

E_6 grand unified theory with three generations from heterotic string theoryMotoharu Ito,^{1,*} Shogo Kuwakino,^{1,†} Nobuhiro Maekawa,^{1,‡} Sanefumi Moriyama,^{2,3,§} Keijiro Takahashi,^{4,||}
Kazuaki Takei,^{1,¶} Shunsuke Teraguchi,^{1,5,**} and Toshifumi Yamashita^{1,††}¹*Department of Physics, Nagoya University, Nagoya 464-8602, Japan*²*Kobayashi Maskawa Institute, Nagoya University, Nagoya 464-8602, Japan*³*Graduate School of Mathematics, Nagoya University, Nagoya 464-8602, Japan*⁴*Department of Electrophysics, National Chiao-Tung University, Hsinchu, Taiwan, Republic of China*⁵*WPI Immunology Frontier Research Center, Osaka University, Osaka 565-0871, Japan*

(Received 13 December 2010; published 19 May 2011)

We construct two more supersymmetric E_6 grand unified models with three generations within the framework of Z_{12} asymmetric orbifold compactification of the heterotic string theory. Such an asymmetric orbifold is missing in the classification in the literature, which concludes that only one E_6 model is possible. In both of the new models, an adjoint Higgs field is obtained in virtue of the diagonal embedding method. This method mods out the three E_6 factors of an even self-dual momentum-lattice by a permutation symmetry. In order to realize the $(E_6)^3$ even self-dual lattice, we utilize the lattice engineering technique. Among the eight possible orbifold actions in our setup, two lead to new E_6 models. Though these models still share the unsatisfactory issues with the known one, our discovery raises hopes that excellent models that solve all the problems in the supersymmetric grand unified models will be found in this framework.

DOI: [10.1103/PhysRevD.83.091703](https://doi.org/10.1103/PhysRevD.83.091703)

PACS numbers: 11.25.Wx, 11.25.Mj, 12.10.Dm

I. INTRODUCTION

Superstring theory is one of the most promising candidates that describe quantum gravity and unify all the four fundamental forces of nature. Usually, however, superstring theory is defined in a ten-dimensional (10D) space-time and its characteristic scale is taken to be the Planck scale, incredibly larger than that of the standard model (SM). If superstring theory truly describes our world, it must be an indispensable subject to find the way to the SM from superstring. Unfortunately, superstring has a lot of perturbative vacua, and so far, the way has not been established. Therefore, it is worthwhile to ask phenomenological studies for hints.

Independently of the developments on superstring, the supersymmetric (SUSY) grand unified theory (GUT) [1] is known as an interesting candidate for the model beyond the SM. It unifies the three gauge groups $SU(3)_C \times SU(2)_L \times U(1)_Y$ in the SM into a single gauge group. This unification is quantitatively supported by experiments which have revealed that the three gauge couplings in the minimal supersymmetric SM meet with a very good accuracy at a very high scale (the GUT scale) close to the Planck scale. Moreover, it unifies one generation of quarks and leptons,

which is dispersed among five multiplets in the SM, into one or two multiplets. This matter unification is qualitatively supported by the measurements of quark/lepton masses and mixings: The pattern of the various hierarchical structures of the masses and the mixings can be explained by a simple assumption that the hierarchies of the Yukawa couplings are induced mainly by the **10** multiplets of $SU(5)$ [2–4].

Among the SUSY-GUTs, the E_6 GUT [5], which unifies all one generation quarks and leptons into a single **27** multiplet, has an advantage that the above assumption for the Yukawa hierarchies, which must be made by hand in the $SU(5)$ unification, is naturally derived [3,4]. This advantage is particularly important since it seems difficult to explain the hierarchical structure in the minimal supersymmetric SM-like models obtained directly from superstring [6,7]. Thus, apart from remaining issues, such as the so-called doublet-triplet (DT) splitting problem and the SUSY-flavor/ CP problem, it is plausible that this E_6 structure is realized.

In addition, consistently with the E_6 structure, it has been shown that the anomalous $U(1)_A$ gauge symmetry [8] and the $SU(2)_H$ (or $SU(3)_H$) family symmetry [4] with a spontaneous CP violation [9] respectively serve solutions to the DT splitting problem and the SUSY-flavor/ CP problem. Interestingly, both of the above two additional symmetries can be simultaneously adopted. Then, the resulting models are really promising, where almost all the phenomenological problems are solved, the realistic quark/lepton mass matrices are obtained naturally and all the three generations are unified into two multiplets (or a single one for $SU(3)_H$).

*mot@eken.phys.nagoya-u.ac.jp

†skuwa@eken.phys.nagoya-u.ac.jp

‡maekawa@eken.phys.nagoya-u.ac.jp

§moriyama@math.nagoya-u.ac.jp

||takahashi-2nyh@jp.nomura.com

¶takei@eken.phys.nagoya-u.ac.jp

**teraguch@ifrec.osaka-u.ac.jp

††yamasita@eken.phys.nagoya-u.ac.jp

Thus, when we seek for the way to the SM from superstring, the above scenario must be valuable to be considered, though we should also keep in mind the possibilities that not all of the above additional symmetries, such as the anomalous $U(1)_A$ symmetry and the $SU(2)_H$ symmetry, are actually realized. Therefore, we assume only the four-dimensional (4D) SUSY E_6 unification in this letter. Namely, our strategy is to construct phenomenological string models with the following minimal requirements,

- (i) E_6 unification group,
- (ii) 4D $\mathcal{N} = 1$ SUSY,
- (iii) adjoint Higgs field,
- (iv) three families,

anticipating to find models with all or some of the additional symmetries mentioned above.

In the literature, despite decades of research, only one model with these requirements has been reported [10]. The authors of the reference claimed that they classified the models with the minimal requirements and a hidden non-Abelian gauge symmetry as well which may be useful to break the SUSY dynamically. Unfortunately, however, the above-mentioned additional symmetries are not realized in their model. Therefore, we would like to search for more 4D $\mathcal{N} = 1$ SUSY E_6 models in string theory, by relaxing the requirement of the hidden non-Abelian gauge symmetry as it is possible to break the SUSY in other ways (for example, in a meta-stable vacuum [11,12]).

II. STRATEGY

Let us work on the heterotic string theory [13], where the E_6 unified group can be realized without much difficulty. In contrast to the F -theory [14] which is another related framework realizing the E_6 group [15] and has attracted attention recently [16], the heterotic string theory has a microscopic description by a Lagrangian and thus any quantity is, at least in principle, calculable.

In the heterotic string, to obtain the 4D effective theory, twenty-two extradimensions in the left-moving bosonic string and six in the right-moving superstring should be compactified. Then, both the left-moving and the right-moving momenta are quantized and compose a lattice. For the consistency of string theory, the partition function, namely, the one-loop vacuum diagram of the closed string must be invariant under the modular transformations $\mathcal{T}: \tau \rightarrow \tau + 1$ and $\mathcal{S}: \tau \rightarrow -1/\tau$, where τ is the moduli of the worldsheet torus. This requires that the lattice is even and self-dual with a (22, 6)-Lorentzian signature. A space-time gauge symmetry G (in particular E_6) is realized when this momentum-lattice contains the appropriately normalized Lie lattice of G in the left-moving part.

To reduce the 4D $\mathcal{N} = 4$ SUSY to $\mathcal{N} = 1$, six components among eight of the massless 10D spinor mode in the right-moving superstring have to be projected out. This

is achieved by an orbifold compactification [17], which identifies the compactified space under an action of a point group that leaves the lattice unchanged. In particular, the compactified right-moving six dimensions should be fully rotated with three nontrivial rotating angles t_{Ri} that satisfy $\sum_{i=1}^3 t_{Ri} = 0 \pmod{2 \times 2\pi}$.

In general, when the heterotic string realizes a spacetime gauge symmetry, the currents of the corresponding world-sheet theory form a Kac-Moody algebra: $[j_m^a, j_n^b] = i f^{ab}_c j_{m+n}^c + km \delta^{ab} \delta_{m+n,0}$. Here, j_m^a is the Laurent coefficient of the world sheet current $j^a(z) = \sum_{m \in \mathbb{Z}} j_m^a z^{-m-1}$, f^{ab}_c is a structure constant and the integer k is a Kac-Moody level. The usual orbifold construction described above realizes the lowest Kac-Moody level ($k = 1$), while it is known [18] that a higher level is necessary to obtain adjoint Higgs fields. A way to increase the level is the so-called diagonal embedding method [19], where K -copies of the current $(j)_I$ with level $k = 1$ are permuted by an orbifold action so that only the diagonal part $j_{\text{diag}} = \sum_{I=1}^K (j)_I$ remains phaseless under the action. It is easy to see that j_{diag} satisfies the Kac-Moody algebra with $k = K$. The other eigenstates have nontrivial phases, and thus do not contribute to the 4D gauge multiplets, while some of them may couple with chiral multiplets in the right-mover to cancel the phases, resulting in adjoint Higgs fields. It is also possible that adjoint Higgs fields appear in twisted sectors.

Unfortunately, it is not easy to clarify the condition in string theory to construct models with three generations, while there is a conjecture that the number of the generations is proportional to the Kac-Moody level [10]. Therefore, we start with the construction of 4D $\mathcal{N} = 1$ SUSY E_6 models with an adjoint Higgs field, leaving the numbers of generations to be determined model-by-model. To summarize, we take the following strategy:

- (1) we prepare a (22,6)-dimensional even self-dual lattice with equivalent K -copies of the left-moving E_6 lattice,
- (2) we consider an orbifold identification that includes
 - (a) a permutation among the E_6 factors,
 - (b) rotations of the right-moving six dimensions with three nonzero angles t_{Ri} satisfying $\sum_{i=1}^3 t_{Ri} = 0 \pmod{2 \times 2\pi}$,
- (3) we find out the number of the generations.

According to the conjecture, $k = 3$ is needed for the three generations, and we take this choice hereafter. In this case, the left-moving $(E_6)^3$ lattice occupies 18 dimensions and cannot be fitted in the 16 extradimensions (with respect to the 10D viewpoint). This means that the usual left-right symmetric treatment of the six extradimensions is not valid and we have to work in the asymmetric orbifold [20] with a Narain compactification [21]. In contrast to the symmetric orbifold, the general rules for consistent models are rather involved in the asymmetric orbifold [10], and thus, we calculate the one-loop partition functions explicitly to check the modular invariance (see Ref. [22] for the details).

III. SETUP

The lattice engineering technique [23] is helpful in constructing desired lattices. The essence of the technique is that a lattice (for example, the A_2 lattice) transforms oppositely as its complement lattice in the Euclidean even self-dual E_8 lattice (the E_6 lattice for the A_2 example) under the modular transformation. Thanks to this, we can always replace the left-moving A_2 lattice by the right-moving \bar{E}_6 lattice (denoted with a bar) and vice versa. Subsequently, since the \bar{E}_6 lattice is decomposed into three \bar{A}_2 lattices, we can construct a left-moving $(E_6)^3$ lattice out of the right-moving $(\bar{A}_2)^3$ lattice using the same technique again. Thus, we can easily convert an A_2 lattice into three equivalent E_6 lattices.

With all these insights in mind, let us pick up the $E_6 \times \bar{E}_6$ lattice as our starting point. After decomposing E_6 into $(A_2)^3$, we end up with $[(A_2)^2 \times (E_6)^3] \times \bar{E}_6$ using the above technique. Though this lattice is the same as the one used in Ref. [10] constructed from the 10D $SO(32)$ heterotic string by a compactification with Wilson lines, our method is more direct and hence many Narain lattices with $(E_6)^3$ symmetry are accessible with their discrete symmetries manifest.

The orbifold action on the three left-moving E_6 factors is chosen to be a permutation among them. It turns out that a shift along the diagonal factor has to be introduced in addition, as a source of the asymmetry between the numbers of the generations and the antigerations in the twisted sectors.

A natural candidate for the action on the right-moving factor is the rotation by the Coxeter element of the right-moving \bar{E}_6 lattice, which is an element of a point group $\mathbb{Z}_{12} = \mathbb{Z}_3 \times \mathbb{Z}_4$. Though it was claimed that \mathbb{Z}_2 is the only

possible symmetry to add to the \mathbb{Z}_3 symmetry for the above permutation [10], we find no reasons to exclude this possibility. Thus, we choose this rotation, which corresponds to the one with three angles $t_R = 2\pi(1, 4, -5)/12$ classified as \mathbb{Z}_{12} -I [24], as part of our setup.

Then, the remaining options are actions on the two left-moving A_2 factors. The allowed choices on each factor, labeled by i , are

- (i) shift s_{Li} , with $12s_{Li} = 0 \pmod{\text{roots}}$,
- (ii) rotation with an angle $t_{Li} = 2\pi/3$,
- (iii) Weyl reflection.

There are a lot of choices of s_{Li} but many of them are related to each other by transformations under the symmetry of the A_2 lattice, leading to identical models. In addition, the modular invariance does not allow arbitrary choices, but only certain combinations. Thus, there remain only a few possible actions:

$$\begin{aligned} &\{(2, 0), (4, 0), \text{“rot”}\} \otimes \{(0, 0), (6, 0)\}, \\ &(1, 0) \otimes (3, 6), \\ &(1, 6) \otimes (3, 0), \end{aligned}$$

where “rot” denotes the $1/3$ rotation while (n, m) represents the shift defined by the vector $s = (n\alpha_1 + m\alpha_2)/12$, with α_i being the simple roots of A_2 . In the first line, we have three options for the action on one of A_2 lattices, and two for the other. Therefore, there are, in total, $3 \times 2 + 1 \times 1 + 1 \times 1 = 8$ consistent models possible in this setup. Note that the order is irrelevant since the two A_2 factors are equivalent.

TABLE I. The massless spectra of the models with three generations: U and T_α denote the untwisted and various twisted sectors, respectively. The quantum numbers of left-handed chiral multiplets and the normalization of the $U(1)$ charges are shown. The gravity and gauge multiplets are omitted.

gauge symmetry	Model 1 $E_6 \times SU(2) \times U(1)^3$	Model 2 $E_6 \times SU(2) \times U(1)^3$	Model 3 $E_6 \times U(1)^4$
U	$(\mathbf{1}, \mathbf{1}, +6, 0, 0)_L$ $(\mathbf{78}, \mathbf{1}, 0, 0, 0)_L$	$(\mathbf{1}, \mathbf{1}, +6, \pm 3, 0)_L$ $(\mathbf{78}, \mathbf{1}, 0, 0, 0)_L$	$(\mathbf{1}, -6, 0, 0, 0)_L$ $(\mathbf{1}, +3, \pm 6, 0, 0)_L$ $(\mathbf{78}, 0, 0, 0, 0)_L$
T_1	$(\mathbf{27}, \mathbf{1}, +1, 0, \pm 1)_L$...	$(\mathbf{27}, -1, -1, +1, 0)_L$
T_2	$(\mathbf{27}, \mathbf{1}, -1, \pm 1, 0)_L$	$(\mathbf{27}, \mathbf{1}, +2, 0, -2)_L$	$(\mathbf{27}, +1, 0, 0, \pm 1)_L$
T_3	$2(\mathbf{1}, \mathbf{1}, -3, 0, \pm 3)_L$	$(\mathbf{1}, \mathbf{1}, -3, \pm 3, -3)_L$	$(\mathbf{1}, +3, -3, +3, 0)_L$ $(\mathbf{1}, +3, +3, -3, 0)_L$
T_4	$(\mathbf{27}, \mathbf{1}, -2, 0, 0)_L$	$(\mathbf{27}, \mathbf{1}, -2, \pm 1, 0)_L$	$(\mathbf{27}, +2, 0, 0, 0)_L$ $(\mathbf{27}, -1, \pm 2, 0, 0)_L$
T_5	$(\mathbf{27}, \mathbf{1}, +1, 0, \pm 1)_L$	$(\mathbf{27}, \mathbf{1}, +1, \pm 1, +1)_L$	$(\mathbf{27}, -1, +1, -1, 0)_L$ $(\mathbf{1}, -3, 0, 0, \pm 3)_L$
T_6	$(\mathbf{1}, \mathbf{2}, 0, 0, \pm 3)_L$ $(\mathbf{1}, \mathbf{1}, +3, \pm 3, 0)_L$	$(\mathbf{1}, \mathbf{2}, 0, \pm 3, 0)_L$ $(\mathbf{1}, \mathbf{1}, -6, 0, +6)_L$	$(\mathbf{1}, 0, +6, -2, 0)_L$ $(\mathbf{1}, 0, -6, +2, 0)_L$
normalization of $U(1)$	$(\frac{\sqrt{2}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6})$	$(\frac{\sqrt{2}}{12}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{12})$	$(\frac{\sqrt{2}}{6}, \frac{\sqrt{6}}{12}, \frac{\sqrt{2}}{4}, \frac{\sqrt{6}}{6})$

IV. MODELS WITH THREE GENERATIONS

Once fixing the orbifold action, one can calculate the partition function (see, for example, Refs. [10,22]), which contains information of the spectrum of the model. It turns out that, among the above eight models, three lead to vanishing net generation numbers, while other two and the remaining three, respectively, have nine and three net generations. Here, we concentrate on the last three,

$$(2, 0) \otimes (6, 0), \quad (1, 0) \otimes (3, 6), \quad (1, 6) \otimes (3, 0),$$

and we call them Model 1, 2, 3, respectively.

Model 1 and Model 2 have the gauge group $E_6 \times SU(2) \times U(1)^3$, and Model 3 has $E_6 \times U(1)^4$. Their massless spectra are listed in Table I. We find five generations and two antigerations in Model 1 and Model 3, while four and one in Model 2. Thus, we obtain three models with three generations. Each model contains an E_6 adjoint Higgs field in the untwisted sector.

Model 1 results in the same massless spectrum as the \mathbb{Z}_6 model in Ref. [10]. The other two, Model 2 and Model 3, are new. Model 3 does not contain any hidden non-Abelian gauge symmetry, which is one of the requirements of the classification in Ref. [10], while Model 1 and Model 2 do. We also find that these models have $(\mathbb{Z}_3)^3$ symmetry which remains unbroken even after all the singlets develop non-vanishing vacuum expectation values and, unfortunately, do not possess the additional symmetries [4,8,9]. Thus, the traditional SUSY-GUT problems, such as the DT splitting problem and the SUSY-flavor/ CP problem, are not resolved in these models.

V. SUMMARY

In this letter, we construct 4D SUSY level-3 E_6 models. The $k = 3$ E_6 gauge symmetry is realized from three copies of $k = 1$ E_6 symmetry via the diagonal embedding. We utilize the lattice engineering technique, instead of the

compactification of the usual 10D heterotic string models with Wilson lines, to construct Narain lattices containing three copies of the E_6 lattices. This technique allows us to construct new even self-dual lattices from a known one in a simple way, and thus, makes it easier to access new models. Though here we work only on the same lattice as the one studied in Ref. [10] where the lattice is obtained through Wilson lines, we show that Narain lattices with desired three copies of E_6 can be immediately constructed from any lattice containing A_2 .

Then, we examine all the possible \mathbb{Z}_{12} actions which are missing in the classification in the literature [10], and we find three models with the minimal requirements. One of them has the same spectrum as the model [10] that has been the only one proposed so far. The other two are new. While one does not have any hidden non-Abelian gauge symmetry, the other does, and thus, should be added into the classification.

The two new models contain neither an $SU(2)_H$ family symmetry nor an anomalous $U(1)_A$ gauge symmetry which make the E_6 models more attractive. Given that we have shown there are E_6 models besides the unique one proposed so far, it is worthwhile to look for more E_6 models, especially the excellent models with the above additional symmetries. For this purpose, our systematic construction of the E_6 models will be useful.

ACKNOWLEDGMENTS

We appreciate Y. Kawamura, K. Hosomichi, H. Kanno, T. Kobayashi, J.C. Lee, S. Mizoguchi, Y. Sugawara for valuable discussions. S.M. would like to thank Yukawa Institute for hospitality. This work was partially supported by the Grant-in-Aid for Nagoya University Global COE Program (G07), by MEXT of Japan [Grant No. 22011004 (N.M.), Grant No. 21740176 (S.M.)] and by JSPS (T.Y.).

-
- [1] H. Georgi and S.L. Glashow, *Phys. Rev. Lett.* **32**, 438 (1974).
 - [2] C. H. Albright, K. S. Babu, and S. M. Barr, *Phys. Rev. Lett.* **81**, 1167 (1998); W. Buchmuller and T. Yanagida, *Phys. Lett. B* **445**, 399 (1999); Q. Shafi and Z. Tavartkiladze, *Phys. Lett. B* **451**, 129 (1999).
 - [3] J. Sato and T. Yanagida, *Phys. Lett. B* **430**, 127 (1998); M. Bando and T. Kugo, *Prog. Theor. Phys.* **101**, 1313 (1999); M. Bando and N. Maekawa, *Prog. Theor. Phys.* **106**, 1255 (2001).
 - [4] N. Maekawa, *Phys. Lett. B* **561**, 273 (2003); *Prog. Theor. Phys.* **112**, 639 (2004).
 - [5] F. Gursey, P. Ramond, and P. Sikivie, *Phys. Lett. B* **60**, 177 (1976).
 - [6] W. Buchmuller, K. Hamaguchi, O. Lebedev, and M. Ratz, *Phys. Rev. Lett.* **96**, 121602 (2006); *Nucl. Phys.* **B785**, 149 (2007).
 - [7] G. Honecker and T. Ott, *Phys. Rev. D* **70**, 126010 (2004); **71**, 069902(E) (2005).
 - [8] N. Maekawa, *Prog. Theor. Phys.* **106**, 401 (2001); N. Maekawa and T. Yamashita, *Prog. Theor. Phys.* **107**, 1201 (2002); **110**, 93 (2003).
 - [9] M. Ishiduki, S. G. Kim, N. Maekawa, and K. Sakurai, *Phys. Rev. D* **80**, 115011 (2009); **81**, 039901(E) (2010); H. Kawase and N. Maekawa, *Prog. Theor. Phys.* **123**, 941 (2010).
 - [10] Z. Kakushadze and S. H. H. Tye, *Phys. Rev. D* **54**, 7520 (1996); **55**, 7878 (1997).

- [11] K. A. Intriligator, N. Seiberg, and D. Shih, *J. High Energy Phys.* **04** (2006) 021.
- [12] S. G. Kim, N. Maekawa, H. Nishino, and K. Sakurai, *Phys. Rev. D* **79**, 055009 (2009).
- [13] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, *Phys. Rev. Lett.* **54**, 502 (1985); *Nucl. Phys.* **B256**, 253 (1985); **B267**, 75 (1986).
- [14] C. Vafa, *Nucl. Phys.* **B469**, 403 (1996).
- [15] C. M. Chen and Y. C. Chung, *J. High Energy Phys.* **03** (2011) 129.
- [16] R. Donagi and M. Wijnholt, [arXiv:0802.2969](https://arxiv.org/abs/0802.2969); C. Beasley, J. J. Heckman, and C. Vafa, *J. High Energy Phys.* **01** (2009) 058.
- [17] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, *Nucl. Phys.* **B261**, 678 (1985); **B274**, 285 (1986).
- [18] D. C. Lewellen, *Nucl. Phys.* **B337**, 61 (1990).
- [19] K. R. Dienes and J. March-Russell, *Nucl. Phys.* **B479**, 113 (1996).
- [20] K. S. Narain, M. H. Sarmadi, and C. Vafa, *Nucl. Phys.* **B288**, 551 (1987).
- [21] K. S. Narain, *Phys. Lett. B* **169**, 41 (1986).
- [22] M. Ito, S. Kuwakino, N. Maekawa, S. Moriyama, K. Takahashi, K. Takei, S. Teraguchi, and T. Yamashita, [arXiv:1104.0765](https://arxiv.org/abs/1104.0765).
- [23] W. Lerche, A. N. Schellekens, and N. P. Warner, *Phys. Rep.* **177**, 1 (1989).
- [24] Y. Katsuki, Y. Kawamura, T. Kobayashi, N. Ohtsubo, Y. Ono, and K. Tanioka, *Nucl. Phys.* **B341**, 611 (1990).