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A linear programming-based method for the network revenue management problem of air cargo

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Abstract

One critical operational issue of air cargo operation faced by airlines is the control over the sales of their limited cargo space. Since American Airlines' successful implementation in the post-deregulation era, revenue management (RM) has become a common practice for the airline industry. However, unlike the air passenger operation supported by well-developed RM systems with advanced decision models, the decision process in selling air cargo space to freight forwarders is usually based on experience, without much support from optimization techniques. This study first formulates a multi-dimensional dynamic programming (DP) model to present a network RM problem for air cargo. In order to overcome the computational challenge, this study develops two linear programming (LP) based models to provide the decision support operationally suitable for airlines. In addition, this study further introduces a dynamic adjustment factor to alleviate the inaccuracy problem of the static LP models in estimating resource opportunity cost. Finally, a numerical experiment is performed to validate the applicability of the developed model and solution algorithm to the real-world problems.

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1. Introduction

Air cargo plays an important role in global trade. For example, more than 30% of internationally traded merchandise, according to value, is transported by air for the U.S. and Japan (Yamaguchi, 2008). In particular, due to world trade liberalization, the air cargo industry has been booming for the past several decades, and the growth

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rate of air cargo has surpassed that of air passengers. According to most forecasts, this trend is projected to continue in the future.

Air cargo is an operation-intensive industry and involves complex procedures and many players. One critical operational issue faced by airlines is the control over the sales of their limited cargo space. Since American Airlines' successful implementation in the post-deregulation era, revenue management (RM) has become a common practice for the airline industry. Based on certain demand forecasting techniques and optimization models, RM has been found to be very effective in generating extra revenue for diversified and uncertain demand, given a fixed capacity of perishable inventory. According to most estimates, the revenue gain from applying RM is about 4%-5%, which is comparable to many airlines' total profitability in a good year (Talluri and van Ryzin, 2004). However, unlike the air passenger operation supported by well-developed RM systems with advanced decision models, the process of selling air cargo space to freight forwarders or shippers is usually not highly automated. The decision process is mainly based on experience, without much support from optimization techniques.

The characteristics of air cargo RM differ from air passenger RM in many respects. One fundamental difference is the nature of the product. For air passenger RM, seats are a well-defined product in terms of the demand initiated by the customer and the capacity provided by the supplier. However, air cargo shipments are categorized by both weight and volume, which can be stochastic in practice. In addition, with the current hub-and-spoke operation of most airlines, the focus of RM research has shifted from the traditional single-leg version to the network version. However, no substantial research work is found to ensure the applicability to real-world problems for air cargo RM.

This study formulates a multi-dimensional dynamic programming (DP) model to present a network RM problem for air cargo, in which the weight, volume, and rate of the shipments can be stochastic. The objective of the DP model is to maximize the expected revenue given the fixed capacities (in both weight and volume) of the airline network. The associated computational load makes it impossible to derive the optimal control decision for a problem of practical size, as the curse of dimensionality is an inherited problem for many DP formulations. In order to provide the decision support for air cargo RM, this study develops two linear programming (LP) based models to generate the control decision with respect to shipment booking requests. In particular, given the concern over the static assumptions in LP formulations, this study further introduces a dynamic adjustment factor to alleviate the inaccuracy problem of opportunity cost estimation due to the gap between resource allocation and realized sales.

The remainder of this paper is organized as follows. Section 2 provides the background to the problem and reviews the related literature. The DP model to illustrate the network RM problem of air cargo and the LP-based methods to generate the control decisions are presented in Section 3. The numerical experiment is described in Section 4. Finally, the findings of this study are summarized and conclusions are drawn in Section 5.

2. Problem Background and Literature Review

There are several important RM research areas, such as demand forecasting, pricing, and overbooking, etc. The focus of this study is on the control of capacity. For the air passenger side, most early seat-inventory control studies relied on the following six assumptions: 1) sequential booking classes, 2) low-before-high fare booking arrival pattern, 3) statistical independence of demands between booking classes, 4) no cancellation or no-shows, 5) single flight leg, and 6) no batch booking (McGill and Van Ryzin, 1999). For example, Belobaba (1989) developed the Expected Marginal Seat Revenue (EMSR) heuristic for the static problem based on the above assumption. In order to incorporate the time-dependent characteristic of demand, Lee and Hersh (1993) developed a DP model in which the request probability based on the Poisson arrival process is used to represent the demand pattern. In addition, Lee and Hersh (1993) further generalized the single-seat booking assumption to batch booking, and thus the request probability turns out to be dependent upon the booking size as well.

Most airlines nowadays operate a hub-and-spoke type of network so as to serve more origin-destination (OD) pairs with fewer flights. For the simple 4-leg network in Figure 1, there are 8 OD pairs (one-way). The level of complexity is significantly increased from the single-leg version to the network version problem as multiple resources are now shared by multiple products. The computational load makes it impossible to derive the optimal control for a problem of practical size. One popular approach for the network RM problem is the bid-price control (Williamson, 1992). A bid price is attached to each leg, and a booking request for a product is accepted if its revenue is greater than the sum of the bid prices of the used legs. The key issue for most bid-price based algorithms is finding

a suitable set of bid prices, supposedly depending on the available leg seats and the demand before departure. Williamson (1992) set the bid prices as the dual prices of the leg capacity constraints in an LP model, which overlooks the stochastic feature of the demand. Several studies have developed improved approaches to generate better bid prices by addressing the issue of demand uncertainty. For example, Talluri & van Ryzin (1999) proposed the RLP (Randomized Linear Program) method, in which the bid prices are determined by averaging the bid prices from a series of deterministic LP problems with simulated demand. In addition, de Boer et al. (2002) provided the PLP (Probabilistic Linear Programming) method to incorporate the distribution of the demand into an LP model with a modified objective function. A general discussion of the problem's nature and solution approach of the network RM problem can be found in the dedicated chapter in Talluri and van Ryzin (2004)

The characteristics and complexities of air cargo RM differ from those of air passenger RM in many respects. Kasilingam (1997), Bilings et al. (2003), Slager and Kapteijns (2004), Becker and Dill (2007), Becker and Kasilingam (2008), Becker and Wald (2010), and Kasilingam (2011) provided the background to air cargo RM and highlighted the unique features and associated complexities. One fundamental difference is the product itself. In air passenger RM, seats are well defined from the viewpoint of supply and demand. However, in air cargo RM, cargo space is much more complicated.

The air cargo shipments that an aircraft can carry are limited by both weight and volume capacities. As a result, the air cargo rating system takes into account both dimensions. The volume of a shipment is converted into the volume weight by dividing it by the constant 6,000 cm³/kg, defined by IATA (the International Air Transport Association). The chargeable weight, on which the airline charges the airfreight forwarder, is the greater of the gross weight and the volume weight. Another related issue is that the density and the size of air cargo shipments vary by a wide range. In Figure 2, the plot based on the shipments of a specific flight operated by a Taiwanese carrier illustrates the variability of these shipment characteristics. Some prior studies have tried to handle the air cargo shipments based on the concept of batch bookings from air passenger RM or by adopting the approach of discretized shipment types. However, it remains a challenge to model the complex features of air cargo shipments.

The other important special feature of air cargo RM comes from its business practice. The pre-committed allotments to specific freight forwarders or shippers account for a significant portion of the overall available capacity. Although some prior works (e.g., Levin et al., 2012) have tried to integrate the decisions for the allotments based on long-term contracts and for the space control regarding the short-term spot markets, this study will focus only on the short-term space control by following the practice of most airlines.

Finally, this study does not consider overbooking, which has been a common practice in countering the effect of cancellation and no-show. In particular, the overbooking issue of air cargo RM is also related to the fact that the weight and volume of a booking request is usually just an estimation and can be significantly different from the actual values realized before the flight departure. For this particular component of air cargo RM, some recent studies (e.g., Luo et al., 2009, Amaruchkul and Sae-Lim, 2011, and Moussawi-Haidar and Cakanyildirim, 2012) are useful for understanding the overbooking models of air cargo. It is assumed that the input data of capacity in the mathematical models for space control in this study can be provided by some overbooking models.

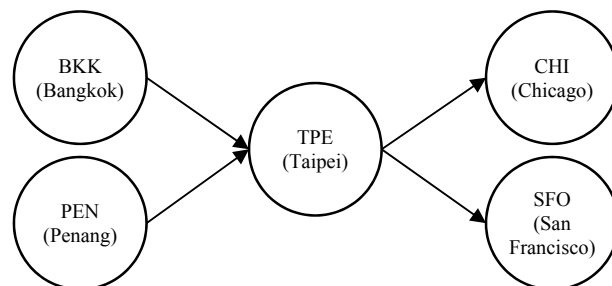


Fig. 1. An illustrative example with a four-leg network

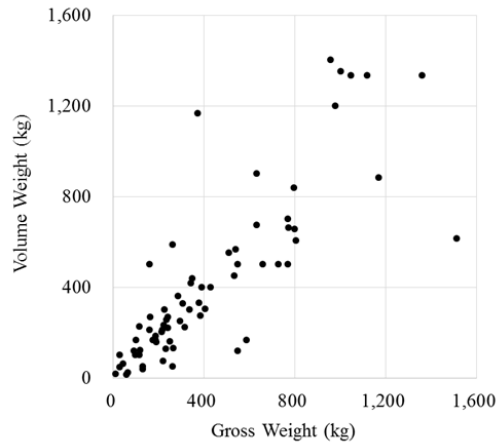


Fig. 2. Example of weight, volume, and density distributions of air cargo shipments

Amaruchkul et al. (2007) made the first attempt to formulate a two-dimensional dynamic model for the single-leg cargo space control problem, in which the weight and volume of various types of shipments are stochastic. They developed several heuristics and bounds by decomposing the problem into one-dimensional sub-problems for weight and for volume. For the similar single-leg problem, researchers have developed various approaches to solve the difficult DP problem. Huang and Chang (2010) proposed a heuristic that jointly estimates the expected revenue from both weight and volume by sampling a limited number of points in the state space. Han et al. (2010) developed a bid-price control policy based on a mixed integer programming (IP) model. Hoffmann (2013a) recently developed an efficient heuristic that exploits the structure of monotone switching curves to reduce the computational load. Lastly, with the focus on the issue of random resource consumption, Zhuang et al. (2012) proposed a general model and two heuristics that consistently outperform heuristics ignoring consumption uncertainty.

In line with the above dynamic single-leg air cargo RM studies, the extension to the network version appears to be very limited. Levina et al. (2011) presented a network dynamic capacity control model considering various sources of uncertainty, including the random booking request arrival, the uncertain capacity requirement of shipments, and the uncertain capacity from the viewpoint of the supply side. The approximate control decisions are derived through an LP and stochastic simulation-based computational method. For a similar problem formulation, Hoffmann (2013b) proposed the upper bound based on linear programming and developed the approach to determine bid prices for both weight and volume units on each leg by approximating the value function. These two studies have nicely advanced the air cargo RM research work to address the need to provide the decision support to airlines under the current operational environment of hub-and-spoke networks. However, we believe that some of their modeling techniques are not consistent with the key characteristics of the air cargo shipments. In addition, the model and the solution approach may lead to the problem of data availability. We have thus developed the method presented in the following section to provide an approach that is more applicable to real-world problems.

3. Mathematical Models and Solution Algorithm

In order to describe the features of the dynamic network RM problem for air cargo, a dynamic programming formulation is presented in the first sub-section, although this DP problem is not directly solved. The second sub-section presents two linear programming models to estimate the opportunity cost of the resources consumed by a booking request. The third sub-section discusses the concept of the dynamic adjustment factor based on the analogy of the Binomial experiment to provide better opportunity cost estimation.

3.1. Basic DP model for network air cargo RM problem

By extending the single-leg model in Amaruchkul et al. (2007) and Huang and Chang (2010), this study presents the following network version of the air cargo RM model. Given the notations, the Bellman equation of the DP model in (1) computes the maximum expected revenue and determines the optimal control policy.

$$V_t(\mathbf{x}, \mathbf{y}) = P_t^0 V_{t-1}(\mathbf{x}, \mathbf{y}) + \sum_{j=1}^J P_j^t \max(V_{t-1}(\mathbf{x} - \mathbf{G}_j, \mathbf{y} - \mathbf{H}_j) + r_j, V_{t-1}(\mathbf{x}, \mathbf{y})) \quad (1)$$

- t : indices of decision periods ($t = 0 \dots T$, assuming $t = 0$ and T , in a count-down fashion, are the end and beginning of the booking process, respectively.)
- i : indices of flight legs ($i = 1 \dots I$)
- j : indices of (shipment) types ($j = 1 \dots J$)
- P_t^j : probability of the booking request for type j in period t (P_t^0 is the probability of no booking request.)
- r_j : revenue of type j
- g_j : *weight* of type j (\mathbf{G}_j is a $1 \times I$ vector representing the *weight* used by type j for all legs.)
- h_j : *volume* of type j (\mathbf{H}_j is a $1 \times I$ vector representing the *volume* used by type j for all legs.)
- x_i : available *weight* capacity of leg i (\mathbf{x} is a $1 \times I$ vector representing the *weight* capacities for all legs.)
- y_i : available *volume* capacity of leg i (\mathbf{y} is a $1 \times I$ vector representing the *volume* capacities for all legs.)
- $V_t(\mathbf{x}, \mathbf{y})$: expected revenue under the available system capacity as (\mathbf{x}, \mathbf{y}) in period t

The above DP model basically inherits the structure of the first single-leg RM model for air passengers in Lee and Hersh (1993). However, in addition to the extension of the network context, several key features of air cargo RM have been introduced into the model. First of all, the two dimensions in weight and volume are considered. Second, the products are now differentiated by the shipment types (indexed by j), unlike the well-defined fare classes to segment the market for air passenger RM. Amaruchkul et al. (2007), as well as Levina et al. (2011) and Hoffmann (2013b), define the shipment types based on the shipment characteristics (e.g., type of cargo, type of packaging, time sensitivity and other possible factors), which may lead to different levels of revenue (denoted by r_j). This way of differentiating is very close to the concept of the fare classes for air passenger RM. However, in order to make the DP model work, the shipment type definition also depends on its weight and volume (denoted by g_j and h_j), which are linked to the resource usage and the state change in (1). Finally, the shipment type definition is of course also related to origin-destination pairs.

This way in which the shipment type is defined gives rise to a serious problem when modeling real world situations. Weight and volume are two continuous dimensions, as shown by the illustrative example in Figure 2. In order to achieve an acceptable level of accuracy, many discretized levels could be needed. Even more importantly, the needed number of differentiations is related to the combination of the two dimensions. In addition, unlike the published fare for each class in air passenger RM, the price of an air cargo shipment is negotiable and variable. It can be affected by many factors, including the relationship between the airline and the airfreight forwarder/shipper. Thus, the needed number of revenue levels creates another challenge for defining the shipment types. Furthermore, even though a huge number of shipment types are used without considering the associated complexity, the availability issue of the request probabilities (denoted by P_t^j) becomes another shortcoming of the DP model (1). It is difficult for most airlines to maintain the historical data with the required detail and to manipulate the quantitative tools to appropriately generate this massive amount of input data for the DP model.

With the attempt originally made to resolve the discrepancy between the space booked and that actually realized upon arrival for a shipment, some prior studies (e.g., Amaruchkul et al., 2007) use random variables, unlike the fixed constants in (1), to model the variation in weight and volume for a shipment type. The introduction of random variables into the DP model partially resolves the issue related to the number of discretization levels needed. However, in addition to the increased complexity from stochastic models, there is the additional challenge of estimating the distributions of the random variables. Thus, in order to provide the decision support operationally suitable for airlines, this study develops two linear programming models, based on the classic approach in the literature on network RM. These two models, presented in the following sub-section, do not differentiate between

the products in terms of types and do not involve random variables. Nonetheless, in the numerical experiment, the variations in weight, volume, and rate for air cargo shipments are incorporated in the experimental design to test the applicability of the developed models for real-world problems that are stochastic by nature.

3.2. LP models for opportunity cost estimation

The first LP model developed in this study is based on the approach in Williamson (1992), a pioneering work for the network RM problem. In particular, the shipments are only differentiated by their OD pairs, which are chosen to share the notation of j for the shipment type as in (1) for the sake of notational simplicity.

$$Z^{DLP}(\mathbf{x}, \mathbf{y}) = \text{Maximize} \sum_j u_j b_j \quad (2)$$

$$s.t. \quad \sum_j a_{ij} b_j \leq x_i \quad \forall i = 1, \dots, I \quad (3)$$

$$\sum_j a_{ij} (b_j / s_j) \leq y_i \quad \forall i = 1, \dots, I \quad (4)$$

$$0 \leq b_j \leq E[D_j] \quad \forall j = 1, \dots, J \quad (5)$$

- b_j : allocation (and sales) of weight capacity for OD j
- u_j : average rate in terms of weight for OD j
- s_j : average shipment density for OD j
- a_{ij} : binary constant = 1, if OD j uses leg i ; = 0, otherwise.
- D_j : random variable representing the demand of OD j ($E[D_j]$ is its expected value.)

The dimension of weight is chosen for the decision variable (b_j), which sets the allocation for each OD pair, as the weight estimation of a shipment is normally much more accurate than the volume estimation when the booking request is made. In this formulation, the capacity allocation and the sales quantity are thought to be equal and are used interchangeably. This assumption is more or less like the situation in the classic product mix problem, in which capacity commitment automatically leads revenue generation. The objective (2) is to then maximize the revenue by multiplying the sales quantity (the capacity allocation) by the associated rate, i.e., the unit revenue. Suppose the rate with respect to the chargeable weight (the greater of the gross weight and the volume weight) is denoted by u'_j (e.g., with the dollar/kg unit). As the decision variable is for the weight allocation, u_j is simply equal to u'_j if the average density (s_j) is larger than 1. On the other hand, for the ODs where the average density (s_j) is smaller than 1, u_j would be set as (u'_j/s_j), after being inflated by the factor of the inverse of the density. The allocation decision is subject to the weight and volume available capacities of the legs in (3) - (4) as well as the average values of the OD demands within the booking horizon in (5).

Williamson (1992) proposed the bid-price approach for the network RM problem based on the dual prices of the capacity constraints. The sum of the bid prices of the used legs, as the estimation of the value or the opportunity cost of the consumed resource, is set as the threshold to determine whether or not a booking request is accepted. In order to handle the dynamic booking requests, the bid prices based on static LP models normally need to be frequently updated so as to take into account the most recent situation regarding demand and resources. Then, the undesired system interruption can become a problem.

In this study, we choose to use the CEC (certainty equivalent control) approach proposed in Bertsimas and Popescu (2003). Given the system capacity as (\mathbf{x}, \mathbf{y}) , the decision concerning a booking request involving the resource (\mathbf{w}, \mathbf{v}) can be made based on the opportunity estimation represented by (6). The CEC approach has a couple of advantages over the bid-price approach. First, the dual prices are sometimes not uniquely defined. In addition, the additive approximation used in the bid-price approach can lead to a larger estimation error as the capacities of the multiple legs are consumed simultaneously. Based on (6), the LP problem must be solved twice so as to derive the opportunity cost estimation. However, the number of the booking requests for air cargo shipments is much less than that in the case of air passengers. Re-solving the LP problems whenever a booking request is made does not cause the frequent system interruption.

$$C_{(x,y)}^{DLP}(\mathbf{w}, \mathbf{v}) = Z^{DLP}(\mathbf{x}, \mathbf{y}) - Z^{DLP}(\mathbf{x} - \mathbf{w}, \mathbf{y} - \mathbf{v}) \tag{6}$$

One important shortcoming of the approach based on deterministic linear programming (DLP) models, such as (2) - (5), is that the stochastic feature of the demand is overlooked, as shown in the constraint (5). For this study, we follow the probabilistic linear programming (PLP) method proposed by de Boer et al. (2002) to develop the following LP model and take demand uncertainty into account. In this particular approach, the demand distribution (D_j) is approximated by a discrete random variable, which has the values denoted by d_j^k ($k = 1, \dots, K$) with the corresponding probability of q_j^k . Based on the numerical experiment in de Boer et al. (2002), the number of segments (K) does not have to be big for achieving a good result when approximating continuous demand distributions by using discrete ones.

$$Z^{PLP}(\mathbf{x}, \mathbf{y}) = \text{Maximize} \sum_j \sum_k u_j^k b_j^k \tag{7}$$

$$s.t. \quad \sum_j \sum_k a_{ij} b_j^k \leq x_i \quad \forall i = 1, \dots, I \tag{8}$$

$$\sum_j \sum_k a_{ij} (b_j^k / s_j) \leq y_i \quad \forall i = 1, \dots, I \tag{9}$$

$$0 \leq b_j^k \leq d_j^k - d_j^{k-1} \quad \forall j = 1, \dots, J \quad k = 1, \dots, K \tag{10}$$

- d_j^k : the k th value of the discretised demand of OD j (d_j^0 is defined as 0.)
- q_j^k : the probability associated with the k th value of the discretised demand of OD j
- u_j^k : rate in terms of weight for the k th segment of OD j
- b_j^k : allocation (sales) of weight capacity for the k th segment of OD j

The original decision variable is further divided into multiple ones to represent the allocation for each segment defined by the discrete demand level (indexed by k). In the objective function value (7), one more summation is taken to consider all segments. For segment k , the rate (unit revenue) depends on the chance of realizing the demand, the right-tail probability, for that particular segment. Thus, u_j^k is set as $(1 - \sum_{i=1}^{k-1} q_j^i) u_j$. The capacity constraints of (8) and (9) are similar to (3) and (4), after taking the summations over segments. In addition, the constraint of (10) specifies that the allocation for each segment must be limited by the segment definition. The control decision now depends on the opportunity cost estimation shown in (11).

$$C_{(x,y)}^{PLP}(\mathbf{w}, \mathbf{v}) = Z^{PLP}(\mathbf{x}, \mathbf{y}) - Z^{PLP}(\mathbf{x} - \mathbf{w}, \mathbf{y} - \mathbf{v}) \tag{11}$$

3.3. Adjustment the factor based on Binomial calibration

The two LP models in the previous sub-section basically assume that the allocations are exactly the same as the sales based on the objective functions for determining the overall revenue. Does capacity commitment imply revenue generation? What happens if the sales during the dynamic booking process are quite different from the allocations? The bottom line is that LP models typically overlook the dynamic features of problems and assume the availability of perfect information when determining the solution as a whole. Thus, some prior RM studies have tried to make use of simulation-based techniques to deal with the demand that is by nature dynamic and stochastic. For example, as the first major work with a simulation-based method for the network RM problem, Bertsimas and de Boer (2005) developed a stochastic gradient algorithm, in which a number of booking request sequences are randomly generated to catch the dynamic and stochastic effect of demand arrival.

In this study, we do not follow the simulation-based approach due to its inherent computational complexity, which is further complicated by the special features of air cargo RM. Instead, we try to model the possible gap between the static allocations and the dynamic sales by adopting a very simple approach, the Binomial experiment with a fair coin, in which the probability of a head (H) or a tail (T) is equal to 0.5. Suppose that H and T represent two types of demand. The producer needs to allocate the capacity to them before a series of demand arrivals

modeled by the Binomial trial. For an experiment with an even number of trials (e.g., n), the allocated capacity for H and for T should be the same, i.e., $n/2$ units. However, the chance with the same number of realized H's and T's is only $(0.5)^n(n!)/[(n/2)!(n/2)!]$, based on the Binomial distribution. When the number of realized H's (or T's) is less than $n/2$ units, there is a loss of sales. If the loss of sales is expressed in terms of a percentage, the expected percentage loss (denoted by L_n) can be computed as (12). In the numerator, the number of lost sales is multiplied by the associated probability based on the Binomial distribution, and the multiplication of the value of 2 is due to the symmetrical assumption between H and T. In addition, we further define $(1-L_n)$ as the actual sales percentage, denoted by A_n . For the case of an odd number of trials, the advance allocation for H can be rounded up, and that for T rounded down. The sales loss percentage can be evaluated by a formula very similar to (12) with some minor modifications.

$$A_n = 1 - L_n = 1 - \frac{2 \sum_{r=1}^{n/2} \binom{n}{r} \frac{n!}{(n-r)!r!} (0.5)^n}{n} \tag{12}$$

The relationship between the actual sales percentage (A_n) and the number of Binomial trials (n) is shown as the curve referred to as “Basic Binomial” in Figure 3. Apparently, for a small number of trials, the actual sales percentage is significantly lower than 1, but the gap can really be overlooked for an experiment with many trials. The two situations in general can be viewed as the end and the beginning of the booking horizon. In addition, the relationship between the actual sales percentage (the revenue or opportunity cost estimation) and the number of trials (the available capacity), to some extent, share a similar implication with the asymptotic optimality characteristics of the bid-price control discussed in Talluri and van Ryzin (1998) and Topaloglu (2009).

When extending the above discussion to the context of an airline network, we first focus on the case of one single leg, for example, the link from BKK to TPE in Figure 1. There are two categories of demand using this leg: the local traffic (from BKK to TPE) and the through traffic (from BKK to other destinations via TPE). If they are represented by H and T, respectively, the relationship of (12) based on the analogy of Binomial experiments can be applied to the capacity allocation problem of this single-leg case. Of course, the shares for the local traffic and the through traffic do not have to be equal. However, we keep the fair-coin assumption, as the trend should be similar for different kinds of demand division.

The next stage is to handle the multiple ODs for the through traffic, for which the revenue is lost again if the allocated capacity does not match the realized demand. For example, the capacity allocated to BKK-TPE-CHI cannot be used to serve the demand of BKK-TPE-SFO. Suppose the two ODs of the through traffic, BKK-TPE-CHI and BKK-TPE-SFO, are represented by T_1 and T_2 , respectively. Their demand realization can once again be analogized by a Binomial experiment. Obviously, the real situation does not have to be the case with only two through-traffic ODs of equal shares. However, in order to make the analysis tractable, we use this simple experiment with two fair coins to link the lost/actual sales percentage to the number of trials for the problem with two stages. Their relationship is illustrated by the curve referred to as “Compound Binomial” in Figure 3. The actual sales percentage is further reduced when considering the mismatch between the ODs of the through traffic.

We consider that the objective function value of the static LP models may give rise to a problem similar to the concept of lost sales in the above discussion. The allocation made by the LP models is much more complicated when compared with the simple capacity allocation based on the Binomial experiment. However, we believe the trend is similar. For a run with many Binomial trials, the sales loss in terms of a percentage is negligible. In the meantime, for an LP-based allocation problem with massive available resources, the objective function value is very close to the revenue achieved at the end of the booking horizon. However, the LP models are probably not a good way to estimate the overall revenue in the case where resources are limited, or toward the end of the booking horizon. Therefore, we further propose the idea of applying the actual sales percentage to adjust the objective function values of the LP models when using them for opportunity cost estimation.

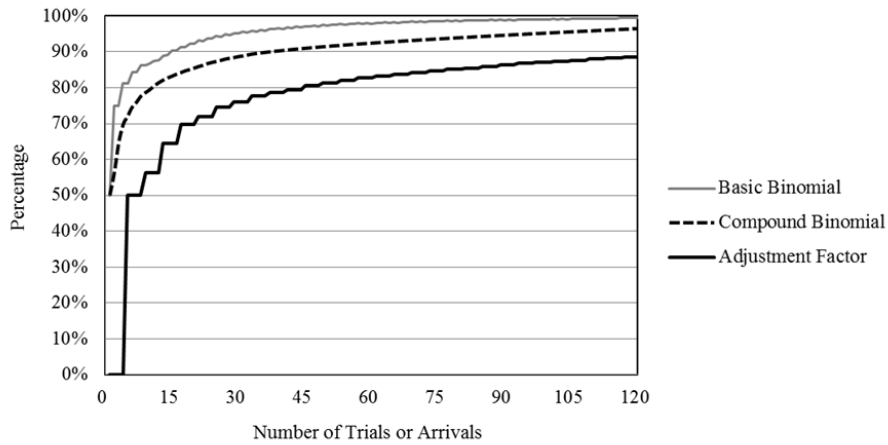


Fig. 3. Opportunity cost estimation adjustment based on Binomial experiment

However, the curves shown in Figure 3 are based on the number of Binomial trials. One step is needed to relate the dynamic booking requests arriving continuously in time to the number of Binomial trials. This transformation can be performed by calculating the average arrival time of each booking request in the ordered demand stream to serve as a yardstick. In addition, as shown in the Bellman equation of the DP model (1), the control decision in one period (t) is based on the opportunity cost estimation for the next period ($t-1$) toward the end of the booking horizon. Thus, when applying the adjustment based on the Binomial experiment, the number of the trials should be decreased by one.

Finally, the discussion above is made with respect to the single-leg case. By assuming that the chance of being used in a booking request is the same for all legs of the network, the curve is further scaled by the factor equal to the number of legs in the system. The resulting curve referred to as the “Adjustment Factor” (denoted by F_n) in Figure 3, based on the particular 4-leg example in Figure 1, can be used in adjusting the opportunity cost estimation from the LP model based on (13). As shown in Figure 3, if the “Compound Binomial” curve is shifted to the right by one unit and then expanded horizontally by the scale factor of four, it becomes the “Adjustment Factor” curve.

$$C_{(x,y)}^{PLP-B,n}(\mathbf{w}, \mathbf{v}) = F_n C_{(x,y)}^{PLP}(\mathbf{w}, \mathbf{v}) \tag{13}$$

4. Numerical Experiment

The small four-leg network in Figure 1 is used to design the test problems, for which it is possible to derive the optimal accept/deny decision for all shipments by using an integer programming (IP) model if the perfect information of the shipments is provided in advance. The first sub-section provides the basic settings for the test problem design. In the second sub-section, the simulation process and the performance evaluation method are presented. In order to better understand the behavior of the developed model and the solution approach, sensitivity analysis, as presented in the third sub-section, is performed.

4.1. Test problem settings

In terms of demand, there are eight OD pairs (routes) in the small network, and the related information regarding the OD pairs is summarized in Table 1. The arrivals of the shipment booking requests are assumed to follow non-homogeneous Poisson (NHP) processes, which have been used in many RM research works (e.g., Klein, 2007, Chen and Homem-de-Mello, 2010, and Huang and Lin, 2014) to simulate the dynamic booking requests in a network RM problem. Within the booking horizon of 30 days, the booking requests are generated based on a triangular-shaped

demand intensity pattern, which begins with zero at the beginning, increases linearly and reaches the maximum on the 28th day (2 days before flight departure), and then decreases linearly and reaches zero at the end. The shipment arrivals of all ODs are assumed to follow a similar triangular-shaped pattern, but the maximum arrival rates, shown in the third column in Table 1, are set differently to reflect different demand levels among the ODs. Given these settings, the number of the generated booking requests, including the shipments of all ODs, is about 130 on average during the 30-day booking horizon.

The shipment weights for all ODs are assumed to follow the same types of distributions, which was chosen to be a Weibull distribution ($\alpha=1.04, \beta=307$) based on the real operational data provided by a Taiwanese carrier. Thus, there are mostly small-sized shipments. The average weight is about 295 kg. The shipment density does not appear to follow any popular distribution. However, it seems to be fine to use a normal distribution ($\mu=-0.155, \sigma=0.25$) to approximate the natural logarithm of the density. The average density is about 0.86. The weight and density of a shipment is generated randomly by assuming that the two distributions are independent, and its volume can be derived by dividing the weight by the density. The rate (unit revenue) of an OD is assumed to be normally distributed, and the mean and the standard deviations of the eight ODs are shown in the fourth and fifth columns of Table 1. For a given randomly generated shipment, the revenue can be computed by multiplying the shipment’s rate by its chargeable weight, which is the larger of its weight and volume.

Table 1. Demand information related to the ODs in the test problem

ID	Route	Maximum Arrival Rate (λ_{Max})	Rate Distribution (TWD/kg)	
			Mean	Standard Deviation
1	BKK-TPE	1.0	40	2.2
2	BKK-TPE-SFO	1.4	190	3.1
3	BKK-TPE-CHI	1.3	172	8.1
4	PEN-TPE	1.1	46	1.8
5	PEN-TPE-SFO	1.2	195	3.2
6	PEN-TPE-CHI	0.8	179	4.8
7	TPE-SFO	1.0	158	3.7
8	TPE-CHI	1.9	139	8.5

For the supply side, the capacities in weight and volume of the four legs are set to maintain a 3-to-2 ratio in terms of average demand to supply for all legs, given the demand levels specified in the previous paragraphs. In general, these settings lead to a case with excessive demand, which provides the room for RM to increase revenue. Nonetheless, sensitivity analysis with respect to various demand-supply ratios is conducted to understand the performance of the proposed method.

4.2. Simulation procedure and performance evaluation

For the base case described in the previous sub-section, the booking requests are randomly generated for 50 simulation runs. In addition to the control policies based on the LP models, two benchmarking methods have been used. The first one is the first-come-first-served (FCFS) policy (effectively with no RM control), under which a booking request is always accepted as long as the system capacity is sufficiently large to accommodate it. The second one is the IP model in (14) - (17), which makes the control decision based on the advance perfect information (PI) of all shipments.

$$Z^{API} = Maximize \sum_i e_i m_i \tag{14}$$

$$s.t. \quad \sum_j \sum_k a_{ij} w_k m_i \leq x_i^C \quad \forall i = 1, \dots, I \tag{15}$$

$$\sum_j \sum_k a_{ij} v_l m_l \leq y_i^C \quad \forall i = 1, \dots, I \tag{16}$$

$$m_l : \text{binary} \quad \forall l = 1, \dots, L \tag{17}$$

- l : indices of shipments ($l = 1 \dots L$)
- e_l : revenue of shipment l
- w_l : weight of shipment l
- v_l : volume of shipment l
- a_{il} : binary constant = 1, if shipment l uses leg i ; = 0, otherwise.
- x_i^C : initial weight capacity of leg i
- y_i^C : initial volume capacity of leg i
- m_l : binary decision = 1, if shipment l is accepted; = 0, otherwise.

The objective (14) is to maximize the total revenue by considering all shipments, for which the weights and volumes are known for each simulation run. The accept/deny decision, a binary variable as specified by (17), must be made according to the capacity limitations in terms of weight and volume specified by (15) and (16). The objective function value of this IP model serves as an upper bound of the revenue for all control policies in any simulation run.

Three RM approaches are examined in this study. The first one uses the deterministic LP (DLP) model in (2) - (6) to estimate the opportunity cost and make the control decision whenever a booking request is received. The second one on the other hand uses the probabilistic linear programming (PLP) model in (7) - (11) to handle the booking requests. Finally, the third one also uses the PLP model, but the opportunity cost estimation is adjusted by the factor from the analogy of the Binomial experiment as shown in (13). This approach is later referred to as the PLP-B. With the DLP method as the example, the major phases and the associated steps of the simulation procedure can be summarized as follows:

- *Phase I - Initialization*: Generate the stream of the booking requests.
 1. For each OD j , generate the dynamic booking requests of the shipments based on the NHP process and their rates based on the normal distribution, given the parameters in Table 1.
 2. Form the demand stream by combining the booking requests of all ODs according to their arrival times, which are denoted by t_l for shipment l .
 3. For each shipment booking request l , generate the weight (w_l) and density (s_l) and then derive the volume (v_l) and revenue (e_l).
- *Phase II - Booking Control*: Determine whether the booking request is accepted and compute the revenue.
 1. Set the time at the beginning of the booking horizon, i.e., $t = T$.
 2. For each shipment booking request l , perform the following operation.
 - STEP 1: Move the time to the shipment arrival time, $t = t_l$.
 - STEP 2: Update the expected demands of the ODs within the interval of $[0, t_l]$ in (5) based on the assumed NHP process.
 - STEP 3: Run the LP model of (2) - (5) twice and estimate the opportunity cost based on (6).
 - STEP 4: Make the accept/deny decision by comparing the revenue (e_l) with the estimated opportunity cost. If the shipment is accepted, decrease the capacities in terms of both weight and volume by w_l and v_l respectively for the associated leg(s).

The same streams of booking request are used for different RM approaches. For the PLP method, the booking control phase is basically the same, except that STEP 2 for the demand update is for the LP model of (7) - (10). Based on the NHP processes assumption, the demand distribution of each OD is determined and further approximated by a 10-pt discrete uniform distribution. Then, the segment-related input data (d_j^k , q_j^k , and u_j^k) are derived. Finally, for STEP 3, the opportunity cost estimation is based on (11).

For the PLP-B method, the steps are almost the same as those for the PLP method. However, for STEP 4, the shipment arrival time (t_l) must be first transformed as the corresponding order of the arrivals (n) based on the

assumed NHP processes of the ODs. Given the difference between the two objective function values of the LP model in (11), the adjustment factor illustrated in Figure 3 is applied to estimate the opportunity cost based on (13).

For this base case, 50 streams of booking requests are generated. The results of the three RM methods as well as the revenues from the IP model of (14) - (16) with the advance perfect shipment information and the FCFS policy are summarized in Table 2.

In general, there are 130 shipments on average for each stream of booking requests. Given that the demand-supply ratio is set as 3:2, only some of the shipments should be accepted. For the IP model with perfect information in advance, 73.8% of the shipments on average are accepted. This is slightly higher than the level of 2/3, suggesting that more small-sized shipments are accepted. For the FCFS policy, the shipments are accepted without considering the opportunity costs of the associated resources. After the booking horizon begins, the capacities in terms of the weight and volume of the legs are quickly consumed, leading to poor capacity utilization. On the other hand, the shipment acceptance rate is greatly raised by the RM control policies.

Table 2. Shipment acceptance rate and revenue for various policies in the simulation

	IP w/ PI	FCFS	RM Control Policy		
			DLP	PLP	PLP-B
Shipment Acceptance Rate	73.8%	47.8%	67.8%	69.2%	70.7%
Revenue Percentage Gap					
Mean		33.0%	15.6%	11.4%	10.6%
Standard Deviation		11.2%	6.1%	4.5%	3.4%

The control performance is evaluated by referring to the objective function value of the IP model (14) - (17), which takes into account all shipments at the same time with exact weight, volume, and revenue information. The achieved value is actually higher than any dynamic control policy and can be thought of as an upper bound of the revenue that can be achieved. For each simulation run, the revenue gaps as percentages of the various control policies, including the FCFS policy, are calculated. The means and standard deviations of the percentage gaps are presented in the 2nd and the 3rd rows of Table 2. For the FCFS policy, the gap is as high as 33.0%. This gap cannot be thought of as the value of RM, as the objective function value of the IP model is more or less like the revenue of a super-optimal control policy. However, the big gap still represents the shortcoming of the FCFS control under this demand-supply level. The revenue gap is greatly reduced by the RM policies. In particular, the probabilistic LP model does outperform the deterministic LP model. In addition, the adjustment factor based on the analogy of the Binomial experiment appears to be effective in raising the revenue, as the mean value of the revenue gap for the PLP-B method is further reduced by 0.8%, and the standard deviation becomes smaller. Meanwhile, the shipment acceptance rate is increased from 69.2% to 70.7%, closer to the value of 73.8% for the super-optimal policy based on the IP model. Although the improvement of the PLP-B method over the PLP method is not particularly impressive, it is statistically significant and can be practically substantial, given the low margin commonly observed in the airline industry.

4.3. Sensitivity analysis

In order to better understand the behaviour of the proposed RM control policies, two kinds of parameters in the problem design are changed. First, the capacities in weight and volume of the four legs are adjusted so as to cause the overall demand-supply ratio to change from 12:5 to 12:11, given that the ratio in the base case is 12:8 (3:2). The results based on 30 simulation runs are summarized in Table 3.

As the demand-supply ratio decreases, the shortcoming of the FCFS policy becomes less serious. When the system capacity is close to the demand level, the FCFS policy naturally becomes a decision rule not too bad, and the value of the RM control turns out to be less significant. Among the RM control policies, the PLP-B method with the adjustment based on the Binomial experiment is apparently better than the plain PLP method. The advantage is particularly significant when the demand-supply ratio is reduced. For example, in the case with the demand-supply

ratio equal to 12:11, the percentage revenue gaps for the FCFS, PLP, and PLP-B methods are 16.5%, 12.3%, and 9.6% respectively. This finding suggests that the inaccuracy problem in opportunity cost estimation associated with the PLP method can be alleviated by the mechanism of the adjustment factor proposed in this study. In particular, the over-estimation of the opportunity cost and the subsequent false rejection of the booking request of the PLP method can be more detrimental when the demand level is lower. On the other hand, this nice feature of the PLP-B method is valuable as airlines normally would adjust their supply by changing the aircraft type and/or the flight schedule in the long run so as to maintain a demand-supply ratio close to one.

Table 3. Effect of demand-supply ratio on the performance of RM policy

Demand-Supply Ratio	FCFS	RM Control Policy		
		DLP	PLP	PLP-B
12:5	43.9%	14.1%	10.4%	10.8%
12:6	39.6%	14.4%	10.8%	10.7%
12:7	35.9%	14.7%	11.3%	10.6%
12:8 (Base Case)	33.0%	15.6%	11.4%	10.6%
12:9	27.0%	15.1%	11.4%	10.0%
12:10	21.0%	15.2%	11.9%	9.9%
12:11	16.5%	15.5%	12.3%	9.6%

The second issue tested in this study is the composition of the direct and through traffic in the network. Among the 8 ODs in Figure 1, 4 of them are single-leg or direct traffic, and the other 4 are through traffic with a transfer at the hub. The ratio between the two broad categories varies from one leg to another, based on the parameters of the maximum arrival rates (λ_{Max}) in the non-homogeneous Poisson processes. However, the average ratio of direct traffic to through traffic for a leg is roughly 1:2. For the analysis of the effect of traffic composition, the maximum arrival rate parameters are adjusted to scale the existing ratios by a factor of 0.8 to 1.2, while the relative importance among the ODs within the same category remains the same. In addition, the overall demand-supply ratio is kept as 3:2 as in the base case in the previous analysis. The results based on 30 simulation runs are summarized in Table 4.

Table 4. Effect of traffic composition on the performance of RM policy

Through-traffic Scaling Factor	FCFS	RM Control Policy		
		DLP	PLP	PLP-B
1.20	33.3%	14.6%	11.2%	10.2%
1.15	32.7%	15.0%	11.6%	10.5%
1.10	32.7%	15.4%	11.0%	10.2%
1.05	32.9%	15.6%	11.4%	10.6%
1.00 (Base Case)	33.0%	15.6%	11.4%	10.6%
0.95	32.8%	14.5%	11.2%	10.2%
0.90	32.3%	14.8%	11.0%	10.4%
0.85	32.4%	14.9%	11.0%	10.3%
0.80	32.2%	14.8%	11.1%	10.1%

For different levels of the through-direct traffic ratios, the results are basically the same. This finding implies that the performance of the RM policies is insensitive to the composition of the traffic. In general, the network model is more crucial for the situations with more through traffic. As the performance is not degraded when the share of the through-traffic is increased, the approaches developed should be suitable for handling the RM problem of air cargo

networks. The other finding from Table 4 is that the consistency for the advantage of the PLP-B method is supported by the increased number of simulation runs with respect to various through-traffic levels.

5. Conclusions

This study formulates a multi-dimensional dynamic programming model to present a network RM problem for air cargo. In order to overcome the computational challenge, the study develops two LP models to estimate the opportunity cost of the resources consumed by a booking request. The first one is a deterministic LP model, which is improved to become the second one by taking into account demand uncertainty. In particular, based on the analogy of the Binomial experiment, this study further introduces an adjustment factor to alleviate the problem of overestimation in terms of the opportunity cost of using the LP models. Based on a numerical experiment, including sensitivity analysis with respect to the ratio of demand to supply and the composition of direct and through traffic, the developed models and solution methods have been found to be effective in raising revenue and are promising in that they provide the decision support that is operationally suitable for airlines.

Future research extensions can be described as follows. The current models do not consider the issue of overbooking, which is assumed to be handled before the shipment booking process of the spot markets. Although this sequential approach is common for many airlines, the interaction between these two types of decisions can be intensive. In particular, air cargo service is a very dynamic business. It is of great importance to develop an integrated model that simultaneously considers the decision regarding the overbooking level and the control decision of the booking requests.

The other possible extension is to incorporate the choice behavior of customers. Unlike the air passenger RM, for which the booking requests are made by the huge numbers of general travelers, air cargo RM deals with a limited set of customers, airfreight forwarders or shippers that normally have long-term relationships. The choice-based models developed for passenger RM are not necessarily applicable to the air cargo industry. With a view to raising the revenue from air cargo operations, a choice-based model that considers the special features of the business environment of air cargo RM should be useful.

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