JOINT SOURCE/RELAY PRECODERS DESIGN IN AMPLIFY-AND-FORWARD RELAY SYSTEMS: A GEOMETRIC MEAN DECOMPOSITION APPROACH

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ABSTRACT

Existing precoder designs for an amplify-and-forward (AF) cooperative system often assume a linear receiver at the destination, and a precoder at the relay. The performance enhancement of such a system is then limited. In this paper, we consider a nonlinear successive interference cancellation (SIC) receiver, and at the same time take the source precoder into consideration. Using the geometric mean decomposition (GMD), we propose a joint source/relay precoders design method, fully exploring information provided by direct and relay links. With our method, the design problem can be transformed to a standard scalar concave optimization problem, and a closed-form solution can be obtained. Simulations show that the proposed design can significantly enhance the performance of a MIMO AF cooperative system.

Index Terms— Amplify-and-forward (AF), successive interference cancellation (SIC), multi-input-multi-output (MIMO), geometric mean decomposition (GMD)

1. INTRODUCTION

Recently, the amplify-and-forward (AF)-based Multi-input-multi-output (MIMO) cooperative communication (CC) system was proposed in [1]-[4]. With the aid of channel state information (CSI), the precoder can then be designed and applied, either for capacity enhancement [1], [2], or for link quality improvement [3], [4]. For analysis simplicity, these works only consider the design of the relay precoder. The works in [1], [3] and [4] even ignore the transmission of the direct link (channel link from source to destination). In addition, the receiver in the destination is assumed to be linear. To the best of our knowledge, the joint source/relay precoders design for AF-based MIMO-CC systems has not been reported in the literatures. Also, nonlinear receivers at the destination have not been addressed either.

In this paper, we aim to propose a joint source/relay precoders design for a QR successive interference cancellation (QR-SIC) receiver. It is well known that when the QR-SIC receiver is adopted, the precoder design using the geometric mean decomposition (GMD) technique in the conventional MIMO system [5], [6] is asymptotically optimal. This motivates us to consider the application of the GMD technique in our design. Given a channel matrix, one can use the GMD method to derive a precoder making the diagonal elements of the corresponding **R** matrix equal. However, unlike the conventional MIMO systems, the equivalent channel matrix in an AF-CC system now is a function of the relay precoder, so is the **R** matrix. Using the GMD approach, we can first derive the source precoder, and reduce the joint design problem to a relay precoder design problem. However, the optimization involves a highly nonlinear function, and a direct solution is difficult to obtain. We then propose a method simplifying the problem as a standard scalar concave optimization problem. With our method, a closed-form solution can be obtained. Simulation shows that the proposed scheme can significantly improve the BER performance as compared to existing schemes.

2. PROPOSED SYSTEM MODEL AND PROBLEM FORMULATION

2.1. Precoders for AF system and QR-SIC receiver

We consider a simple three-node cooperative MIMO AF system. Under this scenario, signals can be transmitted from the source to the destination (direct link), and from the source to the relay, and then the relay to the destination (relay link) [1], [2]. Let N, R, and M denote the number of antennas at the source, the relay, and the destination, respectively. Also, let all channels be flat-fading. The signals received from the source and the relay (at the destination) can be combined into a vector form as [1], [2]:

$$\mathbf{y}_D := \underbrace{\begin{bmatrix} \mathbf{H}_{SD} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix}}_{:=\mathbf{H}} \mathbf{F}_S\mathbf{s} + \underbrace{\begin{bmatrix} \mathbf{n}_{D,1} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2} \end{bmatrix}}_{:=\mathbf{n}} = \mathbf{H}\mathbf{F}_S\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{F}_R \in \mathbb{C}^{R\times R}$ is the relay precoder matrix; $\mathbf{H}_{SR} \in \mathbb{C}^{R\times N}$, $\mathbf{H}_{SD} \in \mathbb{C}^{M\times N}$, and $\mathbf{H}_{RD} \in \mathbb{C}^{M\times R}$ are the channel matrices between the source and the relay, the source and the destination, the relay and the destination, respectively; $\mathbf{n}_{D,1} \in \mathbb{C}^{M\times 1}$, $\mathbf{n}_{D,2} \in \mathbb{C}^{M\times 1}$, and $\mathbf{n}_R \in \mathbb{C}^{R\times 1}$ are the noise vector received at the destination in the first-phase, that at the destination in the second-phase, and that at the relay in the first-phase, respectively. Here, we assume that $L = N \le R, M$, and $\mathbf{R}_{\mathbf{n}_{D,1}} = E\left[\mathbf{n}_{D,1}\mathbf{n}_{D,1}^H\right] = \sigma_n^2\mathbf{I}_M$, $\mathbf{R}_{\mathbf{n}_{D,2}} = E\left[\mathbf{n}_{D,2}\mathbf{n}_{D,2}^H\right] = \sigma_n^2\mathbf{I}_M$, and $\mathbf{R}_R = E\left[\mathbf{n}_R\mathbf{n}_R^H\right] = \sigma_n^2\mathbf{I}_R$, where σ_n^2 is a noise variance. Also, the elements of the signal vectors are i.i.d. with a zero-mean and a covariance matrix $\mathbf{R}_s = \sigma_s^2\mathbf{I}_L$, where σ_s^2 is the power transmitted on a symbol. With the above assumptions, the covariance matrix of the equivalent noise vector is given by

$$\mathbf{R}_n = E\left[\mathbf{n}\mathbf{n}^H\right] = \begin{bmatrix} \sigma_n^2\mathbf{I}_M & \mathbf{0} \\ \mathbf{0} & \sigma_n^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_n^2\mathbf{I}_M \end{bmatrix}. \quad (2)$$

Note that the equivalent noise vector is not white. To facilitate later analysis of QR-SIC receiver, we first apply a whitening operation to the receive vector. Let $\mathbf{W}$ be a whitening matrix. From (1), we can have

$$\tilde{\mathbf{y}}_D \coloneqq \mathbf{W}\mathbf{y}_D = \mathbf{W}\mathbf{H}\mathbf{F}_S\mathbf{s} + \mathbf{W}\mathbf{n} = \tilde{\mathbf{H}}\mathbf{F}_S\mathbf{s} + \tilde{\mathbf{n}}\,, \quad (3)$$

where $\tilde{\mathbf{H}} = \mathbf{W}\mathbf{H}$ and $\tilde{\mathbf{n}} = \mathbf{W}\mathbf{n}$. Due to the whitening, we have $E\left[\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H\right] = E\left[\mathbf{W}\mathbf{n}\mathbf{n}^H\mathbf{W}^H\right] = \sigma_n^2\mathbf{I}_{2M}$. From (2) in (3), we can then obtain the whitening matrix as

$$\mathbf{W} = \begin{bmatrix} \mathbf{I}_M & 0 \\ 0 & \left(\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \mathbf{I}_M\right)^{-1/2} \end{bmatrix}. \quad (4)$$

The equivalent channel matrix after the whitening process can be reformulated as

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H}_{SD} \\ \left(\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \mathbf{I}_M\right)^{-\frac{1}{2}}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix} \quad (5)$$

From (3), we can see that an AF-CC system can be seen as a MIMO system with the channel matrix defined in (5). However, note that the effective channel matrix in (5) is a function of the relay precoder, and this is quite different from the scenario considered in MIMO systems. Since $\mathbf{F}_R$ is unknown, $\mathbf{F}_S$ is not directly solvable when existing precoder design methods are applied.

It is well-known that nonlinear MIMO receivers can have better performance though their complexity may be higher. In this paper, we mainly consider the QR-SIC receiver. In such an approach, the equivalent channel of the precoded system is first represented by the QR decomposition, i.e., $\tilde{\mathbf{H}}\mathbf{F}_S = \mathbf{Q}\mathbf{R}$, where $\mathbf{Q}$ is a $2M \times 2M$ orthogonal matrix, and $\mathbf{R}$ is a $2M \times N$ upper triangular matrix. Equation (3) can then be rewritten as

$$\hat{\mathbf{y}}_D = \mathbf{Q}^H\tilde{\mathbf{y}}_D = \mathbf{Q}^H\mathbf{Q}\mathbf{R}\mathbf{s} + \mathbf{Q}^H\tilde{\mathbf{n}} = \mathbf{R}\mathbf{s} + \hat{\mathbf{n}}\,. \quad (6)$$

Thus, the signal can be detected via a standard QR-SIC procedure.

2.2 Problem formulation

With the QR-SIC as the receiver, [5] and [6] propose a precoder design method such that diagonal elements of $\mathbf{R}$ in (6) can be made equal. This method is referred to as the geometric mean decomposition (GMD). It has been shown that [6] the GMD can minimize the block error rate (BLER), and also maximize the lower bound of channel's free distance. In [5], the GMD detector was proved to be asymptotically optimal for high SNR, in terms of both channel throughput and bit error rate (BER) performance.

Due to its optimality, we then adopt the GMD method in our design. Let $\tilde{\mathbf{H}}$ have a full rank, i.e., $rank(\tilde{\mathbf{H}}) = N$. It was shown in [5] and [6] that $\tilde{\mathbf{H}}$ can be decomposed as

$$\tilde{\mathbf{H}} = \tilde{\mathbf{Q}}\tilde{\mathbf{R}}\tilde{\mathbf{P}}^H\,, \quad (8)$$

where $\tilde{\mathbf{Q}} \in \mathbb{C}^{2M\times 2M}$ and $\tilde{\mathbf{P}} \in \mathbb{C}^{N\times N}$ are unitary matrices; the upper triangular matrix $\tilde{\mathbf{R}} \in \mathbb{C}^{2M\times N}$ has identical diagonal elements given by

$$\tilde{r}_{i,i} = \left(\prod_{k=1}^{N}\sigma_{\tilde{\mathbf{H}},k}\right)^{1/N}, \text{ for all } i = 1,\cdots,N\,, \quad (9)$$

where $\tilde{r}_{i,i}$ is the ith diagonal element in $\tilde{\mathbf{R}}$, and $\sigma_{\tilde{\mathbf{H}},k} > 0$ is the k th singular value of $\tilde{\mathbf{H}}$. The precoder (at the source) in the GMD method is then determined as

$$\mathbf{F}_S = \alpha\tilde{\mathbf{P}}\,, \quad (10)$$

where α is a scalar designed to satisfy the power constraint, i.e., $tr\left(\mathbf{F}_S E\left(\mathbf{s}\mathbf{s}^H\right)\mathbf{F}_S^H\right) = \sigma_s^2 N\alpha^2 \le P_{S,T}$. Here, $P_{S,T}$ is the maximal available power at the source. Thus, our design problem can then be formulated as

$$\max_{\mathbf{F}_S,\mathbf{F}_R} r_{i,i} = \alpha\tilde{r}_{ii} = \alpha\left(\prod_{k=1}^{N}\sigma_{\tilde{\mathbf{H}},k}\right)^{1/N} \quad s.t.$$
$$\mathbf{F}_S = \alpha\tilde{\mathbf{P}},$$
$$tr\left(\sigma_s^2\mathbf{F}_S\mathbf{F}_S^H\right) \le P_{S,T}, \quad tr\left(\mathbf{F}_R E\left[\mathbf{y}_R\mathbf{y}_R^H\right]\mathbf{F}_R^H\right) = , \quad (11)$$
$$tr\left(\mathbf{F}_R\left(\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H + \sigma_n^2\mathbf{I}_R\right)\mathbf{F}_R^H\right) \le P_{R,T}$$

where $P_{R,T}$ is the maximal available power at the relay. Note here that the cost function in (11) relates to singular values $\sigma_{\tilde{\mathbf{H}},i}$, $i = 1,\cdots,N$, of $\tilde{\mathbf{H}}$ which is a complicated nonlinear function of the relay precoder $\mathbf{F}_R$, as shown in (5). A direct maximization of (11) is then difficult. In the next section, we will propose an effective method to solve the precoders $\mathbf{F}_S$ and $\mathbf{F}_R$.

3. PROSED JOINT SOURCE/RELAY PRECODERS DESIGN

3.1. Proposed method

Taking a close look at (11), we see that the optimum $\mathbf{F}_S$ at source is actually easy to obtain. From the first two constraints, we can obtain the optimum source precoder, denoted by $\mathbf{F}_S^*$, as

$$\mathbf{F}_S^* = \sqrt{\frac{P_{S,T}}{\sigma_s^2 N}}\tilde{\mathbf{P}}. \tag{12}$$

Alternatively, the optimum $\mathbf{F}_R$, however, is much more difficult to obtain. Substituting $\mathbf{F}_S^*$ into (11), the joint design problem can then be simplified to a relay precoder design problem, as shown below:

$$\begin{aligned} &\max_{\mathbf{F}_R} \sqrt{\frac{P_{S,T}}{\sigma_s^2 N}}\left(\prod_{k=1}^{N}\sigma_{\tilde{\mathbf{H}},k}\right)^{1/N} \quad s.t. \\ &tr\left(\mathbf{F}_R\left(\sqrt{\frac{P_{S,T}}{N}}\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \sigma_n^2\mathbf{I}_R\right)\mathbf{F}_R^H\right) \le P_{R,T} \end{aligned}, \tag{13}$$

Since singular values of $\tilde{\mathbf{H}}$ are involved, a direct maximization of (13) may be difficult. We then propose an alternative cost function having the same optimum precoder $\mathbf{F}_R^*$, i.e.,

$$\mathbf{F}_R^* = \arg\max_{\mathbf{F}_R} \sqrt{\frac{P_{S,T}}{\sigma_s^2 N}}\left(\prod_{k=1}^{N}\sigma_{\tilde{\mathbf{H}},k}\right)^{1/N}, \tag{14}$$

$$= \arg\max_{\mathbf{F}_R}\left(\prod_{k=1}^{N}\sigma_{\tilde{\mathbf{H}},k}\right)^{2}, \tag{15}$$

$$= \arg\max_{\mathbf{F}_R} \det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right), \tag{16}$$

where

$$\begin{aligned}\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} = \Big[&\mathbf{H}_{SD}^H\mathbf{H}_{SD} + \mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H \times \\ &\left(\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\Big]\end{aligned}. \tag{17}$$

The equality in (15) is due to the cost functions monotonically increasing property in $\prod_{k=1}^{N}\sigma_{\tilde{\mathbf{H}},k}$; (16) follows $\left(\prod_{k=1}^{N}\sigma_{\tilde{\mathbf{H}},k}\right)^2 = \prod_{k=1}^{N}\lambda_{\tilde{\mathbf{H}}^H\tilde{\mathbf{H}},k} = \det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right)$, where $\lambda_{\tilde{\mathbf{H}}^H\tilde{\mathbf{H}},i}$ is the ith eigenvalue of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$. With the cost function in (16), the solution becomes easier to work with. The following lemma gives a hint regarding how (16) can be solved.

Lemma 1: Let $\mathbf{M} \in \mathbb{C}^{N\times N}$ be a positive definite matrix, and $\mathbf{M}(i,j)$ be its ijth entry. Then, we have

$$\det(\mathbf{M}) \le \prod_{i=1}^{N}\mathbf{M}(i.i). \tag{18}$$

The equality in (18) holds when $\mathbf{M}$ is a diagonal matrix [7]. It turns out that when $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ is diagonalized, the cost function in (16) is then maximized. Unfortunately, from (17) we can see that $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ is a summation of two separated matrices and one of them does not depend on $\mathbf{F}_R$, and the diagonalization is still difficult to conduct. The following lemma suggests a feasible way to overcome the problem.

Lemma 2: Let $\mathbf{A} \in \mathbb{C}^{N\times N}$ and $\mathbf{B} \in \mathbb{C}^{N\times N}$ are two positive definite matrices, then [7]

$$\det(\mathbf{A}+\mathbf{B}) = \det(\mathbf{A})\det(\mathbf{I}_N + \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}). \tag{19}$$

From (16) and (19), we can see that if we let $\mathbf{A} = \mathbf{H}_{SD}^H\mathbf{H}_{SD}$ and $\mathbf{B} = \mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}$, we will have

$$\begin{aligned}\mathbf{F}_R^* &= \arg\max_{\mathbf{F}_R}\det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right) \\ &= \arg\max_{\mathbf{F}_R}\det(\mathbf{I}_N + \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}),\end{aligned} \tag{20}$$

where $\det(\mathbf{A})$ is ignored since it is not a function of $\mathbf{F}_R$. From (20), we see that as long as $\mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}$ is diagonalized, $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ will be diagonalized. This suggests a precoder structure as described in next subsection.

3.2 Optimal relay precoder design

Now, the optimization in (13) can be restated as follows

$$\begin{aligned}&\max_{\mathbf{F}_R}\det(\mathbf{M}) \\ &\text{where } \mathbf{M} = \Big(\mathbf{I}_N + \left(\mathbf{H}_{SD}^H\mathbf{H}_{SD}\right)^{-1/2}\mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H \times \\ &\left(\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\left(\mathbf{H}_{SD}^H\mathbf{H}_{SD}\right)^{-1/2}\Big)\end{aligned} \tag{21}$$

$$\begin{aligned}&s.t. \quad \mathbf{M} \text{ is diagonal and} \\ &tr\left(\mathbf{F}_R\left(\frac{P_{S,T}}{N}\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \sigma_n^2\mathbf{I}_R\right)\mathbf{F}_R^H\right) \le P_{R,T}.\end{aligned}$$

The diagonalization requirement motivates us to consider the following singular value decomposition (SVD)

$$\mathbf{H}_{RD} = \mathbf{U}_{rd}\Sigma_{rd}\mathbf{V}_{rd}^H; \tag{22}$$

$$\mathbf{H}'_{SR} := \mathbf{H}_{SR}\left(\mathbf{H}_{SD}^H\mathbf{H}_{SD}\right)^{-1/2} = \mathbf{U}'_{sr}\Sigma'_{sr}\mathbf{V}'^H_{sr}, \tag{23}$$

where $\mathbf{U}_{rd} \in \mathbb{C}^{M\times M}$ and $\mathbf{U}'_{sr} \in \mathbb{C}^{R\times R}$ are the left singular vectors of $\mathbf{H}_{RD}$ and $\mathbf{H}'_{SR}$, respectively; $\Sigma_{rd} \in \mathbb{R}^{M\times R}$ and $\Sigma'_{sr} \in \mathbb{R}^{R\times N}$ are the diagonal singular-value matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}'_{SR}$, respectively; $\mathbf{V}_{rd}^H \in \mathbb{C}^{R\times R}$ and $\mathbf{V}'^H_{sr} \in \mathbb{C}^{N\times N}$ are the right singular vector matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}'_{SR}$, respectively. To have a full diagonalization of $\mathbf{M}$, it turns out that the optimum $\mathbf{F}_R^*$ have the following structure

$$\mathbf{F}_R = \mathbf{V}_{rd}\Sigma_r \mathbf{U}'^{H}_{sr} \,, \tag{24}$$

where Σ_r is a diagonal matrix with its *i*th diagonal element, $\sigma_{r,i}$, yet to be determined. Let $\sigma_{rd,i}$ and $\sigma'_{sr,i}$ be the *i*th diagonal element of Σ_{rd} and Σ'_{sr} , respectively. Substituting (22), (23) and (24) into (21) and taking the log operation to the cost function, we can rewrite (21) as:

$$\max_{p_{r,i},\ 1\le i\le N} \sum_{i=1}^{N} \ln\left(1+\frac{p_{r,i}\sigma^2_{rd,i}\sigma^2_{sr,i}}{p_{r,i}\sigma^2_{rd,i}+1}\right)$$
$$s.t. \tag{25}$$
$$\sum_{i=1}^{N} p_{r,i}\left(\frac{P_{S,T}}{N}\sigma'^2_{sr,i}\mathbf{D}'_{sr}(i,i)+\sigma_n^2\right) \le P_{R,T},\ p_{r,i}\ge 0.$$

where $p_{r,i}=\sigma^2_{r,i}$ and $\mathbf{D}'_{sr}=\mathbf{V}'^{H}_{sr}\left(\mathbf{H}^H_{SD}\mathbf{H}_{SD}\right)\mathbf{V}'_{sr}$. The cost function now is reduced to a function of scalars. Since the cost function and the inequality constraints are all concave [8], (25) becomes a standard concave optimization problem. As a result, the optimal solutions $p_{r,i}\,, i=1,\cdots,N$, can be solved by means of Karush-Kuhn-Tucker (KKT) conditions. After some tedious derivations, we can obtain

$$p_{r,i}=\left[\sqrt{\frac{\mu}{\sigma^2_{rd,i}\left(\frac{P_{S,T}}{N}\sigma'^2_{sr,i}\mathbf{D}'_{sr}(i,i)+\sigma_n^2\right)\left(\sigma'^{-2}_{sr,i}+1\right)}+\frac{1}{4\sigma^4_{rd,i}\left(\sigma'^{-2}_{sr,i}+1\right)^2}}-\frac{1+\frac{1}{2}\sigma'^2_{sr,i}}{\sigma^2_{rd,i}\left(1+\sigma'^2_{sr,i}\right)}\right]^+ , \tag{26}$$

where $[y]^+=\max(0,y)$ and μ is chosen to satisfy the power constraint in (25). Substituting (26) into (24), we can then obtain the optimum relay precoder. Finally, substituting (24) into (5) and conducting the decomposition in (8), we can then obtain the optimum source precoder via (12).

3. SIMULATIONS AND CONCLUSIONS

We consider *N*=*R*=*M*=4 case. Assume that channel state information (CSI) of all links are known at all nodes, and perfect synchronization can be achieved. Furthermore, the elements in each channel matrix are assumed to be i.i.d. complex Gaussian random variables with a zero mean and a same variance. We let the received SNR at each antenna of the relay in the first-phase, and that at each antenna of the destination in the second phase be $15\ dB$, and vary the received SNR at each antenna of the destination in the first-phase. Also, the modulation scheme is QPSK.

Fig. 1 shows the BER comparison for three un-precoded receiver schemes, the optimal relay precoder with MMSE receiver [4], and for two precoded QR-SIC receivers which are the conventional GMD precoder [5], [6] and the proposed precoding scheme. As shown in the figure, the relay-only precoded systems outperform the un-precoded ones (except for MMSE-OSIC). This is because the amplified signal from the relay can somewhat benefit the receiver. The proposed scheme has significant performance improvement compared to the other schemes. Particularly, it outperforms the conventional GMD approach since the proposed method not only makes the diagonal elements of **R** in (6) equal but also maximizes the values, yielding a higher received SNR for each transmitted symbol stream.

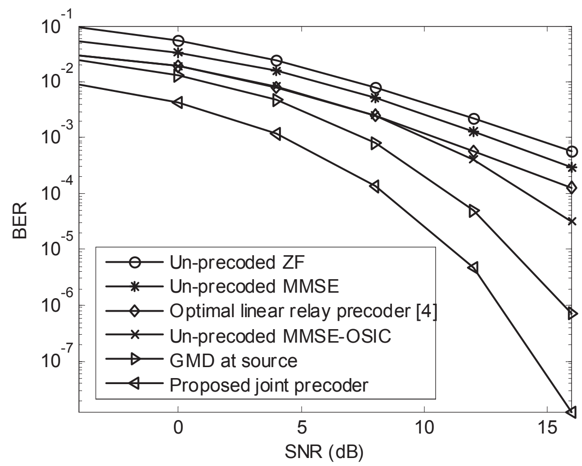

Fig. 1. BER performance for proposed method and other schemes (*N*=*R*=*M*=4).

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