1-\mathbf{F}_{12}\mathbf{d}_2\\ \mathbf{z}_2\end{bmatrix}$, the estimation error is

$$\begin{aligned}\tilde{\mathbf{x}}(t)&=\mathbf{x}(t)-\hat{\mathbf{x}}(t)=\mathbf{T}_1^{-1}\mathbf{T}_2\begin{bmatrix}-\mathbf{F}_{12}\tilde{\mathbf{d}}_2\\ \tilde{\mathbf{z}}_2\end{bmatrix}\\&=\mathbf{T}_1^{-1}\mathbf{T}_2\begin{bmatrix}\mathbf{0}&-\mathbf{F}_{12}\\ \mathbf{I}_{n-l+k}&\mathbf{0}\end{bmatrix}\tilde{\mathbf{w}}(t).\end{aligned}$$

Hence, the term $\tilde{\mathbf{w}}(t)$ dominates the estimation performance. In the following, we show that the pair $((\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\mathbf{H},\mathbf{C}_1)$ in (17) is detectable. From Lemma 1 and

$$\begin{aligned}&\operatorname{rank}\left(\begin{bmatrix}s\mathbf{I}_{n-l+2k}-\left(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1\right)^{-1}\mathbf{H}\\ \mathbf{C}_1\end{bmatrix}\right)\\&=\operatorname{rank}\left(\begin{bmatrix}s\left(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1\right)-\mathbf{H}\\ \mathbf{C}_1\end{bmatrix}\right)\\&=\operatorname{rank}\left(\begin{bmatrix}\mathbf{I}_{n-l+2k}&s\mathbf{K}_1\\ \mathbf{0}&\mathbf{I}_{p-l+k}\end{bmatrix}\begin{bmatrix}s\mathbf{E}-\mathbf{H}\\ \mathbf{C}_1\end{bmatrix}\right)\\&=\operatorname{rank}\left(\begin{bmatrix}s\mathbf{E}-\mathbf{H}\\ \mathbf{C}_1\end{bmatrix}\right),\end{aligned}$$

it follows that

$$\begin{aligned}&\operatorname{rank}\left(\begin{bmatrix}s\mathbf{I}_{n-l+2k}-\left(\mathbf{E}+\mathbf{K}_1\mathbf{C}_1\right)^{-1}\mathbf{H}\\ \mathbf{C}_1\end{bmatrix}\right)\\&=\operatorname{rank}\left(\begin{bmatrix}s\mathbf{E}-\mathbf{H}\\ \mathbf{C}_1\end{bmatrix}\right)=n-l+2k,\quad\forall s\in\mathbb{C}^+.\end{aligned}$$

Hence, the pair $((\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\mathbf{H},\mathbf{C}_1)$ is detectable and there exists the gain matrix N stabilizing $((\mathbf{E}+\mathbf{K}_1\mathbf{C}_1)^{-1}\cdot\mathbf{H}-\mathbf{N}\mathbf{C}_1)$. Furthermore, in order to satisfy the property of disturbance attenuation, choosing a high gain $\mathbf{K}_{12}$ to reduce the effect of $\mathbf{d}_2$ is recommended. The proof is completed.

Remark 1: When the matrix $\bar{\mathbf{F}}$ is full rank, system (1) can directly transfer to the descriptor system form (12) without the coordinate transformation (8). In this special case, the descriptor system form for system (1) is described as

$$\begin{aligned}&\mathbf{E}_1\dot{\mathbf{w}}_1(t)=\mathbf{H}_1\mathbf{w}_1(t)+\mathbf{G}_1\mathbf{u}(t)+\mathbf{M}_1\mathbf{d}(t),\\&\mathbf{y}(t)=\mathbf{C}_2\mathbf{w}_1(t),\end{aligned}$$

where

$$\mathbf{w}_1 = \begin{bmatrix} \mathbf{x} \\ \mathbf{d} \end{bmatrix}, \quad \mathbf{E}_1 = \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \in \mathbb{R}^{(n+l)\times(n+l)},$$
$$\mathbf{H}_1 = \begin{bmatrix} \bar{\mathbf{A}} & \bar{\mathbf{D}} \\ \mathbf{0} & -\mathbf{I}_l \end{bmatrix}, \quad \mathbf{G}_1 = \begin{bmatrix} \bar{\mathbf{B}} \\ \mathbf{0} \end{bmatrix} \in \mathbb{R}^{(n+l)\times m},$$
$$\mathbf{M}_1 = \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_l \end{bmatrix} \in \mathbb{R}^{(n+l)\times l}, \text{ and } \mathbf{C}_2 = \begin{bmatrix} \bar{\mathbf{C}} & \bar{\mathbf{F}} \end{bmatrix}.$$

Follow the same rule mentioned above to design the gain of observer $\mathbf{K}_{12}$ reducing the effect of the unknown input. Then design another parameter matrix $\boldsymbol{N}$ to place the desired eigenvalues of estimated error dynamics and avoid the peaking phenomenon.

Remark 2: For system (1), the conventional UIO observer designs [1-4,9-16] can obtain the perfect estimation if the following condition is satisfied:

$$\text{rank}\left(\begin{bmatrix} s\mathbf{I}_n - \bar{\mathbf{A}} & -\bar{\mathbf{D}} \\ \bar{\mathbf{C}} & \bar{\mathbf{F}} \end{bmatrix} \right) = n + l \quad \forall s \in \mathbb{C}^+.$$

The above condition implies that the system with respect to the relation between the output and the unknown input must be minimum phase. This condition has been altered that the pair $(\bar{\mathbf{A}}, \bar{\mathbf{C}})$ is detectable in our proposed observer.

5. NUMERICAL EXAMPLE

To demonstrate the designed observer, we consider the following system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & 2 & 0 & 1 \\ -1 & -2 & -1 & 0 \\ 1 & 0 & -3 & -3 \\ 1 & -2 & 0 & -2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix},$$
$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix}.$$

The invariant zeros of the system between the output and the unknown input are 1.7890 and $-4.1445 \pm 0.3897i$. Due to the unstable invariant zero 1.7890, the conventional UIO methods [1-4,9-16] can not be implemented in the system. Using the linear transformations proposed in Section 3, the above system can be transformed into the following form:

$$\dot{z}_1(t) = -4z_1(t) + [0.8 \;\; 0.8 \;\; -1.8]\mathbf{z}_2(t) + 0.6d_2(t) + d_1(t),$$
$$\dot{\mathbf{z}}_2(t) = \begin{bmatrix} -2 \\ -3 \\ 3 \end{bmatrix} z_1(t) + \begin{bmatrix} -0.4 & 0.2 & 0.6 \\ -2.6 & -1.2 & -4.6 \\ -3.4 & -1.8 & -2.4 \end{bmatrix} \mathbf{z}_2(t) + [4 \;\; -4 \;\; -4]^T d_2(t),$$
$$y_1(t) = z_1(t) - 0.2d_2(t),$$
$$y_2(t) = [0.8944 \;\; -0.4472 \;\; 0.8944]\mathbf{z}_2(t) + 0.8944d_2(t),$$

where the unknown inputs are set as $d_1(t) = 2\cos(t)$ and $d_2(t) = 0.2\sin(5\pi t) + 0.1\cos(20t)$. Design $K_{12} = 3$ which places the desired eigenvalues of the observer at $\{-4, -3, -2 \pm i\}$. The proposed observer is given by

$$\dot{\eta}(t) = \begin{bmatrix} 1 & 33.25 & 58.5 & 61.5 \\ -2 & -17.75 & -24.5 & -24.5 \\ -2 & -14.25 & -29.5 & -27.5 \\ 3 & 21.0417 & 37.5833 & 35.25 \end{bmatrix} \eta(t) + [-5 \;\; 3 \;\; 1 \;\; -2.5]^T y_1(t) + [-1.1180 \;\; 5.1430 \;\; 0.2236 \;\; -3.1678]^T y_2(t),$$
$$\begin{bmatrix} \hat{\mathbf{z}}_2(t) \\ \hat{d}_2(t) \end{bmatrix} = \eta(t) + [0 \;\; 0 \;\; 0 \;\; 1.118]^T y_2(t),$$

and the estimation state can be attained by

$$\hat{\mathbf{x}}(t) = \begin{bmatrix} 1 & 0 & 0.6 & 0.2 \\ -1 & 1 & 0.4 & -0.2 \\ 1 & 0 & -0.4 & 0.2 \\ 1 & -1 & -0.4 & 1.2 \end{bmatrix} \begin{bmatrix} y_1(t) + 0.2\hat{d}_2(t) \\ \hat{\mathbf{z}}_2(t) \end{bmatrix}.$$

The noise attenuation observer [20] is simultaneously simulated. For the same system, the observer [20] can be designed as

$$\dot{\xi}(t) = \begin{bmatrix} -316.0110 & -238.9735 & -74.0375 & -239.9735 & -315.0110 & -240.9735 \\ 223.7300 & 500.9847 & -279.2548 & 502.9847 & 224.7300 & 502.9847 \\ 31.1998 & 278.7039 & -251.5041 & 275.7039 & 30.7998 & 278.7039 \\ -339.2302 & -889.4069 & 547.1768 & -889.4069 & -340.2302 & -887.4069 \\ 265.1693 & -38.9530 & 305.1223 & -34.9530 & 264.8360 & -36.9530 \\ 143.8446 & 639.4346 & -494.5901 & 634.4346 & 142.8446 & 635.1013 \end{bmatrix} \xi(t)$$
$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & -0.3333 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.3333 \end{bmatrix}^T \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 & 1 & -2 & 1 \\ 3 & 1 & -1 & -1 & -2 & -1 \end{bmatrix}^T \hat{\mathbf{d}}(t),$$
$$\dot{\hat{\mathbf{d}}}(t) = \begin{bmatrix} -398.1891 & -983.7861 & 585.5971 & -983.7861 & -398.1891 & -983.7861 \\ -103.1360 & 32.3694 & -135.5054 & 32.3694 & -103.1360 & 32.3694 \end{bmatrix} \xi(t),$$
$$\begin{bmatrix} \hat{\mathbf{x}}(t) \\ \hat{\omega}(t) \end{bmatrix} = \xi(t) + \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \mathbf{y}(t),$$

where $\hat{\omega}(t)$ is the estimation of the noise in output signals. The eigenvalues of observer are placed at $\{-15, -12, -10, -8, -4, -3, -2 \pm i\}$, and the gain suppressing the disturbances is also three as K_{12}=3 in the proposed method above. Using the initial state $\mathbf{x}(0)=[-0.2 \quad 2 \quad -1 \quad -1]^T$ and $\boldsymbol{\eta}(0)=\xi(0)=\mathbf{0}$ in both observers, Figs. 1-4 illustrate the responses of the real and estimated states in two methods. From these figures, the proposed method can obtain better estimation responses in the steady state and the disturbance attenuation property is evident. Hence, the proposed observer is capable of estimating the state when the underlying system has the unstable zero and does not suffer from the peaking phenomenon.

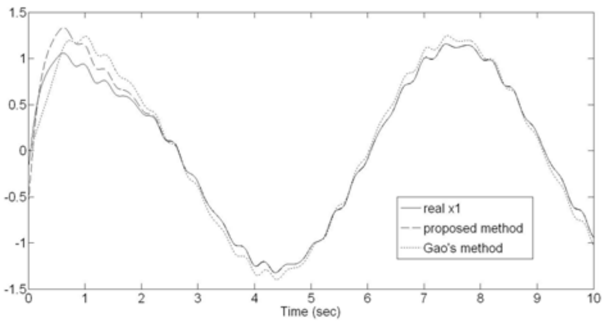

Fig. 1. True and estimation values of x_1.

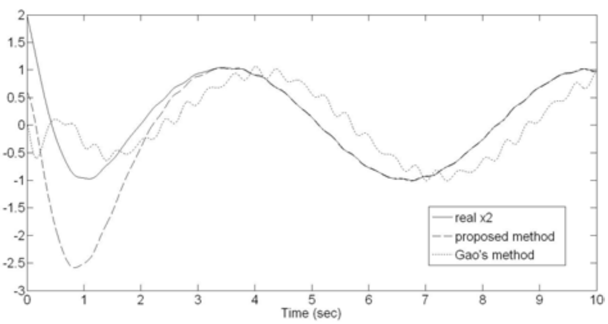

Fig. 2. True and estimation values of x_2.

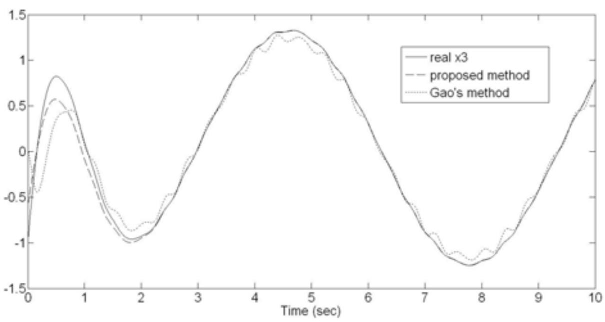

Fig. 3. True and estimation values of x_3.

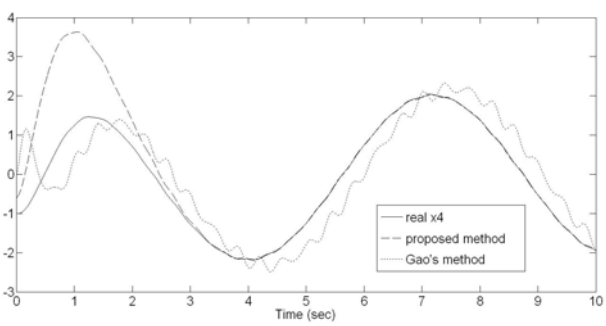

Fig. 4. True and estimation values of x_4.

6. CONCLUSIONS

With respect to an MIMO linear system with the unknown input in the state and output equations, this paper have developed an observer design method using the descriptor system approach. It is shown that the proposed proportional derivative observer is able to reconstruct the system state under the system with respect to the relation between the output and the unknown input is nonminimum phase. The robust stability of the estimation dynamics can be guaranteed and the estimation error is bounded in a small region. Simulation results demonstrate that the present observer scheme exhibits reasonably good estimation performance and avoids the peaking phenomenon.

REFERENCES

[1] Y. Guan and M. Saif, "A novel approach to the design of unknown input observers," *IEEE Trans. Automatic Control*, vol. 36, pp. 632-635, 1991.

[2] M. Darouach, M. Zasadzinaski, and S. J. Xu, "Full-order observers for linear systems with unknown inputs," *IEEE Trans. Automatic Control*, vol. 39, pp. 606-609, 1994.

[3] M. Hou and P. C. Muller, "Design of observers for linear systems with unknown inputs," *IEEE Trans. Automatic Control*, vol. 37, pp. 871-875, 1992.

[4] C. C. Tsui, "A new design approach to unknown input observers," *IEEE Trans. Automatic Control*, vol. 41, pp. 464-468, 1996.

[5] J. Chen, R. J. Patton, and H. Y. Zhang, "Design of unknown input observer and robust fault detection filters," *International Journal of Control*, vol. 63, pp. 85-105, 1996.

[6] R. J. Patton and J. Chen, "On eigenstructure assignment for robust fault diagnosis," *International Journal of Robust Nonlinear Control*, vol. 10, pp. 1193-1208, 2000.

[7] K. Y. Ng, C. P. Tan, Z. Man, and R. Akmeliawati, "New results in disturbance decoupled fault reconstruction in linear uncertain systems using two sliding mode observers in cascade," *International Journal of Control, Automation, and Systems*, vol. 8, no. 3, pp. 506-518, 2010.

[8] K. Zhang, B. Jiang, and V. Cocquempot, "Adaptive observer-based fast fault estimation," *International Journal of Control, Automation, and Systems*, vol. 6, no. 3, pp. 320-326, 2008.

[9] M. Hou and P. C. Muller, "Disturbance decoupled observer design: a unified viewpoint," *IEEE Trans. Automatic Control*, vol. 39, pp. 1338-1341, 1994.

[10] M. Hou, A. C. Pugh, and P. C. Muller, "Disturbance decoupled functional observers," *IEEE Trans. Automatic Control*, vol. 44, pp. 382-386, 1999.

[11] M. Darouach, "Complements to full order observer design for linear systems with unknown inputs,"

Applied Mathematics Letters, vol. 22, pp. 1107-1111, 2009.
[12] S. Mondal, G. Chakraborty, and K. Bhattacharyya, "LMI approach to robust unknown input observer design for continuous systems with noise and uncertainties," *International Journal of Control, Automation, and Systems*, vol. 8, no. 2, pp. 210-219, 2010.
[13] H. Alwi, C. Edwards, and C. P. Tan, "Sliding mode estimation schemes for incipients sensor faults," *Automatica*, vol. 45, pp. 1679-1685, 2009.
[14] N. H. Jo, H. Shim, and Y. I. Son, "Disturbance observer for non-minimum phase linear systems," *International Journal of Control, Automation, and Systems*, vol. 8, no. 5, pp. 994-1002, October 2010.
[15] M. Aldeen and R. Sharma, "Estimation of states, faults, and unknown disturbances in nonlinear systems," *International Journal of Control*, vol. 81, pp. 1195-1201, 2008.
[16] R. Sharma and M. Aldeen, "Estimation of unknown disturbances in nonlinear systems," *Proc. of IEE Control Conference*, University of Bath, UK, 2004.
[17] M. Boutayeb, M. Darouach, and H. Rafaralahy, "Generalized state-space observers for chaotic synchronization and secure communication," *IEEE Trans. Circuits and Systems I: Fundamental Theory and Applications*, vol. 49, pp. 345-349, 2002.
[18] T. Fernando and H. Trinh, "Design of reduced-order state/unknown input observers based on a descriptor system approach," *Asian Journal of Control*, vol. 9, pp. 458-465, 2007.
[19] J. Ren and Q. Zhang, "PD observer design for descriptor systems: an LMI approach," *International Journal of Control, Automation, and Systems*, vol. 8, no. 4, pp. 735-740, August 2010.
[20] Z. Gao and H. Wang, "Descriptor observer approaches for multivariable systems with measurement noises and application in fault detection and diagnosis," *Systems and Control Letters*, vol. 55, pp. 304-313, 2006.
[21] K. K. Busawon and P. Kabore, "Disturbance attenuation using proportional integral observers," *International Journal of Control*, vol. 74, pp. 618-627, 2001.
[22] M. Darouach, M. Zasadzinski, and M. Hayar, "Reduced-order observer design for descriptor systems with unknown inputs," *IEEE Trans. Automatic Control*, vol. 41, pp. 1068-1072, 1996.
[23] Z. Gao, "PD observer parametrization design for descriptor systems," *Journal of the Franklin Institute*, vol. 342, pp. 551-564, 2005.

Huan-Chan Ting received his B.S. degree in Electrical Engineering from National Taipei University of Technology, Taipei, Taiwan in 2003, and his Ph.D. degree in Electrical and Control Engineering from National Chiao-Tung University, Hsinchu, Taiwan in 2011. His major fields of research are sliding mode control theorem, robust control, and power electronics.

Jeang-Lin Chang received his B.S., M.S., and Ph.D. degrees in Electrical and Control Engineering from National Chiao-Tung University, Hsinchu, Taiwan. He is a professor in the Department of Electrical Engineering, Orient Institute of Technology. His researches include variable structure control, robust control, and discrete signal processing.

Yon-Ping Chen received his B.S. degree in Electrical Engineering from National Taiwan University in 1981, and his M.S. and Ph.D. degrees in Electrical Engineering from University of Texas at Arlington, U.S.A. He is a professor in the Institute of Electrical Control Engineering, National Chiao-Tung University. His researches include variable structure control, intelligent control, and image processing.