Systematic Constructions of Zero-Correlation Zone Sequences

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***Abstract*— Sequences with desired autocorrelation (AC) and cross-correlation (CC) properties are often needed in many communication and radar system applications. In a multipath fading channel with a known delay spread bound, one needs a family of sequences that meets these desired correlation requirements only for those correlation lags that lie within a window called zero-correlation zone. This paper presents two classes of new systematic approaches–a transform domain method and a direct (time domain) synthesis method–for generating zero-correlation zone (ZCZ) sequences. The former approach generates sequence families that achieve the upper bounds on the family size and the zero-correlation zone width for a given sequence period. The latter unifies many existing construction methods and both provide more flexibilities in producing new families of sequences. We also suggest a modulation operation that is capable of transforming nonconstant modulus sequences into polyphase sequences.**

I. Introduction

Many communication and radar applications necessitate the use of sets of sequences with good correlation properties. For use either as the training signal in the preamble or as the signature codes of a spread-spectrum multiple-access (SSMA) network, one would prefer to have a family of sequences whose AC function has a single peak at the zeroth delay ($\tau = 0$) and whose CC values are identically zero. Such sequences can be used to avoid or minimize (i) the interference from other users or other antennas if multiple transmit antennas were in place and (ii) the self-interference (e.g., inter-symbol interference) due to multiple propagation paths. Practical considerations also require that the sequence period be arbitrary and the family size be as large as possible while maintaining the desired AC and CC properties.

For periodic sequences, AC peaks at τ equals to multiple of the period are inevitable. Besides these periodic peaks, it is impossible to have zero periodic CC and AC at all other lags simultaneously. In fact, the bounds on the magnitude of CC and AC values derived in [1] and [2] indicate that there is a tradeoff between AC and CC values when designing sequences. In a multipath fading environment, however, the ideal correlation properties are not required to suppress the interference in categories (i) and (ii). As a matter of fact, if the channel's maximum delay spread T_m and the maximum co-channel users' (distance) separation D_m are known, the ideal correlation properties only need to be maintained within the zero-correlation zone $T_m + D_m/c$, where c is the speed of light. In other words, one that adopts the alternate design approach that grants nonzero correlations outside the zero-correlation zone has little or no impact on the system performance.

Many ZCZ sequence families have been proposed. The PS sequences [4] have periodic nonzero AC values and zero CC values cross all lags. Binary, ternary [5], quadriphase, and polyphase sequence sets [6] have been constructed as well. A family of polyphase sequences based on generalized chirp-like sequences was suggested in [7]. However, these sequences are generated in heuristic manners and there is no theorizing as to why they were so constructed.

In this paper, we present two systematic approaches for generating families of sequences whose periodic AC and CC functions satisfy a variety of ZCZ requirements. While some known ZCZ sequence construction methods employ Hadamard or unitary matrices in time domain [7]–[8], our first approach specifies the desired transform domain properties of a sequence set via such matrices. Sets generated by this approach can be proved to achieve the upper bound for family sizes and zero-correlation zone widths.

The second approach begins with a basic binary sequence that satisfies the ZCZ requirement for AC. The use of basic sequences seem naïvely simple and straightforward. But through procedures as proper nonuniform upsampling and modulating, we can obtain desired ZCZ sequences. Both approaches are elementary and simple and, as will be seen later, several earlier ZCZ sequence constructions are actually special cases of our methods.

The rest of this paper is organized as follows. In the next section, some basic definitions and transform domain characterization of AC and CC are given. The transform domain approaches are presented in Section III and the class of "time domain" approaches are discussed in Section IV. We then demonstrate that some existing ZCZ sequences are special cases of ours and show that many new ZCZ sequence families can be generated by judicious choices of design

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parameters.

II. Definitions and Fundamental Properties

Let $\mathbf{X}$ denote a set of K complex-valued sequences of period N, i.e., for every sequence $\{u(n)\} \in \mathbf{X}$, $u(i) = u(i+N)$, $\forall i \in \mathcal{Z}$, where $\mathcal{Z}$ is the set of all integers. For the sake of convenience, $\mathbf{X}$ will be referred to as an (N, K) sequence set.

Definition 1: The periodic CC function of two period-N sequences $u \equiv \{u(n)\}$ and $v \equiv \{u(n)\}$ is defined as

$$\theta_{uv}(\tau) = \sum_{n=0}^{N-1} u(n)v^*(n-\tau). \quad (1)$$

Thus, the periodic AC function of sequence $\{u(n)\}$ is simply $\theta_{uu}(\tau)$, and obviously, $|\theta_{uu}(\ell)| \leq \theta_{uu}(0)$. Since these CC and AC functions are periodic, we assume $\tau \in \{0, 1, \cdots, N-1\}$ to simplify the discussion.

Definition 2: An (N, K) sequence set $\mathbf{X}$ is called an *ideal* (sequence) set if the following two conditions are satisfied:

i. $\theta_{uv}(\tau) = 0,\ \forall\tau$ and $\forall u, v \in \mathbf{X}$, where $u \neq v$;

ii. $\theta_{uu}(\tau) = \theta_{uu}(0)\delta(\tau),\ \forall u \in \mathbf{X}$, where

$$\delta(\tau) = \begin{cases} 1, & \tau = 0; \\ 0, & \tau \neq 0. \end{cases} \quad (2)$$

Moreover, we denote the discrete Fourier transform (DFT) of a period-N sequence $\{x(n)\}$ by $X(k)$ = DFT$\{x(n)\}$ and the inverse DFT (IDFT) of a period-N transform domain sequence $\{X(k)\}$ by $x(n)$ = IDFT$\{X(k)\}$. The following fundamental lemmas and corollaries are needed in subsequent discourse [9].

Lemma 1: The DFT of the periodic CC function $\theta_{xy}(\tau)$ of two period-N sequences, $\{x(n)\}$ and $\{y(n)\}$, is equal to $X(k)Y^*(k)$, where $\{X(k)\}$ and $\{Y(k)\}$ are the DFTs of $\{x(n)\}$ and $\{y(n)\}$, respectively.

Corollary 1: The AC function $\theta_{xx}(n)$ is equivalent to $x(n) \otimes x^*(-n)$ and $\Theta_{xx}(k)$ = DFT$\{\theta_{xx}(n)\}$ = $|X(k)|^2$, where $\otimes$ denotes the cyclic convolution.

Hence a sequence $\{x(n)\}$ has an impulse-like AC function, i.e., $\theta_{xx}(n) = \theta_{xx}(0)\delta(n)$, if and only if $|X(k)|^2$ is constant for all k. We refer to a sequence that has such AC function as a *perfect* sequence.

Corollary 2: Upsampling of $\{x(n)\}$ is equivalent to a repetition of $\{X(k)\}$. Hence, a long perfect sequence can be generated by upsampling a shorter perfect sequence.

Definition 3: A sequence $\{u_v(n)\}$ is said to be derived from *modulating* the sequence $U = \{u(n)\}$ by the sequence $V = \{v(n)\}$ of the same period if

$$u_v(n) \stackrel{def}{=} U \circ V = \theta_{UV}(n) = \sum_{m=0}^{N-1} u(m)v^*(m-n). \quad (3)$$

It can be proved that

Lemma 2: The AC and CC functions of a set of sequences are invariant (up to a scaling factor) to modulation if the modulating sequence V is a perfect sequence.

As we will see later that the above property makes the *modulation* operation very useful in transforming a sequence into one with a desired constellation.

Definition 4: A set of K period-N sequences $\mathbf{C} = \{C_0, C_1, \cdots, C_{K-1}\}$ is called a ZCZ sequence family (or set) if $\theta_{C_iC_j}(\tau) = 0$ and $\theta_{C_iC_i}(\tau) = \theta_{C_iC_i}(0)\delta(\tau), \forall C_i, C_j \in \mathbf{C}, i \neq j$ and $|\tau| \leq T < N$.

As a ZCZ sequence set $\mathbf{C}$ is characterized by the parameters (N, K, T), with N the sequence period, K the family size (i.e., the number of sequences), and T the width of the zero-correlation zone, we call such sequence set an (N, K, T) ZCZ sequence family. In [3], it is shown that

Corollary 3: For a set of K ZCZ sequences of period N, the width T of the zero-correlation zone is upper-bounded by

$$K(T+1) \leq N. \quad (4)$$

In the following sections, several methods are proposed to produce ZCZ sequences. Sequence sets generated by these methods are of finite length; however, they satisfy ZCZ conditions in *Definition 4* in the circular sense. Equivalently, if we extend these sequences periodically, desired ZCZ sequences can be derived. Therefore, without specifying, sequences are analyzed as they have finite length.

III. Transform Domain Approach

A. Basic Construction

Definition 5: The matrices

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (5)$$

and

$$\mathbf{H}_{2^n} = \begin{bmatrix} \mathbf{H}_{2^{n-1}} & \mathbf{H}_{2^{n-1}} \\ \mathbf{H}_{2^{n-1}} & -\mathbf{H}_{2^{n-1}} \end{bmatrix},\ n = 2, 3, \cdots \quad (6)$$

are called standard Hadamard matrices.

Theorem 1: Let $\mathbf{H} = [\mathbf{h}_0^T, \mathbf{h}_1^T, \cdots, \mathbf{h}_{N-1}^T]^T$ be a standard Hadamard matrix of order $\mathbf{N} = 2^n$, where $\mathbf{h}_i$ is the ith row of $\mathbf{H}$. Partition $\mathbf{H}$ into $m = N/K = 2^p, K \times N$ submatrices, $\tilde{\mathbf{A}}_i^T = [\mathbf{h}_{iK}^T, \cdots, \mathbf{h}_{(i+1)K-1}^T] \stackrel{def}{=} [\tilde{\mathbf{a}}_{i,0}^T, \tilde{\mathbf{a}}_{i,1}^T, \cdots, \tilde{\mathbf{a}}_{i,K-1}^T], i = 0, 1, \cdots, m-1$, and call such partition as the regular pth-order partition, i.e., each submatrix is formed by K consecutive rows of $\mathbf{H}$. For all i, the sets of K period-N sequences $\mathbf{A}_i \stackrel{def}{=} \{A_{i,0}, A_{i,1}, \cdots, A_{i,K-1}\}$, are all $(N, K, m-1)$ ZCZ sequence families that achieve the upper bound (4), where $A_{i,j}$ = IDFT$\{\tilde{\mathbf{a}}_{i,j}\}$.

The procedure in *Theorem 1* can be generalized to

Theorem 2: Let $\mathbf{U}_2$ be any 2×2 complex unitary matrix and $\mathbf{U}_{2^n}$ be recursively generated by

$$\mathbf{U}_{2^n} = \begin{bmatrix} \mathbf{U}_{2^{n-1}} & \mathbf{U}_{2^{n-1}} \\ \mathbf{U}_{2^{n-1}} & -\mathbf{U}_{2^{n-1}} \end{bmatrix}, \ n = 2, 3, \cdots. \quad (7)$$

Let $N, K, m \in \mathcal{Z}^+$ (the set of all positive integers) be such that $N = 2^n, m = N/K = 2^p$ for some $n, p \in \mathcal{Z}^+$. An $(N, K, m-1)$ ZCZ sequence family can be obtained by applying the regular pth-order partition on the matrix $\mathbf{U}_{2^n}$ and performing IDFT on the row vectors of any submatrix.

Note that if the entries of $\mathbf{U}_{2^n}$ are constant modulus numbers, each member sequence of the generated ZCZ sequence family has a perfect AC function.

B. New Polyphase Sequences

By invoking *Theorem 1* and *Lemma 2*, we give a simple example for constructing a quadriphase ZCZ sequence family that satisfies (4). We first partition the standard Hadamard matrix $\mathbf{H}_{16}$ into four submatrices $\tilde{\mathbf{A}}_0, \tilde{\mathbf{A}}_1, \tilde{\mathbf{A}}_2$, and $\tilde{\mathbf{A}}_3$ (i.e., $m = 4$). Choosing $\tilde{\mathbf{A}}_0^T = [\tilde{\mathbf{a}}_0^T, \tilde{\mathbf{a}}_1^T, \tilde{\mathbf{a}}_2^T, \tilde{\mathbf{a}}_3^T]$ and taking IDFT on these transform domain sequences, we then obtain

$$\begin{aligned} A_0 &= (1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0) \\ A_1 &= (0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0) \\ A_2 &= (0,0,0,0,\frac{1}{\sqrt{2}}W_8^1,0,0,0,0,0,0,0,\frac{1}{\sqrt{2}}W_8^3,0,0,0) \\ A_3 &= (0,0,0,0,\frac{1}{\sqrt{2}}W_8^3,0,0,0,0,0,0,0,\frac{1}{\sqrt{2}}W_8^1,0,0,0) \end{aligned} \quad (8)$$

where $W_N^k = e^{j2\pi k/N}$. However, the above sequences have too many zeros and nonconstant modulus components. To transform them into sequences of constant moduli we need a perfect sequence U with constant modulus. Such a sequence can be generated by applying the method proposed in [9]. We choose

$$\begin{aligned} U = (&W_4^0W_4^0W_4^0W_4^0W_4^0W_4^3W_4^2W_4^1 \\ &W_4^0W_4^2W_4^0W_4^2W_4^0W_4^1W_4^2W_4^3). \end{aligned} \quad (9)$$

The resulting sequences

$$\begin{aligned} C_0 = A_0 \circ U = (&W_4^0W_4^1W_4^2W_4^3W_4^0W_4^2W_4^0W_4^2 \\ &W_4^0W_4^3W_4^2W_4^1W_4^0W_4^0W_4^0W_4^0), \\ C_1 = A_1 \circ U = (&W_4^0W_4^3W_4^2W_4^1W_4^0W_4^0W_4^0W_4^0 \\ &W_4^0W_4^1W_4^2W_4^3W_4^0W_4^2W_4^0W_4^2), \\ C_2 = A_2 \circ U = (&W_4^0W_4^1W_4^0W_4^1W_4^0W_4^2W_4^2W_4^0 \\ &W_4^0W_4^3W_4^0W_4^3W_4^0W_4^0W_4^2W_4^2), \\ C_3 = A_3 \circ U = (&W_4^0W_4^3W_4^0W_4^3W_4^0W_4^0W_4^2W_4^2 \\ &W_4^0W_4^1W_4^0W_4^1W_4^0W_4^2W_4^2W_4^0) \end{aligned} \quad (10)$$

form a quadriphase (16,4,3) ZCZ sequence family.

These sequences have perfect AC function as they have constant DFT magnitudes. Note that if we use a perfect sequence whose components are drawn from a larger constellation, e.g., the Frank-Zadoff-Chu (FZC) sequence [10]–[11], the resulting ZCZ sequences will also have components from the corresponding constellation. Usually, a small constellation size is desirable in practical applications.

IV. Direct Synthesis Method

A. Preliminaries

The transform domain approach needs to perform IDFT to obtain ZCZ sequences and the sequence periods are limited to powers of 2. We now present alternate approaches which have simple intuitive interpretation and are capable of generating ZCZ sequences of arbitrary non-prime periods.

Definition 6: A binary sequence, consists only of 0's and 1's, of period N satisfies the zero-correlation zone width constraint T on AC is called a basic (N, T) sequence.

A basic sequence can be obtained by the simple rule given in

Lemma 3: A binary sequence $B = (b(0), b(1), \cdots, b(N-1))$ with $b(i) \in \{0, 1\}$ is a basic (N, T) sequence if the minimum run length of 0's between two consecutive 1's is T, where a run refers to a repeating string of symbols. We also call T as the *minimum spacing* of the sequence B.

We start with a decomposition of a basic sequence.

Corollary 4: If a basic (N, T) sequence B, regarding as a real vector of dimension N, can be expressed as the sum of K orthogonal N-dimensional nonnegative integer vectors B^i, with $K \leq w_H(B)$ and $\sum_{i=0}^{K-1} w_H(B^i) = w_H(B)$, the set $\{B^i, \ i = 0, 1, \cdots, K-1\}$ is a binary (N, K, T) ZCZ sequence family, where $w_H(\cdot)$ denotes the Hamming weight of the enclosed vector. We call such decomposition an *orthogonal tone decomposition*.

B. Synthesis Process

Definition 7: Let $\mathbf{V}$ be an N-dimensional vector with Hamming weight $w_H(\mathbf{V}) = k$ and $\mathbf{H} = [h_{m,n}]$ be an arbitrary $k' \times k$ matrix. We define the $k' \times N$ matrix $\mathbf{P}$ via the operation $\mathbf{P} = \mathbf{H} \bigtriangleup \mathbf{V} = [p_{i,j}]$, where

$$p_{m,v(n)} = \begin{cases} h_{m,n}, & v(n) = \text{the coordinate of the } n\text{th} \\ & \qquad \text{nonzero entry of } \mathbf{V}, \\ & \text{where } 0 \leq m < k'; \\ 0, & \text{otherwise}. \end{cases} \quad (11)$$

Using the operation $\bigtriangleup$, which performs nonuniform upsampling on rows of $\mathbf{H}$, we can obtain new ZCZ sequences out of basic sequences. In fact, it can be proved that

Lemma 4: Rows of the upsampled matrix $\mathbf{P} = \mathbf{H} \bigtriangleup \mathbf{V}$ form an $(N, w_H(\mathbf{V}), T)$ ZCZ sequence family if $\mathbf{V}$ is a basic (N, T) sequence and $\mathbf{H}$ is a $k \times k$ unitary matrix.

This immediately leads to

Corollary 5: Let B be a basic (N,T) sequence and $\{B^i,\ i=0,1,\cdots,M-1\}$ be an orthogonal tone decomposition of B. The set of all rows of the M matrices $\{\mathbf{P}^i = \mathbf{H}^i \triangle B^i,\ i=0,1,\cdots,M-1\}$, $\{P_n^i,\ n=0,1,\cdots,m_i-1,\ i=0,1,\cdots,M-1\}$, forms an $(N, w_H(B), T)$ ZCZ sequence family, where $\mathbf{H}^i$ is an $m_i \times m_i$ unitary matrix with $m_i = w_H(B^i)$. Note that for all i, $\mathbf{H}^i$ are not necessarily distinct.

The above discussion suggests that an (N,K,T) ZCZ sequence family can be generated by the following three steps.

1. Let B be a basic (N,T) sequence with $w_H(B)=K$ and $\mathbf{B} \stackrel{def}{=} \{B^i = (b^i(0), b^i(1), \cdots, b^i(N-1)),\ i=0,1,\cdots,M-1\}$ be an orthogonal tone decomposition of B, where obviously $M \le K$.
2. Compute the M nonuniform upsampled matrices $\mathbf{P}^i = \mathbf{H}^i \triangle B^i$, where $\mathbf{H}^i$ is an $m_i \times m_i$ unitary matrix with $m_i = w_H(B^i)$.
3. Let A be a perfect sequence with period N. For all i, modulating each row of $\mathbf{P}^i$ by A, we obtain a set of modulated sequences $\mathbf{C} = \{C_0, C_1, \cdots, C_{K-1}\}$.

Theorem 3: The sequence set obtained at Step 1 is an (N,M,T) ZCZ sequence family, and those obtained at Step 2 and 3 are (N,K,T) ZCZ sequence families.

The set obtained in Step 1 is obviously an (N,M,T) ZCZ sequence family. A larger ZCZ sequence family with size K is derived from this (N,M,T) family in Step 2 by collecting all rows of $\mathbf{P}^i$ for all i. A perfect sequence A is used to modulate the ZCZ sequences into sequences of finite constellation signals in Step 3.

C. Polyphase ZCZ Sequences

Let $A' = \{a'(n)\}$, where $a'(n) \in \{W_{N_{A'}}^{\ell} : 0 \le \ell < N_{A'},\ 2 \le N_{A'} \le N'\}$, be a length-$N'$ polyphase sequence with perfect AC. Upsampling A' by N_r-fold (so that $N = N'N_r$), one obtains another perfect sequence A of length N by *Corollary 2*. *Theorem 3* tells us that we can generate a ZCZ sequence family based on a basic sequence, unitary matrices, and a perfect sequence, thus we can easily come to that

Corollary 6: An $(N, N_r, N'-2)$ or $(N, N_r, N'-1)$ ZCZ polyphase sequence family with elements drawn from the polyphase set $\{W_{L_0}^{\ell} : 0 \le \ell < L_0,\ L_0 = lcm(N_{A'}, N_r)\}$ can be obtained by following the synthesis procedure in *Theorem 3* with the perfect sequence A, and the $N_r \times N_r$ unitary matrix

$$\mathbf{H} = \frac{1}{\sqrt{N_r}} \begin{bmatrix} W_{N_r}^0 & W_{N_r}^0 & \cdots & W_{N_r}^0 \\ W_{N_r}^0 & W_{N_r}^1 & \cdots & W_{N_r}^{N_r-1} \\ \vdots & \vdots & \ddots & \vdots \\ W_{N_r}^0 & W_{N_r}^{N_r-1} & \cdots & W_{N_r}^{(N_r-1)^2} \end{bmatrix}. \quad (12)$$

The basic binary sequences used are of the form $B = (b(0), b(1), \cdots, b(N-1))$ with $w_H(B) = N_r$ and its elements given by

$$b(i) = \begin{cases} 1, & i = kN',\ k = 0,1,\cdots,\frac{\zeta}{N'}-1, \text{ or} \\ & i = \ell\zeta + (\frac{N}{\zeta} - \ell) + kN', \\ & \ell = 1,\cdots,\frac{N}{\zeta}-1,\ k=0,1,\cdots,\frac{\zeta}{N'}-1; \\ 0, & \text{otherwise,} \end{cases} \quad (13)$$

for $N/\zeta > 1$, where $\zeta = lcm(N_r, N')$ or by

$$b(i) = \begin{cases} 1, & i = kN',\ k=0,1,\cdots,N_r-1; \\ 0, & \text{otherwise,} \end{cases} \quad (14)$$

if $N/\zeta = 1$, i.e., N_r and N' are relatively prime.

For fixed N and K, ZCZ sequence families generated from (14) achieve the upper bound (4) and those generated from (13) satisfy the relation $K(T+1) = N-1$. It is worth mentioning that the basic sequences (13) and (14) can be cyclically shifted to generate distinct ZCZ sequence families without affecting properties mentioned above. Furthermore, if N_r is a power of 2, we can use an $N_r \times N_r$ Hadamard matrix instead of $\mathbf{H}$ (12) to reduce $W_{lcm(N_{A'},N_r)}^{\ell}$ to $W_{lcm(N_{A'},2)}^{\ell}$ and thus reduce the needed constellation size.

The new polyphase ZCZ sequences generated from (13) are *generalizations* of sequences reported in [6] where N_r is constrained to be a multiple or a factor of N'. In contrast, our proposed method gives us more flexibility in that we can produce sequences of *any length that is equal to a composite number* $N = N_rN'$, where N' is the length of the polyphase perfect sequence and N_r is any positive integer.

Moreover, some of the sequences generated by the method suggested in [8] are similar to those derived from (14) with the constraint that N_r divides $N'+1$. As a result, one can generate same polyphase ZCZ sequences as those given in [8] by employing *Corollary 6* and a properly rearranged basic sequence B. Note that such sequences have ZCZ widths smaller than those generated by (13) and (14).

V. Application Examples

A. PS-like Sequences

A set of PS-like sequences with family size 3 can be generated by employing *Corollary 6*. More specifically, we let $N_r = 3$, $N' = 4$, and $B = (100010001000)$, and utilize the unnormalized unitary matrix

$$\mathbf{H} = \begin{bmatrix} W_3^0 & W_3^0 & W_3^0 \\ W_3^0 & W_3^1 & W_3^2 \\ W_3^0 & W_3^2 & W_3^1 \end{bmatrix} \quad (15)$$

and the perfect sequence $A = (1,0,0,1,0,0,1,0,0,-1,0,0)$. We first obtain

$$\begin{aligned} P_0 &= (W_3^0 000 W_3^0 000 W_3^0 000) \\ P_1 &= (W_3^0 000 W_3^1 000 W_3^2 000) \\ P_2 &= (W_3^0 000 W_3^2 000 W_3^1 000). \end{aligned} \quad (16)$$

Modulate them by A, we have

$$\begin{aligned} & C_0 = P_0 \circ A \\ = & (W_6^0 W_6^0 W_6^0 W_6^3 W_6^0 W_6^0 W_6^0 W_6^3 W_6^0 W_6^0 W_6^0 W_6^3) \\ & C_1 = P_1 \circ A \\ = & (W_6^0 W_6^2 W_6^4 W_6^3 W_6^2 W_6^4 W_6^0 W_6^5 W_6^4 W_6^0 W_6^2 W_6^1) \\ & C_2 = P_2 \circ A \\ = & (W_6^0 W_6^4 W_6^2 W_6^3 W_6^4 W_6^2 W_6^0 W_6^1 W_6^2 W_6^0 W_6^4 W_6^5). \end{aligned} \tag{17}$$

It can be verified that $\forall i \neq j$

$$\begin{aligned} \theta_{C_i C_j}(\tau) &= 0; \\ |\theta_{C_i C_i}(\tau)| &= 12\delta(|\tau|_4), \end{aligned} \tag{18}$$

where $|k|_N \overset{def}{=} k \bmod N$. Therefore, $\mathbf{C} = \{C_0, C_1, C_2\}$ is a $(12, 3, 3)$ bound-achieving ZCZ sequence family. Sequences in $\mathbf{C}$ are called *PS-like sequences* because they have the same correlation properties as those of PS sequences generated in [4]; however, they require only *half* of the constellation size used by the original PS sequences.

B. Binary and Ternary ZCZ Sequences

While ZCZ sequences presented in the previous example begin with a basic sequence that has equally spaced nonzero entries, one can also build ZCZ sequences based on nonuniformly spaced basic sequences. We provide two examples to demonstrate such generation procedure. Using the basic sequence $B^0 = B = (1000000100100100)$, the standard Hadamard matrix $\mathbf{H}_4$, and the perfect sequence $A = (1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, -1, 0, 0, 0)$, we obtain [5]

$$\begin{aligned} P_0^0 &= (1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0) \\ P_1^0 &= (1, 0, 0, 0, 0, 0, 0, -1, 0, 0, 1, 0, 0, -1, 0, 0) \\ P_2^0 &= (1, 0, 0, 0, 0, 0, 0, 1, 0, 0, -1, 0, 0, -1, 0, 0) \\ P_3^0 &= (1, 0, 0, 0, 0, 0, 0, -1, 0, 0, -1, 0, 0, 1, 0, 0). \end{aligned} \tag{19}$$

These are ternary ZCZ sequences with many 0's. By modulating them with A, we obtain the binary $(16, 4, 2)$ ZCZ sequence family consisting of

$$\begin{aligned} & C_0 = P_0^0 \circ A \\ = & (1, -1, 1, 1, -1, 1, 1, 1, 1, 1, 1, -1, 1, 1, -1, 1) \\ & C_1 = P_1^0 \circ A \\ = & (1, 1, 1, -1, -1, -1, 1, -1, 1, -1, 1, 1, 1, -1, -1, -1) \\ & C_2 = P_2^0 \circ A \\ = & (1, 1, -1, 1, -1, -1, -1, 1, 1, -1, -1, -1, 1, -1, 1, 1) \\ & C_2 = P_3^0 \circ A \\ = & (1, -1, -1, -1, -1, 1, -1, -1, 1, 1, -1, 1, 1, 1, 1, -1). \end{aligned} \tag{20}$$

VI. Conclusion

In this paper, we present two classes of systematic approaches for constructing ZCZ sequences. Our approaches lead to simple and straightforward generations of many existing ZCZ sequences and provide systematic procedures to find new ones with desired parameter values for (N, K, T). These approaches are capable of constructing a ZCZ sequence set that achieves the ZCZ bound with every member sequence possesses either a perfect AC or a perfect CC property. A set of sequences with perfect CC function can be derived by the proposed direct synthesis method, whereas one with ideal AC function can be found by the transform domain approach. We show that the *modulation* operation is both critical and useful in transforming nonconstant modulus sequences into finite constellation-based constant modulus sequences without affecting the ZCZ properties. Some examples are given to demonstrate how new ZCZ sequences are generated. One of the example sequence sets is a new polyphase ZCZ sequence family, each of its member sequences has the perfect AC property in addition to the required CC property.

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