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Analyzing the effects of family-based scheduling rule on reducing capacity loss of single machine with uncertain job arrivals

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ABSTRACT

For a single finite-capacity machine that can process several product types of jobs, uncertainties in job arrival time and product type can make the calculation of required setup time and the setting of output target very complicated. Setup activities may cause wastage in machine capacity and extend job lead time. In such circumstances, the family-based scheduling rule (FSR) can be used to reduce setup frequency and amount of setup time. To efficiently evaluate the effects on capacity-saving, both expected setup time and service time are estimated by the FSR analytic models. The effect of FSR in reducing setup time and capacity loss is explored further by comparing the results with FIFO rule. Finally, the performances of the developed analytic models for estimating setups and setup time are evaluated in the experimental design, and a simulation model is built for accuracy comparisons with the analytic models.

1. Introduction

For a single finite-capacity machine that can process several product types of jobs, the setup is a necessary process change to adjust current machine settings in order to complete a particular product type of job. It was reported that 20% or even as much as 50% loss of available capacity may arise from setup activities (Liu & Chang, 2000; Trovinger & Bohn, 2005). Market demand, uncertainties in job arrival time and types of product, make the estimation of required setup time—especially sequence-dependent setup time—very complicated. Moreover, due to the possible heavy loss of capacity and the difficulty in calculating required setup time, the setting of output targets may have significant errors compared with actual levels. This gap cannot be disregarded. At least three additional factors affect the magnitude of required sequence-dependent setup time: (1) the total arrival rate of all types of incoming jobs, (2) the mix of the arriving rates of various types of jobs, and (3) the dispatching rule applied to select the next job for processing by the machine. If a lengthy setup is required in product type change and peak demand is encountered, then the setup activities may cause wastage in machine capacity apart from extending the job lead time. In such circumstances, the family-based scheduling rule (FSR), which consecutively handles some jobs belonging to the same product family, and which require the same machine setting, can be used to reduce setup frequency and amount of setup time. Hence, developing an analytic model capable of estimating expected setup time under FSR can contribute to an intensive analysis on the exact effects of FSR for the reduction of setup time.

Missbauer (1997) proved that setup time could be saved using FSR for the single-machine system. Jensen, Malhotra, and Philipoom (1998) considered the case of the semiconductor testing facility with parallel machines and dynamic job arrival; FSR has been credited for the reduction of setup time in batch production industries. Chern and Liu (2003) proposed FSR to dispatch wafer lots in the photolithography stage of the wafer fabrication system. Kannan and Lyman (1994) examined the combined effect of lot splitting and family-based scheduling in a manufacturing cell by simulation and showed that FSR can reduce the negative impact on flow time by lot splitting. Nomden, Van Der Zee, and Slomp (2008) extended the existing rules for family-based scheduling by including data on upcoming job arrivals and showed that flow time performance can be improved significantly. Therefore, FSR not only has an effect on savings of setups of the machine, it also indirectly causes reduction in job flow time. In the foregoing investigations, except for Missbauer (1997) and Chern and Liu (2003), the simulation approach is applied to evaluate the effect of FSR on the reduction of setups and flow time. Numerous computer runs are needed to produce reliable results; however, this method is both time-consuming and costly. Thus, the primary focus of this paper is the conduct of an analytic methodology.

Studies on estimations on setup numbers have attracted the attention of some researchers. Vieira, Herrmann, and Lin (2000a, 2000b) developed an analytic model for both single and parallel

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machines to estimate the setup frequency (the average number of setups executed per time unit) under FSR. Under this context, jobs of different product types arrive dynamically; the number of jobs arrivals is a Poisson distribution. However, the authors did not consider the possible differences in the arriving rates of various job types; instead, they simplified the setup probability by categorizing all types of arriving jobs as a constant. Rossetti and Stanford (2003) considered the aforementioned problem on the single machine and presented a case study that examines the use of a heuristic to estimate the expected number of setups. In calculating setup time, both the number and type of setup should be considered. Two types of setup exists: (1) sequence-independent and (2) sequence-dependent. The second type is a generality of the first and is the one considered in this paper. For the first type, the total expected setup time is easily calculated by the product of total expected number of setups and unit setup time. Meanwhile, for the second type, setup time depends on the product type of any two consecutive jobs on machine. Studies on the estimation of sequence-dependent setup time, however, are quiet limited. Missbauer (1997) developed the analytic model to estimate the sequence-dependent setup time of single-machine systems under the first-in first-out (FIFO) rule and FSR. Jobs of different product types arrive dynamically with Poisson distribution, and the same setup time for each product type is assumed to simplify the model. Chern and Liu (2003) extended the result of Missbauer (1997) to the parallel machine problem having multiple re-entrances. Bagherpour, Noghondarian, and Noori (2007) estimated the sequence-dependent setup time for the single machine using the fuzzy approach. However, their fuzzy estimation was significantly lower compared with that of simulated results. Estimation error of the fuzzy setup time cannot be controlled in an acceptable range.

In this paper, FSR analytic models are developed to estimate the number of setups and the setup time for the single-machine problem in order to evaluate the effect of capacity-saving with the adoption of FSR. The inter-arrival time of jobs, assumed as distributed independently and exponentially, is considered to reflect the uncertainty in market demand. Due to the difficulty in directly solving analytical solutions for the expected setup time and service time, a numerical analysis is used. A numerical analysis, a function of work-in-process (WIP), has been studied by Missbauer (1997). In this paper, the numerical solutions of the expected setup time and service time are solved, and the amount of capacity wastage due to changes in the machine setting across several product types are evaluated. Developed models, such as those by Yang, Chung, and Kao (2009), are adopted to estimate the expected setup time under FIFO, and consequently, for comparison with those under FSR. After replacing FIFO with FSR, the effect of the latter on reducing setup time and capacity loss is explored further. To evaluate the accuracy of the analytic models for estimating the number of setups and setup time, a simulation model is built to compare the results with those calculated by analytic models. This paper is organized as follows. Section 2 develops the analytic models to calculate the expected values of the number of setups, setup time, and service time under FSR. Section 3 shows the FSR effects on the reduction of setup time and capacity wastage as compared with FIFO, and then investigates savings in machine utilization rate upon application of FSR into job dispatches as a result of setup time reduction. Section 4 presents the performance analysis for the proposed FSR analytic models. Section 5 gives the conclusions.

2. Development of FSR analytic models

FSR implies following the criterion for selecting jobs that are of the same product type and need the same machine setting, hence those that are processed consecutively. Queued jobs with the same product type as the previous job on the machine indicates higher priority for processing (Missbauer, 1997). In this section, the number of setups, setup time, and service time are estimated for a single machine with inter-arrival time for each job type that is distributed independently and exponentially.

We assume that the number of setups and setup time spent on changing machine settings are observed for a period of time RT, where RT is a positive integer. Beginning time is labeled 0. We then assume that the number of arriving jobs of product type j follows the Poisson distribution with arrival rate λ_j. Inter-arrival time T_j for the arriving jobs of product type j is an exponential distribution with parameter λ_j. Arrival time T_{ij} of the ith arriving job of product type j is the gamma distribution with parameters i and λ_j. Thus, the probability for ith job of product type j arrives at the system at the time interval $(0, RT]$ can be shown as Eq. (1), where $i = 1,2,\ldots,n_j$, $j = 1,2,\ldots,J$, $n_j = \lambda_j RT$, and J is the number of product types.

$$\Pr[T_{ij} \leqslant RT] = \int_0^{RT} \frac{(\lambda_j)^i}{\Gamma(i)} (t_{ij})^{i-1} e^{-\lambda_j t_{ij}} dt_{ij}. \tag{1}$$

The probability of ith job of product type j arriving at the system but out of time interval $(0, RT]$ is denoted by $\Pr[T_{ij} > RT] = 1 - \Pr[T_{ij} \leqq RT]$.

2.1. Probability of requiring setups

When a job of specific product type arrives at the system, it may enter the queue of the batch (i.e., by product type) and wait for processing on machine, as required by FSR. FSR consists of two parts: (1) the assignment of a newly arrived job to a specific batch on queue based on the type of product family, which cannot be dispatched immediately on the machine, and (2) the dispatching of a next candidate job from several batches on queue that should be processed by the busy machine.

The operation executed by FSR is illustrated in Fig. 1. When a job of a specific product type arrives at the system, if the machine is idle, FSR immediately dispatches this newly arrived job on machine. However, if the machine is busy and there is at least one job on queue or on machine, by carrying the same type as the new arrived job, FSR moves the arrived job to the batch with the same product type. If the machine is busy but there are no jobs (i.e., either on queue or on machine), by carrying the same type as the newly arrived job, FSR by itself transforms the arrived job into a new batch. When an arrived job is moved into an existing batch, jobs are sorted according to job arrival time in increasing order. Once the busy machine has completed one job on a specific batch, then the job with the first order in the same batch is processed. After all jobs in this batch are completed, another batch designated as having the earliest arrival time of the first job among all jobs on queue is picked. Then, the first job is dispatched on machine. If FSR cannot find another batch on queue for machine processing, implying that no jobs are waiting on queue, then the machine becomes idle.

Note that before starting the processing of a new job, a setup is required if the type of job is different from the last completed job on machine. Similarly, when a job of specific product type arrives at the system at a time when the machine is busy, a setup is required if there is an additional new batch generated. For this purpose, let $P_{s,ij,FSR}$ be the probability of requiring a setup under FSR, given that ith job of product type j arrives at the system at time interval $(0, RT]$. The probability $P_{s,ij,FSR}$ is given by Eq. (2), where $P_{s,j,FSR}$ is the probability of requiring a setup under FSR, given that product type j job arrives at the system at time interval $(0, RT]$.

$$P_{s,ij,FSR} = \Pr[T_{ij} \leqslant RT] \times P_{s,j,FSR}. \tag{2}$$

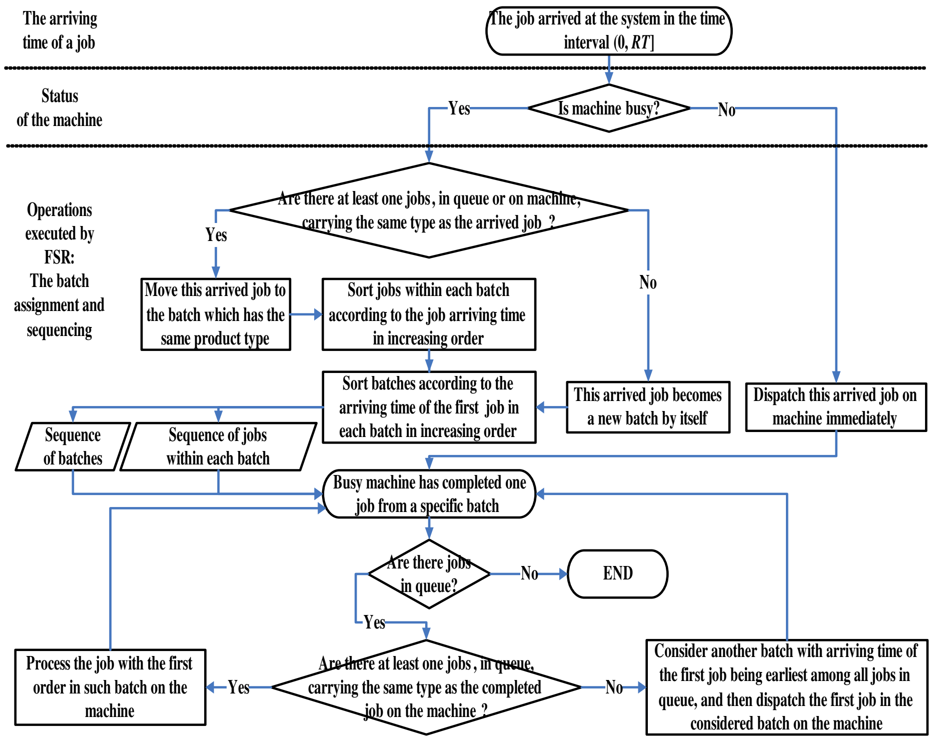

Fig. 1. Flow chart of family-based scheduling rule.

The probability $P_{s,j,FSR}$ should consider the number of jobs queued in the system. This includes two cases: (1) no jobs and (2) n ($n \geqq 1$) jobs. Thus, $P_{s,j,FSR}$ is defined by Eq. (3).

$$P_{s,j,FSR} = p_{0,FSR} P^{n=0}_{setups,FSR} + \sum_{n=1}^{\infty} p_{n,FSR} P^{n\geqslant 1}_{setups,FSR}. \tag{3}$$

In Eq. (3), $p_{0,FSR}$ and $p_{n,FSR}$ are the probabilities under FSR under conditions that there are no jobs and there are n ($n \geqq 1$) jobs in the system, and $P^{n=0}_{setups,FSR}$ and $P^{n\geqslant 1}_{setups,FSR}$ are the probabilities of requiring a setup under FSR for a job of type j arriving at a time when there are no jobs and there are n ($n \geqq 1$) jobs in the system.

The probability $P_{s,j,FSR}$ is presented as follows: For the first condition, the ith job of type j arrives at time interval $(0,RT]$ and there are no jobs in the system. A setup is necessary if this arrived job is different from the job previously completed by the current idle machine. Therefore, $P^{n=0}_{setups,FSR}$ can be expressed as $(1-\lambda_j/\lambda)$, which indicates the probability that the previously completed job on the current idle machine is different from type j. For the second condition, the ith job of type j arrives at time interval $(0,RT]$ and there are n ($n \geqq 1$) jobs in the system. A setup is necessary if there are no jobs in the system belonging to type j. Therefore, $P^{n\geqslant 1}_{setups,FSR}$ is equal to $(1-\lambda_j/\lambda)^n$.

By referring to Eqs. (2) and (3), the probability of requiring a setup for ith job of product type j under FSR ($P_{s,ij,FSR}$) is rewritten as Eq. (4). Note that $P_{ns,ij,FSR}$ is the probability of a setup that is not required by ith job of product type j under FSR, which is given as $(1-P_{s,ij,FSR})$.

$$P_{s,ij,FSR} = \Pr[T_{ij} \leqslant RT]\left[p_{0,FSR}\left(1-\frac{\lambda_j}{\lambda}\right) + \sum_{n=1}^{\infty} p_{n,FSR}\left(1-\frac{\lambda_j}{\lambda}\right)^n\right]. \tag{4}$$

To simplify the calculation of $P_{s,ij,FSR}$, the probabilities ($p_{0,FSR}$ and $p_{n,FSR}$) need to be defined. If $p_{0,FSR}$ and $p_{n,FSR}$ are approximated by the $M/G/1$ formula, then $p_{0,FSR}$ and $p_{n,FSR}$ are approximately set to $(1-\rho_{FSR})$ and $(1-\rho_{FSR})(\rho_{FSR})^n$, respectively, as executed in Missbauer (1997) & Chern & Liu (2003). Subsequently, $P_{s,ij,FSR}$ can be reformulated as Eq. (5), where ρ_{FSR} is the machine utilization rate under FSR for the single machine. It is equal to $\lambda E[ST_{FSR}]$, where λ is the total arrival rate and $E[ST_{FSR}]$ is the expected service time of jobs under FSR.

$$P_{s,ij,FSR} = \Pr[T_{ij} \leqslant RT]\left(1-\frac{\lambda_j}{\lambda}\right)\left\{1-\rho_{FSR}\left[1-\left(1+\frac{\rho_{FSR}}{1-\rho_{FSR}}\frac{\lambda_j}{\lambda}\right)^{-1}\right]\right\}. \tag{5}$$

2.2. Expected number of setups

$P_{s,ij,FSR}$ represents the probability of requiring "one" setup under FSR and given by ith new job of type j; $(1-P_{s,ij,FSR})$ represents the probability of requiring "no" setup under FSR and given by ith new job of type j. The expected number of setups under FSR for the ith arrived job of product type j can be derived as Eq. (6).

$$E[NS_{ij,FSR}] = 1 \times P_{s,ij,FSR} + 0 \times (1-P_{s,ij,FSR}) = P_{s,ij,FSR}. \tag{6}$$

Suppose there arrives n_j independent product type j jobs at time interval $(0,RT]$. Using the summation of $E[NS_{ij,FSR}]$ for all i, the expected number of setups of product type j under FSR is computed as $E[NS_{j,FSR}] = \sum_{i=1}^{n_j} E[NS_{ij,FSR}]$, where $n_j = \lambda_j RT$ and $j = 1,2,\ldots,J$. Finally, using the summation of $E[NS_{j,FSR}]$ for all j, the expected number of setups for all jobs under FSR is calculated as $E[NS_{FSR}] = \sum_{j=1}^{J}\sum_{i=1}^{n_j} E[NS_{ij,FSR}]$.

2.3. Expected setup time

For this purpose, let s_{jr} be the setup time prior to the processing of a job with product type j right after the last completed job belonging to product type r, referred to as predecessor. The length of the required setup time depends on product type change between any two consecutive jobs. We consider the following three cases with the inclusion of job arrival time: (1) The ith job of product type j does not arrive at time interval $(0,RT]$. Then, the setup time should equal 0 with the probability $(1-\Pr[T_{ij} \leqq RT])$. (2) The ith job of product type j arrives at time interval $(0,RT]$ but a setup is not needed. Thus, the setup time s_{jj} would be equal to 0 with the probability $\Pr[T_{ij} \leqq RT](1-P_{s,j,FSR})$. (3) The ith job of product type j arrives at time interval $(0,RT]$ and a setup is needed. This implies that the product type of the arrived job is different from the predecessor. Therefore, the setup time would be equal to s_{jr} with the probability $P_{s,ij,FSR}(\lambda_r/\lambda^c)$, where $r = 1,2,\ldots,J$, $r \neq j$, and $\lambda^c = \sum_{r=1,r\neq j}^{J}\lambda_r$.

Based on the abovementioned three cases, Eq. (7) can be used to estimate the expected setup time for ith job of product type j arriving at time interval $(0,RT]$ under FSR.

$$E[S_{ij,FSR}] = (1-\Pr[T_{ij} \leqslant RT]) \times 0 + \Pr[T_{ij} \leqslant RT](1-P_{s,j,FSR}) \times s_{jj} + P_{s,ij,FSR}\sum_{\substack{r=1\\ r\neq j}}^{J}\frac{\lambda_r}{\lambda^c}s_{jr} = P_{s,ij,FSR}\sum_{\substack{r=1\\ r\neq j}}^{J}\frac{\lambda_r}{\lambda^c}s_{jr}. \tag{7}$$

Then, the expected mean setup time for product type j jobs and the expected mean setup time for a job under FSR are expressed as $E[S_{j,FSR}] = E\left[\sum_{i=1}^{n_j}S_{ij,FSR}/n_j\right]$ and $E[S_{FSR}] = E\left[\sum_{j=1}^{J}\sum_{i=1}^{n_j}S_{ij,FSR}/\sum_{j=1}^{J}n_j\right]$, respectively. Applying Eq. (7) to $E[S_{j,FSR}]$ and $E[S_{FSR}]$ yields Eqs. (8) and (9), where $n_j = \lambda_j RT$ and $j = 1,2,\ldots,J$.

$$E[S_{j,FSR}] = n_j^{-1}\sum_{i=1}^{n_j}P_{s,ij,FSR}\sum_{\substack{r=1\\ r\neq j}}^{J}\frac{\lambda_r}{\lambda^c}s_{jr}, \tag{8}$$

$$E[S_{FSR}] = \left(\sum_{j=1}^{J}n_j\right)^{-1}\sum_{j=1}^{J}\sum_{i=1}^{n_j}P_{s,ij,FSR}\sum_{\substack{r=1\\ r\neq j}}^{J}\frac{\lambda_r}{\lambda^c}s_{jr}. \tag{9}$$

2.4. Expected service time

The service time of a job is equal to the sum of its processing time and its setup time. Therefore, the expected service time for a job also relates to the three cases when estimating the setup time, as mentioned in Section 2.3. Moreover, the processing time of a job depends on its product type.

In this context, let $ST_{ij,FSR}$ be the random variable of service time for ith job of product type j under FSR. The probability mass function of $ST_{ij,FSR}$ can then be shown as Eq. (10). The expected mean service time for specific type j jobs and expected mean service time for a job are defined by $E[ST_{j,FSR}] = E\left[\sum_{i=1}^{n_j}ST_{ij,FSR}/n_j\right]$ and $E[ST_{FSR}] = E\left[\sum_{j=1}^{J}\sum_{i=1}^{n_j}ST_{ij,FSR}/\sum_{j=1}^{J}n_j\right]$, respectively. According to the probability mass function of $ST_{ij,FSR}$, $E[ST_{j,FSR}]$ and $E[ST_{FSR}]$ can be derived as Eqs. (11) and (12), where pt_j is the job processing time of product type j, $n_j = \lambda_j RT$, and $j = 1,2,\ldots,J$.

$$P(ST_{ij,FSR} = st_{ij}) = \begin{cases} 1-\Pr[T_{ij} \leqslant RT], & \text{if } st_{ij} = 0, \\ \Pr[T_{ij} \leqslant RT](1-P_{s,j,FSR}), & \text{if } st_{ij} = pt_j, \\ P_{s,ij,FSR}(\lambda_r/\lambda^c), & \text{if } st_{ij} = pt_j + s_{jr},\ r = 1,2,\ldots,J,\ r\neq j, \end{cases} \tag{10}$$

$$E[ST_{j,FSR}] = n_j^{-1}\sum_{i=1}^{n_j}\Pr[T_{ij} \leqslant RT]\left[pt_j + P_{s,j,FSR}\sum_{\substack{r=1\\ r\neq j}}^{J}\frac{\lambda_r}{\lambda^c}s_{jr}\right], \tag{11}$$

$$E[ST_{FSR}] = \left(\sum_{j=1}^{J}n_j\right)^{-1}\sum_{j=1}^{J}\sum_{i=1}^{n_j}\Pr[T_{ij} \leqslant RT]\left[pt_j + P_{s,j,FSR}\sum_{\substack{r=1\\ r\neq j}}^{J}\frac{\lambda_r}{\lambda^c}s_{jr}\right]. \tag{12}$$

3. Analyzing the effect of FSR on the reduction of setup time and capacity loss

With the analytic models developed in Section 2, the effect of FSR on the reduction of setup time and capacity loss is further explored by comparing the results with the FIFO rule. Relative to FSR, FIFO dispatches jobs even without batching some jobs into the same type in order to process them consecutively. This implies wastage in setup frequency. Based on the FIFO principle, a setup occurs when any two consecutive jobs in the sequence have different product types and the total setup time may take up a large part of the machine capacity. Therefore, selecting FSR instead of FIFO may contribute to a reduction in setup frequency, setup time, and machine capacity utilization rate, and consequently, lessened capacity loss. In this section, we first compare the effect of FSR with FIFO in terms of reduced setup time and machine utilization rate. Second, we provide details on how machine utilization rate is saved by FSR while dispatching jobs as a result of setup time reduction, and then demonstrate how the effect of FSR on reducing utilization rate is related to the level of total arrival rate.

3.1. The effects of FSR

According to Eq. (5) and the definition of $P_{s,ij,FIFO}$ as $P_{s,ij,FIFO} = \Pr[T_{ij} \leqq RT](1\text{-}\lambda_j/\lambda)$ (Yang et al., 2009), the probability of $P_{s,ij,FSR}$ can be rewritten as Eq. (13), where $P_{s,ij,FIFO}$ is the probability of requiring a setup under FIFO, given that the ith job of type j arrives at time interval $(0,RT]$.

$$P_{s,ij,FSR} = P_{s,ij,FIFO}\left\{1-\rho_{FSR}\left[1-\left(1+\frac{\rho_{FSR}}{1-\rho_{FSR}}\frac{\lambda_j}{\lambda}\right)^{-1}\right]\right\}. \tag{13}$$

The following theorems can then be used to state the effect of FSR in relation to FIFO.

Theorem 1. *$P_{s,ij,FSR} \leqq P_{s,ij,FIFO}$, if $J > 0$ and $0 \leq \rho_{FSR} < 1$ with $\lambda_j > 0$ for all j.*

Theorem 2. *$P_{s,ij,FSR} < P_{s,ij,FIFO}$, if $J > 0$ and $0 < \rho_{FSR} < 1$ with $\lambda_j > 0$ for all j.*

The inequality expressed as Eq. (14) can be used to explain the above theorems. In particular, the probability of requiring a setup under FSR is always less than or equal to the probability of requiring a setup under FIFO. Therefore, FSR can be used to reduce the setup frequency by assigning jobs on queue to a specific batch according to their product type. The effect of FSR on reducing setup time, service time, and capacity loss based on Theorem 1 can be expressed as the following.

$$1+\frac{\rho_{FSR}}{1-\rho_{FSR}}\frac{\lambda_j}{\lambda}\begin{cases} = 1, \text{if } \rho_{FSR} = \sum_{n=1}^{\infty}p_{n,FSR} = 0 \text{ with } \lambda_j > 0,\ \forall j, \\ > 1, \text{if } 0 < \rho_{FSR} = \sum_{n=1}^{\infty}p_{n,FSR} < 1 \text{ with } \lambda_j > 0,\ \forall j. \end{cases} \tag{14}$$

Lemma 1.

$E[S_{FSR}] \leqq E[S_{FIFO}]$.

The expected mean setup time under FIFO for jobs arriving at time interval $(0,RT]$, $E[S_{FIFO}]$, has been expressed as Eq. (15) (Yang et al., 2009). According to Theorem 1, the expected mean setup time under FSR in Eq. (9), $E[S_{FSR}]$, is always less than or equal to that under FIFO.

$$E[S_{FIFO}] = \left(\sum_{j=1}^{J}n_j\right)^{-1}\sum_{j=1}^{J}\sum_{i=1}^{n_j}P_{s,ij,FIFO}\sum_{\substack{r=1\\ r\neq j}}^{J}\frac{\lambda_r}{\lambda^c}s_{jr} \tag{15}$$

Lemma 2.

$E[ST_{FSR}] \leqq E[ST_{FIFO}]$.

The expected mean service time of jobs under FIFO, $E[ST_{FIFO}]$, can be given by Eq. (16) (Yang et al., 2009). The expected mean service time of jobs under FSR, $E[ST_{FSR}]$, can be reformulated as Eq. (17) based on Eqs. (9) and (12).

$$E[ST_{FIFO}] = \left(\sum_{j=1}^{J} n_j\right)^{-1} \sum_{j=1}^{J} \sum_{i=1}^{n_j} \Pr[T_{ij} \leqslant RT] pt_j + E[S_{FIFO}], \tag{16}$$

$$E[ST_{FSR}] = \left(\sum_{j=1}^{J} n_j\right)^{-1} \sum_{j=1}^{J} \sum_{i=1}^{n_j} \Pr[T_{ij} \leqslant RT] pt_j + E[S_{FSR}]. \tag{17}$$

Then, $E[ST_{FIFO}] - E[ST_{FSR}] = E[S_{FIFO}] - E[S_{FSR}]$ is derived from Eqs. (16) and (17). Note that $E[ST_{FSR}] \leqq E[ST_{FIFO}]$ is the result of $E[S_{FIFO}] \geqq E[S_{FSR}]$. This means that service time can be reduced by using FSR when dispatching jobs.

Lemma 3.

$\rho_{FSR} \leqq \rho_{FIFO}$.

For a single machine, machine utilization rates under FIFO and FSR are shown as $\rho_{\text{FIFO}} = \lambda E[ST_{FIFO}]$ and $\rho_{FSR} = \lambda E[ST_{FSR}]$, respectively. In accordance with $E[ST_{FSR}] \leqq E[ST_{FIFO}]$, $\rho_{FSR} \leqq \rho_{FIFO}$ if the total arrival rate is given. This implies that machine utilization rate can be reduced by replacing FIFO with FSR when dispatching jobs. Savings in machine utilization rate by replacing FIFO with FSR can be written as $\Delta\rho = \rho_{\text{FIFO}} - \rho_{\text{FSR}} = \lambda(E[S_{\text{FIFO}}] - E[S_{FSR}])$. From Eqs. (9), (13), and (15), $\Delta\rho$ can then be written as Eq. (18) depending on the machine utilization rate under FSR (ρ_{FSR}).

$$\Delta\rho = \lambda\rho_{FSR}\left(\sum_{j=1}^{J} n_j\right)^{-1} \sum_{j=1}^{J} \times \sum_{i=1}^{n_j} P_{s,ij,FIFO}\left(\sum_{\substack{r=1 \\ r\neq j}}^{J} \frac{\lambda_r}{\lambda^c} s_{jr}\right)\left[1 - \left(1 + \frac{\rho_{FSR}}{1-\rho_{FSR}}\frac{\lambda_j}{\lambda}\right)^{-1}\right], \tag{18}$$

$$\frac{d\Delta\rho}{d\rho_{FSR}} = \lambda\left(\sum_{j=1}^{J} n_j\right)^{-1} \sum_{j=1}^{J} \sum_{i=1}^{n_j} P_{s,ij,FIFO}\left(\sum_{\substack{r=1 \\ r\neq j}}^{J} \frac{\lambda_r}{\lambda^c} s_{jr}\right) \times \left[1 - \left(1 + \frac{\rho_{FSR}}{1-\rho_{FSR}}\frac{\lambda_j}{\lambda}\right)^{-1} + \frac{\lambda_j}{\lambda}\frac{\rho_{FSR}}{(1-\rho_{FSR})^2}\left(1 + \frac{\rho_{FSR}}{1-\rho_{FSR}}\frac{\lambda_j}{\lambda}\right)^{-2}\right] \geqslant 0. \tag{19}$$

Prior to the discussion of the influence of ρ_{FSR} on the savings in machine utilization rate, the first derivative of $\Delta\rho$ with respect to ρ_{FSR} is used and given by Eq. (19). Note that $d\Delta\rho/d\rho_{FSR} \geqq 0$ with $0 \leq \rho_{FSR} < 1$ and $\lambda_j > 0$ based on Eq. (14). Let ρ_{FSR1} and ρ_{FSR2} be two different machine utilization rates under FSR and $\rho_{FSR1} \geqq \rho_{FSR2}$. Using ρ_{FSR1} and ρ_{FSR2} in Eq. (18), $\Delta\rho(\rho_{FSR1})$ and $\Delta\rho(\rho_{FSR2})$ can then be computed. Next, $\Delta\rho(\rho_{FSR1}) \geqq \Delta\rho(\rho_{FSR2})$ is set in accordance with Eq. (19), where $0 \leqq \rho_{FSR1}<1$ and $0 \leqq \rho_{FSR2}<1$. Thus, savings in machine utilization rate achieved by replacing FIFO with FSR increases with the rise in utilization rate of the machine. This implies that more savings in machine utilization rate is achieved with high levels of workload on machine.

3.2. Relationship between the reduction of service time and the saving of utilization rate by varying total arrival rate

In earlier discussions, we mentioned that savings in machine utilization rate ($\Delta\rho$) depends on machine utilization rate under FSR (ρ_{FSR}), which also depends on total arrival rate (λ) and reduced service time. Next, we investigate how savings in machine utilization rate can be affected by the changes in total arrival rate and reduction of service time. The result is plotted in Fig. 2.

For a single machine system, by referring to the queuing theory, the expected service time ($E[ST]$) is proportional to the utilization rate of machine (ρ) with gradient $1/\lambda$; this denotes an inverse of total arrival rate (Ross, 2007). Thus, the expected service time behaves as a function of machine utilization rate. In relation, the straight line in Fig. 2 can be depicted, which passes through the origin with the slope equal to the inverse of total arrival rate ($1/\lambda$). In Fig. 2a, a line with slope $1/\lambda^*$ and intercept zero, $E[ST] = \rho/\lambda^*$ can be obtained for a given specific total arrival rate λ^* and the vector of job processing time **PT**. Therefore, the expected service time under FIFO ($E[ST_{FIFO}]|_{\lambda^*,\mathbf{PT}}$) is calculated by Eq. (16) using λ^* and **PT**. The machine utilization rate under FIFO ($\rho_{FIFO}|_{\lambda^*,\mathbf{PT}}$) can then be computed by $\lambda^* E[ST_{FIFO}]|_{\lambda^*,\mathbf{PT}}$.

Similarly, a curve of expected service time under FSR ($E[ST_{FSR}]$) by varying the machine utilization rate under can be seen in Fig. 2a. Based on Eqs. (5), (9), and (17), the estimation of expected service time under FSR ($E[ST_{FSR}]$) is required by the machine utilization rate under FSR (ρ_{FSR}) in order to compute the expected setup time ($E[S_{FSR}]$). However, by referring to the queuing theory, the machine utilization rate under FSR (ρ_{FSR}) also depends on the expected service time under FSR ($E[ST_{FSR}]$). Therefore, it is difficult to solve an analytical solution for $E[ST_{FSR}]$. Instead, a numerical analysis can be used to compute $E[ST_{FSR}]$. The numerical solution of $E[ST_{FSR}]$ can be solved by solving the two equations, $E[ST_{FSR}] = \rho_{FSR}/\lambda$ and $E[ST_{FSR}] = E[ST_{FIFO}] - E[S_{FIFO}] + E[S_{FSR}]$, derived

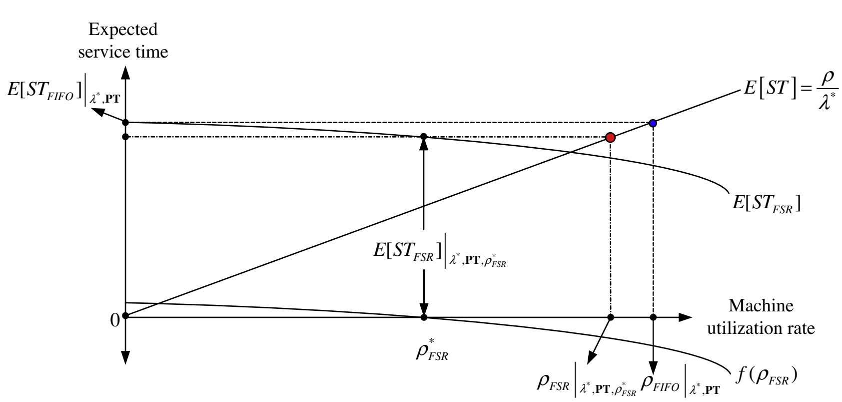

Fig. 2a. The expected service time under FSR for a given total arrival rate λ^*.

from Eqs. (16) and (17). If $E[ST_{FSR}] = \rho_{FSR}/\lambda$ is substituted in $E[ST_{FSR}] = E[ST_{FIFO}] - E[S_{FIFO}] + E[S_{FSR}]$, then a new equation can be written as $f(\rho_{FSR}) = E[ST_{FIFO}] - E[S_{FIFO}] + E[S_{FSR}] - \rho_{FSR}/\lambda = 0$ and then it can be rewritten as Eq. (20) based on Eq. (16), where $E[S_{FSR}]$ can be derived by substituting Eq. (5) with Eq. (9).

$$f(\rho_{FSR}) = \left(\sum_{j=1}^{J} n_j\right)^{-1} \sum_{j=1}^{J} \sum_{i=1}^{n_j} \Pr[T_{ij} \leqslant RT] pt_j + E[S_{FSR}] - \frac{\rho_{FSR}}{\lambda} = 0. \tag{20}$$

As $f(\rho_{FSR})$ is differentiable, the Newton's method can be used to solve the nonlinear equation, $f(\rho_{FSR}) = 0$. According to $f(\rho_{FSR})$ and its derivative with respect to ρ_{FSR}, we begin with a first guess of ρ^0_{FSR} by setting $0 < \rho^0_{FSR} \leqslant 1$. An approximate solution ρ^1_{FSR} can be obtained by calculating $\rho^0_{FSR} - f(\rho^0_{FSR})/f'(\rho^0_{FSR})$, in which ρ^1_{FSR} should be a better approximation to the solution of $f(\rho_{FSR}) = 0$. Once we have ρ^1_{FSR}, the process can be repeated to obtain ρ^2_{FSR}. After n steps, if we have an approximate solution of ρ^n_{FSR}, then the next step is to calculate ρ^{n+1}_{FSR} and $\rho^{n+1}_{FSR} = \rho^n_{FSR} - f(\rho^n_{FSR})/f'(\rho^n_{FSR})$. Note that value of ρ^n_{FSR} moving closer to the value of ρ^{n+1}_{FSR} indicate that the approximate solution of $f(\rho_{FSR}) = 0$ after n steps has been determined.

The curve of the function $f(\rho_{FSR}) = 0$ for various machine utilization rates is plotted in Fig. 2a. The function of $f(\rho_{FSR}) = 0$ is the expected service time under FSR ($E[ST_{FSR}]$) that shifts down with shifts in quantum ρ_{FSR}/λ. Thus, a root of $f(\rho_{FSR}) = 0$; that is, ρ^*_{FSR} is identified using the Newton's method. By giving $\rho_{FSR} = \rho^*_{FSR}$ for Eq. (17) to calculate $E[ST_{FSR}]$, then machine utilization rate under FSR is obtained; that is, $\rho_{FSR}|_{\lambda^*,\mathbf{PT},\rho^*_{FSR}} = \lambda^* E[ST_{FSR}]|_{\lambda^*,\mathbf{PT},\rho^*_{FSR}}$.

In Fig. 2b, by changing the total arrival rate from λ^* to λ^{**}, λ^{**} is found to be smaller compared with λ^* along with the same vector of job processing time **PT**. A line $E[ST] = \rho/\lambda^{**}$ is drawn with slope $1/\lambda^{**}$; that is, the inverse of the total arrival rate and this line is steeper because $1/\lambda^{**}$ is larger compared with $1/\lambda^*$. By repeating the aforementioned steps, the expected service time and the machine utilization under FSR can then be depicted as $E[ST_{FSR}]_{\lambda^{**},\mathbf{PT},\rho^{**}_{FSR}}$ and $\lambda^{**}E[ST_{FSR}]|_{\lambda^{**},\mathbf{PT},\rho^{**}_{FSR}}$. The expected service time and machine utilization rate under FIFO can be computed as $E[ST_{FIFO}]|_{\lambda^{**},\mathbf{PT},\rho^{**}_{FSR}}$ and $\lambda^{**}E[ST_{FIFO}]|_{\lambda^{**},\mathbf{PT},\rho^{**}_{FSR}}$.

The varied total arrival rate from λ^* to λ^{**} with small increment is depicted by the two bold curves in Fig. 2c. They represent the relationships between the expected service time and the machine utilization rate for various total arrival rates under FIFO and FSR, respectively. Fig. 2c also illustrates the effect of varying total arrival rates on the reduction of service time, which corresponds to the pairs of machine utilization rates under FIFO and FSR. These show that the reductions of service time and machine utilization rate become larger as total arrival rate increases. Therefore, FSR can effectively reduce service time and machine utilization rate at peak demand times.

4. Performance analysis for the proposed FSR analytic models

To evaluate the accuracy of the proposed FSR analytic models, a simulation model is built for job inter-arrival time in exponential distribution and for the selection of next jobs on queue to be processed on machine according to FSR. First, the simulation results are collected from a fixed time period of jobs arriving with various arriving rates. Next, results for the number of setups and setup time

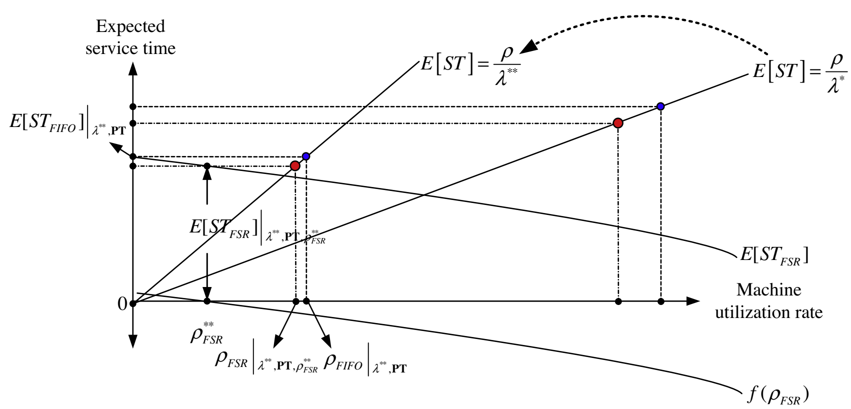

Fig. 2b. The expected service time by changing total arrival rate from λ^* to λ^{**}.

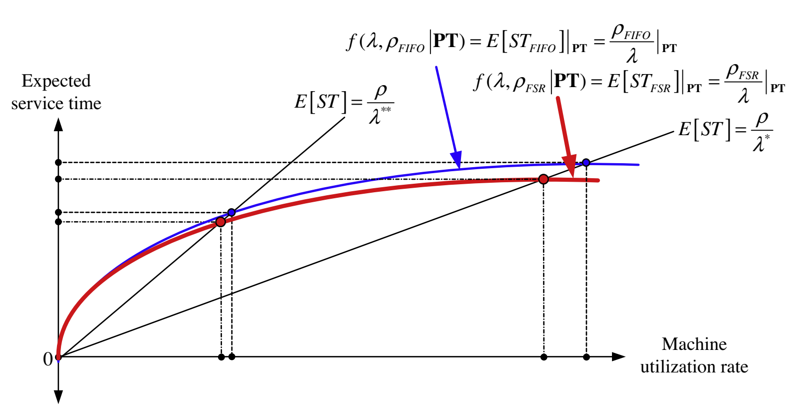

Fig. 2c. Two curves of expected service time for various total arrival rates under FIFO and FSR.

are compared with those calculated by FSR analytic models. Finally, the numerical results of the sensitivity analysis on the reductions of the expected setups and expected setup time for each product type are conducted by replacing FIFO with FSR analytic models.

4.1. Experimental design

The magnitude of required setup time depends on the total arrival rate of various types of incoming jobs and the mix of the arrival rates of various job types. Thus, the total arrival rate (λ) and the coefficient of variations among job arrival rates (*CV*) are considered in the experimental design to implement the simulation. Run time (*RT*) is also considered. The number of setups and the setup time are observed for a period of run time (*RT*).

First, there are six levels of total arrival rate to be considered. The arrival rates among eight product types are defined by $\lambda_j = a\tau_j$. Total arrival rate can be computed as $\lambda = \sum_{j=1}^{8}\lambda_j = a\sum_{j=1}^{8}\tau_j$ jobs in 60 s. The various values of τ_j are shown in Table 1. Constant a has six levels with a = 1.00, 0.95, 0.90, 0.85, 0.80, and 0.75. If a = 0.95, then the total arrival rate is calculated as $\lambda = 0.95\sum_{j=1}^{8}\tau_j = 0.0095$, where the arrival rates among eight product types are equal to $\lambda_j = 0.95\tau_j$. For this reason, the six levels of total arrival rate in the experimental design are given by 0.0100, 0.0095, 0.0090, 0.0085, 0.0080, and 0.0075 for varying the values of a.

Next, the coefficient of variation is set as a percentage and calculated from the mean and the standard deviation of the job arrival rate of product types. This is defined by $CV = (s_\lambda/\bar{\lambda}) \times 100\% = (s_\tau/\bar{\tau}) \times 100\%$, where $\bar{\lambda}$ and $\bar{\tau}$ are the means of λ_j and τ_j and are given by $\sum_{j=1}^{8}\lambda_j/8$ and $\sum_{j=1}^{8}\tau_j/8$, respectively. Additionally, s_λ and s_τ are the standard deviations of λ_j and τ_j and are derived as $\sqrt{\sum_{j=1}^{8}(\lambda_j-\bar{\lambda})^2/7}$ and $\sqrt{\sum_{j=1}^{8}(\tau_j-\bar{\tau})^2/7}$, respectively. According to Table 1, the *CV*s are calculated as 0, 27.9753, and 53.7234, which imply that the dispersion of the job arrival rates among various types increases with the rise in *CV*. Finally, three levels of run time (*RT*) are considered: 8, 16, and 24 h. Therefore, there are 54 combinations.

For each combination, the simulation model uses the same vector of job processing time (**PT**) among eight product types. The matrix of setup time (**ST** = $[s_{jr}]$) is used for switching the machine setting along changing product types, where s_{jr} is the setup time for product type j job after product type r job, and is numbered by the j row/r column position in **ST**. "Second" is the unit of processing time and setup time. Note that the simulation results are collected for each combination after 10,000 independent simulation runs.

$$\mathbf{PT} = [\,15 \quad 75 \quad 85 \quad 45 \quad 55 \quad 10 \quad 80 \quad 125\,],$$

$$\mathbf{ST} = [s_{jr}] = \begin{bmatrix} 0 & 90 & 60 & 15 & 15 & 30 & 45 & 30 \\ 15 & 0 & 75 & 30 & 45 & 75 & 90 & 45 \\ 30 & 60 & 0 & 45 & 90 & 90 & 75 & 60 \\ 45 & 75 & 90 & 0 & 45 & 30 & 60 & 45 \\ 60 & 75 & 45 & 45 & 0 & 45 & 75 & 15 \\ 45 & 30 & 30 & 30 & 75 & 0 & 60 & 75 \\ 60 & 45 & 60 & 15 & 45 & 15 & 0 & 45 \\ 15 & 30 & 15 & 30 & 60 & 30 & 45 & 0 \end{bmatrix}.$$

4.2. Accuracy analysis for the FSR analytic models

4.2.1. Accuracy analysis for the FSR analytic models in estimating number of setups

Fig. 3 shows that the numbers of setups for each product type are observed for a period of run time in the simulation model and the FSR analytic model by varying the *CV*s of job arrival rate, total arrival rates, and run times. The expected number of setups for each product type using FSR depends on its arrival rate (λ_j) according to Eqs. (5) and (6). If the arrival rate parameters among eight product types are the same (*CV* = 0) for a given specific total arrival rate, then the expected numbers of arrived jobs for each product type are fixed and the expected numbers of setups for each product type are equal. However, the numbers of arrived jobs for each product type in the simulation model are not exactly equal when *CV* = 0 because the jobs are generated at random to respond to the reality of market demand. Thus, for a given specific total arrival rate, the numbers of setups for each product type by the simulation model are not all the same as *CV* = 0. These conditions are apparent in Fig. 3(a)–(c).

The dispersion of the number of setups for each product type in the simulation model and the FSR analytic model increases with the rise in *CV*. This implies that the extreme values of the arrival rate parameters among various product type increase and can lead to higher and lower setup frequencies. Moreover, the number of setups in the simulation model and the FSR analytic model increases as a result of the rise in the numbers of arrived jobs accumulated over time. According to earlier discussions, the trends in the numbers of setups for a given specific *CV* in the simulation model and the FSR analytic model are the same when the factors (*RT* and λ) are changed.

To compare the result of the numbers of setups under FSR generated by the simulation model and the analytic model, the error percentage of estimated setups is given by $EP_{setps,j} = |(SNS_{j,FSR} - E[NS_{j,FSR}])/SNS_{j,FSR}| \times 100\%$, where $SNS_{j,FSR}$ and $E[NS_{j,FSR}]$ are the number of setups of product type j jobs under FSR under the simulation model and FSR analytic model, respectively. The mean error percentages of estimated setups for each product type, as compared with the data from simulation model upon varying the *CV*s of job arrival rate and run times, are plotted in Fig. 4(a)–(c). Fig. 4(d)–(f) illustrate the mean error percentages of estimated setups under FSR by varying the *CV*s of job arrival rate, total arrival rates, and run times. In Fig. 4, the overall mean of error percentage of estimated setups for a given specific run time is the arithmetic mean of all individual error percentage of estimated setups, which is obtained in a specific run time for each combination.

Regardless of run times, the *CV* equal to 27.9753% has the smaller error percentage of estimated setups compared with the *CV* equal to 0 and the *CV* equal to 53.7234% (Fig. 4). As the *CV* equals to zero, the expected numbers of setups for each product type using the FSR analytic model become the same. However, the numbers of arrived jobs for each product type by the simulation model are different. Thus, owing to the reality of market demand, uncertainty in the number of jobs can influence the performance of the error percentage of estimated setups. Moreover, the extreme values of the arrival rate parameters among various product types increase with larger *CV*. This can lead to an increase in extreme

Table 1
Coefficient of variations among job arrival rates.

*CV*s (%)	Parameters among eight product types							
	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7	τ_8
0	0.001250	0.001250	0.001250	0.001250	0.001250	0.001250	0.001250	0.001250
27.9753	0.001342	0.001577	0.001804	0.000917	0.001145	0.001443	0.000817	0.000955
53.7234	0.001821	0.001255	0.000297	0.001721	0.000946	0.000363	0.001467	0.002130

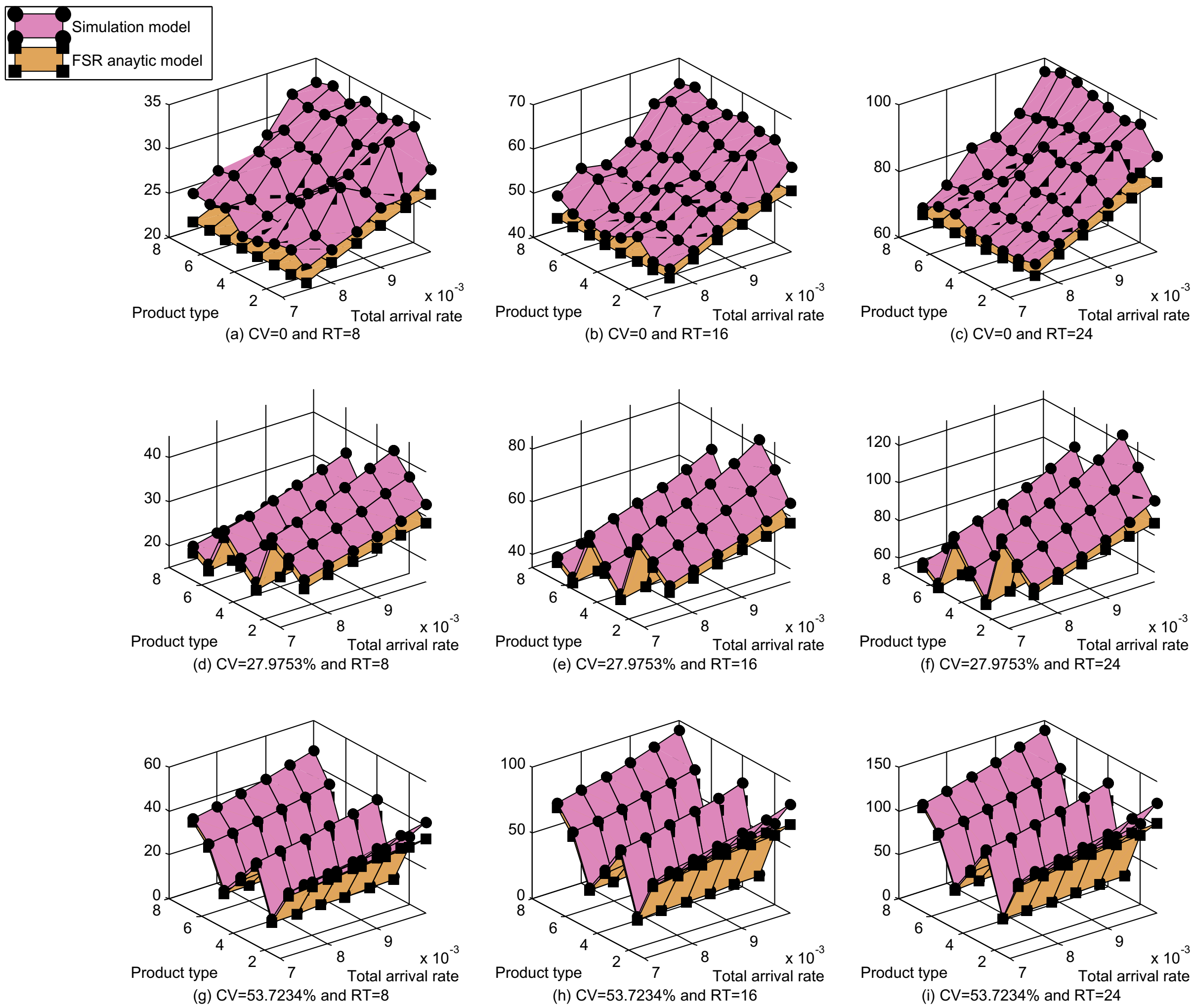

Fig. 3. Number of setups for each product type by simulation model and FSR analytic model.

values in the number of setups and can make the error percentage of estimated setups rise as well. Thus, the moderate dispersion of job arrival rate among various types is related to the accuracy of the proposed FSR analytic model in estimating the number of setups.

The values and dispersions of the error percentage of estimated setups for each *CV* in Fig. 4(a)–(c) decrease with a lengthened *RT*. The larger error percentage of estimated setups occurs at *RT* = 8 h because of the few setups. When the run time becomes longer, the error percentage of estimated setups decreases as a result of the larger setups. The overall mean of the error percentage of estimated setups ranges from 12.1776 to 8.3446%, as *RT* changes from 8 h to 24 h. In addition, when the total arrival rate increases, the mean error percentage increases correspondingly, as shown in Fig. 4(d)–(f). In particular, the larger mean error percentage occurs at a higher level of machine utilization rate.

4.2.2. Accuracy analysis for the FSR analytic models in estimating setup time

Fig. 5 shows the setup times of single job for each product type in the simulation model and the FSR analytic model by varying the *CV*s of job arrival rate, run times, and total arrival rates. The setup time of a single job is defined by the total setup time of all jobs with the same product type at a time interval divided by the total number of jobs arrival specific for that.

According to Eq. (8), the setup time of a single job for each product type depends on its arrival rate and setup time matrix. When *CV* is equal to zero (i.e., the arrival rate parameters among eight product types are the same), the setup time of a single job for each product type only depends on the setup time matrix. The average setup time of product type *j* job ($\bar{s}_j$) can be calculated as the summation of s_{jr} in **ST** for all *r* divided by the number of product type. Product types 3 and 4 have larger average setup time ($\bar{s}_3 = 56.2500$ and $\bar{s}_4 = 48.7500$, respectively), and product type 8 has the minimum value of average setup time ($\bar{s}_8 = 28.1250$). The setup times of a single job for each product type in the simulation model and the FSR analytic model, as shown in Fig. 5(a)–(c), are near its average setup time as *CV* = 0.

When *CV* is equal to 27.9753%, product type 3 obtains the larger arrival rate, whereas product type 4 achieves the smaller arrival rate. The gap between the setup times by the simulation model and the FSR analytic model in Fig. 5(d)–(f) narrows as compared with *CV* = 0 because the setup time of product type 3 is reduced due to its larger arrival rate. In contrast, product type 3 has the smaller arrival rate while product type 4 has the larger arrival rate with *CV* = 53.7234%. Thus, the wider reduction in setup time for product type 4 leads to the larger gap between the setup times in the simulation model and the FSR analytic model as compared with *CV* = 0, which is apparent in Fig. 5(g)–(i).

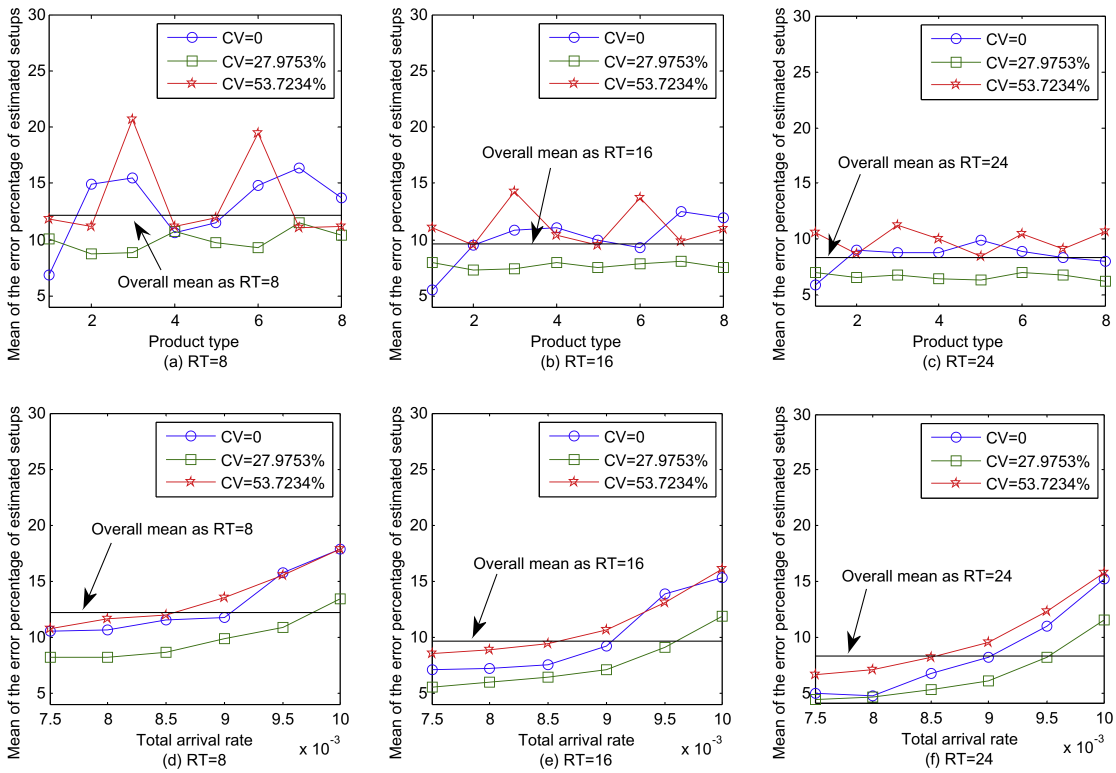

Fig. 4. Mean of the error percentage of estimated setups between FSR analytic model and simulation model.

Product type 8 also has the smaller arrival rate as $CV = 27.9753\%$ and the larger arrival rate as $CV = 53.7234\%$. However, its setup time reduction is limited as $CV = 53.7234\%$ because of the shorter average setup time. Thus, the setup times of product type 8 upon varying the *CV*s are nearly equal, whether to adopt the FSR analytic model or to implement the simulation model. In general, the patterns of the setup time for a given specific *CV* in the simulation model and the FSR analytic model are identical in terms of the changes of the other two factors in the experimental design (*RT* and λ).

A comparison of the results of setup time for single jobs generated by the simulation model and the analytic model suggests that the error percentage of estimated setup time is defined by $EP_{setuptime,j} = |(SST_{j,FSR} - E[ST_{j,FSR}])/SST_{j,FSR}| \times 100\%$, where $SST_{j,FSR}$ and $E[ST_{j,FSR}]$ represent the setup times of a single job of product type *j* under FSR by the simulation model and the analytic model, respectively. The mean error percentages of estimated setup time for each product type between FSR analytic model and simulation model by varying the *CV*s of job arrival rate and the run times are shown by Fig. 6(a)–(c). Meanwhile, Fig. 6(d)–(f) show the mean error percentages of the estimated setup time between FSR analytic model and the simulation model by varying the *CV*s of job arrival rate, total arrival rates, and run times.

As setup time depends on the number of setups, the behavior of the error percentage of estimated setup time in Fig. 6 is similar to that in Fig. 4. From shorter to longer run time, the error percentage of estimated setup time decreases and the lower error percentage of estimated setup time is attained at longer run time, regardless of the *CV*s of job arrival rate and total arrival rates. The overall means of the error percentage of estimated setup time range from 10.9263 to 7.9113% as *RT* changes from 8 h to 24 h. Meanwhile, when *CV* equals 27.9753%, the lowest error percentage of estimated setup time is obtained. Finally, when total arrival rate increases, the error percentage of estimated setup time increases correspondingly; that is, lower error percentage of estimated setup time occurs at lower levels of machine utilization rate.

In general, the number of setups and the setup time can be estimated accurately using our models to a certain extent. Based on the analysis, better accuracy of the proposed FSR analytic models in estimating the number of setups and setup time can be obtained for longer run times, smaller total arrival rates, and moderate dispersion of job arrival rates among various types. This result can be offered to managers as reference for evaluating capacity loss and others.

4.3. Sensitivity analysis for the FSR analytic models

4.3.1. Sensitivity analysis of the reduction of number of setups for each product type

The differences of the expected number of setups between FIFO and FSR are defined by the expected number of setups under FIFO minus the expected number of setups under FSR. The mean of the difference of the expected number of setups between FIFO and FSR for each product type by varying the *CV*s of job arrival rate is illustrated in Fig. 7(a). The mean of the difference of the expected number of setups between FIFO and FSR for each product type is constant when *CV* equals zero. Moreover, the dispersion of the mean of the difference of the expected number of setups between FIFO and FSR increases with *CV*, which implies that the extreme value of arrival rate parameters among various product type increases and can influence the performance of the FSR analytic model in reducing setup frequency. The positive correlation coefficients are calculated as 0.998 and 0.991 when the *CV*s equal 27.9753 and 53.7234%, respectively. These positive correlation coefficients indicate a relationship between the mean of the difference of the expected number of setups between FIFO and FSR and the arrival rate parameters among eight product types. As values for the arrival rate parameters among eight product types increase, the values for reducing setup frequency also increase. Therefore, by replacing FIFO with FSR, the largest reduction of the number of setups occurs at $CV = 53.7234\%$, which is apparent in Fig. 7(b) and (c).

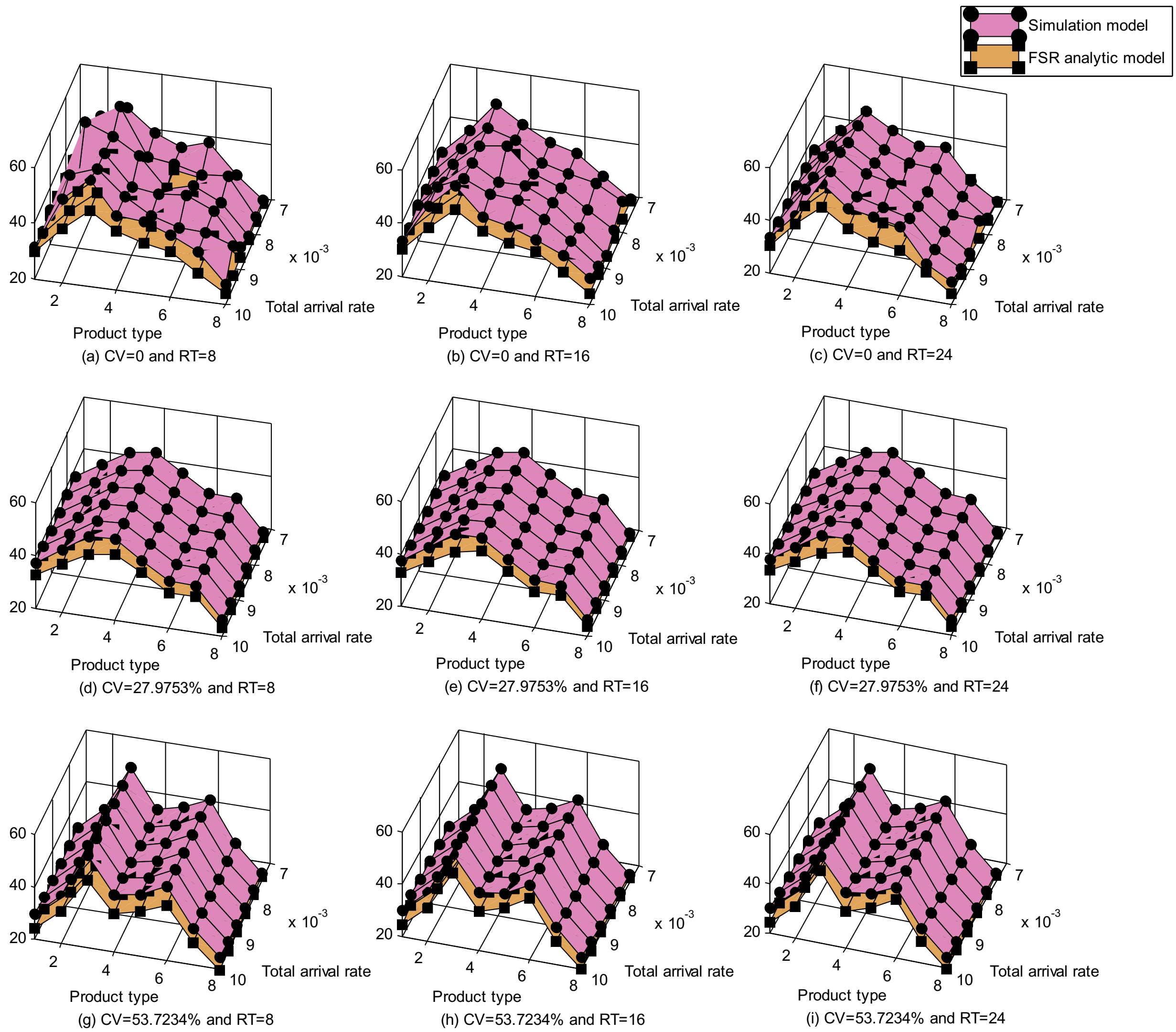

Fig. 5. Setup time of single job for each product type in simulation model and FSR analytic.

4.3.2. Sensitivity analysis of the reduction of expected setup time for each product type

The differences of the expected setup time between FIFO and FSR are defined by the expected setup time under FIFO minus the expected setup time under FSR, and the mean of the difference of the expected setup time between FIFO and FSR for each product type by varying the *CV*s of job arrival rate is displayed in Fig. 8(a). The dispersion of the mean of the difference of the expected setup time between FIFO and FSR increases with *CV*. The correlation coefficients are positive and are calculated as 0.906 and 0.845 when the *CV*s equal 27.9753 and 53.7234%, respectively. Therefore, the arrival rate parameters among eight product types and the mean of the difference of the expected setup time tend to increase and decrease, respectively, along with each other.

Job arrivals tend to concentrate on fewer product types as *CV* increases. The types obtaining high possibilities of setup reduction leading to the largest reductions of the setup time occur at *CV* = 53.7234%, which are showed in Fig. 8(b) and (c).

5. Conclusions

In this paper, we consider a single finite-capacity machine responsible for processing several product types of jobs when setup time is dependent on product type. With uncertainties in job arrival time and types of demand, setting an output target may be significantly different from actual scenarios due to possible heavy capacity loss and difficulty in calculating the required setup time. Thus, FSR analytic models are developed to estimate expected setup time and service time. The effect on capacity wastage due to changes in machine setting among several product types can then be evaluated. Due to the difficulty in obtaining analytical solutions for the expected setup time and service time, the numerical solutions of expected setup time and service time are provided in this paper.

Results of the proposed FSR analytic models are compared with simulation results. Computational results show that error percentages of estimated setups and setup time are larger when *CV* and total arrival rate increase, but they are reduced when run time is lengthened. Generally speaking, the smaller error percentage of estimated setups and setup time can be obtained with longer run time, smaller total arrival rate, and moderate dispersion of job arrival rate among various types. In this paper, we also provide the sensitivity analyses to discuss how the reductions of the setup frequency and the setup time can be affected by the changes of three factors (*CV*, λ, and *RT*). Compared with FIFO, FSR can be used to reduce the frequency of setups and the length of the setup time, hence leading to a reduction in machine utilization rate, especially

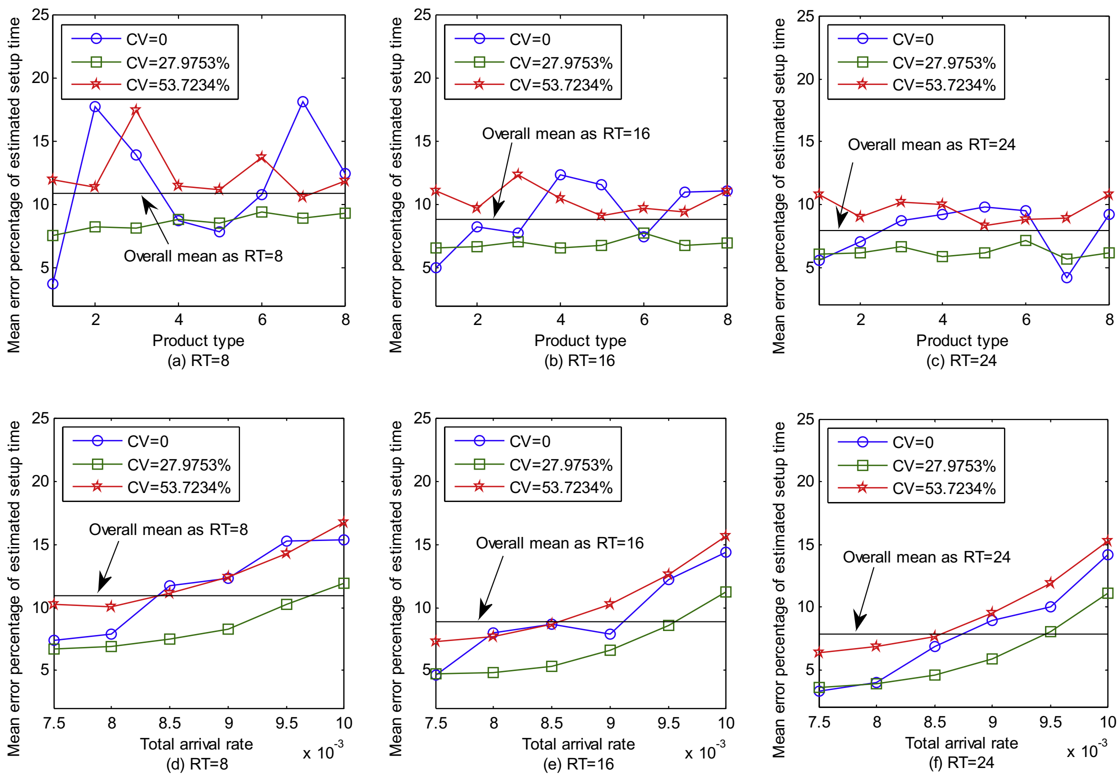

Fig. 6. Mean error percentage of estimated setup time between FSR analytic model and simulation model.

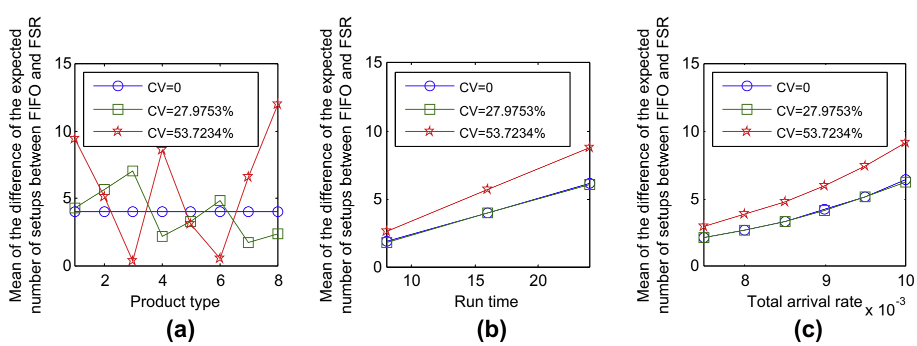

Fig. 7. Mean of the difference of the expected number of setups between FIFO and FSR.

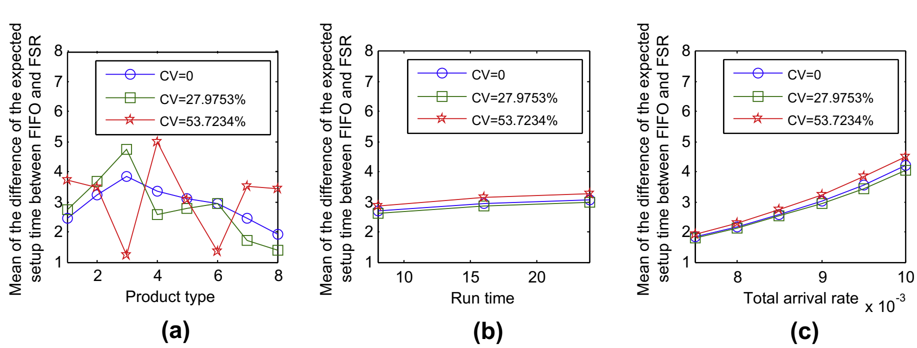

Fig. 8. Mean of the difference of the expected setup time between FIFO and FSR.

at conditions of high total arrival rate and high dispersion of arrival rates among several types of job.

The FSR models can, to some extent, estimate accurately the setup time and evaluate efficiently the capacity of wastage arising from switching the machine setting responding to uncertainties in job arrivals. Managers can utilize the expected setup time as threshold and tolerance during production planning. Moreover, in this paper, the sequence of batches by FSR is sorted according to arrival time of the first jobs in each batch in increasing order. In the future, the rule of sorting batches may change to using the setup time for any two batches in increasing order in order to minimize total setup time.

Acknowledgments

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