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A study of optimal weights of Data Envelopment Analysis – Development of a context-dependent DEA-R model

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ABSTRACT

The weight is one of the main issues of Data Envelopment Analysis (DEA), and relevant theoretical research indicates that many DEA mathematical models include redundant restraints on weight, resulting in underestimated efficiency, pseudo inefficiency, and difficulty in representing specific Input/Output relationships. This study proposes a context-dependent DEA-R model to address shortcomings resulting from redundant restraints on the weights of an efficient decision making unit (DMU), and converts the optimal weight to analyze the influences of redundant restraints on weights. The evaluation results of Taiwan medical centers show that the efficiency of the DMU is underestimated and pseudo inefficiency may occur due to redundant restraints on weight. Moreover, optimal weights are used as variables to conduct cluster analysis in order to determine the information of the weights. The results of cluster analysis indicate that it can assist DMUs in understanding the relationships between DMUs, and contribute to the development of a unique survival strategy for hospitals.

1. Introduction

Discussions of how to improve the efficiency of organizations are very important, and one of the most representative methods of efficiency evaluation is Data Envelopment Analysis (DEA) (Wei, Chen, Li, & Tsai, 2011), which is constructed on the basis on two concepts, namely, non-dominance solutions, as proposed by Italian economist Pareto (1927), and relative efficiency through production frontier evaluation, as proposed by Farrell (1957). Based on these two concepts, Charnes, Cooper, and Rhodes (1978) developed a mathematical programming approach to calculate the efficiency of decision making units (DMUs), where DMUs with efficiency value equal to 1 are considered efficient, and those less than 1 are considered inefficient. This method uses a mathematical programming approach to determine the efficiency of a DMU, which are collectively referred to as DEA, and the first DEA mathematical model is called a CCR. DEA is characterized by each DMU's ability to select its most favorable weight, and evaluate its relative efficiency among a set of DMUs. In addition, DEA is both objective and subjective in its method of efficiency evaluation, as the subjective opinions of experts and decision makers can be incorporated.

In the field of DEA, many researches have discussed the issue of weight. Some research discusses how to incorporate preferences or expert opinions into weight restrictions, such as Dyson and Thanassoulis (1988), Thompson, Langemeier, Lee, Lee, and Thrall (1990), and Wong and Beasley (1990). Other researches have focused on how to modify the models and limit weight within a reasonable range, such as the assurance region (AR) concept proposed by Thompson, Singleton, Thrall, and Smith (1986); the cone ratio concept and its applications by Charnes, Cooper, Wei, and Huang (1989) and Charnes, Cooper, Huang, and Sun (1990); the common weight concept proposed by Roll, Cook, and Golany (1991), and further developed by Roll and Golany (1993). Despić, Despić, and Paradi (2007) pointed out that the CCR model included an imperceptible redundant restraint on weights, making it difficult to represent the weight relationship and the influences of single Input/Output, thus, the novel DEA-R model was proposed by Despić et al. (2007) to avoid such problems. Moreover, other research pointed out that such restraints could lead to underestimations and pseudo inefficiency, namely, an efficient DMU being judged as inefficient. However, when DMUs are evaluated by an ordinary DEA-R model, the efficiency level of all efficient DM is 1 and the optimal weight has multiple solutions. Identical efficiency scores of DMUs make evaluating the influence difficult to understand,

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namely, how redundant restraints of weights affect an efficient DMU, and the degree of advantage of the efficient DMU. Moreover, multiple solutions may lead to analysis errors. Thus, a context-dependent concept is introduced to develop the original DEA-R model into a context-dependent DEA-R model, which is applied to discuss the influence of redundant restraints on weights, as applied in the case study of medical centers. Cluster analysis is also employed to provide an in-depth analysis of the weights.

In sum, the purposes of this research include: (1) develop a context-dependent DEA-R model and the conversion of CCR weights to DEA-R weights, and then, compare the evaluation results to gain an understanding of the influence of redundant restraints on weight for efficient DMUs; (2) conduct cluster analysis for further understanding of the differences of the various models in weight selection; (3) perform a case study to make suggestions for medical organizations, enabling them to offer additional high-quality medical services.

This study is organized as follows. Section 1 describes the motives and purpose of this research; Section 2 introduces the context-dependent CCR and context-dependent DEA-R models, and explains the reasons for selecting each model; Section 3 contains the case description and efficiency evaluation, as well as explaining the reasons for and suitability of the case selection. In addition, by analyzing the evaluation results, the influence of redundant weight restraints is clarified. Section 4 conducts cluster analysis of the optimal weight of the two models in order to provide further insight into the information hidden within the weights; and Section 5 offers conclusions.

2. Description of an efficient evaluation model

In this study, context-dependent CCR and context-dependent DEA-R models are selected for evaluation. This study selects the model based on CCR, as it is the first equation of DEA and one of the most popular models. However, the CCR model includes redundant weight restraints, making it difficult to represent specific Input/Output weight relationships, and possibly resulting in underestimated efficiency or pseudo inefficiency. Hence, the DEA-R model, which is the model without redundant weight restraints, is selected to develop an extended model. To understand the degree of advantage of an efficient DMU, and the influences of redundant weight restraints on an efficient DMU, it must be evaluated through the extended model. Prior to the concept of context-dependency, Andersen and Petersen (1993) proposed a concept for model extension, which boasted super efficiency, to evaluate an efficient DMU. However, because efficient DMUs are not evaluated against the same reference set as the super efficient model, Seiford and Zhu (2003) proposed the concept of context-dependency, which identifies all DMUs through an efficiency frontier, then removes these DMUs and determines a second-level efficiency frontier from the remaining DMUs, and continues until no DMUs remain. Efficient DMUs of a first-level efficiency frontier can be applied to the second-level, or other levels of efficiency frontiers, in order to evaluate their efficiencies. To overcome the shortcomings of the context-dependent CCR model, Morita, Hirokawa, and Zhu (2005) developed a context-dependent SBM model, and evaluated 14 Japanese power companies. This study develops a context-dependent model based on DEA-R to address the possible problems of the context-dependent CCR model.

In addition, input oriented models, which take the reduction of inputs as an improved strategy for the inefficient DMU, are adopted in this study, as increased output does not correspond with increased revenues under Taiwan's medical payment system, which executes a global budget system. In the input oriented model, an efficient DMU has a strong advantage if it can maintain its efficiency with large increases of inputs; otherwise, an efficient DMU would have less advantage. In other words, there are many resources available for the DMU with a strong advantage position. In practice, such resources have not yet been applied to unlimited outputs, such as medical research.

2.1. Context-dependent CCR model

In the context-dependent CCR model, l denotes lth level of the evaluation; o denotes the object DMU; θ_o^l denotes the efficiency of the DMU of the lth level evaluation; x_{ij} or x_{io} denotes the i-th input of the DMU; y_{rj} or y_{ro} denotes the rth output of the DMU; v_{io}^l denotes the ith input weight of the DMU in lth level evaluation; u_{ro}^l denotes the rth output weight of DMU in lth level evaluation; J^l denotes DMU as the reference set in lth level evaluation; and ε is an Archimedean number. In the context-dependent model, the basic model is repetitively used to evaluate the efficiency of each DMU in every level until the completion of efficiency calculations at every level. The input oriented CCR, which is the basic model of the input oriented context-dependent CCR model, is expressed below:

$$\begin{aligned} \max \quad & \theta_o^l = \sum_{r=1}^{s} u_{ro}^l \times y_{ro} \\ s.t. \quad & \sum_{i=1}^{m} v_{io}^l \times x_{ij} \geqslant \sum_{r=1}^{s} u_{ro}^l \times y_{rj} j \in J^l \\ & \sum_{i=1}^{m} v_{io}^l \times x_{io} = 1 \\ & v_{io}^l, u_{ro}^l \geqslant \varepsilon > 0 \end{aligned} \tag{1}$$

The context-dependent CCR model can calculate the efficiency θ_o^l of the various levels, as per the following steps:

Step 1: set l as 1, let all DMUs be contained in J^1, namely, all DMUs are taken as the baseline of the first level. After calculating the efficiency of all DMUs, through the CCR model, incorporate those DMUs with an efficiency value equal to 1 into E^1.

Step 2: let $J^{l+1} = J^l - E^l$. Stop this step when no DMU exists in J^{l+1} otherwise enter into Step 3.

Step 3: take J^{l+1} as the baseline of $l + 1$ level. After calculating the efficiency of all DMUs through the CCR model (1), incorporate those DMUs, with an efficiency value equal to 1, into E^{l+1}, and then enter into Step 4.

Step 4: let $l = l + 1$, and return to Step 2.

2.2. Context-dependent DEA-R model

In the context-dependent DEA-R model, l denotes lth evaluation level; o denotes the object DMU; θ_o^l denotes the efficiency of the DMU in the l-th evaluation level; x_{ij} or x_{io} denotes the ith input of the DMU; y_{rj} or y_{ro} denotes the rth output of the DMU; W_{iro}^l denotes the weight of the ith input versus the rth output ratio of the object DMU in the lth evaluation level; and J^l denotes the DMU as the baseline in the lth level evaluation. In the context-dependent DEA-R model (2), DEA-R is repetitively used to evaluate the efficiency of each DMU in every level until the completion of the efficiency calculations of every level. The input oriented DEA-R model, which the input oriented context-dependent DEA-R is based on, is expressed below:

$$\begin{aligned} \max \quad & \theta_o^l \\ s.t. \quad & \sum_{i=1}^{m} \sum_{r=1}^{s} W_{iro}^l \frac{(X_{ij}/Y_{rj})}{(X_{io}/Y_{ro})} \geqslant \theta_o^l, \quad j \in J^l \\ & \sum_{i=1}^{m} \sum_{r=1}^{s} W_{iro}^l = 1 \\ & W_{iro}^l \geqslant 0, \quad \theta_o^l \geqslant 0 \end{aligned} \tag{2}$$

The context-dependent DEA-R model can calculate the efficiency θ_o^l of the various levels, as per the following steps:

Step 1: set l as 1, let all DMUs be contained in J^1, namely, all DMUs are taken as the baseline of the first level. After calculating the efficiency of all DMUs, through the DEA-R model, incorporate those DMUs with an efficiency value equal to 1 into E^1.
Step 2: let $J^{l+1} = J^l - E^l$. Stop this step when no DMU exists in J^{l+1} otherwise enter into Step 3.
Step 3: take J^{l+1} as the baseline of $l+1$ level evaluation. After calculating the efficiency of all DMUs through the DEA-R model (1), incorporate those DMUs, with an efficiency value equal to 1, into E^{l+1}, and then enter into Step 4.
Step 4: let $l = l + 1$, and return to Step 2.

Next, the two context-dependent models are applied to evaluate the same case, and then, compare their results.

2.3. Corresponding weights

Because the numbers of CCR and DEA-R weights are different, it is not suitable to directly compare CCR weights with DEA-R weights, for instance, in a case where there are 2 input variables and 3 output variables, there are five CCR weights, and six DEA-R weights, thus, the two models' weights are not suited for direct comparison. In order to compare the two models' weights and determine the influence of the redundant weight restraints, this study converts the CCR weights to corresponding weights according to the variable relationships of the weights. Because the input variable i is related to DEA-R weight W_{ir}, and the CCR weight v_i, and the output variable r are related to DEA-R weight W_{ir} and CCR weight u_r, the corresponding CCR weight to the DEA weight could be set as $W'_{ir} = v_i x_i \times u_r y_r \times t$; and, $\sum_{i=1}^{m} v_i \times x_{io} = 1$; and $\sum_{i=1}^{m}\sum_{r=1}^{s} W_{ir} = 1$. Therefore, $t = 1/\sum_{r=1}^{s} u_r y_r$ and the transformation of the CCR weight to its corresponding weight, $w'_{ir} = (v_i x_i \times u_r y_r)/\sum_{r=1}^{s} u_r y_r$, are obtained.

After the corresponding weights are obtained, and each CCR weight with a corresponding DEA-R weight is converted, there are DEA-R weights remaining, as the CCR weights cannot cover all. By analyzing the corresponding weights, it is found that, the corresponding weights include the restraints, of which $w_{11}:w_{21}...:w_{m1} = w_{12}:w_{22}...:w_{m2} = \cdots = w_{1s}:w_{2s}...:w_{ms} = v_1:v_2...:v_m$ are input variables, and $w_{11}:w_{12}...:w_{1s} = w_{21}:w_{22}...:w_{2s}] = \cdots = w_{m1}:w_{m2}...:w_{ms} = u_1:u_2...:u_s$ are output variables. Moreover, these restraints are the mathematical representation of the redundant weight restraints and could explain why the corresponding CCR weights could not cover all DEA-R weights.

Next, corresponding efficiency is calculated by placing the corresponding weights into the DEA-R model, and then, the DEA-R efficiency, with redundant weight restraints, is calculated by placing weight restraints on the DEA-R model weights. Then, the influences of the differences of summing methods, differences of weight selections, and redundant weight restraints are distinguished by calculating four diverse efficiencies: CCR efficiency, corresponding efficiency, DEA-R efficiency with restraints, and DEA-R efficiency without restraints. The difference between the CCR efficiency and the corresponding efficiency could represent the influences of the differences between the CCR summing method, $\frac{\sum_{r=1}^{s} u_r Y_{ro}}{\sum_{i=1}^{m} v_i X_{io}}$, and the DEA-R summing method, $\sum_{i=1}^{m}\sum_{r=1}^{s} W_{ir} \frac{(X_{ij}/Y_{rj})}{(X_{io}/Y_{ro})}$. In addition, the difference between corresponding efficiency and the DEA-R efficiency, with redundant weight restraints could represent the influence of differences between CCR and DEA-R weight selections. Finally, the difference between DEA-R efficiency, with and without redundant restrain, could represent the influence of the redundant restraint on the weight, however, because of the added restraint, it is inferable that the efficiency with the redundant restraint on the weight is no greater than the efficiency without the restraint. To confirm this inference of redundant restraint on weight, medical originations will be evaluated.

3. Comparison of efficiency and optimal weight

3.1. Case description

This study evaluates data of Taiwan's medical centers from 2007, taken from the "The Statistical Annual Report of Medical Care Institutions Status & Hospitals Utilization", as collected by the Department of Health. The medical institutions in Taiwan are worthy of discussion as the coverage of medical insurance has reached 99%, and 95% of medical institutions can deliver high quality medical services to the insurant at reasonable cost. Moreover, the growing demands of the public for medical treatments means, that competent authorities must adopt a global budget system to cover both public health and financial integrity. Therefore, how to offer high-quality medical services with limited public resources has become a hot topic, and DEA has been widely used to evaluate the efficiency of medical institutions at various levels. According to the investigation of the Department of Health, the utilization rate of large medical institutions is higher than its peers. Hu and Huang (2004) pointed out that with the implementation of healthcare insurance, large medical institutions in Taiwan have attracted increasing numbers of patients, thus, only such large high-level medical centers are selected for this study, which consists of 21 medical centers, including 7 state-owned hospitals (33%), and 14 privately owned hospitals (67%).

Sickbeds and doctors are selected as input, while outpatients, inpatients, and surgery are selected as output. The data are listed in Table 1, and the correlation coefficients of the variables are listed in Table 2. The number of Input and Output variables, which is smaller than half the number of DMUs, and the correlation coefficients of Input/Output variables, which is larger than 0.7, and thus, does not violate empirical rules, and variable selection is unquestioned.

3.2. Comparison of the efficiency of context-dependent CCR and context-dependent DEA-R

This study compares the efficiency levels of a context-dependent CCR with a context-dependent DEA-R model to explain why the context-dependent DEA-R is developed. In addition, in Section 3.3, the weight of DMUs with different efficiency levels are analyzed to learn the influence of redundant weight restraints for DMU efficiency. The efficient frontier of the DMU, the first-level efficiency of CCR and DEA-R, the second-level efficiency of CCR and DEA-R, as well as the differences of CCR and DEA-R efficiency are listed in Table 3.

Observations of efficient DMUs show that by definition, DMUs 02, 03, 13, 14, and 21 are first level CCR and DEA-R, with efficiency levels equal to 1 are on the first level of the efficiency frontier, namely, efficient DMUs, as marked in bold in Table 3. All efficient DMUs are removed in order to re-determine the efficient frontier through the remaining DMUs, in order to calculate efficiency. To distinguish various efficient frontiers, the efficiency frontier determined the first time is called the first level efficient frontier, and from the second time is called the second level efficient frontier, and so on. Like an efficient frontier, the relative efficiency obtained from the first level efficient frontier is called the first level efficiency, and that obtained from the second level efficient frontier is called the second level efficiency.

Table 1
Input and output data of Taiwan medical centers in 2007.

DMU	Input1 Sickbed	Input2 Doctor	Output1 Outpatient	Output2 Inpatient	Output3 Surgery	DMU	Input1 Sickbed	Input2 Doctor	Output1 Outpatient	Output2 Inpatient	Output3 Surgery
01	3721	1158	2,319,835	1,009,763	81,855	11	1311	415	1,387,916	364,970	36,209
02	2909	976	2,455,352	854,531	80,085	12	1250	542	1,053,882	318,096	20,846
03	2661	708	1,877,506	691,048	41,424	13	1130	421	1,856,101	329,073	31,196
04	2632	1156	2,104,800	666,980	41,371	14	1053	300	1,367,840	287,960	30,426
05	2062	552	1,646,344	434,422	32,737	15	985	307	598,405	248,012	17,029
06	1771	549	1,677,396	390,950	36,787	16	925	309	341,951	260,572	16,087
07	1676	515	1,388,045	412,189	33,124	17	921	391	1,090,327	213,138	24,911
08	1658	602	1,987,233	409,152	21,573	18	981	310	877,364	242,451	15,016
09	1515	573	1,486,432	364,272	26,273	19	776	326	963,372	174,565	20,203
10	1406	473	1,163,799	346,212	24,143	20	756	329	1,309,539	196,162	14,194
						21	340	167	1,209,475	49,624	6891

Table 2
Correlation coefficients of Input and Output Variables.

	Input1 Sickbed	Input2 Doctor	Output1 Outpatient	Output2 Inpatient	Output3 Surgery
Input1 Sickbed	1.000**				
Input2 Doctor	0.940**	1.000**			
Output1 Outpatient	0.790**	0.793**	1.000**		
Output2 Inpatient	0.986**	0.933**	0.786**	1.000**	
Output3 Surgery	0.897**	0.832**	0.775**	0.932**	1.000**

** *P*-value < 0.01 (two-tails).

Table 3
Efficient Frontier, Efficiency and Efficiency Difference of DMUs.

DMU	CCR			DEA-R			Difference between CCR and DEA	
	L	First level efficiency	Second level efficiency	L	First level efficiency	Second level efficiency	First level efficiency	Second level efficiency
01	3	0.9555	0.9915	3	0.9555	0.9915	0	0
02	1	1	1.0534	1	1	1.0540	0	0.0006
03	1	1	1.1099	1	1	1.1099	0	0
04	5	0.8634	0.9075	5	0.8640	0.9091	0.0006	0.0016
05	3	0.8135	0.8949	3	0.8152	0.8949	0.0018	0
06	3	0.7843	0.8750	3	0.7869	0.8908	0.0026	0.0157
07	3	0.8708	0.9101	3	0.8708	0.9101	0	0
08	3	0.8453	0.9101	3	0.8483	0.9362	0.0030	0.0261
09	4	0.8216	0.8698	3	0.8228	0.8731	0.0012	0.0033
10	4	0.8436	0.8827	4	0.8470	0.8838	0.0034	0.0011
11	2	0.9801	1	2	0.9817	1	0.0016	0
12	4	0.8675	0.9120	4	0.8683	0.9133	0.0008	0.0013
13	1	1	1.1901	1	1	1.2636	0	0.0736
14	1	1	1.3084	1	1	1.3322	0	0.0238
15	4	0.8860	0.9186	4	0.8860	0.9186	0	0
16	2	0.9609	1	2	0.9609	1	0	0
17	2	0.9361	1	2	0.9361	1	0	0
18	3	0.8650	0.8893	3	0.8687	0.8893	0.0037	0
19	3	0.9089	0.9983	2	0.9105	1	0.0016	0.0017
20	2	0.9400	1	2	0.9907	1	0.0507	0
21	1	1	2.0536	1	1	2.0536	0	0

Then ensure that all second-level efficiency levels of efficient DMUs are larger than 1. The efficient DMU with a higher second level efficiency denotes a DMU with obvious advantages; otherwise, the efficient DMU with a lower second level efficiency is a DMU with an insignificant advantage. If the increased input exceeds the second level efficiency, the DMU will drop from a first level to third level efficient frontier, take DMU 13 as an example; the CCR second level efficiency of DMU 13 is 1.1901, which denotes that DMU 13 remains at the second level efficient frontier when sickbeds of DMU 13 increase from 1130 to 1345 (=1130 × 1.1901), and doctors increase from 421 to 501. However, if the increase exceeds this value, then DMU 13 drops to a third level efficient frontier. DMU 21 is an efficient DMU with a maximum second level efficiency of 2.0536. DMU 2 is an efficient DMU with minimum second level efficiency of 1.0540. This result means that although both DMU 21 and DMU 02 are efficient DMUs, their advantages have a large difference. DMUs with fewer advantages may drop from the first level to the third level efficient frontier more easily than DMUs with stronger advantages. Thus, when evaluating according to a context-dependent model, the evaluator must identify the efficient DMUs; moreover, there must be an understanding of the degree of the advantages of the efficient DMUs.

According to the second-level efficiency of inefficient DMUs 11, 16, 17, and 20 are CCR second level, with efficiencies equal to 1; and DMUs 11, 16, 17, 19, and 20 are DEA-R second level, with efficiencies equal to 1. According to the definition, a DMU whose second level efficiency is equal to 1 is on the second level efficient frontier. Dive DMUs, which are on the second level efficient frontier, are found using DEA-R, whereas, only 4 DMUs are found using CCR. The results show that DMU 19, on the second-level efficient frontier, that cannot be found by CCR is similar to pseudo inefficiency that some DMUs of first-level efficient frontier cannot be found. Thus, this study infers that a context-dependent CCR, like CCR, may lead to pseudo inefficiency.

Finally, analysis of the differences between efficiencies of the context-depend CCR and context-depend DEA-R shows that DEA-R could be used to calculate efficiencies equal to or higher than that of CCR. There are 11 DMUs (DMU 04–06, 08–12, 18–20) whose first level DEA-R efficiency is larger than CCR efficiency, and 10 DMUs (DMU 02, 04, 06, 08–10, 12–14, 19) whose second level DEA-R efficiency is larger than CCR efficiency. The differences of efficiency are listed in last two columns of Table 3, and marked with a border. The result of DEA-R efficiency being higher than CCR efficiency confirms the previous theoretical research. The practical meaning of this result is that DEA-R could locate more available resources without affecting the judgment of an efficient frontier. Such results infer that the context-dependent CCR may lead to pseudo inefficient results, and indicates that the context-dependent DEA-R model is a better choice than context-dependent CCR, and developing the context-dependent DEA-R is worthy of research.

3.3. Comparison of optimal weights for context-dependent CCR and context-dependent DEA-R

To explain why the efficiency of a context-dependent DEA-R model is higher than or equal to that of the context-dependent CCR model, the optimal weights of the two models are compared. Due to the difference of the numbers of context-dependent DEA-R weights and context-dependent CCR weights, such as, 6 weights for DEA-R and 5 weights for CCR in this case, the CCR weight must be converted into a corresponding weight. As per the description in Section 2.3, the corresponding weight is $w'_{ir} = (v_i x_i \times u_r y_r)/\sum_{r=1}^{s} u_r y_r$. Next, the corresponding weight is placed into the DEA-R model to calculate the corresponding efficiency. Since the second level efficiency has a greater difference and is capable of evaluating the advantage degree of an efficient DMU, the second level optimal weight is taken for comparison. The optimal CCR and DEA-R weight sets are represented in Table 4. Corresponding weight and corresponding efficiency are listed in Table 5.

3.3.1. DMU with the same efficiencies

Take DMU 21, with the highest efficiency, as an example, where the DEA-R efficiency of DMU 21 is no different than the CCR efficiency, where the corresponding weight set of the CCR weight is $w_{11} = (v_1 x_1 \times u_1 y_1)/\sum_{r=1}^{s} u_r y_r = 1 \times 2.0536/2.0536 = 1$, $w_{21} = 0$, $w_{12} = 0$, $w_{22} = 0$, $w_{13} = 0$, $w_{23} = 0$. In this way, the CCR corresponding weight of DMU 21 is the same as the DEA-R weight, meaning that CCR and DEA-R show consistent viewpoints on the weight selection for DMU 21.When the efficiencies of DMU 01, 03, 05, 15, 18, and 2 are evaluated, neither the efficiency, nor the corresponding weight calculated for these two models show any differences from the DEA-R optimal weight. However, when DMU 11, 16, 17, and 20 are evaluated, the second level efficiencies are equal to 1 and CCR corresponding weights are not consistent with DEA-R optimal weights. Thus, the following conclusions can be drawn: (1) the consistency of efficiency between the two models is not correlated with either the high or low efficiency; (2) when the efficiency is 1, the influence of weight on efficiency is unclear due to the multiple solutions of the optimal weight; and (3) when the context-dependent efficiency is not 1, and the efficiency levels of CCR and DEA-R are consistent, the weights of two models must be consistent.

3.3.2. DMU with different efficiencies

Observe the DMUs which CCR efficiency level differ from DEA-R's efficiency level. Taking DMU 13, with the largest difference of efficiency levels as an example, the corresponding weight of the context-dependent CCR, which is $(w_{11}, w_{21}, w_{12}, w_{22}, w_{13}, w_{23}) = (0, 0.805, 0, 0, 0, 0.195)$, shows an obvious difference with the optimal weight of context-dependent DEA-R, which is $(w_{11}, w_{21}, w_{12}, w_{22}, w_{13}, w_{23}) = (0.590, 0, 0, 0, 0, 0.410)$. As previous argument in Sections 2 and 3 stated, the differences of CCR's and DEA-R's efficiency levels are attributed to three factors: the difference of the summing method, the difference of weight selection, and redundant weight restraints. To distinguish the influences of the three factors, the corresponding weight is first placed into the constraint of context-dependent DEA-R to calculate the corresponding efficiency. Then, the difference of corresponding efficiency, in relation to the CCR efficiency, is caused by the different of summing methods, and the difference of corresponding efficiency, in relation to the DEA-R efficiency, is caused by the difference of weight selection and redundant weight restraints. Take DMU 08 as an example, the difference of the corresponding weight's efficiency of 1.2266, for the CCR efficiency of 1.1901, is caused from the different summing methods, and the difference of the corresponding efficiency with DEA-R's efficiency of 1.2636, is caused from weight. The total difference between the two models is 0.0736, of which 0.0365 (49.7%) is caused by the summing method, and 0.0370 (50.3%) is caused by weight. The influences of the summing method and weight are listed in Table 5. Although the influence of summing method and weight are distinguished, the influences of weight selection and redundant weight restraints are unclear. Therefore, after understanding the mean of the redundant weight restraints, defining the influence of redundant weight restraints will follow.

3.3.3. The influence of redundant restraints on weight

It is found that the corresponding weight of CCR to DEA-R has the following characteristic: if one output variable has no advantage, then, no corresponding weights related to this output will be selected. In other words, CCR includes the assumption that restrains on input weights conform to w_{11}:w_{21}...:w_{m1} = w_{12}:w_{22}...:w_{m2} = ⋯ = w_{1s}:w_{2s}...:w_{ms} = v_1:v_2...:v_m, and output weights conform to w_{11}:w_{12}...:w_{1s} = ⋯ = w_{m1}:w_{m2}...:w_{ms} = u_1:u_2...:u_s. Take the corresponding weight of DMU 13 as an example, if input 1 has no advantage, no advantage exists no matter the input, 1 produces output 1 or output 2, the corresponding DEA-R weight, will not confirm this restraint. Take optimal DEA-R weight of DMU 13 as an example. A doctor attracting an inpatient has an advantage, the weight relating to the doctor and inpatient could be selected even if the outpatient has no advantage. These restraints are the mathematic representation of redundant restraints on weight.

Because corresponding weights, which are the weights selected by the CCR model, are not optimal weights in DEA-R, the DEA-R efficiency and optimal DEA-R weight, with redundant weight restraints, are computed. Moreover, the different between corresponding efficiency and DEA efficiency with redundant restraints on weight is caused by the influence of the different models' weight selections. The difference between DEA efficiency, both with and without redundant restraint, is caused by the influence of the redundant weight restraints. In this case, redundant weight restraints affect the efficiencies of DMU 08 and 13. The weight with redundant restraint on weight, CCR efficiency, corresponding efficiency, DEA-R efficiency with redundant restraint, DEA-R efficiency without redundant restraint, and the influences of three factors are as listed in Table 6.

In sum: (1) the difference between CCR and DEA-R efficiencies are caused by the summing methods, weight selections, and redundant weight restraints. In addition, the influences of the three factors could be distinguished by CCR efficiency, corresponding

Table 4
Second Level CCR and DEA-R Optimal Weight.

DMU	Second level CCR optimal weight					Second level DEA-R optimal weight					
	Input1	Input2	Output1	Output2	Output3	I 1/O 1	I 2/O 1	I 1/O 2	I 2/O 2	I 1/O 3	I 2/O 3
1	0.000	1.000	0.000	0.992	0.000	0.000	0.000	0.000	1.000	0.000	0.000
2	1.000	0.000	0.000	1.022	0.031	0.000	0.000	0.979	0.000	0.021	0.000
3	0.000	1.000	0.000	1.110	0.000	0.000	0.000	0.000	1.000	0.000	0.000
4	1.000	0.000	0.014	0.894	0.000	0.008	0.000	0.992	0.000	0.000	0.000
5	0.000	1.000	0.000	0.895	0.000	0.000	0.000	0.000	1.000	0.000	0.000
6	0.000	1.000	0.574	0.301	0.000	0.000	0.843	0.000	0.000	0.000	0.157
7	0.000	1.000	0.000	0.910	0.000	0.000	0.000	0.000	1.000	0.000	0.000
8	1.000	0.000	0.109	0.801	0.000	0.455	0.000	0.000	0.545	0.000	0.000
9	1.000	0.000	0.089	0.780	0.000	0.149	0.000	0.851	0.000	0.000	0.000
10	1.000	0.000	0.014	0.869	0.000	0.007	0.000	0.993	0.000	0.000	0.000
11	1.000	0.000	0.020	0.406	0.574	0.106	0.000	0.000	0.000	0.098	0.797
12	1.000	0.000	0.014	0.898	0.000	0.007	0.000	0.993	0.000	0.000	0.000
13	0.000	1.000	0.958	0.000	0.232	0.590	0.000	0.000	0.000	0.000	0.410
14	0.000	1.000	0.991	0.000	0.318	0.000	0.845	0.000	0.000	0.000	0.155
15	0.000	1.000	0.000	0.919	0.000	0.000	0.000	0.000	1.000	0.000	0.000
16	1.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
17	1.000	0.000	0.275	0.000	0.725	0.266	0.000	0.000	0.000	0.734	0.000
18	0.000	1.000	0.000	0.889	0.000	0.000	0.000	0.000	1.000	0.000	0.000
19	0.884	0.116	0.422	0.000	0.577	0.575	0.000	0.000	0.000	0.167	0.259
20	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000
21	1.000	0.000	2.054	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000

efficiency, DEA-R efficiency with redundant restraint, DEA-R efficiency without redundant restraint; and (2) The difference of the summing method and weight selection comes from the differences between the CCR and DEA-R models, redundant weight restraints are a shortcoming of the CCR model, as the redundant restraint causes an underestimation of CCR efficiency.

3.4. Restrictions of single Input/Output relationship

After discussing the underestimation of efficiency and pseudo inefficiency caused by redundant weight restraints in the CCR model, which makes it difficult for CCR to represent the relationship of single Input/Output. In the other words, because there is only one CCR weighted input variable, it is difficult to simultaneously represent relationships with several output variables. However, there are equivalent DEA-R weights of input variables for each output variable; therefore, it is easy to simultaneously represent relationships with several output variables by setting the DEA-R weight of the input variables for specific output variables.

Take the following case as an example to explain a single Input/Output relationship, represented by the DEA-R model. For a medical center, a doctor is an important input variable providing inpatients, surgery, and outpatient services; however, sickbeds are the input only contributing to inpatient and surgery, but not outpatient services. Because there is only one CCR weight for the input sickbed variable, it is difficult to simultaneously represent different relationships between a doctor with three output variables and sickbeds with three output variables. Unlike CCR weight, three DEA-R weights of each input variable could represent the relationships of each input variable with inpatient, surgery, and outpatient service, respectively. Therefore, the DEA-R weight of input sickbed for output outpatient service is set as 0 to represent the different relationship between the input variable sickbed and the output

Table 5
Corresponding weight, corresponding efficiency, and influences of summing method and weight.

DMU	Corresponding weight						Corresponding efficiency	Influences			
	I 1/O 1	I 2/O 1	I 1/O 2	I 2/O 2	I 1/O 3	I 2/O 3		Weight	(%)	Summing method	(%)
1	0.000	0.000	0.000	1.000	0.000	0.000	0.9915	–	–	–	–
2	0.000	0.000	0.971	0.000	0.029	0.000	1.0535	0.0005	84%	0.0001	16%
3	0.000	0.000	0.000	1.000	0.000	0.000	1.1099	–	–	–	–
4	0.015	0.000	0.985	0.000	0.000	0.000	0.9080	0.0011	71%	0.0005	29%
5	0.000	0.000	0.000	1.000	0.000	0.000	0.8949	–	–	–	–
6	0.000	0.656	0.000	0.344	0.000	0.000	0.8779	0.0129	82%	0.0029	18%
7	0.000	0.000	0.000	1.000	0.000	0.000	0.9101	–	–	–	–
8	0.120	0.000	0.880	0.000	0.000	0.000	0.9159	0.0203	78%	0.0058	22%
9	0.103	0.000	0.897	0.000	0.000	0.000	0.8702	0.0029	88%	0.0004	12%
10	0.016	0.000	0.984	0.000	0.000	0.000	0.8829	0.0009	81%	0.0002	19%
11	0.020	0.000	0.406	0.000	0.574	0.000	1.0000	–	–	–	–
12	0.016	0.000	0.984	0.000	0.000	0.000	0.9123	0.0010	79%	0.0003	21%
13	0.000	0.805	0.000	0.000	0.000	0.195	1.2266	0.0370	50%	0.0365	50%
14	0.000	0.757	0.000	0.000	0.000	0.243	1.3145	0.0177	74%	0.0061	26%
15	0.000	0.000	0.000	1.000	0.000	0.000	0.9186	–	–	–	–
16	0.000	0.000	1.000	0.000	0.000	0.000	1.0000	–	–	–	–
17	0.275	0.000	0.000	0.000	0.725	0.000	1.0000	–	–	–	–
18	0.000	0.000	0.000	1.000	0.000	0.000	0.8893	–	–	–	–
19	0.373	0.049	0.000	0.000	0.511	0.067	1.0000	0.0000	0%	0.0017	100%
20	1.000	0.000	0.000	0.000	0.000	0.000	1.0000	–	–	–	–
21	1.000	0.000	0.000	0.000	0.000	0.000	2.0536	–	–	–	–

Table 6
Optimal weight and Efficiency with redundant restraint on weight and influences of factors.

DMU	Optimal DEA-R weight with redundant restraint on weight						DEA-R efficiency		CCR efficiency		Influences of factors		
	I 1/O 1	I 2/O 1	I 1/O 2	I 2/O 2	I 1/O 3	I 2/O 3	Without restraint	With restraint	Corresponding	CCR	Restraint on Weight	Model different	Sum
1	0.000	0.000	0.000	1.000	0.000	0.000	0.9915	0.9915	0.9915	0.9915	–	–	–
2	0.000	0.000	0.979	0.000	0.021	0.000	1.0540	1.0540	1.0535	1.0534	16%	84%	0%
3	0.000	0.000	0.000	1.000	0.000	0.000	1.1099	1.1099	1.1099	1.1099	–	–	–
4	0.008	0.000	0.992	0.000	0.000	0.000	0.9091	0.9091	0.9080	0.9075	29%	71%	0%
5	0.000	0.008	0.000	0.993	0.000	0.000	0.8949	0.8949	0.8949	0.8949	–	–	–
6	0.000	0.843	0.000	0.000	0.000	0.157	0.8908	0.8908	0.8779	0.8750	18%	82%	0%
7	0.000	0.000	0.000	1.000	0.000	0.000	0.9101	0.9101	0.9101	0.9101	–	–	–
8	0.191	0.285	0.210	0.314	0.000	0.000	0.9362	0.9263	0.9159	0.9101	22%	40%	38%
9	0.149	0.000	0.851	0.000	0.000	0.000	0.8731	0.8731	0.8702	0.8698	12%	88%	0%
10	0.007	0.000	0.993	0.000	0.000	0.000	0.8838	0.8838	0.8829	0.8827	19%	81%	0%
11	0.006	0.000	0.993	0.000	0.000	0.000	1.0000	1.0000	1.0000	1.0000	–	–	–
12	0.007	0.000	0.993	0.000	0.000	0.000	0.9133	0.913	0.9123	0.9120	21%	79%	0%
13	0.392	0.147	0.000	0.000	0.336	0.126	1.2636	1.2437	1.2266	1.1901	50%	23%	27%
14	0.000	0.845	0.000	0.000	0.000	0.155	1.3322	1.3322	1.3145	1.3084	26%	74%	0%
15	0.000	0.000	0.000	1.000	0.000	0.000	0.9186	0.9186	0.9186	0.9186	–	–	–
16	0.000	0.000	0.848	0.152	0.000	0.000	1.0000	1.0000	1.0000	1.0000	–	–	–
17	0.405	0.034	0.059	0.005	0.459	0.038	1.0000	1.0000	1.0000	1.0000	–	–	–
18	0.000	0.000	0.000	1.000	0.000	0.000	0.8893	0.8893	0.8893	0.8893	–	–	–
19	0.401	0.008	0.214	0.004	0.366	0.007	1.0000	1.0000	1.0000	0.9983	100%	0%	0%
20	0.241	0.172	0.172	0.123	0.170	0.121	1.0000	1.0000	1.0000	1.0000	–	–	–
21	1.000	0.000	0.000	0.000	0.000	0.000	2.0536	2.0536	2.0536	2.0536	–	–	–

variable outpatient service. The second level efficiency and optimal weight, with restrictions on the weights of sickbeds/outpatients, are as listed in Table 7.

The optimal weights of DMUs 01–03, 05–07, 14–16, and 18, as well as the second level efficiency, remain unchanged. The optimal weights of DMUs 04, 08–13, 17, and 19–21, which weights do not conform to restricted conditions, have changed accordingly. Among these 11 DMUs, 9 second level efficiencies are lower than previous efficiencies. The second level efficiencies of DMU 11 and 20 remain unchanged; however, the optimal weights lack of changes is attributed to the multiple solutions of optimal weights. Moreover, since multiple solutions of optimal weights would probably lead to analytical errors, future studies should carefully analyze those DMU with an efficiency of 1. Redundant weight restraints not only cause the underestimation of efficiency, but also the problem of representing the ability of a single Input/Output relationship, thus, the DEA-R based model is worthy of development to avoid underestimates and accurately represent the relationship of single Input/Output.

4. Cluster analysis

Wu, Liang, and Yang (2009) applied cluster analysis by taking individual cross efficiencies of DMUs as variables. When cluster analysis takes cross efficiency as its variables, the DMUs with similar efficiency are classified into the same cluster. That research inspired this study to conduct further analysis on optimal weights by cluster analysis. When cluster analysis takes optimal weight as a variable, the DMUs with similar optimal weight are classified into the same cluster. In addition, the averages of the optimal weights are compared to learn the characteristics of the cluster. The information differences between the two models may be obtained by

Table 7
Efficiency and optimal weight with the restriction on the weight of sickbeds/outpatient.

DMU	Optimal DEA-R weight with the restriction on the weight of sickbeds/outpatient						DEA-R efficiency	
	I 1/O 1	I 2/O 1	I 1/O 2	I 2/O 2	I 1/O 3	I 2/O 3	Without restriction	With restriction
1	0.000	0.000	0.000	1.000	0.000	0.000	0.9915	0.9915
2	0.000	0.000	0.979	0.000	0.021	0.000	1.0540	1.0540
3	0.000	0.000	0.000	1.000	0.000	0.000	1.1099	1.1099
4	0.000	0.010	0.990	0.000	0.000	0.000	0.9091	0.9068
5	0.000	0.000	0.000	1.000	0.000	0.000	0.8949	0.8949
6	0.000	0.843	0.000	0.000	0.000	0.157	0.8908	0.8908
7	0.000	0.000	0.000	1.000	0.000	0.000	0.9101	0.9101
8	0.000	0.699	0.000	0.301	0.000	0.000	0.9362	0.9227
9	0.000	0.006	0.994	0.000	0.000	0.000	0.8731	0.8631
10	0.000	0.007	0.993	0.000	0.000	0.000	0.8838	0.8835
11	0.000	0.009	0.991	0.000	0.000	0.000	1.0000	1.0000
12	0.000	0.009	0.991	0.000	0.000	0.000	0.9133	0.9111
13	0.000	0.805	0.000	0.000	0.000	0.195	1.2636	1.2267
14	0.000	0.845	0.000	0.000	0.000	0.155	1.3322	1.3322
15	0.000	0.000	0.000	1.000	0.000	0.000	0.9186	0.9186
16	0.000	0.000	1.000	0.000	0.000	0.000	1.0000	1.0000
17	0.000	0.000	0.000	0.000	1.000	0.000	1.0000	0.9793
18	0.000	0.000	0.000	1.000	0.000	0.000	0.8893	0.8893
19	0.000	0.000	0.000	0.000	1.000	0.000	1.0000	0.9426
20	0.000	1.000	0.000	0.000	0.000	0.000	1.0000	1.0000
21	0.000	1.000	0.000	0.000	0.000	0.000	2.0536	1.8195

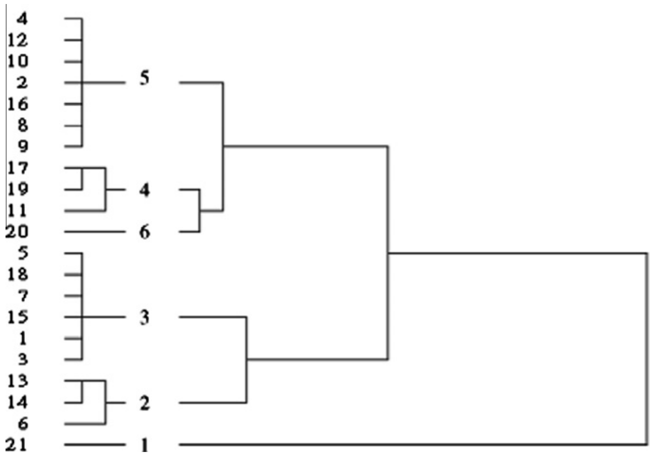

Fig. 1. Cluster analysis diagram with CCR weight as a variable.

Table 8
Clusters of DMUs.

DMU	Cluster with CCR weight	Cluster with DEA-R weight	DMU	Cluster with CCR weight	Cluster with DEA-R weight
1	3	3	11	4	4
2	5	5	12	5	5
3	3	3	13	2	1
4	5	5	14	2	2
5	3	3	15	3	3
6	2	2	16	5	5
7	3	3	17	4	6
8	5	3	18	3	3
9	5	5	19	4	1
10	5	5	20	6	1
			21	1	1

comparing the results of cluster analysis, taking CCR and DEA-R optimal weights as variable. Hence, SPSS is used as a tool to conduct cluster analysis.

4.1. Cluster analysis with optimal weight of context-dependent CCR as a variable

Cluster analysis, with an optimal weighted context-dependent CCR as a variable, is as shown in Fig. 1. When DMUs are divided into six clusters: where the first cluster contains DMU 21; the second cluster contains DMUs 06, 13, and 14; the third cluster contains DMUs 01, 03, 05, 07, 15, and 18; the fourth cluster contains DMUs 11, 17, and 19; the fifth cluster contains DMUs 02, 04, 08–10, 12, and 16; and the sixth cluster contains DMU 20. The clusters to which the DMU belong are listed in Table 8. The average optimal CCR weights of Clusters are listed in Table 9, which result reveals the characteristics of each cluster. Among the input variables, the DMU in first cluster only selects the weight of the sickbeds; among the output variable, the DMU in first cluster only selects the weight of outpatient, whose value is significantly higher than other clusters; hence, the medical institutions in first cluster are characterized by attracting patients according to the scale of sickbeds.

Among the input variable, the DMUs in second cluster only selects the weight of doctors; among the output variables of DMUs in the second cluster, the weight of outpatient is the most important; hence, the medical institutions in second cluster are characterized by attracting patients via doctors. Among the input variables, the DMUs in the third cluster only selects the weight of doctors; among the output variables, the DMUs in third cluster selects only the weight of inpatients whose value is higher than the other clusters; hence, the medical institutions in the third cluster are characterized by attracting inpatients via doctors. Among the input variable of the DMUs in the fourth cluster, the weight of the sickbeds is most important; among the output variables of DMUs in the fourth cluster, the weight of surgery, which value is significantly higher than other clusters, is most important; hence, the medical institutions in the fourth cluster are characterized by transforming sickbeds into surgery. Among the input variable, the DMUs in the fifth cluster only selects the weight of the sickbeds; among the output variable of the DMUs in the fifth cluster, the weight of inpatients is most important; hence the medical institutions in the fifth cluster are characterized by transforming sickbeds into inpatients. Among the weights of the input variables, the DMUs in the sixth cluster selects only the weight of the sickbeds; among the output variables, the DMUs in the sixth cluster selects only the weight of the outpatients, which is lower than that of the first cluster. Hence, the medical institutions in the sixth clusters are characterized by attracting patients according to sickbeds. It is found that most DMUs select one input and one output as their advantage.

4.2. Cluster analysis with optimal weight of context-dependent DEA-R as a variable

Cluster analysis diagram, with the optimal weight of a context-dependent DEA-R as a variable, is as shown in Fig. 2. When DMUs are divided into six clusters, DMUs 13 and 19–21 are in the first cluster; DMUs 06 and 14 are in the second cluster; DMUs 01, 03, 05, 07, 08, 15, and 18 are in the third cluster; DMU 11 is in the fourth cluster; DMUs 02, 04, 09, 10, 12, and 16 are in the fifth cluster; and DMU 17 is in the sixth cluster. The clusters in which the DMUs belong are as listed in Table 8.

The averages of the optimal DEA-R weights of the Clusters are as listed in Table 9. This result reveals the characteristics of each cluster. The average w_{11} of the DMUs in the first cluster is higher than that of the other clusters, thus, the advantages of medical institutions in the first cluster are focused on attracting outpatients according to sickbeds. The average w_{21} of the DMUs in the second cluster is higher than that of other clusters, thus, the medical insti-

Table 9
Average Efficiency and Average Optimal Weight of Clusters.

CCR								DEA-R								
C	N	Efficiency	Average optimal weight					C	N	Efficiency	Average optimal weight					
			I 1	I 2	O 1	O 2	O 3				I 1/O 1	I 2/O 1	I 1/O 2	I 2/O 2	I 1/O 3	I 2/O 3
1	1	2.0536	1.000	0.000	2.054	0.000	0.000	1	4	1.3293	0.791	0.000	0.000	0.000	0.042	0.167
2	3	1.1245	0.000	1.000	0.841	0.100	0.183	2	2	1.1115	0.000	0.844	0.000	0.000	0.000	0.156
3	6	0.9524	0.000	1.000	0.000	0.952	0.000	3	7	0.9501	0.065	0.000	0.000	0.935	0.000	0.000
4	3	0.9994	0.961	0.039	0.239	0.135	0.625	4	1	1.0000	0.106	0.000	0.000	0.000	0.098	0.797
5	7	0.9336	1.000	0.000	0.034	0.895	0.004	5	6	0.9389	0.028	0.000	0.968	0.000	0.003	0.000
6	1	1.0000	1.000	0.000	1.000	0.000	0.000	6	1	1.0000	0.266	0.000	0.000	0.000	0.734	0.000

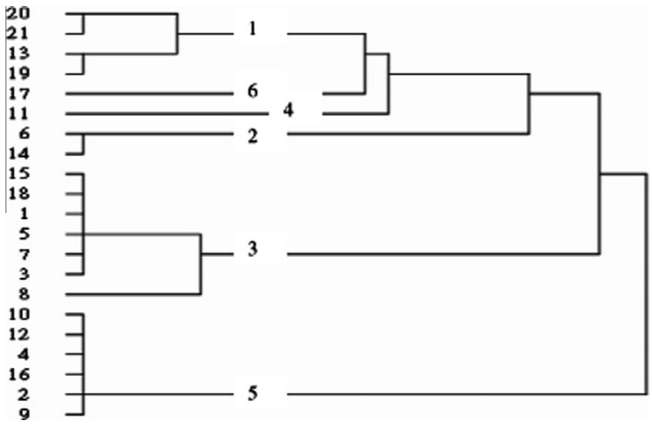

Fig. 2. Cluster analysis diagram with DEA-R weight as a variable.

tutions in the second cluster are characterized by attracting outpatients through doctors. According to the average optimal weight of the DMUs in the third cluster that, the average w_{22} is higher than that of other clusters, thus, the medical institutions in the third cluster are composed of DMUs that have advantages in the management of inpatients and doctors. In the fourth cluster, w_{23} is higher than that of the other clusters, thus, the medical institutions in the fourth cluster are composed of DMUs that have advantages in surgery performed by doctors. In the fifth cluster, w_{12} is higher than that of the other clusters, thus, the medical institutions in the fifth cluster dominate in attracting inpatients according to sickbeds. In the sixth cluster, w_{13} is higher than that of the other clusters, indicating that these medical institutions dominate in surgery according to sickbeds. It is found that most DMUs select one input and one output as their advantage.

When analyzing the correlation of clustering and efficiency of a context-dependent model; it is found from the clustering that highly efficient DMUs could generate a squeezing effect, and other DMUs will try to increase the weight of other outputs to evade the advantages of the efficient DMU. In the case study, such a squeezing effect is particularly obvious in the cluster analysis of context-dependent CCR weights; as almost all hospitals strive to increase the weight of inpatients, and eliminate any significant advantages of DMU 21 on outpatients. Unlike the context-dependent CCR, the context-dependent DEA-R could more flexibly select weights, enabling DMUs without significant advantages to become efficient models, via single Input/Output.

5. Conclusions

As the main feature of DEA, weight selection has been extensively discussed, both in practice and theory. Since 2007, a series of theoretical researches discussed redundant the weight restraints of CCR, and have overcome the shortcomings through DEA-R. This study researched redundant weight restraints by developing an extended DEA-R model, and then, converted the CCR weights to DEA-R weights, based on research. This research obtained the following results: (1) context-dependent DEA-R model was developed by combining the basic DEA-R model with the context-dependent concept; (2) the CCR weight was converted to corresponding DEA-R weights, and then, the DEA-R model was added with weight restraints that discussed the influences of redundant weight restraint restrictions upon the underestimation of efficiency; (3) the weight restriction on single Input/Output was used to represent a single Input/Output relationship; and (4) the weight was further discussed according to cluster analysis in order to learn the relationships between DMUs.

First, a context-dependent DEA-R model was developed to evaluate the efficiency of an efficient DMU, while the original DEA-R cannot analyze the degrees of the advantages contained in an efficient DMU. In this case, the advantage of the highest efficiency DMU of 2.0536, is double that of the advantage of an efficient DMUs of the lowest efficiency, 1.0540. This indicated the practical value in developing a context-dependent DEA-R model, as based on the original DEA-R, which was unable to evaluate the degrees of the advantages of an efficient DMU. Secondly, the difference in efficiency is caused by three factors, which are the different summing methods, the different weight selection methods, and redundant weight restraints. Through the conversion of CCR weights to the corresponding weight, and added weight restraints of DEA-R, the influences of the three factor variables are distinguished. In addition, the redundant restraints on weight tend to cause an underestimation in efficiency. Thirdly, constraints on single Input/Output are added to the DEA-R model in order to represent the single Input/Output relationship. In sum, the second and third results indicate the necessity of developing a context-dependent DEA-R, as based on the context-dependent CCR, including the redundant weight restraints, which tend to cause underestimations in efficiency and fail to represent the ability of an Input/Output relationship.

Finally, the optimal weight is taken as a variable of the cluster analysis in this study. The case study shows the correlation of clustering and efficiency of a context-dependent CCR, where it was found from the clustering that an efficient DMU, with an obviously higher level of efficiency, could generate a squeezing affect; and other DMUs would try to increase the weights of other outputs in order to evade the advantages of an efficient DMU. In the case study, such a squeezing effect was particularly obvious in the cluster analysis of context-dependent CCR weights, as almost all hospitals strive to increase the weights of hospitalization in order to negate the significant advantages of DMU 21 on clinics. Unlike the context-dependent CCR, the context-dependent DEA-R has greater flexibly in selecting weights, enabling DMUs without significant advantages to become efficient, via single Input/Output. Thus, by conducting cluster analysis, weight provides information on the advantages of the DMUs; moreover, it provides information regarding relationships with other DMUs, such as the characteristics of a DMU or the squeezing effect.

In practice, many Taiwanese hospitals have been accredited as medical centers, and as such could acquire a greater global budget, which in turn, would support more research and development. However, governments find it difficult to develop unique hospitals through pooling resources. By using a context-dependent efficiency evaluation model, the competent medical authorities could distinguish the efficient hospitals, and support their unique development for the benefit of the public, while maintaining fiscal integrity. Moreover, exceptional medical centers could not stand out among its peers under general blanketing policies; therefore, in addition to acquiring the most dominate situation through observations of optimal weights, cluster analysis facilitates the correlation of a global context, allowing competent authorities and medical centers to achieve long-lasting success, growth, and development, based on performance evaluations, where exceptional effort is recognized through analysis.

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