Magnetic force acting on a magnetic dipole over a superconducting thin film

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The magnetostatic interaction energy and corresponding magnetic force acting on a magnetic point dipole placed above a type-II thin superconducting film in the mixed state with a single vortex are calculated using electromagnetics coupled with the London theory of superconductivity. If a vortex is trapped by a circular defect of radius $b \ll \Lambda$, the magnetic forces, caused by the vortex, differ from the results of free from defect pinning by the factor $(1-b/\Lambda)$, where Λ is the effective penetration depth. The possibility of formation of the vortex in the thin film only in the field of the magnetic point dipole is investigated. The critical position of the dipole for creating the first vortex under the electromagnetic pinning of a circular defect and that position in the absence of defect pinning are obtained for comparison. In particular, in the limit of $a/\Lambda \gg 1$, where a is the separation between the dipole and the thin film, the only difference between two results is in the cutoff length, i.e., in the case of a circular defect the only difference in the critical position calculation is the cutoff at radius b rather than at coherence length ξ. The pinning force of a single vortex by a circular defect is also calculated. Further, we investigate the conditions of the vortex creation for various cases (including the first, second, and third vortices) for a free of pinning center in the examining region. It is found that the creation of a new single vortex in the thin film causes an abrupt change in vertical levitation force: the force changed discontinuously. [S0163-1829(96)06946-9]

I. INTRODUCTION

The phenomenon of superconducting levitation has always attracted wide attention. The high-T_c oxide superconductor has made it easy to directly observe the magnetic levitation and demonstrated tremendous potential for several fascinating applications such as magnetic bearing, magnetic levitation, magnetic suspension, and other magnetic force related devices.[1–4] Concerning the microscopic feature, the magnetic force microscope[5] (MFM) has been recognized as a powerful noncontact probe method, which originally developed on the basis of atomic force microscopy (AFM).[6] The performance of scanning MFM has been applied to investigate small magnetic structures.[7–9] The MFM image of high-T_c superconductors is now possible,[10] especially to investigate the flux line structure below the transition temperature. A MFM consists of a tiny magnetic tip which interacts with a stray magnetic field of the sample. The force on the scanned tip is measured as a function of the tip position. In a superconducting levitation system, most researchers[11–15] calculated the magnetostatic interaction between a magnetic tip or a point dipole and a type-II superconductor in the Meissner state, based on the London theory. Recently, Xu *et al.*[16] calculated the levitation force acting on a magnet over a semi-infinite type-II superconductor with different pairing symmetries in both Meissner and mixed states to study the pairing symmetry of superconductors. Coffey[17,18] also examined a point dipole above a semi-infinite type-II superconductor in both the Meissner and mixed states based upon axisymmetric models of London theory. Moreover, for stratified type-II superconductors of interest to MFM, the static vortex solutions for both straight line and pancake vortex have been derived[19] by using Fourier transformation and convolution. However, none of these studies correlated the creation of the vortex due to the field of the dipole or magnetic tip and the effect of the pinning center with the magnetic force behaviors, while such correlation could be of practical importance to design MFM systems for experiment and understand physical properties of superconductors.

To obtain further insight into the interaction of a magnetic point dipole over a type-II superconductor in the mixed state and only one vortex in the superconductor, we investigate the interaction energy between a point dipole and the screening currents (or self-interaction energy[11,12,14,16]), $U_{\rm in}$, and the interaction energy between a point dipole and a vortex line, U_v. We find $U_{\rm in}$ can be written in the form

$$U_{\rm in} = -\frac{1}{2}\,\mathbf{m}\cdot\mathbf{B}_{\rm in}, \tag{1.1}$$

where $\mathbf{m}$ is the magnetic moment of the dipole and is located at the origin. $\mathbf{B}_{\rm in}$ is the induced field caused by the screening currents in the superconductor. U_v can be written in the form

$$U_v = -\mathbf{m}\cdot\mathbf{B}_v, \tag{1.2}$$

where $\mathbf{B}_v$ is the magnetic stray field of the vortex. This expression of U_v differs from the expression for $U_{\rm in}$ by a factor 2. The difference between the self-interaction energy $U_{\rm in}$ and the interaction energy U_v comes from the fact that $U_{\rm in}$ is the result of an indirect self-interaction between the magnetic moment and its image.[20] U_v typically represents the work done in establishing the permanent magnetic moment in the magnetic field, which is induced by the vortex. The magnetic force acting on the dipole tip then can be expressed as the negative of the gradient of the interaction energy. The force includes two parts: one is due to the screening current caused by the dipole itself, and the other is due to the presence of the vortex.

The purpose of this study is to theoretically investigate the magnetic levitation force acting on a dipole tip in the presence of a single vortex in the superconducting thin film. The approach is based on the London theory. We also take into account the pinning effect of a circular defect and the vortex creation in the thin film on the process to study the change of the magnetic force acting between the dipole and the vortex. For calculating the free energy of the system, we skipped the self-energy of a point dipole because it is a divergent term.

In Sec. III, consider a single vortex in the thin film; the magnetic force acting on the magnetic dipole is the effect of the superposition of the magnetic field due to the source of the vortex and the screening current in the thin film. We find the vertical force acting on the dipole was contributed by the induced screen current and the vortex. Only the effect of a vortex contributes to the lateral force. We also investigate the influence of electromagnetic pinning of a circular defect on the magnetic force. If the defect radius is larger than the superconducting coherence length ξ, then not only is the vortex energy changed, but the vector potential due to the vortex will also be different. We hope to capture some of the essential properties of microscopic levitation in this MFM field. In Sec. IV we extend our treatment to investigate the situations of vortex creation in the film under the field of the magnetic dipole when the dipole approaches the film surface. This is an attempt to relate the magnetic force acting on the dipole to the vortex creation and the pinning effect. With regard to a single quantum flux creation in the film, we use the critical condition of the system transition before and after the formation of the first vortex equals in free energy to decide the critical position of the dipole, the critical position at which a single quantum flux forms in the film. The change of the magnetic forces after the creation of the first vortex essentially depends on the existence of the pinning effect. We then study the creation of the vortex for a free of pinning center. The conditions of critical positions of the dipole for creating the vortex in the thin film and the equilibrium distances between vortices for a various number of vortex created in sequence in the film are expressed. The equilibrium position of the vortex can be determined by the condition of force equilibrium. Finally, Sec. V includes discussions and conclusions.

II. CALCULATION OF THE INTERACTION ENERGY

Let us first consider a magnetic point dipole with a moment $\mathbf{m}$ placed outside a type-II superconductor in the Meissner state. In London approximation, the free energy of this system is

$$U = \frac{1}{2\mu_0}\int \mathbf{B}^2 dv_1 + \frac{1}{2\mu_0}\int [\mathbf{B}^2 + \lambda^2|\nabla\times\mathbf{B}|^2] dv_2, \tag{2.1}$$

where μ_0 is the vacuum permeability and λ is the London penetration depth. The integral of the first term is taken out of the superconductor except for the volume of the dipole and the integral of the second term is taken over the sample volume. We transform Eq. (2.1) into a surface integral. Using the London equation and divergence theorem,[21,22] we obtain

$$U = \frac{1}{2\mu_0}\int_{\text{(dipole and supercond)}} [\mathbf{A}\times\mathbf{B}]\cdot d\sigma, \tag{2.2}$$

where $\mathbf{A}$ is the vector potential and $\mathbf{B} = \nabla\times\mathbf{A}$. The solution $\mathbf{A}$ and $\mathbf{B}$ can be written as

$$\mathbf{A} = \mathbf{A}_{\rm dip} + \mathbf{A}_{\rm in}, \quad \mathbf{B} = \mathbf{B}_{\rm dip} + \mathbf{B}_{\rm in}, \tag{2.3}$$

where $\mathbf{A}_{\rm dip}$ is the vector potential directly contributed from the dipole, and $\mathbf{A}_{\rm in}$ is the induced vector potential due to the screening current of the superconductor. $\mathbf{B}_{\rm dip}$ and $\mathbf{B}_{\rm in}$ are magnetic fields corresponding to the vector potentials $\mathbf{A}_{\rm dip}$ and $\mathbf{A}_{\rm in}$. The surface integral includes the surface of the dipole and the surface (inner and outer surface) of the superconductor. The boundary conditions of $\mathbf{A}$ and $\mathbf{B}$ are continuous at the sample surfaces, but the inner surface and outer surface are facing in opposite directions; hence the integral on the surface of the superconducting sample vanishes. Equation (2.2) becomes

$$U = \frac{1}{2\mu_0}\int_{\text{(dipole)}} (\mathbf{A}_{\rm dip}\times\mathbf{B}_{\rm dip} + 2\mathbf{A}_{\rm dip}\times\mathbf{B}_{\rm in} + \mathbf{A}_{\rm in}\times\mathbf{B}_{\rm in})\cdot \mathbf{d}\sigma. \tag{2.4}$$

The surface integral $\int_{\rm dipole}\mathbf{d}\sigma$ is taken in the limit the radius of the dipole $r\rightarrow 0$. The first term represents the self-energy of the dipole $U_{\text{self}(d)}$. The surface integral of the last term vanishes when $r\rightarrow 0$. If the dipole is at the origin of the coordinate system, we rewrite Eq. (2.4) as

$$U = U_{\text{self}(d)} + \lim_{r\rightarrow 0}\frac{-1}{4\pi}\int\left[\frac{\mathbf{m}\times\mathbf{r}}{r^3}\times\mathbf{B}_{\rm in}(r)\right]\cdot\hat{r}r^2 d\Omega, \tag{2.5}$$

where $\hat{r}$ is a unit vector in the r direction and $d\Omega$ is the element of solid angle subtended at the origin by the area element $d\sigma$. The second term of Eq. (2.5) gives the interaction energy between the screening current of the superconductor and the dipole, $U_{\rm in}$,

$$U_{\rm in} = -\frac{1}{2}\,\mathbf{m}\cdot\mathbf{B}_{\rm in}(0). \tag{2.6}$$

Next we introduce a vortex into this system. A single vortex embedded in the type-II superconductor. It is easy to

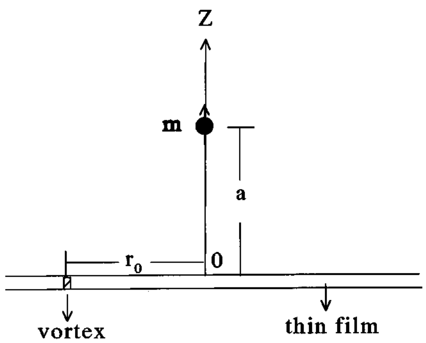

FIG. 1. The diagram of a magnetic point dipole (**m**) placed above a thin superconducting film, and a single vortex embedded in it.

find the interaction energy between the vortex and the dipole. The vortex line produces a magnetic field $\mathbf{B}_v$. The magnetic dipole interacts with the stray magnetic field $\mathbf{B}_v$ which emerges from the superconducting sample. Thus the magnetic interaction energy of the permanent magnetic moment in the field of the vortex, U_v, is

$$U_v = -\mathbf{m}\cdot\mathbf{B}_v(0). \tag{2.7}$$

Finally, the total interaction energy is the superposition of Eqs. (2.6) and (2.7):

$$U_{\text{int}} = U_{\text{in}} + U_v,$$
$$= -\frac{1}{2}\,\mathbf{m}\cdot\mathbf{B}_{\text{in}} - \mathbf{m}\cdot\mathbf{B}_v. \tag{2.8}$$

III. INTERACTION BETWEEN A POINT DIPOLE WITH A SINGLE VORTEX IN A THIN FILM

We consider a single vortex embedded in an infinite type-II thin superconducting film and directed perpendicularly to its surface. The basic dimensionless parameter, which is assumed to be small, is the ratio of the thickness of the film d to the penetration depth λ. A magnetic dipole with moment **m** normal to the plane of the thin film is placed at a distance a above the film surface, as shown in Fig. 1. The film lies in the x-y plane with the vortex located at distance r_0 from the origin of the coordinate system. It can be analyzed by using the London theory of vortex in the superconductor, assuming the radius of the vortex core is very small. From Maxwell's and London's equations, the vector potential **A** can be expressed as

$$\nabla\times\nabla\times\mathbf{A} = -\mu_0\mathbf{m}\times\nabla[\delta(r)\delta(z-a)], \quad z>\frac{d}{2}, \tag{3.1}$$

$$\nabla\times\nabla\times\mathbf{A} + \frac{\mathbf{A}}{\lambda^2} = \frac{\phi_0}{2\pi\lambda^2}\frac{1}{|\mathbf{r}-\mathbf{r}_0|}\,\hat{\theta}', \quad |z|\leqslant\frac{d}{2}, \tag{3.2}$$

$$\nabla\times\nabla\times\mathbf{A} = 0, \quad z<-\frac{d}{2}, \tag{3.3}$$

where ϕ_0 is the flux quantum $h/2e$ and $\hat{\theta}'$ is a unit vector in the θ' direction when the origin is located at the center of the vortex.

By the superposition principle, the vector potential **A** is the superposition of the vector potential $\mathbf{A}_1$ and $\mathbf{A}_2$ due to the source of the vortex and the magnetic dipole, respectively. Now, we solve the vector potential $\mathbf{A}_1$ and $\mathbf{A}_2$ individually. In the case of $d\ll\lambda$, the vector potential $\mathbf{A}_1$ satisfies the equation[23,24]

$$\nabla\times\nabla\times\mathbf{A}_1 = \frac{d\,\delta(z)}{\lambda^2}\left[\frac{\phi_0\hat{\theta}'}{2\pi R} - \mathbf{A}_1\right] \quad \text{for } z=0, \tag{3.4}$$

$$\nabla\times\nabla\times\mathbf{A}_1 = 0 \quad \text{elsewhere} \tag{3.5}$$

where $R=\sqrt{r^2+r_0^2-2rr_0\cos\theta}$ and θ is the angle between **r** and $\mathbf{r}_0$. Using the angular symmetry of geometry, the vector potential $\mathbf{A}_1$ can be written as

$$\mathbf{A}_1 = f_1(R,z)\,\hat{\theta}', \tag{3.6}$$

which leads to an equation for the scalar function f_1

$$\frac{\partial^2 f_1}{\partial z^2} + \frac{\partial}{\partial r}\,r^{-1}\,\frac{\partial}{\partial r}\,(rf_1) = \left[\frac{2f_1}{\Lambda} - \frac{\phi_0}{\Lambda\pi r}\right]\delta(z), \tag{3.7}$$

where $\Lambda=2\lambda^2/d$ is the effective penetration depth. Equation (3.7) was solved by expanding in the first-order Bessel function

$$f_1 = \int_0^\infty dk\; J_1(kR)e^{-k|z|}\,\frac{\phi_0}{2\pi}\,\frac{1}{k\Lambda+1}. \tag{3.8}$$

Similarly, the solution of $\mathbf{A}_2$ is obtained

$$\mathbf{A}_2 = \left\{\frac{\mu_0 m}{4\pi}\,\frac{r}{[r^2+(z-a)^2]^{3/2}} - \frac{\mu_0 m}{4\pi}\int_0^\infty dk\,\frac{k}{k\Lambda+1}\,J_1(kr)e^{-k(z+a)}\right\}\hat{\theta}, \quad z>0, \tag{3.9}$$

$$\mathbf{A}_2 = \frac{\mu_0 m}{4\pi}\int_0^\infty dk\,\frac{k^2\Lambda}{k\Lambda+1}\,J_1(kr)e^{k(z-a)}\hat{\theta}, \quad z<0. \tag{3.10}$$

The first term of Eq. (3.9) comes from the dipole and the second term is the induced vector potential due to the screening current. The magnetic induction **B** can thus be calculated by taking the curl of **A**, i.e., $\mathbf{B}=\nabla\times\mathbf{A}$. The cylindrical coordinates (r,θ,z) with center on the origin are used to express **B**

$$\mathbf{B}=\frac{\mu_0 m}{4\pi}\left\{\frac{3r(z-a)}{[r^2+(z-a)^2]^{5/2}}-\int_0^\infty dk\,\frac{k^2}{k\Lambda+1}\times J_1(kr)e^{-k(z+a)}\right\}\hat{r}+\frac{\mu_0 m}{4\pi}\left\{\frac{2(z-a)^2-r^2}{[r^2+(z-a)^2]^{5/2}}-\int_0^\infty dk\,\frac{k^2}{k\Lambda+1}J_0(kr)e^{-k(z+a)}\right\}\hat{k}+\frac{\phi_0}{2\pi}\times\int_0^\infty dk\,\frac{k}{k\Lambda+1}e^{-kz}[C_r\hat{r}+C_\theta\hat{\theta}+C_k\hat{k}],\quad z>0, \tag{3.11}$$

$$\mathbf{B}=-\frac{\mu_0 m}{4\pi}\int_0^\infty dk\,\frac{k^2\Lambda}{k\Lambda+1}e^{k(z-a)}[J_1(kr)\hat{r}-J_0(kr)\hat{k}]-\frac{\phi_0}{2\pi}\int_0^\infty dk\,\frac{k}{k\Lambda+1}e^{kz}[C_r\hat{r}+C_\theta\hat{\theta}-C_k\hat{k}],\quad z<0, \tag{3.12}$$

where

$$C_r=J_1(kr)J_0(kr_0)+\sum_{n=1}^{\infty}[J_{n+1}(kr)-J_{n-1}(kr)]\times J_n(kr_0)\cos n\theta, \tag{3.13}$$

$$C_\theta=\sum_{n=1}^{\infty}[J_{n+1}(kr)+J_{n-1}(kr)]J_n(kr_0)\sin n\theta, \tag{3.14}$$

$$C_k=J_0(kr)J_0(kr_0)+2\sum_{n=1}^{\infty}J_n(kr)J_n(kr_0)\cos n\theta, \tag{3.15}$$

and $J_n(x)$ is the nth order Bessel function.

Using the result as we obtained in the Sec. II we can find the force which acts on the dipole tip. From Eqs. (2.8), (3.11), and (3.12), we get the interaction energy

$$U_{\rm int}=U_{\rm in}(a)+U_v,(a,r_0)=\frac{\mu_0 m^2}{8\pi}\int_0^\infty dk\,\frac{k^2}{k\Lambda+1}e^{-2ka}-\frac{m\phi_0}{2\pi}\int_0^\infty dk\,\frac{k}{k\Lambda+1}J_0(kr_0)e^{-ka}. \tag{3.16}$$

The first term in Eq. (3.16) represents the self-interaction energy caused by the screening current and the second term in Eq. (3.16) represents the interaction energy between the dipole and the vortex. The magnetic force components acting on the dipole can be obtained as follows:

$$F_z=-\frac{\partial U_{\rm int}}{\partial a}=\frac{\mu_0 m^2}{4\pi}\int_0^\infty dk\,\frac{k^3}{k\Lambda+1}e^{-2ka}-\frac{m\phi_0}{2\pi}\int_0^\infty dk\,\frac{k^2}{k\Lambda+1}J_0(kr_0)e^{-ka}, \tag{3.17}$$

$$F_{r_0}=-\frac{\partial U_{\rm int}}{\partial r_0}=-\frac{m\phi_0}{2\pi}\int_0^\infty dk\,\frac{k^2}{k\Lambda+1}J_1(kr_0)e^{-ka}, \tag{3.18}$$

where F_z is the vertical force (in the z direction) and F_{r_0} is the lateral force. The m^2 term in Eq. (3.17) is the component of vertical force due to the superconductor in the Meissner state, and the $m\phi_0$ term in Eq. (3.17) is the component of vertical force contributed by the vortex. In this system, the interaction force between the dipole and the screening current is repulsive, and the interaction force between the dipole and the vortex is attractive.

We consider the simplest case to examine the vertical force, i.e., the dipole is placed just above the vortex. In this case, $r_0=0$, we get $J_0(kr_0)=1$ and $J_1(kr_0)=0$. Therefore we obtain immediately from Eq. (3.17)

$$F_z=F_1+F_2,=\frac{\mu_0 m^2}{4\pi\Lambda^4}\left[\frac{\Lambda}{a}-\frac{1}{4}\left(\frac{\Lambda}{a}\right)^2+\frac{1}{4}\left(\frac{\Lambda}{a}\right)^3-e^{2a/\Lambda}E_1\left(\frac{2a}{\Lambda}\right)\right]-\frac{m\phi_0}{2\pi\Lambda^3}\left[-\left(\frac{\Lambda}{a}\right)+\left(\frac{\Lambda}{a}\right)^2+e^{a/\Lambda}E_1\left(\frac{a}{\Lambda}\right)\right], \tag{3.19}$$

where F_1 and F_2 are respectively caused by the screening current and the vortex, and $E_1(z)$ are the exponential integral. The exponential integral $E_1(z)$ are defined as follows:

$$E_1(z)=\int_z^\infty dt\,e^{-t}t^{-1}. \tag{3.20}$$

Taking two limitations in Eq. (3.19), we have

$$F_z\approx\frac{3\mu_0 m^2}{32\pi a^4}\left(1-2\frac{\Lambda}{a}\right)-\frac{m\phi_0}{\pi a^3}\left(1-3\frac{\Lambda}{a}\right)\quad\text{for } a\gg\Lambda, \tag{3.21}$$

$$F_z\approx\frac{\mu_0 m^2}{16\pi}\frac{1}{a^3\Lambda}-\frac{m\phi_0}{2\pi}\frac{1}{a^2\Lambda}\quad\text{for } a\ll\Lambda. \tag{3.22}$$

This result is similar to the results of Refs. 16 and 17 for the semi-infinite superconductor case, which is given as

$$F_z\approx\frac{3\mu_0 m^2}{32\pi a^4}\left(1-4\frac{\lambda}{a}\right)-\frac{m\phi_0}{\pi a^3}\left(1-3\frac{\lambda}{a}\right)\quad\text{for } a\gg\Lambda. \tag{3.23}$$

In Fig. 2 we present the dependence of the levitation force F_1 and F_2 on the vertical distance a (normalized in units of the effective penetration depth) for $r_0=0$. We see that these two force curves intersect at the point $a_0=0.25\ \Lambda$, which means the vertical levitation force F_z becomes equal to zero if $m=2\phi_0\Lambda/\mu_0$. Figure 3 shows the force components F_z and F_{r_0} acting on the dipole, as a function of lateral distance. The forces have been calculated at $a=\Lambda$. The value of the vertical force due to the screening current is constant because we have not considered the edge effect of the sample. This force is independent of lateral distance r_0 and can be seen from the first term on the right-side of Eq. (3.17). The vertical force, caused by the vortex, decreases with increasing lateral distance from the vortex center. Figure 4 presents the lateral force as a function of vertical distance. The lateral force has

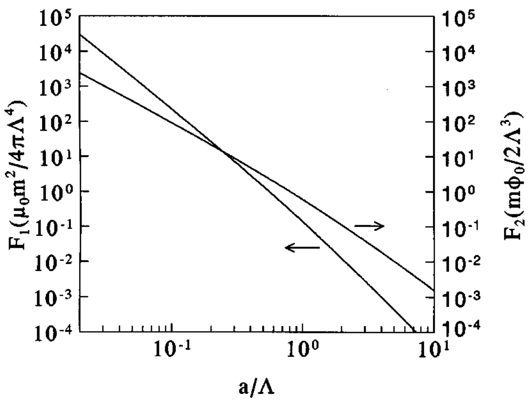

FIG. 2. The vertical force components F_1 and F_2 as a function of the vertical distance r at distance $a_0=0$. Both lines are calculated from Eq. (3.19).

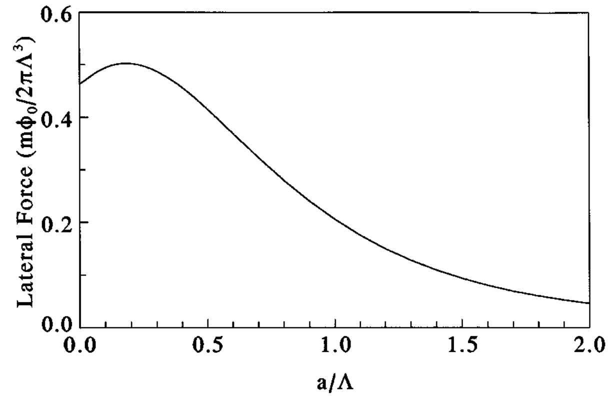

FIG. 4. The lateral force F_{r_0} as a function of the vertical distance a at distance $r_0=\Lambda$.

been calculated at $r_0=\Lambda$. The lateral force show its maximum some distance away from the center of the vortex.

It is important to know F_{r_0} since it might be possible for a vortex to become depinned. How does the lateral force compete with local pinning force? If the vortex becomes depinned when the dipole is scanning it, then the vortex would be attracted and moved near the bottom of the dipole, i.e., $r_0 \to 0$, and hence the lateral force $F_{r_0} \to 0$. The local pinning force for a vortex may be estimated by the difference of the lateral force for a vortex with and without pinning. In the limit of $r_0 \ll \Lambda$, Eq. (3.18) can be reduced to a simple form for $a \gg \Lambda$ and $a \ll \Lambda$

$$F_{r_0} = -\frac{3m\phi_0}{2\pi a^4} r_0 \left(1-4\frac{\Lambda}{a}\right) \quad \text{for } a \gg \Lambda, \tag{3.24}$$

$$F_{r_0} = -\frac{m\phi_0}{2\pi a^3}\frac{r_0}{\Lambda} \quad \text{for } a \ll \Lambda. \tag{3.25}$$

The lateral force varies linearly with the lateral distance for both situations.

If the single vortex is trapped by a pinning center, and the vortex core will coincides with the pinning center, this effect

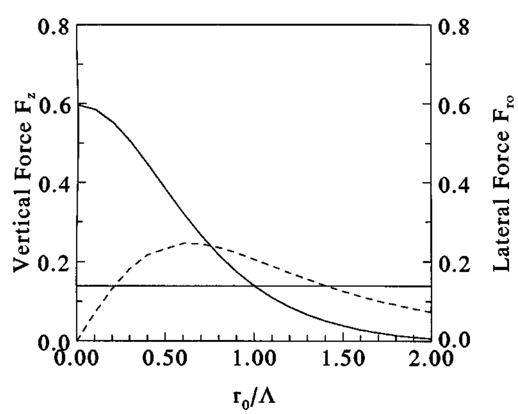

FIG. 3. The vertical force F_z and lateral force F_{r_0} as a function of the lateral distance r_0 at distance $a=\Lambda$. The curves (solid line) of the vertical force are calculated from Eq. (3.17), and the curve (dashed line) of the lateral force is calculated from Eq. (3.18). The value of the vertical force due to the screening current is constant.

would influence the magnetic force acting on the dipole. We consider the pinning center as a circular defect with radius b. We assume $b \ll \Lambda$, but larger than the coherence length ξ. Under this condition, a proper correction for the vortex energy and the vector potential should be made.

The self-energy of the vortex in the superconducting thin film is[24,25]

$$E_v = \frac{\phi_0^2}{2\pi\mu_0\Lambda}\ln\left(\frac{\Lambda}{\xi}\right). \tag{3.26}$$

In the case of the circular defect the only difference in the vortex energy calculation is the cutoff at b rather than at ξ, and the self-energy of the defect with one quanta of the magnetic flux is

$$E_{vd} = \frac{\phi_0^2}{2\pi\mu_0\Lambda}\ln\left(\frac{\Lambda}{b}\right). \tag{3.27}$$

Now we find the vector potential, $\mathbf{A}_{1d}$, due to the trapped vortex. Let $\mathbf{A}_{1d}$ have the form

$$\mathbf{A}_{1d} = \beta \mathbf{A}_1, \tag{3.28}$$

where β is the coefficient which relates to the effect of the defect. The coefficient β can be determined by considering the condition of flux quantization. The total magnetic flux trapped in the thin film (including the defect) is equal to the flux quantum Φ_0. Integrating vector potential $\mathbf{A}_{1d}$ along the circular contour of the defect in the superconducting film just as that done in Ref. 26, we have

$$\pi b^2 B_z(r=b) + \oint_{r\to\infty} \mathbf{A}_{1d}\cdot d\mathbf{l}_1 - \oint_{r\to b} \mathbf{A}_{1d}\cdot d\mathbf{l}_2 = \phi_0$$
$$\text{for } z=0, \tag{3.29}$$

where B_z is the z component of magnetic field $\mathbf{B}$ in the region of the defect core. B_z is distributed uniformly in the region of $r<b$ and $z=0$ because $\nabla\times\mathbf{B}=\mu_0\mathbf{J}_s=0$. It is convenient to use the London equation to obtain B_v and we have

$$B_z \approx \frac{\beta\phi_0}{2\pi\Lambda r} \quad \text{for } r \ll \Lambda. \tag{3.30}$$

Substituting Eqs. (3.28) and (3.30) into Eq. (3.29), we obtain

$$\beta = 1 - \frac{b}{\Lambda}. \tag{3.31}$$

The vector potential $\mathbf{A}_{1d}$

$$\mathbf{A}_{1d} = \left(1 - \frac{b}{\Lambda}\right)\mathbf{A}_1 = \frac{\phi_0}{2\pi}\left(1 - \frac{b}{\Lambda}\right)\int_0^\infty dk\, \frac{J_1(kR)e^{-k|z|}\hat{\theta}'}{k\Lambda + 1}, \tag{3.32}$$

which differs from $\mathbf{A}_1$ by the factor $(1-b/\Lambda)$. We proceed as above to see in place of the interaction energy of the second term on the right-hand side of Eq. (3.16) is the product of the original form times the factor β and similarly of magnetic force in Eq. (3.17) with relation to the vortex term and in Eq. (3.18). Since in our calculations we assume $b \ll \Lambda$, the factor β is a small amount of correction. Here we propose that the distance between the dipole and the defect is large, so the interaction between the dipole and the small defect core need not be considered.

Let us estimate the interaction force between the vortex and the defect, i.e., the pinning force. The interaction energy between the vortex and the defect, V_p, has the form[27]

$$V_p = -\frac{\phi_0^2}{2\mu_0\Lambda}\sum_{n=1}^{\infty}\gamma_n(r)\cdot\left[H_{-n}\left(\frac{r}{\Lambda}\right) - Y_{-n}\left(\frac{r}{\Lambda}\right)\right]J_{-n}\left(\frac{b}{\Lambda}\right) \quad \text{for } b \ll \Lambda, \tag{3.33}$$

and

$$\gamma_n = H_{-n}\left(\frac{r}{\Lambda}\right) - Y_{-n}\left(\frac{r}{\Lambda}\right) \Big/ H_{-n}\left(\frac{b}{\Lambda}\right) - Y_{-n}\left(\frac{b}{\Lambda}\right), \tag{3.34}$$

where $H_n(x)$ are the Struve function, and $Y_n(x)$ are the Bessel function of the second kind.

The asymptotic behavior is given by

$$V_p \approx -\frac{\phi_0^2\Lambda}{2\pi\mu_0}\frac{b^2}{r^4} \quad \text{for } r \gg \Lambda \gg b, \tag{3.35}$$

$$V_p \approx \frac{\phi_0^2}{2\pi\mu_0\Lambda}\ln\left(1 - \frac{b^2}{r^2}\right) \quad \text{for } b < r \ll \Lambda. \tag{3.36}$$

It is easy to see from Eq. (3.36) that the vortex-defect interaction is attractive; indeed, the capture of a vortex by the defect is always favored. On the basis of Eq. (3.36), we have

$$f_p = -\frac{\partial V_p}{\partial r}\bigg|_{r=b+\xi} \approx \frac{\phi_0^2}{2\pi\mu_0\Lambda\xi}. \tag{3.37}$$

To compare Eq. (3.37) with Eq. (3.18), we can estimate the force necessary to overcome the electromagnetic pinning by using the calculation in the lateral force of the dipole tip effect on the pinned vortex.

IV. CREATION OF THE VORTEX IN THE THIN FILM

In the previous section, we discussed the interaction between a single vortex being in the thin film and a dipole above the film. A natural question would arise. Are any extra vortices formed in the thin film when the dipole is near the film surface? Of course, it is possible to allow a vortex to form in the thin film in the field of magnetic dipole moment. We now investigate the interaction between a dipole moment and one or more vortices which are created in the thin film by the field of dipole itself. The conditions of creation of the flux quantum in the film will be found. The behavior of magnetic force before and after the vortex creation in the film are also examined.

A. Pinning effect of a circular defect

Let a point dipole be placed just above the circular defect. Initially, if the point dipole is so far from the thin film that the film is in the Meissner state, the persistent currents in the film are established to screen the magnetic field of the dipole. In this situation, the levitation force acting on the dipole is obtained as

$$F_z = \frac{\mu_0 m^2}{4\pi}\int_0^\infty dk\, \frac{k^3}{k\Lambda + 1}\, e^{-2ka}. \tag{4.1}$$

Here the dipole is at the point $(0,0,a)$ and the defect center is at the origin. Equation (4.1) is equal to F_1 which was obtained in Eq. (3.19).

The point dipole is lowered to a position a_1, the position at which the first vortex is created in the thin film, and then trapped by the defect. In order to find the critical position a_1, we need to compare the free energy of no vortex existence and the free energy of the appearance of the first vortex instead of none in the film. The free energy of the system before, U_0, and after, U_1, the creation of the first vortex, respectively, are written as

$$U_0 = U_{\text{in}}(a_1), \tag{4.2}$$

$$U_1 = U_{\text{in}}(a_1) + \left(1 - \frac{b}{\Lambda}\right)U_v(a_1,0) + E_{vd}. \tag{4.3}$$

The critical position of the dipole a_1 can be found by setting $U_0 = U_1$. Then a_1 can be determined using the following equation:

$$\left[\frac{\Lambda}{a_1} - e^{a_1/\Lambda}E_1\left(\frac{a_1}{\Lambda}\right)\right]\left(1 - \frac{b}{\Lambda}\right)\alpha = \ln\left(\frac{\Lambda}{b}\right), \tag{4.4}$$

where $\alpha = \mu_0 \mathrm{m}/\phi_0\Lambda$ reflects the normalized strength of the magnetic moment, and $E_1(a_1/\Lambda)$ is an exponential integral. Considering the case $a_1 \gg \Lambda$, we have

$$a_1 = \sqrt{(1 - b/\Lambda)\alpha/\ln(\Lambda/b)}\;\Lambda. \tag{4.5}$$

It is clear that the critical position a_1 for creating the first vortex depends on the strength of the magnetic moment and the defect radius. The levitation force acting on the dipole can be obtained easily by using the result of the last section

$$F_z = F_1(a_1), \tag{4.6}$$

$$F_{z_1} = F_1(a_1) + F_2(a_1)(1 - b/\Lambda), \tag{4.7}$$

where F_z and F_{z_1} represent the vertical force on the dipole with none and one quanta of the flux in thin film, respectively. The expressions of F_1 and F_2 have been expressed in an earlier section. Equation (4.7) shows clearly how the vertical force changes as a vortex is captured by the defect. The force F_1 caused by the dipole itself is still the same after the creation of the vortex. An additional force F_2, caused by the pinned vortex, appears after the creation of the vortex. Hence, from Eqs. (4.6) and (4.7), the difference of the vertical force, $|\Delta F_z|$, is

$$|\Delta F_z| = \left| \left(1-\frac{b}{\Lambda}\right) F_2(a_1) \right| = \frac{m\phi_0}{2\pi\Lambda^3}\left(1-\frac{b}{\Lambda}\right)\left[-\left(\frac{\Lambda}{a_1}\right)+\left(\frac{\Lambda}{a_1}\right)^2+e^{a_1/\Lambda}E_1\left(\frac{a_1}{\Lambda}\right)\right]. \tag{4.8}$$

If the dipole is a large distance from the film, Eq. (4.8) becomes

$$|\Delta F_z| = \frac{m\phi_0}{\pi a_1^3}\left(1-\frac{b}{\Lambda}\right)\left(1-3\frac{\Lambda}{a_1}\right). \tag{4.9}$$

B. Without pinning effect

We continue to make the assumption that the vortex is formed perpendicularly through the thin film one at a time when the dipole is vertically approaching the film surface. Sometimes it is not necessary to have a pinning center in the examining region, as we did in Sec. IV A. Here we investigate the system free of defect pinning.

In the Meissner state, the levitation force acting on a dipole is the same as that given in Eq. (4.1). As the dipole is lowered to a position a_1 the creation of the vortex appears in the film. In this case, the third term in Eq. (4.3) has to be replaced by the self-energy of the vortex $\phi_0^2 \ln(\Lambda/\xi)/2\pi\mu_0\Lambda$ and the factor β of the second term in Eq. (4.3) has to be removed; then the critical position a_1 satisfies

$$\left[\frac{\Lambda}{a_1} - e^{a_1/\Lambda}E_1\left(\frac{a_1}{\Lambda}\right)\right]\alpha = \ln\left(\frac{\Lambda}{\xi}\right). \tag{4.10}$$

This result differs from the result of Eq. (4.4) by the radius b which is replaced by the coherence length ξ and the factor $(1-b/\Lambda)$. In the limiting case $a_1 \gg \Lambda$, we have

$$a_1 = \sqrt{\alpha/\ln(\Lambda/\xi)}\ \Lambda. \tag{4.11}$$

After the creation of the first vortex, as we know, a change in the vertical force is responsible for the vortex creation. The difference of the vertical force between the first vortex created and no vortex existing in the film has the form

$$|\Delta F_z| = \frac{m\phi_0}{\pi a_1^3}\left(1-3\frac{\Lambda}{a_1}\right) \quad \text{for } a_1 \gg \Lambda. \tag{4.12}$$

When the dipole continues to approach the film, increasing in magnetic field, the possibility of creation of the second vortex appears. To perform the corresponding calculations we should know the interaction energy between two vortices in the case of infinitely thin superconducting film, which was obtained in Ref. 25. Now for simplicity to analysis, we pay our attention to the case of the distance between two vortices r small enough to satisfy $r \ll \Lambda$.

If the dipole is at the point $(0,0,a_2)$, the creation of the second vortex appears. The free energy of the systems before, U_1, and after, U_2, the creation of the second vortex are

$$U_1 = U_{\text{in}}(a_2) + U_v(a_2,0) + E_v, \tag{4.13}$$

$$U_2 = U_{\text{in}}(a_2) + 2U_v(a_2,r_1) + 2E_v + \frac{\phi_0^2}{\pi\mu_0\Lambda}\ln\left(\frac{\Lambda}{r}\right), \tag{4.14}$$

where $r = 2r_1$ is the distance between two vortices and r_1 is the equilibrium position of the vortex from the origin (or equilibrium center). The last term of Eq. (4.14) is the interaction energy between two vortices.[25] In this derivation, $r_1 \ll \Lambda$ is assumed. We do not consider these two vortices coinciding because they need more energy to form two-quanta vortex. In the following, we express the procedures to find a_2 and r_1:

The condition for equilibrium of vortices is

$$F_r = -\frac{\partial U_2}{\partial r_1} = -\frac{m\phi_0}{2\pi}\int_0^\infty dk\,\frac{k^2}{k\Lambda+1}\,e^{-ka_2}\left(\frac{1}{2}kr_1\right) + \frac{\phi_0^2}{\pi\mu_0\Lambda}\frac{1}{r_1} = 0. \tag{4.15}$$

The critical condition of the system before and after the creation of the second vortex occurs as U_1 is equal to U_2; we have

$$-\frac{m\phi_0}{2\pi}\int_0^\infty dk\,\frac{k}{k\Lambda+1}\,e^{-ka_2} + \frac{\phi_0^2}{2\pi\mu_0\Lambda}\ln\left(\frac{\Lambda}{\xi}\right) + \frac{\phi_0^2}{\pi\mu_0\Lambda}\ln\left(\frac{\Lambda}{2r_1}\right) = 0. \tag{4.16}$$

Applying these two conditions to determine a_2 and r_1, we rewrite Eqs. (4.15) and (4.16)

$$\left[\frac{\Lambda}{a_2} - \left(\frac{\Lambda}{a_2}\right)^2 + 2\left(\frac{\Lambda}{a_2}\right)^3 - e^{a_2/\Lambda}E_1\left(\frac{a_2}{\Lambda}\right)\right]\alpha = 2\left(\frac{\Lambda}{r_1}\right)^2, \tag{4.17}$$

$$\left[\frac{\Lambda}{a_2} - e^{a_2/\Lambda}E_1\left(\frac{a_2}{\Lambda}\right)\right]\alpha = \ln\left(\frac{\Lambda}{\xi}\right) + 2\ln\left(\frac{\Lambda}{2r_1}\right). \tag{4.18}$$

Figure 5 shows the critical positions a_1, a_2 and the equilibrium position r_1 versus x. The height of the critical position a_2 is lower than that of the critical position a_1 for the same reduced magnetic moment α. The vertical force on the dipole before, F_{z_1}, and after, F_{z_2}, the creation of the second vortex is obtained as follows, respectively:

$$F_{z_1} = F_1(a_2) - \frac{m\phi_0}{2\pi}\int_0^\infty dk\,\frac{k^2}{k\Lambda+1}\,e^{-ka_2}, \tag{4.19}$$

$$F_{z_2} = F_1(a_2) - \frac{m\phi_0}{\pi}\int_0^\infty dk\,\frac{k^2}{k\Lambda+1}\,e^{-ka_2}\left(1-\frac{1}{4}k^2r_1^2\right). \tag{4.20}$$

The difference in the vertical force is

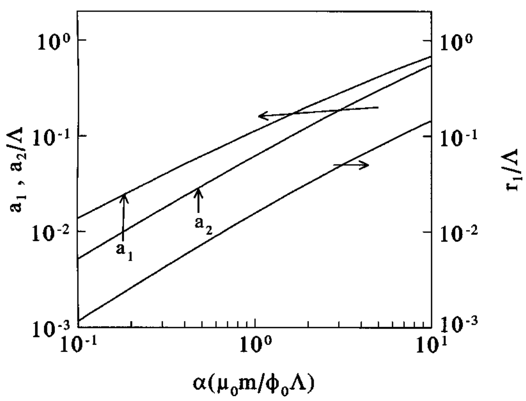

FIG. 5. The critical positions a_1, a_2 and the equilibrium position r_1 of the vortices as a function of α for the parameter $\Lambda/\xi=10^3$.

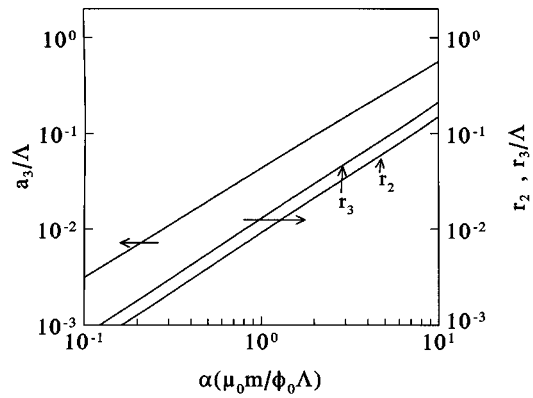

FIG. 6. The critical position a_3 and the equilibrium positions r_2 and r_3 as a function of α for the parameter $\Lambda/\xi=10^3$, where r_2 and r_3 specify the equilibrium positions of the vortex before and after the creation of the third vortex, respectively.

$$|\Delta F_z|=|F_{z_2}-F_{z_1}|$$
$$=\frac{m\phi_0}{2\pi}\frac{1}{\Lambda^3}\left[-\left(\frac{\Lambda}{a_2}\right)+\left(\frac{\Lambda}{a_2}\right)^2+e^{a_2/\Lambda}E_1\left(\frac{a_2}{\Lambda}\right)\right]$$
$$-\frac{m\phi_0}{4\pi}\frac{r_1^2}{\Lambda^5}\left[-\left(\frac{\Lambda}{a_2}\right)+\left(\frac{\Lambda}{a_2}\right)^2-2\left(\frac{\Lambda}{a_2}\right)^3+6\left(\frac{\Lambda}{a_2}\right)^4+e^{a_2/\Lambda}E_1\left(\frac{a_2}{\Lambda}\right)\right]. \quad (4.21)$$

From the result of Fig. 5, we see that most values of the critical position a_2 are much smaller than the effective penetration depth Λ. We use here the condition $a_2/\Lambda \ll 1$; from Eq. (4.21) we obtain

$$|\Delta F_z|=\frac{m\phi_0}{2\pi}\frac{1}{\Lambda^3}\left(\frac{\Lambda}{a^2}\right)^2\left[1-3\left(\frac{r_1}{a_2}\right)^2\right]. \quad (4.22)$$

The values of r_1/a_2 are about 0.21 to 0.26 for a range of α from 0.1 to 10.

In the following, let us treat the problem of three vortices. We consider these vortices to form an equilateral triangle. Following similar procedure, we have

$$\left[\frac{\Lambda}{a_3}-\left(\frac{\Lambda}{a_3}\right)^2+2\left(\frac{\Lambda}{a_3}\right)^3-e^{a_3/\Lambda}E_1\left(\frac{a_3}{\Lambda}\right)\right]\alpha=2\left(\frac{\Lambda}{r_2}\right)^2, \quad (4.23)$$

$$\left[\frac{\Lambda}{a_3}-\left(\frac{\Lambda}{a_3}\right)^2+2\left(\frac{\Lambda}{a_3}\right)^3-e^{a_3/\Lambda}E_1\left(\frac{a_3}{\Lambda}\right)\right]\alpha=4\left(\frac{\Lambda}{r_3}\right)^2, \quad (4.24)$$

$$\left[\frac{\Lambda}{a_3}-e^{a_3/\Lambda}E_1\left(\frac{a_3}{\Lambda}\right)\right]\alpha=\ln\left(\frac{\Lambda}{\xi}\right)+6\ln\left(\frac{\Lambda}{\sqrt{3}r_3}\right)-2\ln\left(\frac{\Lambda}{2r_2}\right), \quad (4.25)$$

where a_3 is the position of the dipole for creating the third vortex in the film and r_2 and r_3 specify the equilibrium positions of the vortex from the origin of the system before and after the creation of the third vortex, respectively. Figure 6 shows the critical position a_3 of the creation of the third vortex and the equilibrium positions r_2 and r_3 versus α. It is clear that the equilibrium position of the vortex (or the distance between vortices) becomes farther than the position it was after the creation of the third vortex. The change of the vertical force acting on the dipole after the creation of the third vortex in the film is

$$|\Delta F_z|=\frac{m\phi_0}{2\pi}\frac{1}{\Lambda^3}\left[-\left(\frac{\Lambda}{a_3}\right)+\left(\frac{\Lambda}{a_3}\right)^2+e^{a_3/\Lambda}E_1\left(\frac{a_3}{\Lambda}\right)\right]$$
$$-\frac{m\phi_0}{8\pi}\frac{(3r_3{}^2-2r_2{}^2)}{\Lambda^5}\left[-\left(\frac{\Lambda}{a_3}\right)+\left(\frac{\Lambda}{a_3}\right)^2-2\left(\frac{\Lambda}{a_3}\right)^3+6\left(\frac{\Lambda}{a_3}\right)^4+e^{a_3/\Lambda}E_1\left(\frac{a_3}{\Lambda}\right)\right]. \quad (4.26)$$

Again, in the $a_3/\Lambda \ll 1$ limit, we find

$$|\Delta F_z|=\frac{m\phi_0}{2\pi}\frac{1}{\Lambda^3}\left(\frac{\Lambda}{a_3}\right)^2\left[1-\frac{3}{2}\left[3\left(\frac{r_3}{a_3}\right)^2-2\left(\frac{r_2}{a_3}\right)^2\right]\right]. \quad (4.27)$$

The values of r_3/a_3 and r_2/a_3 are about 0.24 to 0.38 and 0.18 to 0.26, respectively, for a range of α from 0.1 to 10. From the above results, the change in the vertical force originates from the vortex creation. In the free of pinning case, there is no lateral force signal owing to the symmetry of the vortex geometry.

V. DISCUSSION AND CONCLUSION

We have shown the interaction energy between a dipole moment and the screening current of the superconductor using a direct calculation of the free energy of the system and compare the result with the image method.[20] The interaction energy between a dipole moment and a single vortex also have been obtained from energy consideration. The magnetic force interacting among a magnetic dipole, screening current, and a single vortex in a type-II superconducting thin film are calculated. The vertical component of the magnetic force is attractive or repulsive depending on the strength of the magnetic moment and the distance between the dipole and the vortex. We also have found that there exists a simple relationship between the vertical levitation force and effect penetration depth $\Lambda=2\lambda^2/d$ for the special case of a dipole placed just above the vortex. In the limit of $a/\Lambda \gg 1$, the vertical levitation force varies linearly with Λ. This result gave a more complicated characterization of the force as a function of London penetration depth λ and thickness d than that obtained in Ref. 16 for the levitation force for a half-space superconductor. Clearly, the asymptotic result at large dipole-sample distance depends on the thickness of the superconducting sample, which could be important for analyzing pairing symmetry in superconductors. The lateral force varies linearly with r_0 for both large and small dipole-sample distances, in the limit of $r_0 \ll \Lambda$. If a vortex becomes depinned, the dipole tip will feel a finite change in the lateral force. The local pinning force for a single vortex then can be estimated by the difference of lateral force for a vortex with and without pinning. If a vortex is trapped by a circular defect, the vector potential due to the source of the trapped vortex is reduced by a factor $(1-b/\Lambda)$ as compared with that of vector potential free of defect pinning. This factor will appear as long as the interaction with relation to the trapped vortex. Both the effective penetration depth Λ and coherence length ξ have an inverse proportionality to the electromagnetic pinning force of a vortex by a circular defect. Our main results also contain further generality to include angular dependence. Generalization to the case with more vortices is straightforward.

As concerns vortex creation in the film when the dipole holds vertically at a certain height to scan the film surface, the behaviors of the magnetic force acting on the dipole and the conditions of the vortex creation are quit different for different pinning mechanisms. A physically important situation such as the well studied limit $a \gg \Lambda$ is readily apparent. We have found that the critical position of the vortex creation depends on the strength of the magnetic moment and the radius of the circular defect. A vortex creation in the film means a change in magnetic force which acts on the dipole. These results provide a method of investigating pinning nature in thin films through the measurements of the magnetic force on the dipole tip.

As concerns vortex creation in free of pinning effect, we have obtained at what position a created vortex is energetically favorable for a various number of vortices, including the first, second, and third vortices. The results have shown that by increasing the number of vortex creations in the superconducting thin film the position of the dipole comes near to the film surface. For convenience, we have proposed that the distance between vortices is much smaller than the effective penetration depth, and we have found a change in vertical force acting on the dipole when a new single vortex is created in the film. The force changed discontinuously as the creation of the vortex occurred.

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