



Route-based data envelopment analysis models

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ABSTRACT

This paper proposes two novel route-based data envelopment analysis (DEA) models that jointly measure the route-level and company-level efficiencies amongst transport carriers. The core logics comprise a three-stage procedure that determines company efficiency, route efficiency and optimal allocation ratios for the common inputs. We prove that the ranking order of company performance determined by the route-based DEA model is identical to that determined by the company-based DEA model. An empirical case demonstrates the superiority of the proposed models in identifying the less efficient routes/companies as well as in reducing the input slacks without subjective conjectures.

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1. Introduction

Data envelopment analysis (DEA) is a well-known nonparametric method for measuring the relative efficiencies of decision-making units (DMUs), such as different organizations in the same industry or different departments in the same organization. DEA employs linear programming technique without the need of imposing any subjective parameters (or weights) upon the inputs and outputs (Bouyssou, 1999). In the past, a considerable number of studies have employed DEA approaches to evaluating the efficiency/effectiveness of different transport practices, including airline (e.g. Schefczyk, 1993; Charnes et al., 1996; Sengupta, 1999; Alder and Golany, 2001; Chiou and Chen, 2006), air-express courier (e.g. Lin et al., 2010), airport (e.g. Peck et al., 1998; Salazar de La Cruz, 1999; Chiang and Tzeng, 2000; Sarkis, 2000; Martin and Roman, 2001; Adler and Berechman, 2001; Barros and Dieke, 2007), maritime (e.g. Tongzon, 2001; Cullinane et al., 2006), transit (e.g. Fielding et al., 1984; Fielding, 1987; Nolan, 1996; Kerstens, 1996; Viton, 1998; Odeck and Alkadi, 2001; Nolan et al., 2002; Karlaftis, 2003; Karlaftis, 2004; Sheth et al., 2007; Margari et al., 2007; Chiou et al., 2010), and railway (e.g. Oum and Yu, 1994; Coelli and Perelman, 1999; Lan and Lin, 2003; Lan and Lin, 2005). Of these studies, a great deal of effort has been made on measuring the efficiency/effectiveness at company level by treating each transport carrier as a DMU. What seems to be lacking, however, is jointly measuring the relative performance amongst different carriers (i.e., company-level efficiency) and the detailed performance of all routes within each carrier and across different carriers (i.e., route-level efficiency).

In practice, an efficient carrier may operate some inefficient routes; likewise, an inefficient carrier may run some efficient routes. Such phenomena are particularly common for the franchised carriers who grant the concession to provide services on a fixed-route fixed-schedule basis. By regulation the carriers are required to provide a minimum level of service (at least a certain number of scheduled frequencies) in less developed areas (e.g., rural, mountainous or offshore areas), resulting in some inefficient or ineffective routes. Using a company-based DEA (hereinafter, CDEA) approach can identify the inefficient companies but it cannot reveal the problematic routes. For instance, if overstaffing were the key problem to an inefficient carrier, the slack analysis based on CDEA approach would provide a clue about extra staff for the whole company. However, it did not provide any information about the exact labor in each route. Improper reduction in specific route inputs may deteriorate

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the overall performance of the whole company. Here arises a need to develop a route-based DEA (hereafter, RDEA) approach, which can clearly indentify the inefficient routes and propose more rational countermeasures accordingly.

It is imperative for the less efficient companies to level up their overall performance by benchmarking the high performers. More importantly, it is crucial for the efficient companies to further identify and ameliorate the less efficient routes in order to sustain their competitive advantages in marketplace. On the other hand, the government may need the detailed company-based and route-based performance information for the purposes of granting new routes, renewing old routes, subsidizing deficit routes, among others. In this circumstance, route-based information would certainly facilitate the regulator to make decisions that are more favorable. Consequently, simultaneously measuring the route-based and company-based performance becomes crucial to both carriers and regulator.

Recently, Chiou and Chen (2006) proposed RDEA models by adopting subjective allocation of common inputs among different routes within a transport carrier. Subjectively allocating the common inputs into different routes, however, can probably result in misleading outcomes. To rectify this shortage, this study proposes a novel RDEA approach that decomposes company efficiency into route efficiency by simultaneously optimizing the allocation of common inputs. The core logics of the proposed RDEA approach contains the following three stages. The first stage uses a CDEA model to acquire a set of optimal input/output multipliers in an objective manner to correct the shortage of subjective allocation of common inputs. The second stage uses the corresponding objective multipliers to determine its optimal allocation ratios of common inputs among different routes within a company to maximize the overall efficiency of all routes in that company. The third stage further uses a RDEA model (treating each route as a DMU) to determine the efficiency scores of all routes across all companies, upon the determination of the optimal allocation ratios. In other words, the proposed three-stage procedure can jointly determine the efficiency values on a company- and route-level, together with the optimal allocation ratios of common inputs among routes. To demonstrate the proposed approach, this paper conducts an empirical study of 37 Taiwanese intercity bus companies currently operating 1035 routes. It is hoped that the results can provide the managers with in-depth insights to propose practical strategies to ameliorate the individual route efficiency as well as to level up the overall company performance.

The rest of this paper is organized as follows. Section 2 elaborates the formulations of the proposed RDEA models. The empirical study is conducted in Section 3. Based on the results, a Tobit regression is further performed to analyze the estimated results in Section 4. Finally, the conclusions and recommendations for future research are presented.

2. The proposed models

DEA is a method for measuring the relative efficiency of DMUs that perform similar tasks. A DEA model with constant-returns-to-scale (CRS) technologies was proposed by Charnes, Cooper and Rhodes (1978; CCR model hereinafter). Similar model under variable-returns-to-scale (VRS) context was later developed by Banker, Charnes and Cooper (1984; BCC model hereinafter). Accordingly, this paper proposes two similar RDEA models under CRS and VRS contexts, termed as route-based CCR (RCCR) model and route-based BCC (RBCC) model, respectively. Both of the proposed RCCR and RBCC models comprise a three-stage procedure, which decomposes company efficiency into individual route efficiencies and simultaneously optimizes the allocation of common inputs. The formulations of the proposed RCCR and RBCC models are elaborated as follows:

2.1. RCCR model

The first stage uses the following company-based CCR model to determine a set of optimal input/output multipliers:

$$[\text{CCR}] \quad \underset{u, v}{\text{Max}} \quad h_q = \sum_{r=1}^R u_r y_{qr} \quad (1)$$

$$\text{s.t.} \quad \sum_{r=1}^R u_r y_{ir} - \sum_{j=1}^J v_j x_{ij} \leq 0, \quad i = 1, 2, \dots, I \quad (2)$$

$$\sum_{j=1}^J v_j x_{sq} = 1 \quad (3)$$

$$v_j \geq 0, \quad j = 1, 2, \dots, J \quad (4)$$

$$u_r \geq 0, \quad r = 1, 2, \dots, R \quad (5)$$

where h_s is the efficiency score of company q . Supposed that there are totally I companies to be evaluated, each of which utilizes J types of inputs and produces R kinds of outputs. u_r and v_j are the multipliers corresponding to output r and input j of company q , respectively. From the above [CCR] model, the optimal input/output multipliers can be determined.

The second stage then uses the solved multipliers to determine its optimal allocation ratios for the common inputs among the routes within a company to maximize the overall efficiency of all routes. In practice, however, some portions of the inputs can be clearly attributed to only a specific route; some other portions should be regarded as common inputs for all routes of a company. For instance, the drivers are responsible for and should be reasonably attributed to a specific route; the administrative staff and the managers, however, are the common inputs—not readily attributed to any specific route.

In determining the optimal allocation ratios, the following develops two models: [AR1] and [AR2]. [AR1] model is for the case when all the route attributed inputs cannot be identified, whereas [AR2] model is for the case when a portion of the route attributed inputs can be identified.

The [AR1] model is expressed as follows:

$$[AR1] \quad \underset{s}{Max} \quad h_i = \frac{1}{L_i} \left(\sum_{l=1}^{L_i} \frac{\sum_{r=1}^R u_r y_{lr}^i}{\sum_{j=1}^J v_j s_{lj}^i x_j^i} \right) \tag{6}$$

$$s.t. \quad \frac{\sum_{r=1}^R u_r y_{lr}^i}{\sum_{j=1}^J v_j s_{lj}^i x_j^i} \leq 1, \quad l = 1, 2, \dots, L_i \tag{7}$$

$$\sum_{l=1}^{L_i} s_{lj}^i = 1, \quad j = 1, 2, \dots, J \tag{8}$$

$$s_{lj}^i \geq 0, \quad l = 1, 2, \dots, L_i; \quad j = 1, 2, \dots, J \tag{9}$$

where h_i is the average of efficiency scores for all routes of company i which operates totally L_i routes and each route utilizes J types of inputs and produces R kinds of outputs. u_r and v_j are the multipliers determined by the [CCR] model. Besides, each output is assumed route attributable, i.e., can be clearly identified as a route output (y_{lr}^i). Since all the routes attributed inputs cannot be identified, the inputs (x_j^i) to be allocated is based on the optimally solved ratio (s_{lj}^i)—an allocation ratio of route l for input j of company i . Eq. (8) ensures that each common input is completely allocated to all routes.

On the other hand, the [AR2] model is expressed as follows:

$$[AR2] \quad \underset{s}{Max} \quad h_i = \frac{1}{L_i} \left(\sum_{l=1}^{L_i} \frac{\sum_{r=1}^R u_r y_{lr}^i}{\sum_{j=1}^J v_j (x_{lj}^i + s_{lj}^i x_{cj}^i)} \right) \tag{10}$$

$$s.t. \quad \frac{\sum_{r=1}^R u_r y_{lr}^i}{\sum_{j=1}^J v_j (x_{lj}^i + s_{lj}^i x_{cj}^i)} \leq 1, \quad l = 1, 2, \dots, L_i \tag{11}$$

$$\sum_{l=1}^{L_i} s_{lj}^i = 1, \quad j = 1, 2, \dots, J \tag{12}$$

$$s_{lj}^i > 0, \quad l = 1, 2, \dots, L_i; \quad j = 1, 2, \dots, J \tag{13}$$

where the input j of company i is divided into two parts: the attributable part (x_{lj}^i) and the common part (x_{cj}^i); $x_j^i = \sum_{l=1}^{L_i} x_{lj}^i + x_{cj}^i$. Only the common part (x_{cj}^i) requires an optimally solved allocation ratio (s_{lj}^i) to assign to route l . To determine the optimal allocation ratio of common input, however, only the routes operated by the same company are considered; namely, the route allocation ratios for one company are irrelevant to the routes operated by other companies. With the optimal allocation ratios (s_{lj}^i), the inputs of route l under evaluation can be computed by $x_{lj}^i = x_{lj}^i + s_{lj}^i x_{cj}^i$.

Finally, based on the computed inputs, the third stage is to optimally determine the route efficiency by treating each route (could be operated by different companies) as a DMU, expressed as follows:

$$[RCCR] \quad \underset{u,v}{Max} \quad h_k^i = \sum_{r=1}^R u_{kr}^i y_{kr}^i \tag{14}$$

$$s.t. \quad \sum_{r=1}^R u_{lr}^i y_{lr}^i - \sum_{j=1}^J v_j^i x_{lj}^i \leq 0, \quad i = 1, 2, \dots, I; \quad l = 1, 2, \dots, L \tag{15}$$

$$\sum_{j=1}^J v_{kj}^i x_{kj}^i = 1 \tag{16}$$

$$v_j^i \geq 0, \quad j = 1, 2, \dots, J; \quad l = 1, 2, \dots, L \tag{17}$$

$$u_{lr}^i \geq 0, \quad r = 1, 2, \dots, R; \quad l = 1, 2, \dots, L \tag{18}$$

where h_k^i is the efficiency score of route k operated by company i . u_{lr}^i and v_j^i are the multipliers corresponding to output r and input j for route l operated by company i , respectively. There are a total of L routes under evaluation, $L = L_1 + L_2 + \dots + L_I$. Unlike [AR1] or [AR2] model wherein the routes sequence are ordered only within the same company, the routes sequence of [RCCR] here are ordered among all routes across all companies.

2.2. RBCC model

Following the same vein of the above [RCCR] modeling procedures, the [RBCC] model simply adds a convexity constraint. In the first stage, the following company-based BCC model is used to determine the optimal multipliers.

$$[\text{BCC}] \quad \text{Max}_{u,v} \quad h_q = \sum_{r=1}^R u_r y_{qr} - u \quad (19)$$

$$\text{s.t.} \quad \sum_{r=1}^R u_r y_{ir} - u - \sum_{j=1}^J v_j x_{ij} \leq 0, \quad i = 1, 2, \dots, I \quad (20)$$

$$\sum_{j=1}^J v_j x_{qj} = 1 \quad (21)$$

$$v_j \geq 0, \quad j = 1, 2, \dots, J \quad (22)$$

$$u_r \geq 0, \quad r = 1, 2, \dots, R \quad (23)$$

where u is efficiency scale of company q . In the second stage, the corresponding allocation ratio models can be expressed as follows:

$$[\text{AR1}'] \quad \text{Max}_s \quad h_i = \frac{1}{L_i} \left(\sum_{l=1}^{L_i} \frac{\sum_{r=1}^R u_r y_{lr}^i - u}{\sum_{j=1}^J v_j s_{lj}^i x_{lj}^i} \right) \quad (24)$$

$$\text{s.t.} \quad \frac{\sum_{r=1}^R u_r y_{lr}^i - u}{\sum_{j=1}^J v_j s_{lj}^i x_{lj}^i} \leq 1, \quad l = 1, 2, \dots, L_i \quad (25)$$

$$\sum_{l=1}^{L_i} s_{lj}^i = 1, \quad j = 1, 2, \dots, J \quad (26)$$

$$s_{lj}^i \geq 0, \quad l = 1, 2, \dots, L_i; \quad j = 1, 2, \dots, J \quad (27)$$

and

$$[\text{AR2}'] \quad \text{Max}_s \quad h_i = \frac{1}{L_i} \left(\sum_{l=1}^{L_i} \frac{\sum_{r=1}^R u_r y_{lr}^i - u}{\sum_{j=1}^J v_j (x_{lj}^i + s_{lj}^i x_{cj}^i)} \right) \quad (28)$$

$$\text{s.t.} \quad \frac{\sum_{r=1}^R u_r y_{lr}^i - u}{\sum_{j=1}^J v_j (x_{lj}^i + s_{lj}^i x_{cj}^i)} \leq 1, \quad l = 1, 2, \dots, L_i \quad (29)$$

$$\sum_{l=1}^{L_i} s_{lj}^i = 1, \quad j = 1, 2, \dots, J \quad (30)$$

$$s_{lj}^i > 0, \quad l = 1, 2, \dots, L_i; \quad j = 1, 2, \dots, J \quad (31)$$

In the third stage, the corresponding [RBCC] model can be written as follows:

$$[\text{RBCC}] \quad \text{Max}_{u,v} \quad h_k^i = \sum_{r=1}^R u_{kr}^i y_{kr}^i - u_k^i \quad (32)$$

$$\text{s.t.} \quad \sum_{r=1}^R u_{lr}^i y_{lr}^i - u_k^i - \sum_{j=1}^J v_{lj}^i x_{lj}^i \leq 0, \quad i = 1, 2, \dots, I; \quad l = 1, 2, \dots, L \quad (33)$$

$$\sum_{j=1}^J v_{kj}^i x_{kj}^i = 1 \quad (34)$$

$$v_{lj}^i \geq 0, \quad j = 1, 2, \dots, J; \quad l = 1, 2, \dots, L \quad (35)$$

$$u_{lr}^i \geq 0, \quad r = 1, 2, \dots, R; \quad l = 1, 2, \dots, L \quad (36)$$

where u_k^i is the scale of route k of company i .

2.3. Propositions

2.3.1. Slack analysis

Definition. The slack value of the route is the difference between the shared input of the route and that of its benchmark routes.

The following two slack analyses should be used depending on whether or not the attributed inputs are known:

- Case (1) When attributed inputs are unknown, the shared input value $s_{ij}^i x_j^i$ determined by the [AR1] or [AR1'] model are used as the inputs of the [RCCR] or [RBCC] model to evaluate the route efficiency and to determine the corresponding benchmark routes.
- Case (2) When attributed inputs are known, with the allocation ratios determined by the [AR2] or [AR2'] model, the shared input value $x_{ij}^i + s_{ij}^i x_{cj}^i$ is used as the inputs of the [RCCR] or [RBCC] model to evaluate the route efficiency and to determine the corresponding benchmark routes. For instance, if route r is benchmarked by route i , the slack value for the attribute part of input j is $x_{rj}^i - x_{ij}^i$ and for the common part is $s_{rj}^i x_{cj}^i - s_{ij}^i x_{cj}^i$.

2.3.2. Consistency of ranking order

Property. *the ranking order of company's performance represented by the efficiency value determined by the company-based DEA model is identical to the average of route efficiency values determined by the route-based DEA model.*

Proof. Without loss of generality, consider two companies—company 1 and company 2, each operates two routes. According to Charnes et al. (1978), the company efficiency can be defined as $E_i = \frac{y_c^i}{y_R^i}$, $y_R \geq y_c^i$, where y_R is the maximum outputs produced from given inputs and y_c^i is the actual outputs rated from the same inputs for company i . We use this concept to derive the company efficiency with the company-based DEA model as follows:

Let u_c^{i*} , v_c^{i*} represent the optimal set of corresponding values. Then $x_R = x_c^i$ implies $v_c^{i*} x_c^i = v_c^{i*} x_R$. By definition, the efficiency score of the benchmark company is equal to 1, implying $v_c^{i*} x_R = u_c^{i*} y_R$. Thus, the following relationship holds:

$$E_i = \frac{u_c^{i*} y_c^i}{v_c^{i*} x_c^i} = \frac{u_c^{i*} y_c^i}{v_c^{i*} x_R} = \frac{u_c^{i*} y_c^i}{u_c^{i*} y_R} = \frac{y_c^i}{y_R} \tag{37}$$

Without loss of generality, assuming company 1 performs better than company 2, then we obtain the result: $\frac{y_c^1}{y_R} = E_1 > E_2 = \frac{y_c^2}{y_R}$, implying $y_c^1 - y_c^2 > 0$. □

Similarly, the route efficiency can be defined as $E_l^i = \frac{y_l^i}{y_r^i}$, $y_r \geq y_l^i$, where y_r is the maximum outputs of the benchmark route produced by the given inputs and y_l^i is the actual outputs rated from the same inputs for route l in company i . We use this concept to derive the route efficiency with the route-based DEA model as follows:

Let u_l^{i*} and v_l^{i*} represent the optimal set of corresponding values. $x_r = x_l^i$ implies $v_l^{i*} x_l^i = v_l^{i*} x_r$. By definition, the efficiency score of benchmark route is equal to 1, implying $v_l^{i*} x_r = u_l^{i*} y_r$. Thus, the following relationship holds:

$$E_{route}^i = \frac{u_1^{i*} y_1^i}{v_1^{i*} x_1^i} + \frac{u_2^{i*} y_2^i}{v_2^{i*} x_2^i} = \frac{u_1^{i*} y_1^i}{v_1^{i*} x_r} + \frac{u_2^{i*} y_2^i}{v_2^{i*} x_r} = \frac{u_1^{i*} y_1^i}{u_1^{i*} y_r} + \frac{u_2^{i*} y_2^i}{u_2^{i*} y_r} = \frac{y_1^i + y_2^i}{y_r} = \frac{y_c^i}{y_R} \tag{38}$$

From the company-based DEA model, $y_c^1 - y_c^2 > 0$, therefore we can further derive $\frac{y_c^1}{y_R} = E_{route}^1 > E_{route}^2 = \frac{y_c^2}{y_R}$. Namely, the ranking order of company performance represented by the efficiency value determined by the company-based DEA model is identical to the average of route efficiency values determined by the route-based DEA model.

3. An empirical study

To implement the proposed RDEA models, an empirical study on 1035 routes currently operated by 37 intercity bus companies in Taiwan is conducted. We note that some of these routes are operated in the freeway/expressway contexts, which have enjoyed little traffic interruption and thus are fuel economy; whereas some other routes are operated in the surface highway contexts, which can be interrupted by traffic signals, pedestrian or vehicle crossing, curbside parking or loading activities, etc. Referring to relevant literature (e.g., Gillen and Lall, 1997a,b; Lan and Lin, 2005; Chiou and Chen, 2006; Bhadra, 2009; Greer, 2009; Lin and Lan, 2009), we utilize fuel cost, number of employees (hereinafter, labor), and number of buses (hereinafter, bus) as the input variables; operating revenue and passenger-km as the output variables.

In current practice, some buses are exclusively used in a specific route, but some others may be used flexibly in different routes. It suggests that the input of bus fleet contains two parts: attribute part and common part. The other input variables are regarded as unknown attributed inputs.

3.1. Data

Our dataset came from the annual report published by the Institute of Transportation, Ministry of Transportation and Communications in 2005. It contained the above-mentioned detailed inputs and outputs information for the 1035 routes operated by 37 companies. To save space, the detailed information for each of the 1035 routes is not presented here. Table 1 displays the correlation coefficients among input and output variables at the company level. Note that all correlation coefficients between input and output variables are significantly positive, suggesting that the dataset satisfies the isotonicity property.

To ensure the selected input/output variables important and relevant, regression analyses are further conducted and Table 2 presents the results. Note that all the explanatory variables show positive and significant effects on at least one of the associated dependent variables, suggesting the appropriateness of the above selected variables.

The carriers' operating revenue in Taiwan mainly includes fare box revenue, which have direct correlation with passenger-km. Besides, most of the bus transit carriers also enjoy other business revenues (e.g., real estate rent, advertisements, etc.) which may not be closely related to passenger-km. Due to the competition, it is common for the freeway routes to offer concession fares to attract passengers during weekdays or off-peak periods. Some other freeway routes also issue round-trip or multiple-trip tickets with different amounts of discount to retain the loyal passengers. Such differential fare strategies, however, are seldom found in the highway routes. In light of this, two output variables—operating revenue and passenger-km—are considered in this study.

To further show the difference between these two output variables, a ratio of operating revenue per passenger-km for each bus company is computed in Table 3. Note that the values of operating revenue per passenger-km vary remarkably, ranging from NT\$1.07 to NT\$153.96. To avoid evaluation bias, it is imperative to take these two distinct variables into account.

Table 1
Correlation coefficients among input and output variables.

Variable	Output		Input		
	Operating revenue	Passenger-km	Fuel cost	Labor	Bus
Operating revenue	1.00				
Passenger-km	0.86	1.00			
Fuel cost	0.98	0.89	1.00		
Labor	0.97	0.77	0.96	1.00	
Bus	0.94	0.69	0.90	0.95	1.00

Table 2
Regression results for input and output variables.

Dependent variables	Independent variables		
	Fuel cost	Labor	Bus
Operating revenue	2.311 (8.734)	340118.244 (2.184)	518999.382 (2.827) $R^2 = 0.987$
Passenger-km	5.457 (8.815)	790009.511 (2.169)	776740.479 (1.809) $R^2 = 0.889$

Note: *t* values in parentheses.

Table 3
Operating revenue per passenger-km of 37 bus companies.

Company	Operating revenue/passenger km	Company	Operating revenue/passenger km
1	1.90	20	1.90
2	1.36	21	37.17
3	9.92	22	9.98
4	3.88	23	1.86
5	25.25	24	1.32
6	90.55	25	6.76
7	33.07	26	10.51
8	5.83	27	9.52
9	1.91	28	5.59
10	1.07	29	61.35
11	1.43	30	12.98
12	1.16	31	12.90
13	1.47	32	1.93
14	1.15	33	4.18
15	153.96	34	4.99
16	24.71	35	1.57
17	1.14	36	110.13
18	7.63	37	39.18
19	1.36		

3.2. Results

In the first stage, a CDEA model is used to evaluate the company-level efficiency. The efficiency scores of 37 companies are summarized in Table 4. Note that only four companies are evaluated as efficient. Most of the inefficient companies, characterized as increasing-returns-to-scale (irs), need to enlarge their scales.

In the second stage, the optimal allocation ratios among different routes within each company are determined. For brevity, Table 5 only illustrates the detailed allocation ratios for the 17 routes operated by company 1.

Fig. 1 displays the allocation ratios of inputs and shares of outputs for the 17 routes of company 1. The detailed allocation ratios for the routes operated by the remaining 16 companies are not presented here.

It should be noted that the number of buses is the only input variable that has both attributed and common parts. The route with low allocation ratio of buses in the common part is not necessarily associated with low allocation ratio of fuel cost or labor force because the route still has an attributed part—the buses exclusively used in that route. Our results indicate that the total number of buses of a route (including both attributed and allocated common buses) is in effect proportional to

Table 4
Efficiency scores and scale efficiencies of 37 bus companies.

Company	CRS	VRS	Scale	Company	CRS	VRS	Scale		
1	0.564	0.565	0.999	irs	20	0.586	0.594	0.987	irs
2	0.938	1.000	0.938	drs	21	0.965	1.000	0.965	irs
3	0.879	0.919	0.957	drs	22	0.875	0.943	0.928	irs
4	1.000	1.000	1.000	crs	23	0.686	0.901	0.762	drs
5	0.787	0.912	0.863	drs	24	0.465	0.491	0.949	drs
6	0.812	0.921	0.881	drs	25	0.559	0.585	0.956	irs
7	0.741	0.833	0.889	drs	26	0.464	0.479	0.970	drs
8	0.439	0.449	0.978	irs	27	0.417	0.445	0.937	irs
9	0.387	0.397	0.973	irs	28	0.581	0.604	0.962	irs
10	0.678	0.828	0.820	irs	29	0.645	0.774	0.833	irs
11	0.902	1.000	0.902	irs	30	0.525	1.000	0.525	irs
12	0.877	1.000	0.877	irs	31	0.320	0.342	0.933	irs
13	0.995	0.996	0.999	irs	32	0.457	0.467	0.980	irs
14	1.000	1.000	1.000	crs	33	1.000	1.000	1.000	crs
15	0.837	1.000	0.837	drs	34	0.464	0.506	0.917	drs
16	0.828	0.954	0.867	drs	35	0.468	0.531	0.881	drs
17	0.554	0.763	0.726	irs	36	1.000	1.000	1.000	crs
18	0.958	0.981	0.977	irs	37	0.487	0.500	0.974	drs
19	0.769	0.911	0.844	drs					

Note: crs, irs and drs represent constant-, increasing- and decreasing-returns-to-scale, respectively.

Table 5
Optimal allocation ratios for the 17 routes operated by company 1.

Route	Fuel cost (%)	Labor (%)	Bus (%)		
			Common part	Attribute part	Total
1	14.28	16.59	11.03	14.00	13.68
2	11.87	9.48	7.40	8.69	8.55
3	0.07	0.28	2.01	0.71	0.85
4	13.86	11.98	8.12	10.67	10.39
5	0.07	0.52	1.81	0.70	0.82
6	0.07	0.58	2.34	0.71	0.89
7	0.07	0.32	1.78	0.70	0.82
8	0.08	3.16	4.63	0.78	1.20
9	9.13	4.02	5.51	6.03	5.97
10	0.06	0.24	0.48	0.67	0.65
11	0.07	3.51	4.57	0.78	1.19
12	21.60	25.79	27.68	37.30	36.26
13	9.39	3.81	3.15	2.70	2.75
14	0.07	3.98	5.04	0.80	1.26
15	10.38	6.82	4.01	6.63	6.35
16	0.10	4.98	6.01	3.43	3.71
17	8.83	3.94	4.43	4.69	4.66
Total	100.00	100.00	100.00 (23 buses)	100.00 (189 buses)	100.00 (212 buses)

Note: The allocation ratio of attribute part of bus is computed according to the data (i.e. the number of buses exclusively used in the route) not determined by the model.

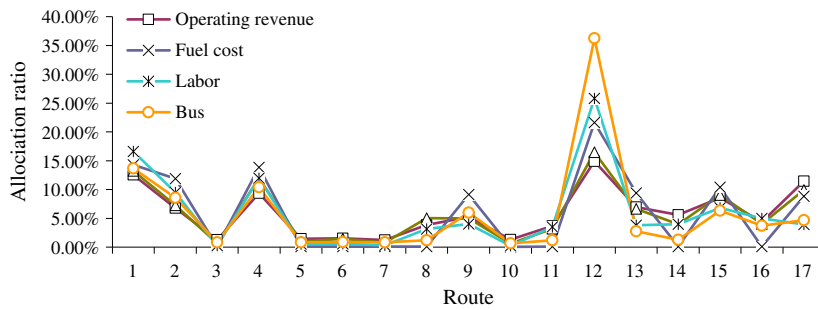


Fig. 1. Allocation ratios of inputs and shares of outputs for all routes of company 1.

the fuel cost and labor force allocated. As shown in Table 5 and Fig. 1, the allocation ratios of three inputs exhibit similar patterns to the shares of two outputs, suggesting that our proposed model tends to allocate larger amount of inputs to those routes with larger amount of outputs, such as routes 1, 4 and 12. This rationale is logical because the route with larger production generally requires more inputs. In other words, the proposed model will not allocate more inputs to lower productive routes. The correlation coefficients of allocated fuel cost and labor associated with allocated bus (combined with common and attributed parts) are 0.85 and 0.95, respectively, further suggesting reasonability of the determined allocation ratios—more buses used in a route requires more fuel cost and labor.

The optimal allocation ratios in Table 5 depict the operational characteristics of routes. Taking route 12 as an example, it is the most profitable route, so-called “golden route”, of company 1 (contributing 31.5% of the total operating revenue). More importantly, route 12 has the following unique characteristics, making the number of buses not comparable to fuel or labor costs: (1) It has the largest number of attribute buses (37.3%). (2) It has the highest service frequencies (112) daily. (3) Its route length is relatively short, which increases the bus-fleet turnover. The average route length of 17 routes operated by company 1 is 28.0 km, and the length of route 12 is only 20.1 km. (4) Most importantly, route 12 is a “freeway route”, which enjoys less traffic interruption, more fuel economy than the “highway routes” operated on surface roads. Similar to route 12, route 16 is also a short-length and high-frequency route. In contrast, route 13 has relatively different characteristics as follows. (1) It has small contribution to the total operating revenue (only 0.2%). (2) It is a long route (60.5 km). (3) It runs very few frequencies (only 16 daily). (4) Most importantly, it is a “highway route” subject to interrupted traffic ubiquitously (pedestrian/vehicle crossing at junctions/exits, curbside parking/loading activities, etc.), which is more fuel consumption than the “freeway routes”.

In the third stage, the proposed RCCR and RBCC models are used to determine the route-level efficiency for all routes, under CRS and VRS contexts, within an individual company and across all companies. For brevity, Table 6 only illustrates the results for the 17 routes operated by company 1. Details of the route-level efficiency scores for the remaining 16 companies are not presented here.

It is interesting to note from Table 4 that the results based on company-based DEA model have revealed that company 1 is in effect inefficient due to its overall scale of (irs). However, it does not mean that all of its subordinated routes require

Table 6
Efficiency scores for the 17 routes operated by company 1.

Route	CRS	VRS	Scale	
1	0.596	0.600	0.993	drs
2	0.557	0.561	0.993	irs
3	0.233	0.656	0.355	irs
4	0.605	0.613	0.987	irs
5	0.302	0.486	0.621	irs
6	0.356	0.625	0.570	irs
7	0.235	0.676	0.348	irs
8	0.800	0.801	0.999	irs
9	0.426	0.437	0.975	irs
10	0.613	1.000	0.613	irs
11	0.897	0.911	0.985	drs
12	0.385	0.455	0.846	drs
13	0.662	0.701	0.944	irs
14	0.960	0.998	0.962	drs
15	0.503	0.516	0.975	irs
16	1.000	1.000	1.000	crs
17	0.408	0.454	0.899	irs
Average	0.561	0.676	–	–

Table 7

Slack values for inputs of the 17 routes operated by company 1.

Route	Fuel cost (%)	Labor (%)	Bus (%)	
			Attributed part	Common part
1	12.42	16.60	12.05	7.26
2	11.33	10.41	8.19	7.96
3	0.05	0.24	0.49	6.25
4	11.66	11.60	8.88	7.02
5	0.08	0.67	0.74	9.34
6	0.06	0.54	1.08	6.81
7	0.05	0.26	0.46	5.88
8	0.03	1.57	2.00	3.61
9	11.18	5.66	7.27	10.20
10	0.00	0.00	0.00	0.00
11	0.01	0.78	0.89	1.62
12	25.60	35.16	43.78	9.89
13	6.11	2.85	1.72	5.43
14	0.00	0.02	0.02	0.04
15	10.93	8.26	6.94	8.79
16	0.00	0.00	0.00	0.00
17	10.48	5.38	5.48	9.91
Total	100.00	100.00	100.00	100.00

scaling up. By further looking into the details of the route efficiencies obtained from the RCCR and RBCC models (Table 6), we can scrutinize the insights: of the 17 routes, only twelve with (irs) need to be scaled up; one with (crs) should remain unchanged, and four with (drs) even require downsizing. This evidence manifestly indicates the importance of jointly evaluating the company-level and route-level performance for the carriers at the same time. It would facilitate the managers to exercise more accurate tactics to improve the performance for the inefficient individual routes and for the whole company.

To propose the improvement tactics for the inefficient companies or inefficient routes, slack values for each of the input variables are computed. Taking company 1 as an example, the results are reported in Table 7. For those inputs (such as bus) that can be distinguished into attributed and common parts, two slack values will be generated; in contrast, for those inputs (such as fuel cost, labor) that cannot be separated from attributed to common part, the proposed model will determine an overall improvement for those inputs. For instance, route 9 has used too much input resource; one should reduce the fuel cost by 11.18%, labor force by 5.66%, and bus fleet by 17.47% (the attributed part takes only 7.27% while the common part takes 10.20%) so as to achieve the efficiency frontier.

4. Discussion

To further identify the external factors affecting the route efficiency, a Tobit regression is conducted. We choose the following four factors as the explanatory variables: load factor (*LF*), subsidy from government (*SG*), freeway route (*FW*), and connection to major cities (*CM*). Where, *LF* is defined as seat-km/passenger-km. The route with higher *LF* is anticipated to have higher route efficiency (a positive sign is expected). *SG* is a binary variable representing whether the route is being subsidized by the government. If yes, *SG* = 1; otherwise, *SG* = 0 (a positive sign is expected). *FW* is a binary variable indicating that the route is operated on the freeways (*FW* = 1) or on the ordinary surface roadways (*FW* = 0). The freeway buses are more fuel efficient than those on the surface roadways (a positive sign is expected). *CM* is also a binary variable representing whether the route connects the major cities (If yes, *CM* = 1; otherwise, *CM* = 0). The five major cities in Taiwan include Taipei City, New Taipei City, Taichung City, Tainan City and Kaohsiung City, which cover approximately 27% of the total island area but inhabit about 60% of the total population. Generally, the bus routes connecting the populated areas can attract more patronage (a positive sign is expected).

Tobit model allows us to incorporate only one bound of the dependent variable while DEA efficiency score is constrained to fall between zero and one. Therefore, by taking the logarithm of the DEA efficiency scores, one could convert the dependent variable so that it has only one bound (Oum and Yu, 1994). For ease of interpretation, however, the signs of the regression coefficients are reported in accordance with the original form. By regressing the logarithm of route efficiency scores on the above four explanatory variables, the estimation result is shown below:

$$\ln(\text{Efficiency score}) = 0.0365 + 0.0867 LF + 0.0585 SG + 0.2899 FW + 0.2704 CM \quad (39)$$

(9.791) (5.962) (2.572) (3.304) (7.173)

$$R^2 = 0.7256$$

where $\ln(\text{Efficiency score})$ denotes the logarithm of the route efficiency score (*t*-values in parentheses). From Eq. (39), all estimated parameters are statistically significant with positive values as anticipated, suggesting that these variables have positive

contributions to route efficiency. On average, one unit increased in load factor (*LF*) will lead to an increase of the route efficiency score by 0.0867%. The remaining three explanatory variables are binary. According to their associated estimated parameters, *FW* has the largest contribution to route efficiency, followed by *CM*, then by *SG*.

Basically, *FW* can be viewed as a proxy variable for better service quality in terms of speedy, smooth and reliable services. Therefore, to enhance the operation efficiency for the ordinary surface roadway routes, providing bus exclusive lanes with preemption signals in congested urbanized areas can be an effective strategy to improve the route efficiency. *CM* is a proxy variable for higher transportation demand. Thus, a concept of transit-oriented development (TOD) land use or traffic management would invite more public transport patronage. Meanwhile, the government should grant the carriers more concession to run the freeway routes connecting the major cities. Finally, *SG* represents the government financial subsidy. The result shows that government subsidy can raise the route efficiency but its effect is relatively small in comparison with both *FW* and *CM*.

5. Concluding remarks

The present study has proposed two route-based DEA models—RCCR and RBCC, respectively, for constant-returns-to-scale and variable-returns-to-scale contexts. The proposed two novel models have contributed to the literature with several merits. First, the proposed DEA models can jointly measure the route- and company-level efficiency at the same time, which is superior to the previous DEA modeling approaches. Next, we prove that the ranking order of company performance determined by the route-based DEA model is identical to that determined by the route-based DEA model, and this adds a significant contribution to the DEA theories. Third, the empirical study results supported the argument that an efficient carrier may operate some inefficient routes and that an inefficient carrier may run some efficient routes. Based on the empirical results, one can easily pinpoint the less efficient routes and/or less efficient companies and exercise more accurate improvement tactics. Forth, the route-based allocation ratios of all common inputs are optimally determined without subjective conjectures. It also greatly contributes to the practices. For instance, when downsizing an inefficient company, conventionally the managers may proportionally reduce the inputs amongst the routes in a subjective manner. With the optimal route-based allocation ratios, however, the managers can now curtail the route inputs in a fairer (objective) manner. Last, the Tobit regression results also provide useful information to the regulation agencies for better decision making to help improve the carriers' efficiencies.

It is inevitable that the present study has some limitations and requires further research. First, this study proposes a three-stage approach to separately determine the optimal multipliers (at stage 3) and the optimal allocation ratios (at stage 2). One may argue that it would be more logical to start the performance evaluation at the route level and to end at the company level by simply averaging the efficiency values of routes operated by the corresponding company. With this rationale, an integrated modeling approach that simultaneously determines the optimal multipliers of input/output variables and optimal allocation ratios has to be developed. However, the integrated model may involve with a greater number of constraints, increasing the complexity in modeling. Moreover, the integrated model is in essence nonlinear due to the multiplication terms of allocation ratios and the multipliers in the denominator of Eqs. (6), (7), (10), and (11), making the integrated model rather difficult to solve. Nonetheless, the formulation and solving algorithm for such an integrated model deserves further exploration. Second, it is interesting to compare the optimal allocation ratios determined by the proposed RDEA model with those determined by other common-cost allocation principles. Last, application of the proposed RDEA models to other transport practices or service industries are also calling for further studies.

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References

- Adler, N., Berechman, J., 2001. Measuring airport quality from the airlines' viewpoint: an application of data envelopment analysis. *Transport Policy* 8, 171–181.
- Alder, N., Golany, B., 2001. Evaluation of deregulated airline networks using data envelopment analysis combined with principal component analysis with an application to Western Europe. *European Journal of Operational Research* 132, 260–273.
- Banker, R.D., Charnes, A., Cooper, A.A., 1984. Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science* 30, 1078–1092.
- Barros, C.P., Dieke, P.U.C., 2007. Performance evaluation of Italian airports: a data envelopment analysis. *Journal of Air Transport Management* 13, 184–191.
- Bhadra, D., 2009. Race to the bottom or swimming upstream: performance analysis of US airlines. *Journal of Air Transport Management* 15, 227–235.
- Bouyssou, D., 1999. Using DEA as a tool for MCDM: some remarks. *Journal of the Operational Research Society* 50, 974–978.
- Charnes, A., Cooper, W.W., Rhodes, E., 1978. Measuring the efficiency of decision making units. *European Journal of Operational Research* 2, 429–444.
- Charnes, A., Gallegos, A., Li, H., 1996. Robustly efficient parametric frontiers via multiplicative DEA for domestic and international operations of the Latin American airline industry. *European Journal of Operational Research* 88, 525–536.
- Chiang, C.I., Tzeng, G.H., 2000. A new efficiency measure for DEA: efficiency achievement measure established on fuzzy multiple objective programming. *Journal of Management* 17, 369–388 (in Chinese).
- Chiou, Y.C., Chen, Y.H., 2006. Route-based performance evaluation of Taiwanese domestic airlines using data envelopment analysis. *Transportation Research Part E* 42, 116–127.

- Chiou, Y.C., Lan, L.W., Yen, T.H., 2010. A joint measurement of efficiency and effectiveness for non-storable commodities: integrated data envelopment analysis approaches. *European Journal of Operational Research* 201, 477–489.
- Coelli, T., Perelman, S., 1999. A comparison of parametric and non-parametric distance functions: with application to European railways. *European Journal of Operational Research* 117, 326–339.
- Cullinane, K., Wang, T.F., Song, D.W., Ji, P., 2006. The technical efficiency of container ports: comparing data envelopment analysis and stochastic frontier analysis. *Transportation Research Part A* 40, 354–374.
- Fielding, G.J., 1987. *Managing Public Transit Strategically*. Jossey-Bass, Inc., San Francisco.
- Fielding, G.J., Timlynn, T.B., Brenner, M.E., 1984. Performance evaluation for bus transit. *Transportation Research* 19, 73–82.
- Gillen, D., Lall, A., 1997a. Competitive advantage of low-cost carriers: some implications for airports. *Journal of Air Transport Management* 10, 41–50.
- Gillen, D., Lall, A., 1997b. Developing measures of airport productivity and performance: an application of data envelopment analysis. *Transportation Research E* 33, 261–273.
- Greer, M., 2009. Is it the labor unions' fault? Dissecting the causes of the impaired technical efficiencies of the legacy carriers in the United States. *Transportation Research Part A* 43, 779–789.
- Karlaftis, M.G., 2003. Investigating transit production and performance: a programming approach. *Transportation Research Part A* 37, 225–240.
- Karlaftis, M.G., 2004. A DEA approach for evaluating the efficiency and effectiveness of urban transit systems. *European Journal of Operational Research* 152, 354–364.
- Kerstens, K., 1996. Technical efficiency measurement and explanation of French urban transit companies. *Transportation Research Part A* 30, 431–452.
- Lan, L.W., Lin, E.T.J., 2003. Technical efficiency and service effectiveness for railways industry: DEA approaches. *Journal of the Eastern Asia Society for Transportation Studies* 5, 2932–2947.
- Lan, L.W., Lin, E.T.J., 2005. Measuring railway performance with adjustment of environmental effects, data noise and slacks. *Transportmetrica* 1, 161–189.
- Lin, E.T.J., Lan, L.W., 2009. Accounting for accidents in the measurement of transport inefficiency: a case of Taiwanese bus transit. *International Journal of Environment and Sustainable Development* 8, 365–385.
- Lin, E.T.J., Lan, L.W., Hsu, C.S.T., 2010. Assessing the on-road route efficiency for an air-express courier. *Journal of Advanced Transportation* 44, 256–266.
- Margari, B.B., Erbetta, F., Petraglia, C., Piacenza, M., 2007. Regulatory and environmental effects on public transit efficiency: a mixed DEA-SFA approach. *Journal of Regulation Economics* 32, 131–151.
- Martin, J.C., Roman, C., 2001. An application of DEA to measure the efficiency of Spanish airports prior to privatization. *Journal of Air Transportation Management* 7, 149–157.
- Nolan, J.F., 1996. Determinants of productive efficiency in urban transit. *Logistic and Transportation Review* 32, 319–342.
- Nolan, J.F., Ritchie, P.C., Rowcroft, J.E., 2002. Identifying and measuring public policy goals: ISTE and the US bus transit industry. *Journal of Economic Behavior & Organization* 48, 291–304.
- Odeck, J., Alkadi, A., 2001. Evaluating efficiency in the Norwegian bus industry using data envelopment analysis. *Transportation* 28, 211–232.
- Oum, T.H., Yu, C., 1994. Economic efficiency of railways and implications for public policy. *Journal of Transport Economics and Policy* 28, 121–138.
- Peck, M.W.J.R., Scheraga, C.A., Boisjoly, R.P., 1998. Assessing the relative efficiency of aircraft maintenance technologies: an application of data envelopment analysis. *Transportation Research* 32, 261–269.
- Salazar de La Cruz, F., 1999. A DEA approach to the airport production function. *International Journal of Transport Economics* 26, 255–270.
- Sarkis, J., 2000. An analysis of the operational efficiency of major airports in the United States. *Journal of Operations Management* 18, 335–351.
- Schefczyk, M., 1993. Operational performance of airlines: an extension of traditional measurement paradigms. *Strategic Management Journal* 14, 301–317.
- Sengupta, J.K., 1999. A dynamic efficiency model using data envelopment analysis. *International Journal of Production Economics* 62, 209–218.
- Sheth, C., Triantis, K., Teodorovic, D., 2007. Performance evaluation of bus routes: a provider and passenger perspective. *Transportation Research Part E* 43, 453–478.
- Tongzon, J., 2001. Efficiency measurement of selected Australian and other international ports using data envelopment analysis. *Transportation Research Part A* 35, 113–128.
- Viton, P.A., 1998. Changes in multi-mode bus transit efficiency, 1988–1992. *Transportation* 25, 1–21.