On Tile-and-Energy Allocation in OFDMA Broadband Wireless Networks

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***Abstract*—The *Orthogonal Frequency Division Multiple Access (OFDMA)* technique has been widely applied in broadband wireless networks, such as IEEE 802.16 (WiMAX) and 3GPP Long Term Evolution (LTE). Existing studies have targeted at improving network throughput by increasing the transmission rates of *mobile stations (MSs)*. Nevertheless, this is at the cost of higher energy consumption of MSs. In the letter, we consider the tile-and-energy joint allocation problem for uplink transmissions in an OFDMA wireless network. We formulate this problem as a mixed integer programming, where the objective is to minimize MSs' energy consumption subject to satisfying their traffic demands. We show this to be NP-hard and develop a heuristic taking advantage of the water-filling technique. Simulation results show that the performance of our heuristic is close to the optimum, especially when the network is under non-saturated condition.**

***Index Terms*—3GPP LTE, energy conservation, IEEE 802.16, OFDMA, resource management, WiMAX.**

I. Introduction

Both IEEE 802.16 (WiMAX) and 3GPP Long Term Evolution (LTE) have employed OFDMA, which can support multiuser diversity and dynamic power adaptation, thus significantly improving the spectral efficiency. Existing studies [1]–[4] for OFDMA networks consider only multiuser diversity and power allocation for higher throughput but neglect the energy conservation issue of MSs. The work [5] does try to reduce MSs' energy consumption by modeling it as a multiple-choice knapsack problem, but it does not take the multiuser diversity and power constraint of MSs into account. In this letter, we consider the allocation of tiles and energy of MSs in the uplink direction of an OFDMA network. We model it as an optimization problem to minimize MSs' energy consumption subject to satisfying their demands. We formulate this problem as a mixed integer programming problem, which has been shown to be NP-hard [6], and propose a low complexity, energy-efficient heuristic. The heuristic first allocates tiles to MSs in a greedy way to satisfy more demands. If there are remaining resources, a new water-filling technique is applied to adjust MSs' transmission power to save their energy. The performance of the heuristic will be shown by simulations.

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II. System Model

The resource in an OFDMA network is *frames*, where a frame is a two-dimensional (subchannel × time slot) array. Each frame is composed of a *downlink subframe* and an *uplink subframe*. The resource unit to be allocated to MSs is *tiles*, where a tile is a time slot on a subchannel where an MS can transmit at a certain rate with a proper transmission power. This letter considers the resource allocation of uplink subframes. Each uplink subframe is modeled by N subchannels and M time slots. There are K MSs. Each $\text{MS}_k, k = 1..K$, has a maximum transmission power of P_k^{MAX} mW at any instant and an uplink traffic demand of D_k bits per frame granted by the BS. Let $x_k^{i,j}$ be the binary indicator such that $x_k^{i,j} = 1$ if the jth time slot of subchannel i (or called tile (i, j)) is allocated to MS_k. Since each tile is allocated to at most one MS, we have

$$\sum_{k=1..K} x_k^{i,j} \leq 1, \quad \forall i, j. \tag{1}$$

Let $p_k^{i,j}$ be the power that MS_k uses for tile (i, j). It follows that

$$\sum_{i=1..N} p_k^{i,j} \leq P_k^{MAX}, \quad \forall k, j. \tag{2}$$

Assuming that the channel condition remains unchanged during a frame, we let $G_k^i > 0$ and $N_k^i > 0$ be the channel gain and the noise power of subchannel i from MS_k to the BS, respectively. Here, we assume the background noise to be additive white Gaussian noise. Given a bandwidth of B (in Hz) per subchannel, according to the *Shannon capacity*, MS_k can transmit at rate $x_k^{i,j} \cdot B \log_2(1+p_k^{i,j} \cdot G_k^i/N_k^i)$ at tile (i, j). Below, we let $f_k^i = G_k^i/N_k^i$ be the channel gain to noise ratio. To meet MS_k's demand, we impose

$$\sum_{i=1..N} \sum_{j=1..M} x_k^{i,j} \cdot B \log_2(1 + p_k^{i,j} \cdot f_k^i) \geq D_k, \quad \forall k. \tag{3}$$

The energy consumption of MS_k, denoted by E_k, is the summation of its energy costs in all tiles, i.e., $E_k = \sum_{i=1..N} \sum_{j=1..M} x_k^{i,j} \cdot p_k^{i,j} \cdot T_s$, where T_s is the time slot length (in second). Our goal is to minimize the total energy consumption of all MSs:

$$\min_{p_k^{i,j}, x_k^{i,j}, \forall i,j,k} \sum_{k=1..K} E_k, \tag{4}$$

by allocating tiles (i.e., $x_k^{i,j}$) and the corresponding power (i.e., $p_k^{i,j}$) subject to Eqs. (1), (2), and (3). This problem is a *mixed integer programming (MIP)* problem, which is known to be NP-hard and thus intractable [6].

III. An Energy-Efficient Heuristic

The proposed heuristic has two phases. The first phase tries to satisfy MSs' demands by selecting the best tiles and power for MSs such that the use of uplink frame space is minimized. If there are free tiles left from the first phase, the second phase will try to allocate them to MSs, which can help MSs lower down their transmission rates, and thus energy costs.

A. Phase 1: Meeting MSs' Demands

To meet more MSs' demands, phase 1 will try to utilize MSs' largest power at all instances, i.e., $\sum_{i=1..N} x_k^{i,j} \cdot p_k^{i,j} = P_k^{MAX}$ if $\sum_{i=1..N} x_k^{i,j} \geq 1$ for any j. We define a *reward function* to calculate the benefit of assigning a tile to an MS and apply a greedy approach for MSs to compete for tiles by this function iteratively. In this iterative process, we use a binary flag $\mathcal{I}_k$ to reflect whether MS$_k$'s demand is satisfied or not and $d_k^{i,j}$ to reflect the amount of data that MS$_k$ transmits in tile (i,j).

1) Initially, set all $\mathcal{I}_k = 0$, all $d_k^{i,j} = 0$, and all $x_k^{i,j} = 0$.
2) Define the reward function for tile (a,b) to be assigned to MS$_k$ as:

$$w((a,b),k) = \frac{\Delta R((a,b),k)}{A(b,k)}, \quad (5)$$

where $A(b,k)$ is the maximal rate that MS$_k$ can transmit if all tiles in time slot b are allocated to MS$_k$,

$$A(b,k) = \sum_{i=1..N} B\log_2(1+\tilde{p}_k^{i,b} \cdot f_k^i).$$

Note that $\tilde{p}_k^{i,b} = (\lambda - \frac{1}{f_k^i})^+$ can be found using the *rate-optimum water-filling technique* [7], where $z^+ = \max\{z,0\}$ and λ is a water level constant subject to $\sum_{i=1..N} \tilde{p}_k^{i,b} = P_k^{MAX}$. The numerator $\Delta R((a,b),k)$ is the additional rate that MS$_k$ can transmit in time slot b if we assign tile (a,b) to MS$_k$. Since MS$_k$ has no tile initially, we have $\Delta R((a,b),k) = B\log_2(1+P_k^{MAX} \cdot f_k^a)$. Intuitively, $w((a,b),k)$ is to compare the importance of tile (a,b) to MS$_k$. A higher ratio $\frac{\Delta R((a,b),k)}{A(b,k)}$ means that tile (a,b) is more important to MS$_k$.
3) For each free tile (a,b) and each unsatisfied MS$_k$, compare their reward value $w((a,b),k)$. Pick the one, say, $w((\hat{a},\hat{b}),\hat{k})$ with the maximum positive value and assign tile $(\hat{a},\hat{b})$ to MS$_{\hat{k}}$. Then we set $x_{\hat{k}}^{\hat{a},\hat{b}} = 1$. Note that this assignment has implied that MS$_{\hat{k}}$ has to redistribute its total power $P_{\hat{k}}^{MAX}$ to all tiles in time slot $\hat{b}$ assigned to it. Again, this is obtained by the water-filling technique. This means that we need to update all $p_{\hat{k}}^{*,\hat{b}}$ and $d_{\hat{k}}^{*,\hat{b}}$ accordingly. Also, we need to update $\mathcal{I}_{\hat{k}}$ if MS$_k$'s demand is already satisfied.
4) Since MS$_{\hat{k}}$ has won tile $(\hat{a},\hat{b})$ in time slot $\hat{b}$, we need to update the numerator parts of all MSs' reward functions properly.
   a) For each MS$_k \neq$MS$_{\hat{k}}$, set $\Delta R((\hat{a},\hat{b}),k) = 0$.
   b) For MS$_{\hat{k}}$, since its power distribution in time slot $\hat{b}$ has been changed, we need to recompute $\Delta R((a,\hat{b}),\hat{k})$, $a \neq \hat{a}$, for each free tile $(a,\hat{b})$ in advance by finding the additional rate that MS$_{\hat{k}}$ can transmit.

   The above updates will change MSs' reward functions accordingly. Note that we do not change the denominator in Eq. (5) throughout the process.
5) If there is any remaining free tile and any unsatisfied MS, go to step 3; otherwise, terminate phase 1.

B. Phase 2: Reducing MSs' Energy Costs

Phase 1 aims at reducing the frame usage. If there are remaining free tiles, we use "spread" the data transmissions of the allocated tiles to these free tiles. This normally imposes lower transmission rates on MSs, and thus lower energy consumption. Toward this goal, we define another *energy reward function* for MSs to compete for free tiles. Phase 2 will repeat this process until all free tiles are allocated.

First, we build our model of rewarding. Consider any free tile (a,b) and any MS$_k$. Let $x_k^{i,j}$, $p_k^{i,j}$, and $d_k^{i,j}$ be the parameters reflecting the current allocation to MS$_k$. The energy reward function of assigning tile (a,b) to MS$_k$ is defined as

$$e((a,b),k) = \max\{E_{st}((a,b),k), E_{ch}((a,b),k)\}.$$

$E_{st}((a,b),k)$ and $E_{ch}((a,b),k)$ are the conserved energy if we spread some of the data of MS$_k$ to be transmitted in time slot b and subchannel a to tile (a,b), respectively. In particular, $E_{st}((a,b),k)$ and $E_{ch}((a,b),k)$ can be presented as the following equations.

$$E_{st}((a,b),k) = \Big(\sum_{i=1..N} x_k^{i,b} \cdot p_k^{i,b}\Big) - \Big(\tilde{p}_k^{a,b} + \sum_{i=1..N} x_k^{i,b} \cdot \tilde{p}_k^{i,b}\Big)$$
$$E_{ch}((a,b),k) = \Big(\sum_{j=1..M} x_k^{a,j} \cdot p_k^{a,j}\Big) - \Big(\tilde{p}_k^{a,b} + \sum_{j=1..M} x_k^{a,j} \cdot \tilde{p}_k^{a,j}\Big),$$

where $\tilde{p}_k^{i,b}, i = 1..N$, and $\tilde{p}_k^{a,j}, j = 1..M$, are the allocated power after assigning the free tile (a,b) to MS$_k$ to spread the data transmitted in time slot b and subchannel a, respectively. We derive the allocating power $\tilde{p}_k^{i,b}$ and $\tilde{p}_k^{a,j}$ by the proposed *energy-optimum water-filling technique*, where

$$\tilde{p}_k^{i,b} = \begin{cases} x_k^{i,b} \cdot (\lambda - \frac{1}{f_k^i})^+, & \forall i = 1..N, i \neq a \\ (\lambda - \frac{1}{f_k^i})^+, & i = a \end{cases}, \quad (6)$$

subject to $\sum_{i=1..N} x_k^{i,b} \cdot d_k^{i,b} = (Blog_2(1+\tilde{p}_k^{a,b} \cdot f_k^a) + \sum_{i=1..N} x_k^{i,b} \cdot Blog_2(1+\tilde{p}_k^{i,b} \cdot f_k^i))$. In the end of the section, we will prove that the proposed energy-optimum water-filling technique can save the most energy when assigning an MS an additional free tile. Note that it is possible that the new allocated power is greater than the original one except for that of tile (a,b), especially when a free tile is used to spread the data on a subchannel. To prevent this, we will recalculate $E_{ch}((a,b),k)$ and $\tilde{p}_k^{a,j}$ by omitting this tile when this case happens.

Below, we present the operations of phase 2 in detail.

1) Calculate $e((a,b),k)$ for each free tile (a,b) for MS$_k$.
2) From all $e((a,b),k)$, pick the one, say, $e((\hat{a},\hat{b}),\hat{k})$ with the maximum positive value assigning tile $(\hat{a},\hat{b})$ to MS$_{\hat{k}}$.
   a) Set $x_{\hat{k}}^{\hat{a},\hat{b}} = 1$. Update the allocated power ($p_{\hat{k}}^{*,\hat{b}}$ and $p_{\hat{k}}^{\hat{a},*}$) and the amount of allocated data ($d_{\hat{k}}^{*,\hat{b}}$ and

$d_{\hat{k}}^{\hat{a},*}$) accordingly. Finally, set $e((\hat{a},\hat{b}),k)=0$ for all MS$_k$ and recompute $e((a,\hat{b}),\hat{k})$ and $e((\hat{a},b),\hat{k})$ for those free tiles on subchannel $\hat{a}$ and time slot $\hat{b}$, respectively.

b) If there is any remaining free tile, go to step 2. Otherwise, terminate phase 2.

Below, we first show that the proposed energy-optimum water-filling technique can minimize the total power of the allocated tiles in a time slot for an MS by keeping to deliver the fixed amount of data. Then, we will prove that spreading the data transmissions of the allocated tiles to an additional free tile can further reduce an MS's total energy consumption. Based on these two properties, we can minimize the total power when assigning an additional tile for an MS in a time slot.

Theorem 1: Consider the MS$_k$ has been allocated tiles with amount of data D in time slot b, the power allocated by the proposed water-filling technique can incur the minimum energy.

Proof: Without loss of generality, we assume that MS$_k$ has been allocated n tiles in time slot b and the channel gain to noise ratios of these tiles are $f_k^i, i=1..n$, and the allocated power of each tile is $p_k^{i,b}$ (we shorten f_k^i as $f^{(i)}$ and $p_k^{i,b}$ as $p^{(i)}$ for ease of presentation). The allocation problem to incur the minimum total energy on these n tiles subject to the amount of data D in time slot b can be formulated as the following optimization: $\min_{p^{(i)}}\sum_{i=1..n}p^{(i)}$ subject to $\sum_{i=1..n}B\log_2(1+p^{(i)}\cdot f^{(i)})=D$. By the method of Lagrange multiplier, we can use y as the Lagrange multiplier and rewrite the optimization as: $\min_{p^{(i)}}L(y,p^{(1)},..,p^{(n)})=\sum_{i=1..n}p^{(i)}+y\cdot(D-\sum_{i=1..n}B\log_2(1+p^{(i)}\cdot f^{(i)}))$. By differentiating $L(y,p^{(1)},..,p^{(n)})$ respect to y and $p^{(i)}, i=1..n$, and setting the results equal to zero yield $p^{(i)}=(\lambda-\frac{1}{f^{(i)}})^+, i=1..n$, where λ is the water level constant such that $D=\sum_{i=1..n}B\log_2(1+p^{(i)}\cdot f^{(i)})$. ■

Theorem 2: With the proposed water-filling technique, under the same amount of allocated data in time slot b, we can save an MS's energy by spreading the data transmissions of the allocated tiles to the additional free tile in time slot b.

Proof: Without loss of generality, we assume that we have allocated MS$_k$ n tiles in time slot b with the channel gain to noise ratios of $f_k^i, i=1..n$ (we shorten f_k^i as $f^{(i)}$ for ease of presentation). The allocating power of each tile (i,b) is assumed to be $(\lambda_A-\frac{1}{f^{(i)}})>0, i=1..n$, where λ_A is the current water-level determined by the proposed water-filling technique. Now, given one more free tile, denoted as the $(n+1)$th tile, from time slot b with the channel gain to noise ratio of $f^{(n+1)}$ to the MS$_k$. Here, we assume the tile's channel gain to noise ratio is good enough, i.e., $(\lambda_A-\frac{1}{f^{(n+1)}})>0$; otherwise, no power will be allocated to the free tile and the total energy allocation will be the same as the original. Let λ_B be the new water level conducted by the proposed water-filling technique for those $n+1$ tiles. The reduced energy $\Delta\mathbb{E}$ by spreading the data transmissions of the n tiles to the $(n+1)$ tiles can be written as

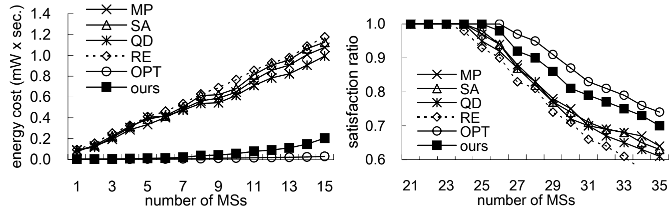
$$\Delta\mathbb{E}=\sum_{i=1..n}(\lambda_A-\frac{1}{f^{(i)}})-\sum_{i=1..n+1}(\lambda_B-\frac{1}{f^{(i)}}). \quad (7)$$

Since we have to maintain the same amount of data delivery, we have $\sum_{i=1..n}B\log_2(1+(\lambda_A-\frac{1}{f^{(i)}})\cdot f^{(i)})=\sum_{i=1..n+1}B\log_2(1+(\lambda_B-\frac{1}{f^{(i)}})\cdot f^{(i)})$. This equation yields $\lambda_B=\sqrt[n+1]{\frac{(\lambda_A)^n}{f^{(n+1)}}}$. Thus, bring it to Eq. (7), we can rewrite Eq. (7) as

$$\Delta\mathbb{E}=\frac{1}{f^{(n+1)}}\cdot(n\cdot\lambda_A\cdot f^{(n+1)}-(n+1)\cdot\sqrt[n+1]{(\lambda_A)^n\cdot(f^{(n+1)})^n}+1). \quad (8)$$

Let $x=\lambda_A\cdot f^{(n+1)}$ (and thus $x>0$). Eq. (8) can be further rewritten as $\Delta\mathbb{E}=\frac{1}{f^{(n+1)}}(n\cdot x-(n+1)\cdot x^{\frac{n}{n+1}}+1)=\frac{1}{f^{(n+1)}}(x^{\frac{1}{n+1}}-1)(n\cdot x^{\frac{n}{n+1}}-x^{\frac{(n-1)}{n+1}}-x^{\frac{n-2}{n+1}}-...-1)=\frac{1}{f^{(n+1)}}(x^{\frac{1}{n+1}}-1)(x^{\frac{n-1}{n+1}}\cdot(x^{\frac{1}{n+1}}-1)+x^{\frac{n-2}{n+1}}\cdot(x^{\frac{2}{n+1}}-1)+...+(x^{\frac{1}{n+1}}-1)(x^{\frac{n-1}{n+1}}+x^{\frac{n-2}{n+1}}+...+1))=\frac{1}{f^{(n+1)}}(x^{\frac{1}{n+1}}-1)^2(x^{\frac{n-1}{n+1}}+x^{\frac{n-2}{n+1}}\cdot(x^{\frac{1}{n+1}}+1)+...+(x^{\frac{n-1}{n+1}}+x^{\frac{n-2}{n+1}}+...+1))>0$. Thus, we can ensure that the MS$_k$'s energy consumption in time slot b can be saved by spreading the data transmissions of the allocated tiles to the additional free tile in time slot b. ■

Above two properties can be applied for the tiles on the same subchannel a, since the channel gain to noise ratios of the tiles on the same subchannel are the same, i.e., $f^{(1)}=f^{(2)}=...=f^{(n)}=f^{(a)}$, which is a special case of Theorems 1 and 2.



(a) energy consumption (b) satisfaction ratio

Fig. 1. The simulation results under different numbers of MSs.

IV. Performance Evaluation and Conclusion

In the simulation, we consider the scenario the same as [1], which is for the uplink of a single-cell OFDMA network. The cell radius is 1 km and the MSs are uniformly distributed over the cell. In particular, the uplink subframe duration is 2.5 ms with $N=15$ subchannels and $M=15$ time slots. The subchannel bandwidth B is 180 KHz. The path loss follows the *modified Hata urban propagation model*, including the frequency correlation and multipath fading. Each MS$_k$ has a traffic demand up to 2.56 Kbits/frame (= 512 Kbits/s) [4] in average and a maximum power per time slot $P_k^{MAX}=50$ mW.

We compare our heuristic against the *maximal-rate pair (MP)* scheme [1], the *sequential-allocation (SA) scheme* [2], the *quota-determination (QD) scheme* [3], the *resource-efficient (RE)* scheme [4], and the *optimal (OPT) scheme*. **MP** scheme iteratively finds the MS-subchannel pair with the maximal rate to allocate. **SA** scheme sequentially allocates each subchannel to the unsatisfied MS which has the maximum incremental rate on the subchannel. **QD** scheme assigns each MS a quota in terms of number of subchannels based on its demand. Then, **QD** allocates an MS subchannels when it has the best subchannel gains on them. **RE** scheme dynamically adjusts the number of subchannels allocated to the MSs over frames to reduce the amount of resources. **OPT** exploits *Lingo software* [8] to find the optimum results, but this incurs high computational cost. Note that we can not compare the scheme in [5] because it neglects the power constraint. Fig. 1(a) compares the energy consumption per frame of all schemes under different numbers of MSs. Our heuristic can approximate **OPT** scheme and save up to 70% of energy in average as compared to **MP**, **SA**, **QD**, and **RE** schemes, because our heuristic utilizes the free tiles to spread the allocated data to lower down MSs' transmission power, which is neglected by above schemes. Fig. 1(b) investigates the number of MSs on *satisfaction ratio*, which is the ratio of the amount of *satisfied* demands to the total amount of demands per frame. Our heuristic has higher satisfaction ratio because it allocates the subchannels by consulting both the incremental rate and the maximum rate of each MS. This can utilize the deviation of subchannels for each MS and thus further exploit multiuser diversity to satisfy more MSs' demands.

To conclude, this letter addressed the energy conservation issue in uplink transmissions of an OFDMA wireless network. This problem is formulated by an MIP problem and an energy-efficient heuristic is proposed to satisfy MSs' demands while saving their energy. By employing multiuser diversity and efficient power allocation in additional free tiles, simulation results verified that, compared with the existing schemes, our heuristic can save more MSs' energy and increase their satisfaction ratios.

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