Sub-Poisson photoelectron statistics in saturated light absorption

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We show that sub-Poisson photoelectrons can result from the finite relaxation time of the fundamental photoelectron-emitting entities, which could be individual atoms, or valence-band electrons, etc., depending on the type of the detectors. After identifying the precise quantum-mechanical criterion for a light without the intrinsic antibunching property, it is demonstrated that the variance to mean ratio of photoelectron counts within a fixed time period can be reduced to as small as $\frac{1}{2}$, even for such light. The condition for strong fluctuation reduction is found to be the saturation of light absorption. Semiconductor photodetectors are considered in relative details, and their photoelectron statistics are expressed in terms of the intrinsic material parameters.

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Since its earliest experimental realizations [1], sub-Poisson photoelectron statistics has acquired substantial attention [2]. In addition to serving as one of the fundamental manifestations of various nonclassical feature of light, the generation of photoelectrons with fluctuations below the standard quantum limit has potential applications on coherent light communications, as well as precision measurements [2]. In the study of the properties of light, photoelectric effect is probably the most commonly used way to perform practical measurements, due to the fact that a photon number of a quantum state of light is not subject to direct detection. However, the generation of photoelectrons is a quantum-mechanical process, and is therefore inherently probabilistic. The ways to recover the precise properties of the light under study, from the outcomes of the stochastic photoelectric process, remains a subject of interest. In particular, the states of light that are able to generate sub-Poisson photoelectrons have been under intensive investigations. On the other hand, how to reduce the quantum fluctuations introduced by the transformation of optical signals to electric signals, due to the probabilistic nature discussed above, is crucial from the point of view of applications. The noise reduction below the standard quantum limit [3] achieved in the various nonclassical light has therefore been one of the main motivations for the study of this subject.

In this paper, we shall take a somewhat different approach to the generation of sub-Poisson photoelectrons. Obviously, the statistics of photoelectrons is not only a property of the light that generates them, but also depends on the reaction of the photodetector to the light. In fact, it is the detector that emits electric current carriers which ultimately give rise to the photoelectron counts. In most of the previous theoretical studies, the detector dependence is so idealized that the final result depends on the properties of the light only. This idealization is in fact the underlining basis for the interpretation of optical coherence functions as the joint photoelectron detection probabilities [4]. In our study, however, we found that the statistics of photoelectrons is in general a function of the response of the detecting materials to the incoming light waves, and could differ substantially for the same state of light. In particular, we found that sub-Poisson photoelectron statistics emerges naturally in the circumstance of absorption saturation. In other words, in order to get photocurrents with signal-to-noise ratio below the standard quantum limit, one can simply run the phototube in the saturation regime.

Since our main purpose is to emphasize the detector dependence, we shall consider only the kinds of light that would generate Poisson photoelectrons in the cases without absorption saturations. Such lights are usually described as given classical electromagnetic waves, and their quantum-mechanical dynamics is neglected. In order to be more precise, before we calculate the photoelectron statistics in saturated light absorption, we shall discuss first the condition for such light in terms of single-mode quantum optics.

It is well known that a stochastic point process without memory of its past forms a Poisson process [5], which gives a Poisson distribution for the number of counts (or points) during a given time period. Therefore, for any sub-Poisson process, a certain memory effect must be present. This memory effect of the photoelectron counts is implicitly the basic theme of the current theories for antibunched light [6]. To be precise, a photoelectron count between time t and $t+dt$, with dt an infinitesimal time interval, corresponds to two quantum-mechanical measurements on the detector state, performed at t and $t+dt$, respectively. If the second measurement results in one more ionized electron in the detector than the first one, then we register a photoelectron count at time t. Otherwise, there is no count registered in the same interval. According to quantum mechanics, any measurement corresponds to a projection of the quantum state vector to one of the eigenspace of the Hermitian operator associated with the measurement. This projection in the Hilbert space will in general alter the state of the system in a significant way, and effect the probability distribution of the outcome of a subsequent measurement, associated with both the same, or a different, Hermitian operator. Applying the above observation to the case of photoelectron counts, we conclude that the detection of a photoelectron at time t corresponds to a projection of the state of the coupled system of the light and the detector. Thus the state of the light alone could also be altered significantly. From elementary quantum mechanics we know that the photoelectron generation probabilities are proportional to the expectation value of the photon number operator n, which is equal to $a^{\dagger}a$ for single

mode light. Here a and $a^\dagger$ are the annihilation and creation operators of the mode, respectively. As far as photoelectron generation probability is concerned, the question is whether $\langle n\rangle$ will differ significantly before and after the projection. Let $|\alpha\rangle$ be the state of the light at time t. It is well known that after the detection of a photoelectron during the small interval dt, the state of the light $|\alpha\rangle$ will be projected into another state $a|\alpha\rangle/\langle\alpha|a^\dagger a|\alpha\rangle^{1/2}$, which we call $|\beta\rangle$. Both $|\alpha\rangle$ and $|\beta\rangle$ are chosen to be normalized. For the immediate small interval after $t+dt$, i.e., from $t+dt$ to $t+2dt$, the probability for a photoelectron count to occur is therefore proportional to $\langle\beta|n|\beta\rangle$, given the fact that there is a count occurring in the previous interval between t and $t+dt$. On the other hand, if there were no count registered between t and $t+dt$, the projection would bring the state $|\alpha\rangle$ back itself, and the probability to have a count immediately afterwards would be proportional to $\langle\alpha|n|\alpha\rangle$. Since $\langle\beta|n|\beta\rangle$ and $\langle\alpha|n|\alpha\rangle$ are in general different, there is in general a memory effect. In other words, the photoelectron counts in the past will have certain effect on the counts in the future, and the photoelectron statistics during a given time period cannot be strictly Poisson. A natural measure for the degree of this memory effect is the ratio between the expectation values of n for $|\alpha\rangle$ and $|\beta\rangle$. Let us define such a parameter G as $\langle\beta|n|\beta\rangle/\langle\alpha|n|\alpha\rangle$. Then $G>1$ (<1) means the enhancement (reduction) of photoelectron counts by a previous count, and therefore implies bunched (antibunched) photoelectron counts, which give rise to super-Poisson (sub-Poisson) photoelectron count distribution within a fixed time period. The relation between the state of light and the bunching-antibunching property is revealed by the observation of the fact that $G<1$ if and only if the variance to mean ratio, which is uaully referred to as the Fano factor F_n, of the photon number distribution is smaller than 1. This can be seen by definition of F_n:

$$F_n \equiv \frac{\langle\alpha|\Delta n^2|\alpha\rangle}{\langle\alpha|n|\alpha\rangle} = \frac{\langle\alpha|n^2|\alpha\rangle - \langle\alpha|n|\alpha\rangle^2}{\langle\alpha|n|\alpha\rangle} .$$

$F_n<1$ implies $\langle\alpha|n^2-n|\alpha\rangle < \langle\alpha|n|\alpha\rangle^2$, and $G<1$ subsequently. States with $F_n<1$ are usually referred to as photon number squeezed states. The connection between squeezed state and antibunched photoelectron counts is thus established. For coherent light, $G=1$, and there is no memory effect intrinsically in the light. Poisson photoelectrons will be generated in an ideal photodetector.

In the cases of $G<1$, let us remark briefly on the conditions for the observation of antibunched photoelectron statistics in practice. The disturbance (projection) caused by the detection of a count at time t on the quantum state of light will not, in fact, last forever. This is because the light is coupled not only to the detector, but also to its source, for example, a laser cavity with population inversion. Therefore, the mean photon number will relax from $\langle\beta|n|\beta\rangle$ to a stationary value within a characteristic time, which we call T_c. T_c is in fact the second order coherence time of the light. It is clear that the antibunching effect can be observed only if the mean temporal separation between successive photoelectron counts is comparable to, or smaller than, T_c [7]. If this were not the case, the disturbance caused by one photoelectron count would be washed out before the next count, and no memory effect would last.

In the following we will concentrate on either a coherent light with $G=1$, or on the case where the mean time intervals between photoelectron counts are much longer than T_c for $G\neq 1$. We will show how sub-Poisson photoelectron statistics can still result naturally from the realistic consideration of the detection process. Let us consider first the case where the detector is composed of a collection of identical atoms. Each atom can be ionized by the incoming light and emit a photoelectron. Concentrating on one particular atom, it will be eventually ionized and become an ion. If it relaxes back to an atom, by capturing an electron, immediately after the ionization, then the sequences of times $t_1, t_2, \ldots$, at which ionizations occur, will form a Poisson point process [5]. However, an ion does not relax immediately in practice. It will remain in the state of an ion for a finite amount of time before relaxation. This finite relaxation time suppresses the occurrences of tightly bunched photoelectron clusters and makes the final statistics sub-Poisson. One would argue, however, that the suppression is valid for the photoelectrons emitted by only one atom, and will be invisible if we superimpose the photoelectron emission times from a large number of atoms. Our calculations below not only confirm the sub-Poisson character for one atom quantitatively, but also show that this character survives in the case of many atoms.

As stated before, an atom will alternate between two states, neutral atom and positively charged ion, when interacting with the external light. We call these two states 1 and 2, respectively. Let the transition probability from 1 to 2 during a small time interval dt be equal to $\lambda_1 dt$, and the transition probability from 2 to 1 $\lambda_2 dt$. Thus the lifetimes of state 1 and 2 are $1/\lambda_1$ and $1/\lambda_2$, respectively. A photoelectron count is registered when a transition from 1 to 2 happens. Let m be the number of such transitions during a given time period T. We need to calculate $P_m(T)$, the probability distribution of m within a time period T. To do so, we first introduce two quantities $Q_m^i(t)$, $i=1,2$. They are defined to be the probability densities that the total time period spent in m of state i intervals combined together is between t and $t+dt$. They are given by [5]

$$Q_m^i(t) = \frac{(\lambda_i t)^{m-1}}{(m-1)!}\lambda_i e^{-\lambda_i t}.$$

When m is much larger, they can be approximated by normal distributions:

$$Q_m^i(t) \simeq \frac{1}{\sigma_i\sqrt{2\pi}} e^{-(t-\mu_i)^2/(2\sigma_i^2)} \quad , \quad m \gg 1,$$

with variances and means given by $\mu_i = m/\lambda_i$, $\sigma_i^2 = m/\lambda_i^2$. Now we define another quantity $Q_m(t)$ to be the probability density that the mth photoelectron count happens between t and $t+dt$. Clearly, $Q_m(t)dt$ is equal to the probability that, after m of 1 and 2 state intervals (they are alternating), the total time spent is between t and $t+dt$. Therefore, we have the convolution

$$Q_m(t)=\int_0^t Q_m^1(t_1)Q_m^2(t-t_1)dt_1 .$$

This is again a normal distribution. The mean μ is equal to $\mu_1+\mu_2$, and the variance σ^2 is $\sigma_1^2+\sigma_2^2$. Finally, the desired photoelectron distribution $P_m(T)$ is given by

$$P_m(T)=\int_0^T Q_m(t)F(T-t)dt,$$

where $F(t)$ is the probability that there is no photoelectron count in an interval of t. It can be easily calculated and is equal to $[\lambda_2\exp(-\lambda_1 t)-\lambda_1\exp(-\lambda_2 t)]/(\lambda_2-\lambda_1)$. For T much longer than the mean photoelectron counting intervals, we have

$$P_m(t)\simeq\int_0^\infty F(t)dt\ Q_m(T)=\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)Q_m(T).$$

The corresponding variance and mean of the random variable m are

$$\langle m\rangle=\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)^{-1}t,$$

$$\langle\Delta m^2\rangle=\left(\frac{1}{\lambda_1^2}+\frac{1}{\lambda_2^2}\right)\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)^{-3}t.$$

The Fano factor F_m for $P_m(T)$ is therefore equal to

$$F_m=\frac{\langle\Delta m^2\rangle}{\langle m\rangle}=\frac{1+\left(\frac{\lambda_1}{\lambda_2}\right)^2}{1+\left(\frac{\lambda_1}{\lambda_2}\right)^2+2\frac{\lambda_1}{\lambda_2}}.$$

One can clearly see that F_m is always smaller than 1, as anticipated before. In fact, the range of F_m is $1/2\leq F_m\leq 1$. The lower limit is achieved when $\lambda_1=\lambda_2$, while the upper limit is achieved when either $\lambda_1\to\infty$ or $\lambda_2\to\infty$, which corresponds to the cases where one of the two states relaxes infinitely fast. They reduce to the case of a Poisson process whose Fano factor is known to be 1.

As emphasized before, the observed photoelectron counts actually result from the superposition of counts emitted by many atoms. One would think that after the superposition, the suppression of photoelectron clusters will be washed out and Poisson statistics recovered. This intuition turns out to be wrong, and the sub-Poisson character does preserve, as shown below. Let the total number of atoms be N; the total number of counts M from all the N atoms is equal to $m_1+m_2+\cdots m_N$. Here $m_1,\ldots,m_N$ are independent random variables with the identical distribution function $P_m(T)$. From the central limit theorem [8] we have that for $N\gg 1$, M is a normal random variable with variance $\langle\Delta M^2\rangle$ equal to $N\langle\Delta m^2\rangle$, and mean $\langle M\rangle$ equal to $N\langle m\rangle$. Therefore,

$$F_M\equiv\frac{\langle\Delta M^2\rangle}{\langle M\rangle}=F_m .$$

In other words, the sub-Poisson character of one atom statistic is transferred exactly to the statistics of a large number of atoms. It is interesting to note that even though M is a sub-Poisson random variable, there is no antibunching effect between successive counts after the superposition. This is because for the majority of the cases, two successive counts are from different atoms, and there should be no correlations between them. Therefore, many atom photoelectron counts and a Poisson process with equal intensity share the same intercount interval probability distribution. The distinction lies in the fact that the first has memory, while the latter has not. We also note that the Fano factor is significantly smaller than 1 only when λ_1 and λ_2 are comparable. This is precisely the condition for absorption saturation to occur, since the absorption cross section of the incoming light is proportional to the fraction of neutral atoms, which is equal to $\lambda_2/(\lambda_1+\lambda_2)$.

In the following, we will apply the results of the above calculations to the more commonly used semiconductor detectors. In a semiconductor material under photoillumination, a valence-band electron would make a direct transition to the conduction-band level with the same crystal momentum, and leave a hole in the valence band. Assuming that the photodetector does not exihibit gain [9], then each such transition will result in one photoelectron count. After a certain moment, which we call the hole lifetime, the empty valence state (hole) will capture an electron in the conduction band through various recombination processes. Therefore, each valence-band level interacting with the incoming light will alternate between two states, occupied and empty, analogous to an atom considered before. We can then identify the transition rates λ_1,λ_2 discussed above as the rate at which a valence electron makes a upward direct transition, and the rate at which an empty valence state captures an electron, respectively. We shall obtain λ_1 and λ_2 in terms of the intrinsic material parameters of the semiconductor.

Since the crystal momentum k is conserved in optical transitions, the only possible final state to which a valence electron can make a transition by a photon absorption is the one in the conduction band with the same k. We denote the valence and conduction states $|vk\rangle$ and $|ck\rangle$, respectively. The resonance frequency of the transition is assumed to be ω_0. Let $\phi(\omega)$ be the photon flux density at frequency ω. We have [9]

$$\lambda_1=\phi(\omega_0)\frac{\pi c^2}{2\tau_r\omega_0^2},$$

where c is the light velocity and τ_r is the spontaneous radiative decay lifetime from $|ck\rangle$ to $|vk\rangle$, which can be expressed in terms of the momentum matrix elements between these two states. Here we make a simplification by assuming that the photon flux density $\phi(\omega)$ takes a square form, with a narrow full width Δ and central frequency ω^*. In other words, $\phi(\omega)$ is equal to ϕ for $\omega^*-\Delta/2<\omega<\omega^*+\Delta/2$, and zero otherwise. Then N, defined as the total number of valence-band states which interact with the photon flux and which could make upward transitions, is equal to

$$N=\rho_J(\omega^*)\Delta V,$$

where $\rho_J(\omega)$ is the optical joint density of states per unit volume and unit frequencies. Let us define further a quantity f as the fraction of the N states that remains in the valence band under illumination. In stationary states, the electron-hole generation rate has to be equal to their recombination rate. The recombination rate in the whole material is equal to $rnpV$, where r is an intrinsic material parameter which includes both radiative and nonradiative (usually the case) recombination processes [9]. V is the volume under illumination and n, p are the electron and hole concentrations. Without doping, it easy to see that

$$n=p=N(1-f)\frac{1}{V}.$$

Equating the generation and recombination rates in the whole volume, we have....

$$Nf\lambda_1=r\frac{N^2}{V^2}(1-f)^2V.$$

Solving it as an equation for f, we have

$$f=1+\frac{\gamma}{2}-\sqrt{\gamma+\frac{\gamma^2}{4}},$$

with

$$\gamma\equiv\frac{\lambda_1 V}{rN}=\frac{\phi}{\phi^*}$$

and

$$\phi^*\equiv\frac{2\tau_r\omega^{*2}r\rho_J(\omega^*)\Delta}{\pi c^2}.$$

Here γ represents the dimensionless photon flux density. One can easily see that f satisfies the requirement that $0\leq f\leq 1$ for all γ. In fact, $f=1$ when $\gamma=0$, and $f\to 0$ when $\gamma\to\infty$. Absorption saturation will occur if f decreases significantly from 1. Population inversion results when $\phi(\omega^*)$, or equivalently λ_1, is so strong that f is smaller than 1/2. This is actually what happens in a semiconductor laser with an optical pump. The fraction of time in which a valence state is occupied is f, by definition. In other words, $f=\lambda_2/(\lambda_1+\lambda_2)$. Therefore we have the ratio between λ_1 and λ_2, once f is know, by the relation

$$\frac{\lambda_1}{\lambda_2}=\frac{1-f}{f}.$$

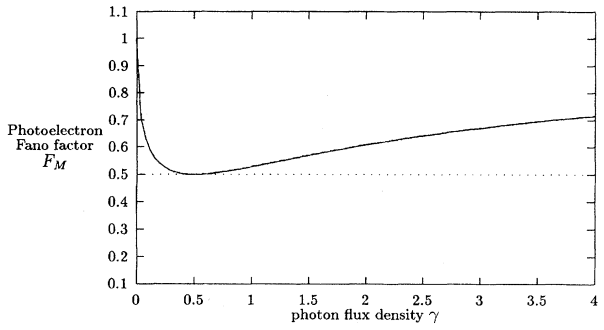

FIG. 1. The variance to mean ratio $F_M\equiv\langle\Delta M^2\rangle/\langle M\rangle$ of photon electron number distribution within an arbitrary fixed time period is plotted as a function of the dimensionless incoming photon flux density $\gamma\equiv\phi/\phi^*$ in a semiconductor photodetector. When $\gamma=1/2$, the transition rates λ_1 and λ_2 are equal to one another, and the minimal F_M is achieved. Poisson process with $F_M=1$ is recovered when $\gamma\to 0$ or $\to\infty$, because one of the transition rates diverges in these cases.

Finally we recall that the Fano factor of the photoelectron generated by the photon flux is completely determined by this ratio, which is now expressed in terms of three fundamental material parameters: the spontaneous emission lifetime τ_r of the conduction electron, the intrinsic recombination rate r, and the joint density of state at illumination peak frequency $\rho_J(\omega^*)$. In Fig. 1, we plot the Fano factor F_M against the dimensionless photon flux density γ. The maximal noise reduction $F_M=1/2$ is achieved when $\gamma=1/2$.

In conclusion, the photoelectron statistics of a photodetector with finite relaxation time is derived. It is shown that the variance to mean ratio of photoelectron counts during an arbitrary time period can be significantly reduced below the standard quantum limit when the light absorption is saturated. In this paper we assumed, however, that the second order coherence time T_c is much smaller than the mean temporal separation between two successive photoelectron counts. In other words, we consider the cases where Poisson statistics would be obtained if the relaxation is infinitely fast. This restriction allows us to treat the incoming light as a classical plane wave implicitly in the calculation of various transition rates. A full quantum theory that deals with the effect of projections of the quantum state of light caused by the measurement processes demands further investigations.

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