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## Photon recycling effect on electroluminescent refrigeration

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We study electroluminescent refrigeration in an AlGaAs/GaAs double heterostructure by a self-consistent calculation with photon recycling considered. To gain insight, we investigate the influence of the recycling on the carrier density and the current components due to various recombination mechanisms in the device under different bias voltages. The photon recycling is a feedback process, which behaves as an internal source of generating electron-hole pairs in the active region and causes an effective feedback current to compensate the driving current from the external source. Consequently, it reduces the driving current, improves the external quantum efficiency, and loosens the requirement on the photon extraction efficiency for refrigeration. For the device with a 1  $\mu$ m GaAs active layer operating at 300 K, the minimum required extraction efficiency is less than 20% if the trapped photons are completely recycled and remains a feasible value of 45% if the recycling efficiency is 90%, which is not difficult to achieve. In addition, photon recycling eases the problem of the drastic deterioration of the cooling power and the external efficiency as the extraction efficiency reduces. These results reveal a good possibility of realizing electroluminescent refrigeration in semiconductors. © 2012 American Institute of Physics. [doi:10.1063/1.3676249]

#### I. INTRODUCTION

Optical refrigeration in solids has attracted significant interest since the first demonstration of photoluminescent (PL) refrigeration in 1995. To date, it has been shown that an Ybdoped LiYF<sub>4</sub> crystal is capable of reducing the temperature down to 155 K with a cooling power of 90 mW.<sup>2</sup> Further reduction of the temperature below 100 K seems difficult to achieve in such a rare-earth doped system. This is because the cooling process becomes inefficient as the thermal energy approaches the energy difference between levels of the ground-state manifold.<sup>3,4</sup> Compared to the rare-earth doped cooler, a semiconductor luminescent cooler has no such limitation and is expected to have the potential of lower operation temperature and higher cooling power density. Additionally, it can be integrated directly with other semiconductor devices. These attractive features have stimulated extensive research in semiconductor PL refrigeration.<sup>5–9</sup> However, experimental demonstration of semiconductor PL cooling has not been realized, mainly due to the harsh requirement of a nearly 100% external quantum efficiency (EQE) for semiconductors with bandgap energies larger than 1 eV.

Recently, semiconductor luminescent refrigeration using an electrical pumping scheme has received growing attention because of its looser requirement on EQE compared with semiconductor PL refrigeration. Several theoretical studies have been carried out to investigate the cooling feasibility and capability of semiconductor electroluminescent (EL) refrigeration. Mal'shukov and Chao studied the influence of the Auger recombination on the cooling capability of a GaAs embedded double heterostructure (DH). They

It was shown that photon recycling can significantly reduce the extraction efficiency required for semiconductor PL refrigeration from 98% to 5%. However, the photon recycling effect on EL refrigeration has not yet been studied thoroughly. Recently, Heikkilä *et al.* performed a self-consistent calculation and used a semiquantitative model to study the ultimate efficiency of AlGaAs/GaAs DH light-emitting diodes with the consideration of the photon recycling effect. Their works mainly focused on a specific case of constant extraction efficiency and optical parasitic loss. It was left open how the photon recycling influences the operation of the EL refrigerator under various external bias voltages through various microscopic electric and optical processes.

In this work, we investigate the photon recycling effect by a self-consistent calculation on the cooling capability,

showed that a net cooling power density of several W/cm<sup>2</sup> is achievable, even if the Auger recombination coefficient is as high as  $4 \times 10^{-29}$  cm<sup>6</sup>/s. Later, Wang et al. performed a detailed self-consistent calculation with the consideration of various recombination mechanisms. 12 Their analysis showed an impressive cooling efficiency of 35% for a DH with a p-GaAs active layer. Recently, we studied the influence of the current leakage and the active layer thickness on EL refrigeration. 10 We found that the leakage current can be eliminated almost completely by carrier blocking layers. The study gave a limiting cooling power density of 97 W/cm<sup>2</sup> for AlGaAs/GaAs DH at 300 K. However, the aforementioned studies did not take into account the photon recycling effect. 15 In the absence of photon recycling, the extraction efficiency required for cooling would be at least 75%. <sup>10,12</sup> Achieving such a tight requirement is a challenge to the state-of-the-art technology, and the difficulty has been obstructing the realization of semiconductor EL cooling.

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the internal and external efficiencies, and the electrical properties of an AlGaAs/GaAs DH EL refrigerator. We find that the photon recycling works as an internal pumping source to supply electron-hole pairs, which mediate as the refrigerant for extraction of internal energy through thermalization and radiative recombination in the active layer. The effect can be modeled as an effective feedback current, which compensates the driving current from the external source. As a consequence, the photon recycling can reduce considerably the driving current for a device biased at a given voltage and, hence, improve the EQE. Additionally, the recycling alleviates the requirement on the extraction efficiency for refrigeration, because it does not dissipate the photon energy, but transforms the photons into the workable potential energy in the form of electron-hole pairs. The minimum required extraction efficiency is less than 20% if the trapped photons were all reincarnated into electron-hole pairs and remains a feasible value of about 45% if the photon recycling efficiency is taken as a more practical value of 90%.

This paper is organized as follows. In Sec. II, we will first explain about the operation principle of the EL refrigerator and then describe the self-consistent method for the calculation of the work. Calculated results together with analysis and discussion will be given in Sec. III. Finally, we draw a conclusion in Sec. IV.

### **II. THEORETICAL APPROACHES**

We consider a p-i-n heterojunction structure, whose band diagram is shown in Fig. 1. It was formed, in order, with a 100-nm p-Al $_{0.25}$ Ga $_{0.75}$ As cladding layer (with an acceptor concentration  $N_a=10^{18}~{\rm cm}^{-3}$ ), a 50-nm p-Al $_{0.4}$ Ga $_{0.6}$ As electron blocking layer (with  $N_a=10^{18}~{\rm cm}^{-3}$ ), a 50-nm undoped Al $_{0.25}$ Ga $_{0.75}$ As spacer layer, a 1- $\mu$ m undoped GaAs active layer, a 50-nm undoped Al $_{0.25}$ Ga $_{0.75}$ As spacer layer, a 50-nm n-Al $_{0.4}$ Ga $_{0.6}$ As hole blocking layer (with a donor concentration  $N_d=10^{18}~{\rm cm}^{-3}$ ), and a 100-nm n-Al $_{0.25}$ Ga $_{0.75}$ As cladding layer (with  $N_d=10^{18}~{\rm cm}^{-3}$ ). Figure 1 also shows the quasi-Fermi level profiles,  $E_{Fc}$  and  $E_{Fv}$ , as well as the energy band profiles,  $E_c$  and  $E_v$ , of the conduction and the

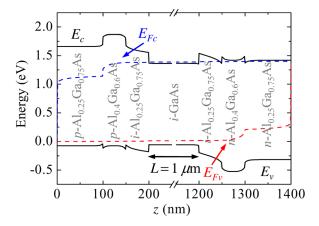


FIG. 1. (Color online) The energy band profiles,  $E_c$  and  $E_v$ , and the quasi-Fermi level profiles,  $E_{Fc}$  and  $E_{Fv}$ , of the conduction and the valence bands, respectively, along the growth direction z. The active layer thickness L is 1  $\mu$ m. The profiles are a result of our self-consistent calculation at a forward bias of 1.4 V.

valence bands, respectively, along the growth direction (defined as the axis z) under a forward bias of 1.4 V. These profiles are obtained from the self-consistent calculation to be described. The calculation approach is similar to that in our previous work, <sup>10</sup> except that we consider the photon recycling in the present calculation.

# A. Operation principle of EL refrigerators with photon recycling

Photon recycling is a feedback process. The operation of an EL refrigerator with photon recycling can be figured out with the aid of the diagram in Fig. 2. Electrons and holes injected from the electrodes under a forward bias cause an electric current J in the device. Most of the electrons and holes are intended to be injected into the active region, recombine therein, and then result in a major part of the current, called the injection current  $J_{\rm inj}$ . The injection efficiency  $\eta_{\rm inj}$  is defined as the ratio  $J_{\rm inj}/J$ . The other small part of the current, called the leakage current  $J_{\text{leak}}$ , arises from recombinations of electrons and holes residing out of the active region. There are two means of injecting carriers to the active region. One is by external electric injection, as described previously. The other is by internal optical excitation if photons of sufficient energy are present. The generation of electron-hole pairs by optical excitation is modeled as an effective current  $J_G$ , while the electric injection results in

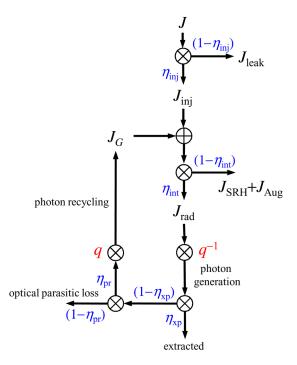


FIG. 2. (Color online) The diagram for illustration of the operation of an electroluminescent refrigerator with photon recycling as a feedback. It indicates the relationships among various currents, including the driving current J, the leakage current  $J_{\text{leak}}$ , the SRH recombination current  $J_{\text{SRH}}$ , the Auger recombination current  $J_{\text{Aug}}$ , the radiative recombination current  $J_{\text{rad}}$ , and the optical generation current  $J_G$ . Symbol  $\oplus$  stands for an addition operation which sums its inputs, indicated by inward arrows with the tips pointing at the symbol, and gives its output, indicated by an outward arrow with the tip pointing away from the symbol. Symbols  $\otimes$  stand for multiplication operations, which give their outputs, indicated by outward arrows, by multiplying their inputs, indicated by inward arrows, by the factors along with the individual outward arrows.

the injection current  $J_{\rm inj}$ . As a result, the sum  $J_{\rm inj}+J_G$  accounts for the total effective injection rate of electrons and holes to the active region. In the steady-state condition, the injection rate is balanced by the recombination rate in the active region. Hence,

$$J_{\text{ini}} + J_G = J_{\text{rad}} + J_{\text{SRH}} + J_{\text{Aug}},\tag{1}$$

where  $J_{\rm rad}$ ,  $J_{\rm SRH}$ , and  $J_{\rm Aug}$  are the current components due to the radiative, the Shockley-Read-Hall (SRH), and the Auger recombinations, respectively, in the active region. The interquantum efficiency  $\eta_{int}$  is defined as the ratio  $J_{\rm rad}/(J_{\rm rad}+J_{\rm SRH}+J_{\rm Aug})$ . Each radiative recombination generates a photon of energy equal to or slightly higher than the bandgap energy of the active region. The resulting photon generation rate is  $J_{\rm rad}/q$ , where q is the elementary charge. All the generated photons have a probability of escaping from the device. Such a probability is called the extraction efficiency of photons and denoted by  $\eta_{xp}$ . Hence, the emission rate of photons from the device is  $\eta_{xp}J_{rad}/q$ . Photons that are trapped within the device are destined for reabsorption. Of the trapped photons, the majority are intended to be recycled with electron-hole pairs created in the active region; the others are reabsorbed parasitically and transformed into thermal energy, leading to the so-called optical parasitic loss. We define the photon recycling efficiency  $\eta_{\rm pr}$  as the percentage of the trapped photons that are eventually recycled. The photon recycling provides one of the two means of injecting carriers to the active region, as has been described. It gives rise to an effective feedback current  $J_G$ , which is

$$J_G = \eta_{\rm pr} (1 - \eta_{\rm xp}) J_{\rm rad}. \tag{2}$$

In conventional optoelectronic devices, the flux of photons emitted from the devices,  $\eta_{\rm xp}J_{\rm rad}/q$ , is regarded as the output and the current in the devices, J, is the input. The external quantum efficiency  $\eta_{\rm ext}$  is naturally defined as the ratio of the photon flux out of the devices to the carrier flux injected into the devices, i.e.,  $\eta_{\rm ext} = \eta_{\rm xp}J_{\rm rad}/J$ . Equations (1) and (2) combined with the definitions of the various efficiencies allow us to express  $\eta_{\rm ext}$  in terms of the other efficiencies,

$$\eta_{\text{ext}} = \frac{\eta_{\text{xp}} \eta_{\text{inj}} \eta_{\text{int}}}{1 - \eta_{\text{pr}} \eta_{\text{int}} (1 - \eta_{\text{xp}})}.$$
 (3)

Equation (3) reveals the enhancement of the external efficiency by photon recycling. In the absence of photon recycling ( $\eta_{\rm pr}=0$ ),  $\eta_{\rm ext}$  would be simply the product  $\eta_{\rm xp}\eta_{\rm inj}\eta_{\rm int}$ . The enhancement factor  $\gamma$ , defined as the ratio of the external efficiency with to without the photon recycling, is thus

$$\gamma = [1 - \eta_{\rm pr} \eta_{\rm int} (1 - \eta_{\rm xp})]^{-1} \tag{4}$$

if  $\eta_{\rm xp}\eta_{\rm inj}\eta_{\rm int}$  is independent of  $\eta_{\rm pr}$ . In practice, Eq. (4) is a good approximation, because  $\eta_{\rm xp}\eta_{\rm inj}\eta_{\rm int}$  is generally insensitive to  $\eta_{\rm pr}$ .

The above analysis implies three paths of energy loss that reduces the output. They are the current leakage, the nonradiative electron-hole recombinations in the active region, and the optical parasitic absorption. In the subsequent analysis, we assume that the three loss paths convert the electric energy of carriers or the optical energy of photons into thermal energy in the devices.

Energy can be extracted out of the devices by photon emission and heat transfer. The extracted power is required by energy conservation to equal the input power into the devices at steady state. The input power is JV, where V is the applied voltage. The radiative power  $P_{\rm rad}$ , which is the power extracted by photon emission, is the product of the emitted photon flux and the average photon energy  $\langle \hbar \omega \rangle$ ,

$$P_{\rm rad} = \langle \hbar \omega \rangle \eta_{\rm xp} J_{\rm rad} / q. \tag{5}$$

The difference  $JV - P_{\rm rad}$  is, therefore, the power extracted out by heat transfer. If the difference is negative, the heat transfer causes a net heat flow into the devices; that is, the devices have the capability of extracting thermal energy from their surroundings to the devices. Accordingly, the cooling power of the devices is defined as

$$P_c = P_{\text{rad}} - JV = \eta_c JV, \tag{6}$$

where  $\eta_c \equiv P_c/JV$  is known as the cooling efficiency or the coefficient of performance (COP). The notion of EL refrigeration is to set the devices in the cooling mode, where the cooling power  $P_c > 0$  or  $\eta_c > 0$ . Using Eq. (5) and the definition of  $\eta_{\rm ext} = \eta_{\rm xp} J_{\rm rad}/J$ , we can rewrite the cooling efficiency as

$$\eta_c = \frac{\eta_{\text{ext}}}{\eta_{\text{ext,cr}}} - 1,\tag{7}$$

where  $\eta_{\rm ext,cr} \equiv qV/\langle \hbar\omega \rangle$  is the critical external efficiency for the devices to operate in the cooling mode. From the relation in Eq. (3), we express the critical extraction efficiency as

$$\eta_{\rm xp,cr} = \frac{1 - \eta_{\rm pr} \eta_{\rm int}}{\eta_{\rm int} \left( \eta_{\rm inj} / \eta_{\rm ext,cr} - \eta_{\rm pr} \right)},\tag{8}$$

for  $\eta_{\rm inj}/\eta_{\rm ext,cr} > \eta_{\rm pr}$ .

### B. Self-consistent model

In Subsection II A, we have introduced various terms for describing the operation principle of the EL refrigerator with photon recycling. The operation involves electric processes and optical processes. In this subsection, we present a self-consistent model for quantitative analysis of the device performance through the terms. The model is concerned mainly with the electric conduction in the devices with the electric processes considered in detail. Some of the optical processes, such as photon extraction and photon recycling, are sensitive to the surface, geometry, structure, and material of the devices. We shall not involve ourselves in treating the details of the optical processes in this work, but instead use the extraction efficiency  $\eta_{xp}$  and the photon recycling efficiency  $\eta_{pr}$  as the input parameters.

With  $\eta_{xp}$  and  $\eta_{pr}$  known, the central problem in the calculation turns out to be solving a set of coupled equations for

the profiles  $E_{Fc}$ ,  $E_{Fv}$ ,  $E_c$ , and  $E_v$ , as shown in Fig. 1, since, in the quasi-equilibrium approximation, the profiles can determine the quantities for analyzing the device performance. These quantities include (1) the various concentrations, such as the electron concentration n, the hole concentration p, the ionized donor concentration  $N_d^+$ , and the ionized acceptor concentration  $N_a^-$ , (2) the various recombination rates, such as the radiative recombination rate  $R_{\rm rad}$ , the Auger recombination rate  $R_{\rm Aug}$ , and the SRH recombination rate  $R_{\rm SRH}$ , (3) the various current components, such as  $J_{\rm inj}$ ,  $J_G$ ,  $J_{\rm rad}$ ,  $J_{\rm Aug}$ ,  $J_{\rm SRH}$ , and so forth, and (4) other quantities, such as  $\langle \hbar \omega \rangle$ ,  $P_c$ , and the various efficiencies. <sup>10</sup> Refer to Ref. 10 for the relations of these quantities, except  $J_G$ , to the profiles  $E_{Fc}$ ,  $E_{Fv}$ ,  $E_c$ , and  $E_v$ .

Since the difference  $E_c(z) - E_v(z)$  is the bandgap  $E_g(z)$ , which is given as an input in the calculation, the bending of the conduction band is the same as that of the valence band. The band bending is described by the electric potential  $\Phi(z)$ ,

$$E_c = E_{c0} - q\Phi, E_v = E_{v0} - q\Phi, \tag{9}$$

where  $E_{c0}$  and  $E_{v0}$  are the profiles of the conduction and the valence flat bands, respectively. Consequently, we need three equations with proper boundary conditions to solve for the three independent functions  $E_{Fc}(z)$ ,  $E_{Fv}(z)$ , and  $\Phi(z)$ . These equations are the one-dimensional Poisson equation,

$$\frac{d}{dz}\epsilon\frac{d}{dz}\Phi = -q(p-n+N_d^+-N_a^-),\tag{10}$$

and the continuity equations of electrons and holes in the steady state,

$$dJ_n/dz = q(R - G), (11)$$

$$dJ_p/dz = -q(R - G), (12)$$

where  $\epsilon$  is the electric permittivity, R is the electron-hole recombination rate minus the generation rate due to thermal excitation, G is the electron-hole generation rate due to optical excitation, and  $J_n$  and  $J_p$  are the drift-diffusion currents of electrons and holes, respectively. The currents  $J_n$  and  $J_p$  can be expressed by  $E_{Fc}$ ,  $E_{Fv}$ , and  $\Phi$ ,  $^{10}$  and, as we shall see, all the terms on the right sides of the equations are also determined by  $E_{Fc}$ ,  $E_{Fv}$ , and  $\Phi$ . Accordingly, Eqs. (10)–(12) can be regarded as a set of coupled differential equations for  $E_{Fc}$ ,  $E_{Fv}$ , and  $\Phi$ , which will be solved self-consistently.

For the recombination processes, we consider the radiative, the Auger, and the SRH recombinations in the active region and only the SRH recombination outside the active region, as in Ref. 10. The generation rate G is used to account for the photon recycling effect. It is the point of the work. We assume the electron-hole generation associated with photon recycling to occur only in the active region, because the photons are originally created by electron-hole radiative recombinations in the active region, which has a bandgap narrower than the other regions. Furthermore, we assume the optical excitation to generate electron-hole pairs uniformly over the active region. This is justifiable for an active region of thickness no more than 1  $\mu$ m. As a result,

the effective current  $J_G$  caused by optical excitation can be simply related to G by

$$J_G = qLG, (13)$$

where L is the thickness of the active region. Comparison between Eqs. (2) and (13) allows expressing

$$G = \frac{1}{qL} \eta_{\rm pr} (1 - \eta_{\rm xp}) J_{\rm rad}. \tag{14}$$

We see that G is determined by  $E_{Fc}$ ,  $E_{Fv}$ , and  $\Phi$ , because so is  $J_{\text{rad}}$ , as has been mentioned.

The boundary conditions for the differential equations (10)–(12) are the same as in Ref. 10. At the interfaces between semiconducting layers, the condition of thermionic emission is used to determine the discontinuity of the quasi-Fermi levels if the band offset is more than twice the thermal energy  $k_BT$ , where  $k_B$  is the Boltzmann constant and T is the temperature. At the outmost surfaces, we assume an infinite surface recombination velocity, so that the two quasi-Fermi levels merge into the Fermi level of thermal equilibrium. The Fermi levels at the two surfaces are pinned at fixed positions relative to the band edges. Their difference is set at qV, which can be understood as the electric work done on each carrier by the external bias across the device.

### **III. RESULTS AND DISCUSSION**

In this section, we present and analyze the results of the self-consistent calculation for EL refrigeration with photon recycling. In the calculation, the temperature is set at T=300 K. All the material parameters used in this work are the same as in our previous work<sup>10</sup> except for some of those listed in Table I.

In the absence of photon recycling, the performance of the EL refrigerator is sensitive to the photon extraction efficiency, because the photons trapped within the device turn out to be totally absorbed and transformed into thermal energy. The situation is revealed in Fig. 3(a), which shows the curves of the cooling power  $P_c$  of the device without photon recycling ( $\eta_{\rm pr}=0$ ) as a function of the applied voltage V for five values of the extraction efficiency  $\eta_{\rm xp}=0.8$ , 0.85, 0.9, 0.95, and 1. The curves exhibit a peak for  $\eta_{\rm xp}=0.9$ , 0.95, and 1, but no peak for  $\eta_{\rm xp}=0.8$  and 0.85. As expected, the height of the peak diminishes drastically from 79 to 2 W/cm² as  $\eta_{\rm xp}$  reduces slightly from unity to 0.9. The voltage at which the curve peaks moves from 1.41 V for  $\eta_{\rm xp}=1$  to 1.28 V for  $\eta_{\rm xp}=0.9$ . The cooling power falls rapidly from the peak value to negative as the voltage increases.

TABLE I. Some of the material parameters used in this work. The others can be found from Ref. 10.

Parameters	GaAs	$Al_xGa_{1-x}As, x \neq 0$
$\tau_n$ (s)	$1.3 \times 10^{-6}$	$10^{-8}$
$\tau_p$ (s)	$10^{-6}$	$10^{-8}$
$C_n$ (cm <sup>6</sup> s <sup>-1</sup> )	$1.9 \times 10^{-31}$	_
$C_p  (\text{cm}^6 \text{s}^{-1})$	$1.2 \times 10^{-30}$	_

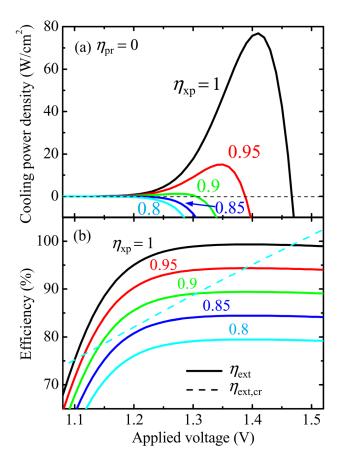


FIG. 3. (Color online) (a) The cooling power density  $P_c$  and (b) the external efficiency  $\eta_{\rm ext.cr}$  as functions of the applied voltage V for the extraction efficiency  $\eta_{\rm xp}=0.8, 0.85, 0.9, 0.95$ , and 1. The photon recycling efficiency is set at  $\eta_{\rm pr}=0$ . The dashed lines of  $\eta_{\rm ext.cr}$  for the different values of  $\eta_{\rm xp}$  are overlapping.

These behaviors can be understood from Eqs. (6) and (7) with the help of the relationships between the curves of the corresponding external efficiencies  $\eta_{\text{ext}}$  and the critical external efficiencies  $\eta_{\text{ext,cr}}$  shown in Fig. 3(b). The curves of  $\eta_{\rm ext,cr}$  for the different  $\eta_{\rm xp}$  values merge into a single line, since  $\eta_{\rm ext,cr} \equiv qV/\langle \hbar \omega \rangle$  is nearly proportional to V and independent of  $\eta_{xp}$ . The curves of  $\eta_{ext}$  for the different  $\eta_{xp}$  values behave similarly and separate almost equidistantly from their neighbors. As V increases, the  $\eta_{\rm ext}$  value increases and then almost reaches saturation at a level  $\eta_{\rm ext} \approx \eta_{\rm xp}$ , with a slight decline in the high-voltage region. These features can be explained by the simple relation  $\eta_{\text{ext}} = \eta_{\text{xp}} \eta_{\text{inj}} \eta_{\text{int}}$  for  $\eta_{\text{pr}} = 0$ , where the injection efficiency  $\eta_{\rm inj}$  is almost unity for the device containing current blocking layers. 10 Therefore, for  $\eta_{xp} = 1$ , the variation of  $\eta_{ext}$  with V simply follows the variation of the internal efficiency  $\eta_{\rm int}$ . The curve of  $\eta_{\rm ext}$  for  $\eta_{\rm xp}$  < 1 is just the curve for  $\eta_{\rm xp}$  = 1 scaled by the factor  $\eta_{\rm xp}$ . The internal efficiency  $\eta_{\text{int}}$  is low at low V, because the SRH recombination is important when the carrier concentration is low. As V increases, the carrier concentration in the active region increases and the radiative recombination becomes increasingly more important than the SRH recombination, leading to the increase in  $\eta_{int}$ . The increase continues until  $\eta_{\rm int}$  reaches a value of nearly unity. The slight decrease of  $\eta_{\rm ext}$  or  $\eta_{\rm int}$  in the high-voltage region is caused by the growing importance of the Auger recombination. From Fig. 3(b), we see that there exist operation regions of refrigeration for  $\eta_{xp} = 0.9, 0.95,$  and 1, but no operation region for  $\eta_{xp} = 0.8$ and 0.85, where the operation region of refrigeration is a range of voltage in which the device operates in the cooling mode ( $\eta_c > 0$  or  $\eta_{\rm ext} > \eta_{\rm ext,cr}$ ). This explains the feature in Fig. 3(a) that only the three curves for  $\eta_{xp} = 0.9$ , 0.95, and 1 exhibit a peak of positive  $P_c$ . According to Eq. (7), we can estimate from Fig. 3(b) the voltages at which the  $\eta_c$ -V curves peak for  $\eta_{XD} = 0.9$ , 0.95, and 1. These voltages are about 1.2 V because, at the voltage, the tangents to the curves of  $\eta_{\rm ext}$ -V are parallel to the line of  $\eta_{\rm ext,cr}$ . Since the cooling power  $P_c = \eta_c JV$ , the  $P_c$ -V curves peak at voltages higher than the  $\eta_c$ -V curves for  $\eta_{xp} = 0.9, 0.95$ , and 1. The peaks of  $P_c$ -V curves occur at voltages near the critical voltages. The critical voltage is the point where  $P_c = 0$  ( $\eta_c = 0$ ), that is, the point where the  $\eta_{\rm ext}$ -V curve crosses the  $\eta_{\rm ext,cr}$ -V line. This explains that a slight increase of V causes the drastic reduction of  $P_c$  from the peak value and may turn the operation of the device from the cooling mode to the heating

In reality, most of the trapped photons are reincarnated into electron-hole pairs rather than into the thermal energy in the high-quality GaAs/AlGaAs heterostructure. The photon recycling, which is neglected above, alleviates considerably the requirement of extraction efficiency for EL refrigeration. To demonstrate the photon-recycling effect, we consider another extreme case by setting  $\eta_{\rm pr}=1$  in the calculation. That is, we assume that the trapped photons are all recycled to generate electron-hole pairs in the active region. The resulting cooling power  $P_c$  as a function of V is shown in Fig. 4(a) for five values of the extraction efficiency  $\eta_{\rm xp}=0.2,~0.4,~0.6,~0.8,~{\rm and}~1.$  All the curves in the figure behave as an asymmetrical peak of positive  $P_c$ , even for the low extraction efficiency of 0.2. The height of the peak now diminishes gently as  $\eta_{xp}$  reduces. These peaks occur almost at the same voltage (V = 1.41 V), except for  $\eta_{XD} = 0.2$ , for which the curve peaks at 1.39 V. Similarly, these behaviors can be understood from Eqs. (6) and (7) and the relationships between the corresponding  $\eta_{\text{ext}}$  and  $\eta_{\text{ext,cr}}$  shown in Fig. 4(b). Compared with those in Fig. 3(b), the external efficiencies  $\eta_{\rm ext}$  in Fig. 4(b) still vary with V in the manner that they first rise, reach a maximum, and then slightly decline. The difference is that  $\eta_{\rm ext}$  is now much less sensitive to  $\eta_{\rm xp}$ , especially in the voltage range where the internal efficiency  $\eta_{int}$  is near unity. In the voltage range between 1.3 V and 1.5 V, where  $\eta_{\rm int}$  is close to unity [Fig. 4(c)], the external efficiency  $\eta_{\rm ext}$ can be approximated by

$$\eta_{\text{ext}} \approx \eta_{\text{inj}} \left[ 1 - \frac{1}{\eta_{\text{xp}}} (1 - \eta_{\text{int}}) \right].$$
(15)

Equation (15) explains that, as  $1 - \eta_{\text{int}}$  is small, the  $\eta_{\text{ext}}$  is insensitive to  $\eta_{\text{xp}}$  and has a value close to  $\eta_{\text{inj}}$ , which is nearly unity, as shown in Fig. 4(c).

In principle, the variation of  $\eta_{\rm ext}$  basically follows the variation of  $\eta_{\rm int}$  modified by photon recycling. This is because the only important loss is the nonradiative recombination in the active region when we set  $\eta_{\rm pr}=1$  and  $\eta_{\rm inj}\approx 1$ .

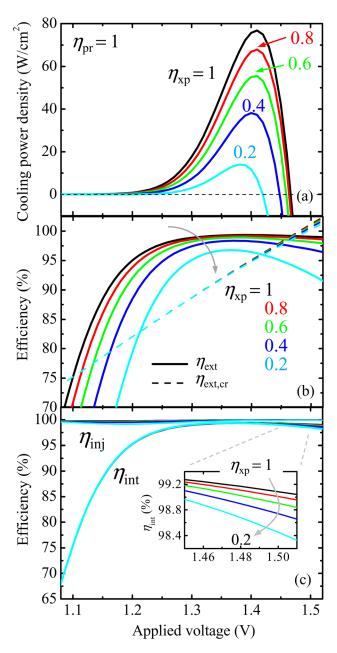


FIG. 4. (Color online) (a) The cooling power density  $P_c$ , (b) the external efficiency  $\eta_{\rm ext}$  and the critical external efficiency  $\eta_{\rm ext,cr}$ , and (c) the injection efficiency  $\eta_{\rm inj}$  and the internal efficiency  $\eta_{\rm int}$  as functions of the applied voltage V for  $\eta_{\rm xp}=0.2,\,0.4,\,0.6,\,0.8,\,{\rm and}\,1$ . The photon recycling efficiency is set at  $\eta_{\rm pr}=1$ . The inset of (c) is a magnification of the  $\eta_{\rm int}$  curves in the high-voltage region.

For  $\eta_{\rm xp}=1$ , all the generated photons are emitted out of the device and there is no photon recycling, regardless of the value of  $\eta_{\rm pr}$ , as can also be seen from Eq. (3). The photon recycling effect becomes manifest when  $\eta_{\rm xp}$  deviates from unity. It causes an effective feedback current  $J_G$  and boosts the carrier concentration in the active region. The increasing carrier concentration degrades  $\eta_{\rm int}$  and, hence,  $\eta_{\rm ext}$  in the high-voltage region. This effect is particularly important for a low  $\eta_{\rm xp}$  [see Fig. 4(b) and the inset in Fig. 4(c)]. The lines of  $\eta_{\rm ext,cr}$  in Fig. 4(b) do not merge as completely as those in Fig. 3(b). This is a consequence of the photon-recycling effect, which causes a higher average photon energy  $\langle \hbar \omega \rangle$ 

and, hence, a lower  $\eta_{\rm ext,cr}$  for a lower  $\eta_{\rm xp}$ , due to a higher carrier concentration.

The condition that  $P_c = 0$  occurs at critical voltages of about 1.47 V for  $\eta_{\rm xp} = 0.4$ , 0.6, 0.8, and 1. The voltages are close to each other because of the crowding of the  $\eta_{\rm ext}$ -V curves in the high-voltage region, which cross the  $\eta_{\rm ext}$ -V curves in the vicinity of 1.47 V. This implies that the  $\eta_{\rm ext}$ -V curves peak almost at the same voltage.

Under the condition that  $\eta_{\rm pr}=1$  and  $\eta_{\rm inj}\approx 1$ , the critical extraction efficiency  $\eta_{\rm xp,cr}$  for the device to operate in the cooling mode can be written as

$$\eta_{\rm xp,cr} \approx \frac{(1/\eta_{\rm int}) - 1}{(1/\eta_{\rm ext,cr}) - 1}.$$
 (16)

The above expression implies that the  $\eta_{\rm xp,cr}$  can be very small when  $\eta_{\rm int}$  approaches unity, while  $\eta_{\rm ext,cr}$  does not. An EL refrigerator having a very low  $\eta_{\rm xp}$  could operate in the cooling mode if there were no optical parasitic loss ( $\eta_{\rm pr}=1$ ) and no current leakage ( $\eta_{\rm inj}=1$ ).

We now know that the internal efficiency  $\eta_{int}$  is an important factor in determining  $\eta_{\rm ext}$  and, hence, to the performance of the device. It is influenced by photon recycling through the feedback generation of electron-hole pairs in the active region. To gain insight into the effect, we set  $\eta_{\rm pr}=1$  and investigate the average carrier density in the active region and the various current components ( $J_{\text{rad}}$ ,  $J_{\text{SRH}}, J_{\text{Aug}}, J_{\text{inj}}, \text{ and } J_G)$  as functions of  $\eta_{\text{xp}}$  when the device is biased at V = 1.2 V and 1.4 V (Fig. 5). As shown in Fig. 5(a), the average carrier density is low ( $\approx 3 \times 10^{16}$ cm<sup>-3</sup>) and nearly independent of  $\eta_{xp}$  for the device biased at 1.2 V, but is more than  $5\times 10^{17}$  cm<sup>-3</sup> and increases as  $\eta_{xp}$  reduces for the device biased at 1.4 V. This difference arises from the difference in the electric potential energy of the electrons (holes) in the active region relative to that in the *n*-type (*p*-type) cladding layer, from which the electrons (holes) are injected. When the device is biased at a low voltage, such as V = 1.2 V, the electric potential energy of the electrons (holes) is higher in the active region than in the n-type (p-type) cladding layer. This drop in electric potential energy causes the excess carriers, due to optical generation, to leak out of the active region, leading to an effective current  $J_G$  opposite the injection current  $J_{inj}$ . Consequently, accumulation of carriers in the active region is negligible, and the carrier density is basically independent of the photon recycling (and, hence,  $\eta_{xp}$ ). The drop in electric potential energy diminishes as the bias increases and, eventually, the situation reverses. At V = 1.4 V, the electric potential energy of the electrons (holes) is lower in the active region than in the n-type (p-type) cladding layer. The potential profile now forms a potential well for the carriers and prohibits most of the excess carriers in the active region from leaking out. As a result, the optically generated carriers are mostly accumulated within the active region, recombined therein, and contribute an additional current, known as  $J_G$ . The carrier accumulation depends on the photon recycling and becomes particularly important for low  $\eta_{xp}$ , as shown in Fig. 5(a).

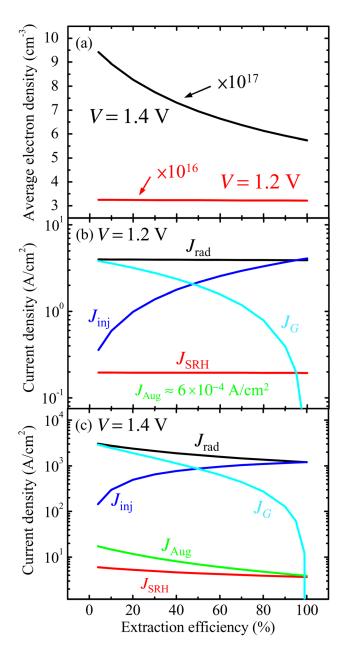


FIG. 5. (Color online) (a) The average electron density in the active layer as a function of the extraction efficiency  $\eta_{\rm xp}$  for the applied voltages V=1.2 and 1.4 V. The current density components  $J_{\rm rad}$ ,  $J_{\rm SRH}$ ,  $J_{\rm Aug}$ ,  $J_{\rm inj}$ , and  $J_G$  as functions of the extraction efficiency  $\eta_{\rm xp}$  for (b) V=1.2 V and (c) V=1.4 V. The photon recycling efficiency is set at  $\eta_{\rm pr}=1$ .

For the device biased at V=1.2 V, the current components  $J_{\rm rad}$ ,  $J_{\rm SRH}$ , and  $J_{\rm Aug}$  are also nearly independent of  $\eta_{\rm xp}$ , as shown in Fig. 5(b), because they depend simply on the carrier density in the active region. At such a not-so-low voltage, the current  $J_{\rm rad}$  due to radiative recombination predominates overwhelmingly over the other two ( $J_{\rm rad}\approx 4$  A/cm²,  $J_{\rm SRH}\approx 0.2$  A/cm²,  $J_{\rm Aug}\approx 6\times 10^{-4}$  A/cm²), leading to the internal efficiency of 95% shown in Fig. 4(c). The Auger recombination is negligibly small because of the low carrier density. In contrast, the feedback current  $J_G$  and the injection current  $J_{\rm inj}$  depend strongly on the photon recycling. The former increases almost linearly with  $1-\eta_{\rm xp}$  [Eq. (2)], while the latter decreases almost linearly with  $1-\eta_{\rm xp}$  according to the condition of Eq. (1) that  $J_G$  compensates  $J_{\rm inj}$  for the total

generation rate of carriers in the active region to balance the total recombination rate  $(J_{\rm rad}+J_{\rm SRH}+J_{\rm Aug})$ , which is nearly independent of  $\eta_{\rm xp}$ . The injection current  $J_{\rm inj}$  is much smaller than  $J_G$  and  $J_{\rm rad}$  for low  $\eta_{\rm xp}$ . Namely, the photon recycling reduces the driving current J, which approximately equals  $J_{\rm inj}$  and, hence, enhances the external efficiency  $\eta_{\rm ext}$ . The enhancement factor can be as high as  $\gamma=4.2$  for  $\eta_{\rm xp}=0.2$ , according to Eq. (4).

For the device biased at V = 1.4 V, the current components  $J_{\rm rad}$ ,  $J_{\rm SRH}$ , and  $J_{\rm Aug}$  increase as  $\eta_{\rm xp}$  reduces, in accordance with the variation of the carrier density in the active region. As  $\eta_{xp}$  reduces from 100% to 4%,  $J_{rad}$ ,  $J_{SRH}$ , and  $J_{\text{Aug}}$  increase from 1200, 4, and 4 A/cm<sup>2</sup> to 3100, 6, and 20 A/cm<sup>2</sup>, respectively. Accordingly, the internal efficiency  $\eta_{\text{int}}$  changes slightly from 99.34% to 99.17%. The Auger recombination now exceeds the SRH recombination because of the high carrier density, while the radiative recombination remains predominant over the other two. As has been described, the extremely high  $\eta_{\rm int}$  enhances the benefit from the photon recycling. It not only improves the value of  $\eta_{ext}$ , but also eases the deterioration of the device performance due to the reduction in  $\eta_{xp}$ . The external efficiency  $\eta_{ext}$ decreases from 99% to 96% as  $\eta_{xp}$  reduces from 1 to 0.2 for V = 1.4 V, as shown in Fig. 4(b), but from 95% to 79% for V=1.2 V, where  $\eta_{\rm int}=95\%$ . The high  $\eta_{\rm int}$  of more than 99% keeps the device in the cooling mode, even when the  $\eta_{xp}$  is reduced to 0.2 (Fig. 4).

We have demonstrated the photon recycling effect by setting a complete recycling ( $\eta_{pr} = 1$ ) and comparing the calculated results to those with a null recycling ( $\eta_{\rm pr}=0$ ). In fact, it is impossible for a device to have a complete photon recycling. A fraction  $(1 - \eta_{xp})(1 - \eta_{pr})$  of the photons generated by radiative recombination turn out to be absorbed and transformed into thermal energy. For the device to operate in the cooling mode, parasitic loss of each photon must be covered by emission of a large number of photons, because emission of a photon reduces an amount of internal energy  $\langle \hbar \omega \rangle - qV$  in average in the device, which is much smaller than the internal energy generated by parasitic absorption of a photon, qV. As a result, the parasitic absorption is a fatal factor to EL refrigeration and the device performance is expected to be drastically degraded with increasing the factor  $(1 - \eta_{xp})(1 - \eta_{pr})$ . To manifest the effect of the parasitic absorption, we show in Figs. 6(a) and 6(b) the external efficiency  $\eta_{\rm ext}$  and the enhancement factor  $\gamma$ , respectively, as functions of  $\eta_{\rm pr}$  for  $\eta_{\rm xp}=0.2,\,0.5,\,{\rm and}\,0.8.$ The curves are obtained from Eqs. (3) and (4) with  $\eta_{\rm ini}=1$ and  $\eta_{\text{int}} = 0.99$ , corresponding to the applied voltage ranging approximately between 1.3 and 1.45 V [see Fig. 4(b)]. As expected,  $\eta_{\text{ext}}$  has a high value predictable from Eq. (15) at  $\eta_{\rm pr}=1$  and then decreases as  $\eta_{\rm pr}$  reduces. The decrease of  $\eta_{\rm ext}$  is more notable for lower  $\eta_{\rm xp}$ , particularly in the region of high  $\eta_{pr}$ . Only in the high  $\eta_{pr}$  region can  $\eta_{ext}$  remain sufficiently high for EL refrigeration. For example, if the device with a high  $\eta_{pr}$  is biased at 1.4 V such that the cooling power reaches approximately at the peak value [Fig. 4(a)],  $\eta_{\text{ext}}$  has to exceed the critical value of  $\eta_{\rm ext,cr}=0.95$ , indicated by the dashed line in Fig. 6(a) for EL refrigeration. Correspondingly,  $\eta_{pr}$  has to be above 0.85, 0.96, and 0.99 for  $\eta_{xp} = 0.8$ ,

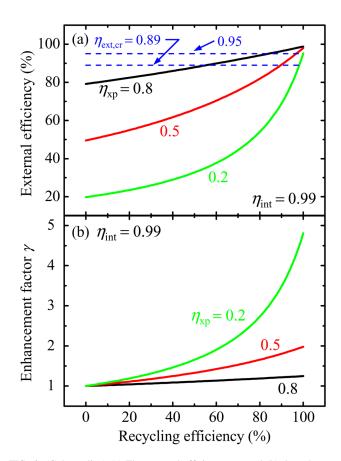


FIG. 6. (Color online) (a) The external efficiency  $\eta_{\rm ext}$  and (b) the enhancement factor  $\gamma$  as functions of the photon recycling efficiency  $\eta_{\rm pr}$  for the extraction efficiency  $\eta_{\rm xp}=0.2,\ 0.5,\$ and 0.8. The data are obtained from Eqs. (3) and (4), with the internal efficiency set at  $\eta_{\rm int}=0.99$  and the injection efficiency set at  $\eta_{\rm inj}=1$ . The two horizontal dashed lines in (a) denote the critical external efficiencies of  $\eta_{\rm ext,cr}=0.89$  and 0.95, which correspond to the applied voltages of V=1.3 and 1.4 V, respectively.

0.5, and 0.2, respectively. The tight tolerance can be loosened at the price of the cooling power. If the device is now biased at 1.3 V, the critical external efficiency  $\eta_{\rm ext,cr}$  reduces to 0.89, accompanied by a significant reduction in cooling power (Fig. 4). The lower bound of the  $\eta_{\rm pr}$  region for EL refrigeration now extends to 0.55, 0.88, and 0.97 for  $\eta_{\rm xp}=0.8$ , 0.5, and 0.2, respectively.

The behavior of the enhancement factor  $\gamma$  in Fig. 6(b) can be understood from the fact that the  $\gamma$  curves can be obtained simply from the  $\eta_{\rm ext}$  curves in Fig. 6(a) normalized by the values of  $\eta_{\rm ext}$  at  $\eta_{\rm pr}=0$  according to the definition of  $\gamma$ . As expected, photon recycling plays a more profound role in improving the performance of the device with a lower  $\eta_{\rm xn}$ .

The quality of the materials making up the device is also important for achieving refrigeration. It is concerned with the nonradiative recombination rate and the optical parasitic absorption and, hence, directly influences  $\eta_{\rm int}$  and  $\eta_{\rm pr}$ . If the material quality is not as high as in the study and gives  $\eta_{\rm int} < \eta_{\rm ext,cr}$ , it is impossible for the device to operate in the cooling mode as a result of the inequality  $\eta_{\rm ext} < \eta_{\rm int}$ , regardless of the values of the other efficiencies. In this case, the corresponding  $\eta_{\rm ext}$  curves will lie totally below the dashed line of  $\eta_{\rm ext,cr}$ , as in Fig. 6(a).

Finally, we show in Figs. 7(a) and 7(b) the cooling power  $P_c$  and the cooling efficiency  $\eta_c$ , respectively, as

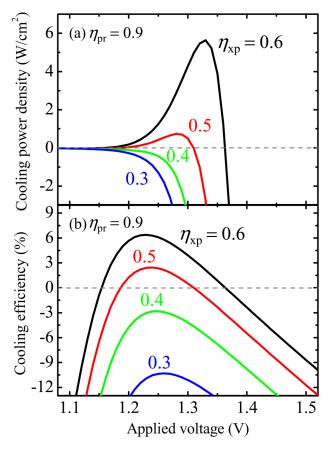


FIG. 7. (Color online) (a) The cooling power density  $P_c$  and (b) the cooling efficiency  $\eta_c$  as functions of the applied voltage V for the extraction efficiency  $\eta_{\rm xp}=0.3,0.4,0.5,$  and 0.6. The recycling efficiency is set at  $\eta_{\rm pr}=0.9$ .

functions of applied voltage V for cases more practical than the previous cases of Figs. 3 and 4. For the device containing a 1- $\mu$ m active layer,  $\eta_{xp}$  is at most 60%.<sup>20</sup> We therefore consider  $\eta_{xp} = 0.3$ , 0.4, 0.5, and 0.6 and set  $\eta_{pr} = 0.9$ , which is not difficult to reach for high-quality GaAs material. The resulting  $P_c$ -V relationship changes with  $\eta_{xp}$  to a degree milder than for  $\eta_{pr} = 0$  [Fig. 3(a)] but more severe than for  $\eta_{pr} = 1$  [Fig. 4(a)]. The peak cooling power is now 5.6 W/cm<sup>2</sup> at V = 1.33 V for  $\eta_{xp} = 0.6$ , which is only 10% of the corresponding value when  $\eta_{\rm pr}=1$ . At the voltage of 1.33 V, the cooling efficiency  $\eta_c$  is small (2%) for  $\eta_{xp} = 0.6$ and falls negative as  $\eta_{xp}$  reduces to 0.5, associated with the severe degradation of  $P_c$  from 5.6 to -3 W/cm<sup>2</sup>. The situation can be cured by biasing the device at a lower voltage for an improved  $\eta_c$ . For example, at V = 1.24 V,  $\eta_c$  is 6.4%for  $\eta_{xp} = 0.6$  and remains positive (about 2%) for  $\eta_{xp} = 0.5$ . The device can operate in the cooling mode for  $\eta_{xp} > 0.45$ . However, the cooling power is sacrificed. It is now only 0.9 W/cm² for  $\eta_{xp}=0.6$  and 0.3 W/cm² for  $\eta_{xp}=0.5$ .

### **IV. CONCLUSION**

We have investigated the photon recycling effect on electroluminescent refrigeration by self-consistent calculation. The photon recycling can alleviate considerably the requirement for refrigeration. It behaves as an additional means of generating carriers, reduces the driving current in the device, and improves the external efficiency. A photon recycling efficiency of 90% can improve the external efficiency by a factor of 4 for the device with an extraction efficiency of  $\eta_{\rm xp}=20\%$  and an internal efficiency of  $\eta_{\rm int}=99\%$ . Consequently, the device can operate in the cooling mode, even if the extraction efficiency is not high. For example, for a device that has a 1- $\mu$ m GaAs active layer and operates at T=300 K, the condition of  $\eta_{\rm xp}=60\%$  and  $\eta_{\rm pr}=90\%$  gives a peak cooling power of 5.6 W/cm² at V=1.33 V associated with a low cooling efficiency of 2% and a tight tolerance to the extraction efficiency. The tolerance can be loosened at the cost of cooling power. The device can operate in the cooling mode for  $\eta_{\rm xp}>45\%$  when biased at 1.24 V. The results reveal a good possibility of experimentally achieving electroluminescent refrigeration in semiconductors.

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- <sup>3</sup>M. Sheik-Bahae and R. I. Epstein, Nat. Photonics 1, 693 (2007).
- <sup>4</sup>M. Sheik-Bahae and R. I. Epstein, Laser Photonics Rev. 3, 67 (2009).
- <sup>5</sup>M. Sheik-Bahae and R. I. Epstein, Phys. Rev. Lett. **92**, 247403 (2004).
- <sup>6</sup>D. Huang, T. Apostolova, P. M. Alsing, and D. A. Cardimona, Phys. Rev. B **70**, 033203 (2004).
- <sup>7</sup>G. Rupper, N. H. Kwong, and R. Binder, Phys. Rev. B **76**, 245203 (2007).
- <sup>8</sup>G. Rupper, N. H. Kwong, R. Binder, C.-Y. Li, and M. Sheik-Bahae, J. Appl. Phys. **108**, 113118 (2010).
- <sup>9</sup>C. Wang, M. P. Hasselbeck, C.-Y. Li, and M. Sheik-Bahae, Proc. SPIE **7614**, 76140B (2010).
- <sup>10</sup>S.-T. Yen and K.-C. Lee, J. Appl. Phys. **107**, 054513 (2010).
- <sup>11</sup>A. G. Mal'shukov and K. A. Chao, Phys. Rev. Lett. **86**, 5570 (2001).
- <sup>12</sup>J.-B. Wang, D. Ding, S.-Q. Yu, S. R. Johnson, and Y.-H. Zhang, in Conference on Lasers and Electro-Optics/Quantum Electronics and Laser Science and Photonic Applications Systems Technologies, Technical Digest (CD) (Optical Society of America, Washington, DC, 2005), paper OTh17.
- <sup>13</sup>S.-Q. Yu, J.-B. Wang, D. Ding, S. R. Johnson, D. Vasileska, and Y.-H. Zhang, Solid-State Electron. 51, 1387 (2007).
- <sup>14</sup>J. Oksanen and J. Tulkki, J. Appl. Phys. **107**, 093106 (2010).
- <sup>15</sup>I. Schnitzer, E. Yablonovitch, C. Caneau, and T. J. Gmitter, Appl. Phys. Lett. 62, 131 (1993).
- <sup>16</sup>J.-B. Wang, S. R. Johnson, D. Ding, S.-Q. Yu, and Y.-H. Zhang, J. Appl. Phys. **100**, 043502 (2006).
- <sup>17</sup>O. Heikkilä, J. Oksanen, and J. Tulkki, J. Appl. Phys. **105**, 093119 (2009).
- <sup>18</sup>O. Heikkilä, J. Oksanen, and J. Tulkki, J. Appl. Phys. **107**, 033105 (2010).
- <sup>19</sup>R. Windisch, C. Rooman, B. Dutta, A. Knobloch, G. Borghs, G. H. Döhler, and P. Heremans, IEEE J. Sel. Top. Quantum Electron. 8, 248 (2002).
- <sup>20</sup>P. Asbeck, J. Appl. Phys. **48**, 820 (1977).

<sup>&</sup>lt;sup>1</sup>R. I. Epstein, M. I. Buchwald, B. C. Edwards, T. R. Gosnell, and C. E. Mungan, Nature (London) 377, 500 (1995).

<sup>&</sup>lt;sup>2</sup>D. V. Seletskiy, S. D. Melgaard, S. Bigotta, A. D. Lieto, M. Tonelli, and M. Sheik-Bahae, Nat. Photonics 4, 161 (2010).