New type of liquid refractometer

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Abstract. In a new type of liquid refractometer, the phase difference variation between *s* and *p* polarizations produced by the total internal reflection is measured using a heterodyne interferometer with a specially designed probe. Substituting the phase difference variation into the Fresnel's equation, the refractive index can be calculated and displayed. The prototype is set up to demonstrate its feasibility. *© 1998 Society of Photo-Optical Instrumentation Engineers.* [S0091-3286(98)02510-0]

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1 Introduction

Liquid refractometers are often used in the optical industry. Although there are some liquid refractometers available, such as, the Pulfrich refractometer,[1] the Abbe refractometer,[1] and the Hilger-Chance refractometer,[1] all of them are based on the measurement of the critical angle or the deviation angle using a microscope or a telescope. The measuring process is tedious. Some other techniques for measuring the refractive index of a liquid include, for example, the interferometric method[2,3] and the Brewster angle method.[4] The latter is related to the measurement of the light intensity variations. Thus, the stability of a light source, the scattering light, the internal reflection, and other factors influence the accuracy of the measurements. Because the former is easily influenced by air turbulence, the fringe counting becomes ambiguous and the resolution decreases.

In this paper, we propose a new type of liquid refractometer based on the effect of the total internal reflection and heterodyne interferometry. In this refractometer, the phase difference variation between *s* and *p* polarizations produced by the total internal reflection is measured using a heterodyne interferometer with a specially designed probe in contact with the test liquid. Substituting this phase difference variation into the Fresnel equation,[5] and with an electronic signal processing unit, the refractive index can be calculated and displayed. The merits of the refractometer include a simple structure, easier operation, rapid measurement, and high stability because of its common path configuration. The prototype has been built to demonstrate its feasibility.

2 Principle

Figure 1 shows a schematic diagram of the new type of liquid refractometer. For convenience, the $+z$ axis is chosen to be along the light propagation direction and the y axis is along the direction perpendicular to the paper plane. A light coming from a heterodyne light source having a frequency difference f between *s* and *p* polarizations is incident on a beamsplitter BS, and is divided into two parts: the reflected light and the transmitted light. The reflected light passes through an analyzer AN_r with the transmission axis at 45 deg to the x axis, then enters a photodetector D_r. The intensity measured by D_r can be written as

$$I_r=\tfrac{1}{2}\,[1+\cos\,(2\pi f t+\phi_r)], \tag{1}$$

where I_r is the reference signal, and ϕ_r is the phase difference between *s* and *p* polarizations produced by the reflection at the BS. The transmitted light enters a specially designed probe, and it is totally reflected at *B*, *D*, and *F*. After these total internal reflections, the light passes through an analyzer AN_t with the transmission axis at 45 deg to the x axis and is detected by another photodetector D_t. The intensity I_t measured by D_t is the test signal.

This specially designed probe is made of a glass material with refractive index n_p. The configuration of the probe and the light path inside it are shown in Fig. 2. The light is normally incident on the upper surface *AG*, which is parallel to the bottom surface *CE*, where the tested liquid will be located. With the conditions that $\angle AGE=\angle GAC=60$ deg and $\angle ACE=\angle CEG=120$ deg, all of the incident angles at *B*, *D*, and *F* are 60 deg, which is larger than the critical angle. Thus, the light is totally reflected at *B*, *D*, and *F*, and the phase differences produced by the total internal reflections between *s* and *p* polarizations are ϕ_B, ϕ_D, and ϕ_F, respectively. Hence the test signal is given by

$$I_1=\tfrac{1}{2}\,[1+\cos\,(2\pi f t+\phi_1+\phi_M)], \tag{2}$$

where

$$\phi_1=\phi_B+\phi_D+\phi_F=3\phi_B=6\tan^{-1}(1/3-4/9n_p^2)^{1/2}, \tag{3}$$

and ϕ_M is the phase difference between *s* and *p* polarizations produced by the reflection at *M*. As the bottom surface *CE* is in contact with the test liquid, the phase difference ϕ_D changes to ϕ_D'. Consequently, the test signal becomes

$$I_2=\tfrac{1}{2}\,[1+\cos\,(2\pi f t+\phi_2+\phi_M)], \tag{4}$$

where

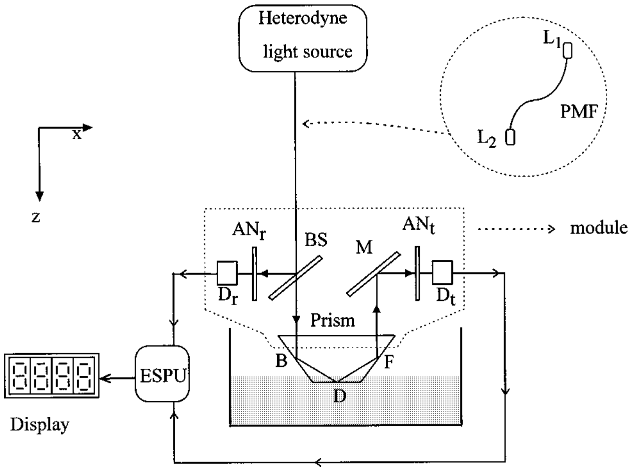

Fig. 1 Schematic diagram of the new type of liquid refractometer: BS, beamsplitter; *M*, mirror; AN, analyzer; *D*, photodetector; ESPU, electronic signal processing unit; PMF, polarization maintaining fiber; and *L*, graded-refractive-index (GRIN) lens.

$$\begin{aligned}\phi_2=\phi_B+\phi'_D+\phi_F=2\phi_B+\phi'_D\\ =4\tan^{-1}(1/3-4/9n_p^2)^{1/2}\\ +2\tan^{-1}(1/3-4n^2/9n_p^2)^{1/2}.\end{aligned}\tag{5}$$

As these two test signals I_1 and I_2 are sent to the ESPU to compare with the reference signal I_r, respectively, the phase difference variation between I_1 and I_2,

$$\begin{aligned}\psi&=(\phi_2+\phi_M)-(\phi_1+\phi_M)\\ &=2\tan^{-1}(1/3-4n^2/9n_p^2)^{1/2}\\ &\quad-2\tan^{-1}(1/3-4/9n_p^2)^{1/2},\end{aligned}\tag{6}$$

can be obtained. Equation (6) can be written as

$$n=\frac{n_p}{2}\left\{3-9\tan^2\left[\tan^{-1}(1/3-4/9n_p^2)^{1/2}+\frac{\psi}{2}\right]\right\}^{1/2}.\tag{7}$$

It is seen from Eq. (7) that if n_p is specified, then n can be calculated with the measurement of the phase difference variation ψ. These calculations are performed by hardware in the ESPU and the calculated value of the refractive index of the test liquid is digitally displayed for direct reading.

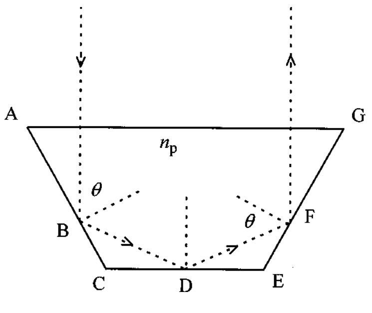

Fig. 2 Configuration of the probe and the light path.

Table 1 The experimental results and the corresponding reference indices.

Liquid	ψ (deg)	n_{exp}	n_{ref}
Water	−15.02	1.331	1.3317
Saltwater (c=1%)	−15.17	1.333	1.3352
Saltwater (c=2%)	−15.78	1.342	1.3405
Acetone	−16.77	1.356	1.3578
Ethanol	−16.91	1.358	1.3604
Toluene	−31.43	1.494	1.4939

Note ψ, measured phase difference; n_{exp}, experimental data; n_{ref}, reference data; and c, concentration of saltwater solutions.

3 Experiments and Results

To show the validity of the system, a prototype of the new type of liquid refractometer was set up as shown in Fig. 1. The heterodyne light source[6] with a 2-kHz difference frequency was used. It consists of a He-Ne laser at 632.8 nm wavelength and an electro-optic modulator modulated by a function generator. The probe is made of SF11 glass with a 1.77862 refractive index, and its size is $AG=26$ mm, $AC=13$ mm, $CE=13$ mm, and $EG=13$ mm. The refractive indices of several liquids were measured. The experimental results and their corresponding reference indices[7,8] are summarized in Table 1. It is clear that they show good correspondence.

4 Discussion

We constructed the BS, the probe, the mirror *M*, the analyzers $AN_{r,t}$, and the photodetectors $D_{r,t}$ in a rigid module, as shown in Fig. 1. If a light-guiding module is added, which consists of a PMF and two GRIN lenses L_1 and L_2, as shown in the upper-right circle of Fig. 1, then the whole probe module can be moved conveniently to the test liquid, and this liquid refractometer can be operated easily. Since the effect of total internal reflection is necessary for the test, the measurable range is limited by the refractive index of the probe. In our prototype, the measurable range is from 1 to 1.5. If the liquid level is over *BF*, then Eqs. (6) and (7) should be modified as

$$\psi=6\tan^{-1}(1/3-4n^2/9n_p^2)^{1/2}-6\tan^{-1}(1/3-4/9n_p^2)^{1/2}\tag{8}$$

and

$$n=\frac{n_p}{2}\left\{3-9\tan^2\left[\tan^{-1}(1/3-4/9n_p^2)^{1/2}+\frac{\psi}{6}\right]\right\}^{1/2},\tag{9}$$

respectively. In addition, from Eq. (7) we can get

$$\begin{aligned}\Delta n\cong&\left|\frac{\sqrt{3}n_p^2}{n}\left[\frac{5n^2}{3n_p^2}-\frac{9\tan u\sec^2 u}{4(3n_p^2-1)}\,\frac{n_p(5-3n_p^2)}{(3n_p^2-4)^{1/2}}-1\right]\right|\\ &\times\Delta\theta+\left|\frac{-9n_p^2}{8n}\tan u\sec^2 u\right|\times\Delta\psi,\end{aligned}\tag{10}$$

where $u=\psi/2+\tan^{-1}(1/3-4/9n_p^2)^{1/2}$. Here, Δn, $\Delta\theta$ and $\Delta\psi$ are the errors of the refractive index, the incident angle,

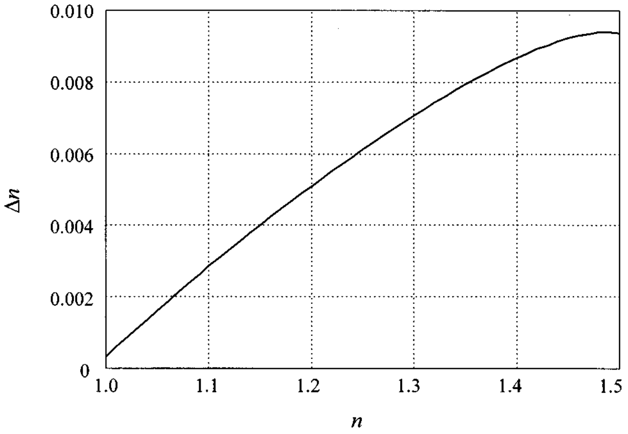

Fig. 3 Relation curve of Δ*n* versus *n*.

and the phase difference, respectively. We see that the resolution of the refractive index is dependent on the refractive index of the test liquid. In our prototype, the angular resolutions of the incident angle and the electric signal processing unit are 0.1 and 0.01 deg, respectively. Consequently, the curve of Δn versus n can be obtained by substituting $\Delta\theta = 0.1$ deg and $\Delta\psi = 0.01$ deg into Eq. (10), as shown in Fig. 3. From the figures, we see that its resolution is better than 10^{-2}.

5 Conclusion

This paper proposed a new type of liquid refractometer based on the effect of the total internal reflection and the heterodyne interferometry. Because of the introduction of the ESPU and its common path configuration, the refractometer has such merits as a simple structure, easier operation, rapid measurement, and high stability. The prototype was set up to demonstrate its feasibility. It has a resolution better than 10^{-2}.

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