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Theory of Nernst Effect in Layered Superconductors

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Abstract. We calculate, using the time-dependent Ginzburg-Landau (TDGL) equation with thermal noise, the transverse thermoelectric conductivity α_{xy}, describing the Nernst effect, in type-II superconductor in the vortex-liquid regime. The method is an elaboration of the Hartree-Fock. An often made in analytical calculations additional assumption that only the lowest Landau level significantly contributes to α_{xy} in the high field limit is lifted by including all the Landau levels. The resulting values in two dimensions ($2D$) are significantly lower than the numerical simulation data of the same model, but are in reasonably good quantitative agreement with experimental data on La_2SrCuO_4 above the irreversibility line (below the irreversibility line at which α_{xy} diverges and theory should be modified by including pinning effects).

1. Introduction

The electric field is induced in a metal under magnetic field by the temperature gradient ∇T perpendicular to the magnetic field $\mathbf{H}$, phenomenon known as Nernst effect [1] (direction of the electric field being perpendicular to both ∇T and $\mathbf{H}$). Recently the Nernst effect in high T_c superconductors attracted attention both theoretically [1, 2, 3, 4, 5, 6] and experimentally [7, 8, 9, 10, 11, 12, 13, 14]. In these materials effect of thermal fluctuations is very strong leading to depinning of Abrikosov vortices created by the magnetic field in type II superconductor below second critical field $H_{c2}(T)$. In the mixed state the Nernst effect is large due to vortex motion, while in the normal state and in the vortex lattice or glass states it is typically smaller. The Nernst effect therefore is a probe of thermal fluctuations phenomena in the vortex matter, but in principle could shed some light on the underlying microscopic mechanism of superconductivity in cuprates.

In low critical temperature superconductors no sign of superconducting fluctuation was reported as the temperature was raised above $T_{c2}(H)$ [15]. In sharp contrast, the appearance of a fluctuation tail above the critical temperature in the Nernst signal was observed in several different high-temperature superconductors [8, 9, 10, 11, 14]. The related Ettignhausen effect was detected as well [7]. At the same time thermal fluctuations in high T_c materials lead to many other remarkable phenomena, most notably vortex lattice melting and thermal depinning well studied both experimentally and theoretically over the last two decades, so that the theory of the Nernst effect should be consistent with the theory of these phenomena.

Theory of the electronic and heat transport (including the Nernst effect) starting from the phenomenological TDGL equation strongly fluctuating superconductors was developed long ago [1, 2]. More recently within the same framework Ussishkin *et al.* [3] calculated perturbatively the low-field the Nernst effect for $T > T_c$ due to contribution of Gaussian fluctuations and obtained results in agreement with microscopic Aslamazov-Larkin [1] calculation. Mukerjee *et al.* [5] numerically simulated two dimensional TDGL equation with Langeven thermal noise for $T < T_c$ and obtained results in reasonable agreement with experimental data on $LaSCO$ at lower temperature, but the thermoelectric conductivity became independent of magnetic field at higher temperatures in contrast to experiment. The simulation of this system, even in 2D, is difficult and it was one of our goals to supplement it with a reliable analytical expression in the region of the vortex liquid, see Fig. 1. Recent understanding of the vortex matter phase diagram is summarized in Fig. 1. There are four phases separated by two transition lines [16]:

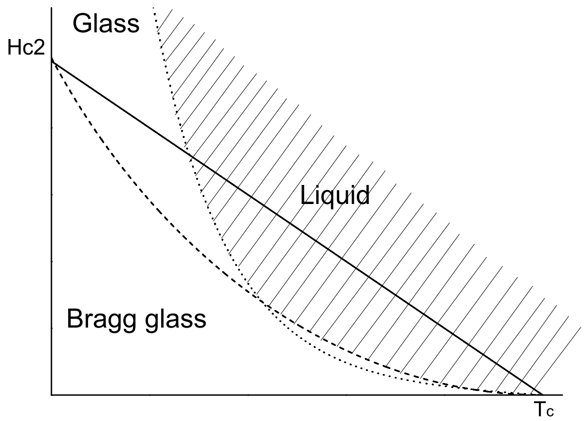

Figure 1. The thermodynamic phase diagram

the first order melting line (sometimes called the order - disorder line at lower temperatures, dashed line in Fig. 1) and the irreversibility (or glass) continuous transition. The melting line separates crystalline phases from a homogeneous phases, while the glass line (dotted line in Fig. 1) separates pinned phases from the unpinned ones. The mean field $H_{c2}(T)$ line (solid line in Fig. 1) in strongly fluctuating superconductors becomes a crossover. Both pinning and crystalline order lead to a strong reduction of the Nernst signal and will not be considered here. Therefore we will concentrate on the vortex liquid phase (dashed area in Fig. 1) and discuss the melting line and disorder only as limits of applicability of the theory. The quantitative theory of the vortex liquid have been developed recently and it was established that the Hartree-Fock approach for the thermodynamic is close to the convergent Borel-Pade one in the wide region of the vortex liquid phase [17].

In this paper we revisit the Hartree-Fock calculation in TDGL originally done in Ref. [2] to obtain explicit expressions for the transverse thermoelectric conductivity α_{xy} in $2D$. Typically only the lowest Landau level contribution was investigated. We extend it to higher Landau levels necessary for exploring the experimentally accessible parameter region and find range of applicability of the results due to approximations made, disorder and crystallization. In this theory the strength of the thermal fluctuations is described by just one dimensionless adjustable parameter η (closely related to the Ginzburg number Gi). The value of the parameter is consistent with the melting line calculated in [18].

2. The Ginzburg-Landau Model in 2D

2.1. Relaxation dynamics and thermal fluctuations

To describe fluctuation of order parameter in thin films or layered superconductors one can start with the Ginzburg-Landau free energy:

$$F = s \int d^2 x \frac{\hbar^2}{2m^*} |\mathbf{D}\psi|^2 + a|\psi|^2 + \frac{b'}{2}|\psi|^4, \tag{1}$$

where $\mathbf{A} = (-By, 0)$ describes a constant and practically homogeneous magnetic field (we generally neglect small fluctuations of the magnetic field due to magnetization of order $1/\kappa^2 << 1$ in the region of interest) in Landau gauge and covariant derivative is defined by $\mathbf{D} \equiv \nabla - i(2\pi/\Phi_0)\mathbf{A}$, with $\Phi_0 = hc/e^*$, $e^* = -2e > 0$. For simplicity we assume $a(T) = \alpha(T - T^\Lambda)$, although the temperature dependence can be easily modified to better describe the experimental coherence length. The "mean field" critical temperature T^Λ depends on the ultraviolet (UV) cutoff Λ specified later. It is higher than measured critical temperature due to thermal fluctuations on the mesoscopic scale. The thickness of a layer is s. We apply this model to describe experiments on overdoped $LaSCO$ [12].

Since we are interested in transport phenomena, it is necessary to introduce some kind of dynamics for the order parameter. The simplest is a gauge-invariant version of the "type A" relaxational dynamics,

$$\tau \left(\frac{\partial}{\partial t} + i \frac{e^*}{\hbar} \phi \right) \psi = -\frac{\delta F}{\delta \psi^*} + \zeta, \tag{2}$$

called in the present context TDGL equation. Explicitly the TDGL equation for the superconducting order parameter is

$$\tau \left(\frac{\partial}{\partial t} + i \frac{e^*}{\hbar} \phi \right) \psi = \frac{\hbar^2}{2m^*} \mathbf{D}^2 \psi - a\psi - b'|\psi|^2 \psi + \zeta, \tag{3}$$

where $\phi(\mathbf{x})$ is the scalar potential describing electric field. To incorporate the thermal fluctuations via Langeven method, the noise term $\zeta(\mathbf{x}, t)$, having Gaussian correlations

$$s\langle \zeta^*(\mathbf{x}, t) \zeta(\mathbf{x}', t') \rangle = 2T\tau \delta(\mathbf{x} - \mathbf{x}') \delta(t - t'), \tag{4}$$

introduced. Here $\delta(\mathbf{x} - \mathbf{x}')$ is the two-dimensional δ function of the in-plane coordinates.

2.2. The heat and the electric transport

We start from the definition of the transport coefficients. Generally the electric and heat transport current densities, $\mathbf{j}^{(e)}$ and $\mathbf{j}^{(h)}$, in metal are related to the applied (sufficiently weak) electric field and the temperature gradient by

$$j_{tr}^{(e)i} = \sigma^{ij} E^j - \alpha^{ij} \nabla^j T, \tag{5}$$

$$j_{tr}^{(h)i} = \widetilde{\alpha}^{ij} E^j - \kappa^{ij} \nabla^j T, \tag{6}$$

where, σ, α, $\widetilde{\alpha}$, and κ are the electrical, the thermoelectric, the electrothermal, and the thermal conductivity components of the conductivity tensor $(i, j = x, y)$. The Onsager relations implies $\widetilde{\alpha} = T\alpha$. The Nernst coefficient (ν_N), under the condition $\mathbf{j}_{tr}^e = 0$, is expressed in terms of the above coefficients as

$$\nu_N = \frac{E_y}{(-\nabla T)_x B} = \frac{1}{B} \frac{\alpha_{xy}\sigma_{xx} - \alpha_{xx}\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}. \tag{7}$$

If the system shows no significant Hall effect (only such systems will be considered), then $\sigma_{xy}=0$ and the expression simplifies:

$$\nu_N = \frac{\alpha_{xy}}{B\sigma_{xx}}. \tag{8}$$

Since near $H_{c2}(T)$ magnetization is very small (of order $1/\kappa^2 \simeq 10^{-4}$ for high T_c materials and in most of the cases the magnetization contributions vanish), we replace the transport currents by the total currents

$$\left\langle \mathbf{j}^h \right\rangle = -\frac{\hbar^2}{2m^*}\left\langle \left(\frac{\partial}{\partial t} - i\frac{e^*}{\hbar}\phi\right)\psi^* \left(\nabla - i\frac{2\pi}{\Phi_0}\mathbf{A}\right)\psi \right\rangle + c.c. \tag{9}$$

These assumptions were extensively discussed in a textbook [1] and [5].

3. The transverse thermoelectric conductivity in the vortex liquid phase

At low temperatures vortex matter organizes itself into a (usually, but not always) hexagonal vortex lattice. When disorder can be effectively neglected (either in very clean materials or when thermal depinning occurs), one can consider transport of the vortex lattice as a whole. Expressions for the electric and the thermal conductivities near $H_{c2}(T)$ neglecting thermal fluctuations were obtained in [2], and according to results the Nernst effect is generally very small compared to one in the vortex liquid. This can be qualitatively understood as a result of rigidity of the lattice. *Below* the melting line the situation in this respect does not change much. Moreover due to unavoidable presence of disorder, the vortex lattice is pinned forming a Bragg glass in most of its domain [16]. However in high T_c superconductors thermal fluctuations are strong enough (especially for high anisotropy and high magnetic fields) to destroy the expectation value of the condensate $\langle\psi\rangle = 0$. We always assume that thermal fluctuations melted away and in addition temperature is high enough to thermally depin the vortex liquid (avoiding the "vortex glass"). As a consequence impurities in the vortex liquid are neutralized.

Due to thermal fluctuations the expectation value of the order parameter in vortex liquid is zero $\langle\psi(\mathbf{x},t)\rangle = 0$. Therefore contribution to the expectation values of physical quantities like the electric and the heat current come exclusively from the correlations. The most important is the quadratic one

$$C(\mathbf{x},t;\mathbf{x}',t') = \langle \psi(\mathbf{x},t)\psi^*(\mathbf{x}',t')\rangle\,, \tag{10}$$

called the correlation function of the order parameter.

In particular the superfluid density is

$$\langle|\psi(\mathbf{x},t)|^2\rangle = C(\mathbf{x},t;\mathbf{x},t). \tag{11}$$

A simple approximation which captures the most interesting fluctuations effects in the Hartree approximation, in which the cubic term in the GL equation Eq. (3) $b'|\psi|^2\psi$ is replaced by a linear one $b'\left\langle|\psi|^2\right\rangle\psi$

$$\tau\frac{\partial}{\partial t}\psi(\mathbf{x},t) = \left(\frac{\hbar^2}{2m^*}\mathbf{D}^2 - \widetilde{a}\right)\psi(\mathbf{x},t) + \zeta(\mathbf{x},t), \tag{12}$$

leading the "renormalized" value of the coefficient:

$$\widetilde{a} = a + b'\langle|\psi|^2\rangle. \tag{13}$$

The formal solution of this equation is

$$\psi(\mathbf{x},t) = \int d\mathbf{x}'\int dt' G_0(\mathbf{x},t;\mathbf{x}',t')\zeta(\mathbf{x}',t'), \tag{14}$$

where G_0 is the equilibrium Green function which is most easily accomplished by expanding G_0 in terms of the Landau eigenfunctions.

The evaluation of $\langle|\psi(\mathbf{x},t)|^2\rangle$ gives

$$\epsilon_b = \widetilde{\epsilon}_b - \frac{b'T}{2\pi s}\frac{m^*\omega_B}{\hbar\left(\alpha T^\Lambda\right)^2}\sum_{n=0}^{N_f}\frac{1}{\widetilde{\epsilon}_b + 2nb}, \tag{15}$$

where the reduced temperature is defined as $\epsilon = a/\alpha T^\Lambda$, $\epsilon_b = \epsilon + b$, (with similar expression for $\widetilde{\epsilon}$ and $\widetilde{\epsilon}_b$) with $b = B/Hc2(0)$ being the scaled magnetic field, $Hc2(0) = \Phi_0/2\pi\xi^2$ the zero-temperature critical field and $\xi = (\hbar^2/2m^*\alpha T_c)^{1/2}$ the zero-temperature coherence length. The UV cutoff was introduced. It effectively limits the number of Landau levels to $N_f = \frac{\Lambda}{b} - 1$. The "bubble" sum, which diverges logarithmically, can be performed:

$$\frac{b}{\pi}\sum_{n=0}^{N_f}\frac{1}{2nb + \widetilde{\epsilon}_b} = \frac{1}{2\pi}\log\Lambda + u', \tag{16}$$

where the function u' is related by

$$u'(\widetilde{\epsilon}_b, b) = \frac{1}{2\pi}[f_s'(\widetilde{\epsilon}_b/2b) - \log(2b)], \tag{17}$$

to the polygamma function f_s':

$$f_s'(x) = \sum_{n=1}^{\infty}\left[\frac{1}{n+x} - \int_{n-1/2}^{n+1/2}\frac{1}{(y+x)}dy\right] + \left[\frac{1}{x} - \log\left(x + 1/2\right)\right]. \tag{18}$$

Thus the critical temperature T_c is significantly renormalized:

$$\epsilon_b^r = \epsilon_b + \frac{b'T}{4\pi s}\frac{m^*\omega_b}{\hbar\left(\alpha T^\Lambda\right)^2}\log\Lambda = \widetilde{\epsilon}_b - \frac{\eta\xi^2 T e^* H_{c2}(0)}{2\hbar c T_c}u'(\widetilde{\epsilon}_b, b), \tag{19}$$

where η is a dimensionless fluctuation parameter

$$\eta = \frac{b'T_c}{\xi^2\left(\alpha T_c\right)^2 s}, \tag{20}$$

introduced in [5]. The relation between η and more often used two dimensional Ginzburg number [1, 17], $Gi_{2D} \equiv \frac{1}{2}\left(8e^2\kappa^2\xi^2 T_c/\pi c^2\hbar^2 s\right)^2$, is

$$\eta = 4\sqrt{2Gi_{2D}}\pi^2. \tag{21}$$

Let us assume that the weak electric field $\mathbf{E}$ is along the y axis, generated by the scalar potential $\phi = -E_y y$. The heat current in the vortex liquid phase is given by [2]

$$\left\langle\mathbf{J}^h\right\rangle = -\frac{\hbar^2}{2m^*}\left[\mathbf{D}\left(\mathbf{x}\right)\left(\frac{\partial}{\partial t'} - i\frac{e^*}{\hbar}\phi\left(\mathbf{x}'\right)\right) + \mathbf{D}^*\left(\mathbf{x}'\right)\left(\frac{\partial}{\partial t} + i\frac{e^*}{\hbar}\phi\left(\mathbf{x}\right)\right)\right]C(\mathbf{x},t;\mathbf{x}',t')|_{\mathbf{x}=\mathbf{x}';t=t'}, \tag{22}$$

where

$$C(\mathbf{x},t;\mathbf{x}',t') = \frac{2\tau T}{s}\int_{\mathbf{x}_1,t_1}G(\mathbf{x},t;\mathbf{x}_1,t_1)G^*(\mathbf{x}',t';\mathbf{x}_1,t_1), \tag{23}$$

with G is the Green function of the linearized TDGL equation in the presence of the scalar potential. One finds correction to the Green function to linear order in the electric field

$$G(\mathbf{x},t;\mathbf{x}',t') = G_0(\mathbf{x},t;\mathbf{x}',t') - i\frac{e^*\tau}{\hbar}\int_{\mathbf{x}_1,t_1}\phi(\mathbf{x}_1)G_0(\mathbf{x},t;\mathbf{x}_1,t_1)G_0(\mathbf{x}_1,t_1;\mathbf{x}',t'). \tag{24}$$

In order to determine the transverse thermoelectric conductivity, we need to compute the x component of the heat current to first the electric field. In the chosen gauge, the electrothermal conductivity $\widetilde{\alpha}_{xy} = \frac{\overline{j_x^h}}{E_y}$ (averaged over $\mathbf{x}$) takes a form

$$\widetilde{\alpha}_{xy} = \frac{e^*Tb}{\hbar\pi s}\sum_{n=0}^{N_f}\left[\frac{n+1/2}{2nb+\widetilde{\epsilon}_b} - \frac{n+1}{2(n+1/2)b+\widetilde{\epsilon}_b}\right] = \frac{e^*T(b-\widetilde{\epsilon}_b)}{2\hbar bs}\Big[u'(\widetilde{\epsilon}_b,b) - u'(\widetilde{\epsilon}_b+b,b)\Big], \tag{25}$$

where function u' was defined in Eq. (17). Using the Onsager relation one obtains the transverse thermoelectric conductivity $\alpha_{xy} = \widetilde{\alpha}_{xy}/T$. The vortex liquid energy gap $\widetilde{\epsilon}_b$ as a function of ϵ_b, and substituting the results into Eq. (25). Equations (19) and (25) agree with the calculation of Ullah and Dorsey [2].

4. Comparison with experiment and MC simulation

The experiment results of Y. Wang *et al.* [12] obtained from the Nernst effect and resistivity measurements on an overdoped $LaSCO$ sample with $x = 0.2$ and $T_c = 28K$. The comparison is presented in Fig. 2 (low temperatures in (a) and close to T_c in (b)). The parameters used in the calculation are (see definitions above) $H_{c2}(0) = 45T$ (thus $\xi = 27A^o$) and layer spacing $s = 16A^o$. The fluctuation parameter is $\eta = 0.25$ and provides a reasonable quantitative agreement between theory and experiment. Below irreversibility line where the theory should be modified including both pining and crystalline phase in Fig. 2(a). The deviation develops roughly at the location of the irreversibility line. However, our results are in good quantitative agreement with experiment data for temperature close to T_c in Fig. 2(b), where the numerical simulation gives a nearly constant α_{xy}, while the experiment shows more variation.

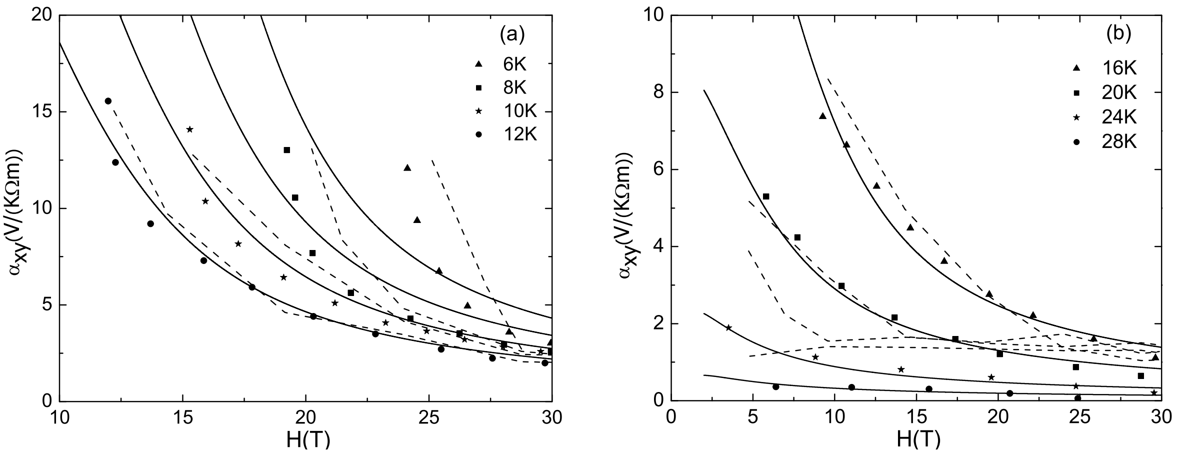

Figure 2. Points are α_{xy} for different temperatures of $LaSCO$ Ref. [12], with x=0.2 (overdoped, $T_c = 28K$). The dashed line is the simulation value of α_{xy} Ref. [5]. The solid line is the theoretical value of α_{xy}, using $H_{c2}(0) = 45T, s = 16A^o, \eta = 0.25$.

5. Conclusion

We obtained, using TDGL equation with thermal noise, explicit expressions for the transverse thermoelectric conductivity α_{xy} in $2D$ including all Landau levels in type-II superconductor in the vortex-liquid regime. The method is the Hartree-Fock. We also obtained the relation between the strength of the thermal fluctuation η and the Ginzburg number Gi. We compared the results to the $2D$ simulation and the experiment results. Our results in $2D$ are significantly lower than the simulation and experiment data below the irreversibility line at which theory should be modified by including both pinning and crystalline effects, but are in reasonably good quantitative agreement with experimental data on La_2SrCuO_4 for temperature close to T_c. In the future work, we will compare with other materials.

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