# New Miniaturized Dual-Band Rat-Race Coupler With Microwave C-Sections

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Abstract—Based on microwave C-sections, rat-race coupler is designed to have a dual-band characteristic and a miniaturized area. The C-section together with two transmission line sections attached to both of its ends is synthesized to realize a phase change of 90° at the first frequency, and 270° at the second passband. The equivalence is established by the transmission line theory, and transcendental equations are derived to determine its structure parameters. Two circuits are realized in this presentation; one is designed at 2.45/5.2 GHz and the other at 2.45/5.8 GHz. The latter circuit occupies only 31% of the area of a conventional hybrid ring at the first band. It is believed that this circuit has the best size reduction for microstrip dual-band rat-race couplers in open literature. The measured results show good agreement with simulation responses.

*Index Terms*—Microwave *C*-section, dual-band, microstrip line, miniaturization, rat-race coupler.

## I. INTRODUCTION

In the RF frond-end of a modern communication system, hybrid couplers are one of the most important devices [1]. They are widely used in several microwave and millimeter wave sub-systems such as balanced amplifiers and mixers. In the past few decades, a series of innovative synthesis and design have been proposed for these couplers [2]-[5]. In [2], simple formulation is devised to design the ring coupler. In [3], a rigorous synthesis procedure is demonstrated for branch-line couplers with equal power division. In [4], a rat-race coupler consisting of cascade structure of alternative high- and lowimpedance transmission lines is presented to achieve a large power split ratio over a wide bandwidth. On the basis of design equations in [4], a miniaturized periodic steppedimpedance rat-race coupler with arbitrary power division ratio can be realized [5]. Note that all couplers in [2]-[5] are designed for operation at a single band.

Recent rapid progress in wireless communications has created a need of dual-band operation for RF devices, such as the global systems for mobile communication systems (GSM) at 0.9/1.8 GHz and wireless local area network (WLAN) at 2.4/5.2 GHz. Recently, numerous research papers on dualband hybrid couplers have been published [6]-[13]. In [6], a planar dual-band branch-line coupler with a compact circuit area is proposed. Each quarter wavelength ( $\lambda$ /4) section is replaced by a short transmission line section with a pair of shunt short-circuited or open stubs attached to its ends. Based on a similar circuit structure, a rat-race coupler with two arbitrary operation frequencies is presented in [7]. An alternative branch-line coupler for dual-band operation is introduced in [8]. The circuit consists of cross branches that offer extra degrees of freedom to achieve the design. In [9], a tapped open stub is used to contrive a dual-band branch-line coupler. The two bands can be arbitrarily designated by adequately tuning the length and characteristic impedance of the stub. In [10], a stepped-impedance line section with two open stubs is proposed to establish a dual-band rat-race coupler. In [11], four identical open stubs are devised for design of a novel dual-band rectangular patch hybrid coupler.

The metamaterial approach [12]-[13] can also be used to implement couplers with the dual-band operation. In [12], arbitrary dual-band components such as branch-line and ratrace couplers are realized by composite right- and left-handed transmission line (CRLH TL) sections. Dual-band devices are also carried out on the basis of the simplified CRLH TL [13]. So far, however, it is still a challenge to design a miniaturized dual-band rat-race coupler. In [14], dual-band couplers with arbitrary power division ratios are presented. Each transmission line section consists of a stepped-impedance line section with open stubs tapped with its both ends. Rigorous design procedure is described and several circuits operating at 2.45/5.2 GHz are realized. To the best of our knowledge, it is the first miniaturized dual-band coupler with arbitrary power divisions at two designated bands in open literature.

In this paper, a miniaturized dual-band rat-race coupler is implemented with the microwave *C*-sections. Each  $\lambda/4$  section of a conventional rat-race coupler is replaced by the proposed elementary two-port which consists of two line sections with a *C*-section in between. Design equations are formulated by establishing the equivalence between the two-port and the  $\lambda/4$ section at two frequencies. To facilitate the circuit design, some design graphs are provided.



Fig. 1. The proposed elementary two-port with a C-section.

## II. ELEMENTARY TWO-PORT FOR DUAL-BAND OPERATION

Figure 1 shows the proposed elementary two-port network for the dual-band operation. The two-port is built with two transmission line segments of lengths  $\theta_1$  and  $\theta_2$  and characteristic impedances  $Z_1$  and  $Z_2$  with a microwave *C*section of length  $\theta_c$  in the middle. Since the phase-shift of the *C*-section is nonlinear with respect to frequency [15], it is suitable for realizing the dual-band function.

Let the two designated operation frequencies be  $f_1$  and  $f_2 = nf_1$ . In our approach, the six  $\lambda/4$  sections of the traditional coupler are replaced by the proposed two-port. For simplicity, let  $Z_1 = Z_2 = \sqrt{Z_{oe}Z_{oo}}$ , where  $Z_{oe}$  and  $Z_{oo}$  are the even- and odd-mode characteristic impedances of the coupled-line, respectively. By enforcing the *ABCD* matrix of the two-port to be equal to those of a uniform transmission line section of 90° and 270° at  $f_1$  and  $f_2$ , respectively, the following four equations can be readily obtained:

$$P(\theta_c)\cos\theta + 2Z_1\sin\theta = 0 \tag{1a}$$

$$P(\theta_c)\sin\theta - 2Z_1\cos\theta = Q(\theta_c) \times Z_T/Z_1$$
(1b)

$$P(n\theta_c)\cos n\theta + 2Z_1\sin n\theta = 0 \tag{1c}$$

$$P(n\theta_c)\sin n\theta - 2Z_1\cos n\theta = -Q(n\theta_c) \times Z_T/Z_1$$
(1d)

where

$$P(\theta_c) = Z_{oo} \tan \theta_c - Z_{oe} \cot \theta_c$$
(2a)

$$Q(\theta_c) = -(Z_{oo} \tan \theta_c + Z_{oe} \cot \theta_c)$$
(2b)

In (1) and (2),  $\theta = \theta_1 + \theta_2$ ,  $Z_o$  is the reference impedance, and  $Z_T = \sqrt{2}Z_o$  is the characteristic impedance of the six  $\lambda/4$  sections of a conventional hybrid ring. After some algebraic manipulations, the following equations can be obtained for determination of  $Z_{oe}$  and  $Z_{oo}$ :

$$Z_T \times Z_{oe}(f_1) = Z_1^2 \sec\theta \tan\theta_c + Z_1 \tan\theta \tan\theta_c$$
(3a)

$$Z_T \times Z_{oo}(f_1) = Z_1^2 \sec\theta \cot\theta_c - Z_1 \tan\theta \cot\theta_c$$
(3b)

$$Z_T \times Z_{oe}(f_2) = Z_T Z_1 \tan n\theta \tan n\theta_c - Z_1^2 \sec n\theta \tan n\theta_c \qquad (3c)$$

$$Z_T \times Z_{oo}(f_2) = -Z_T Z_1 \tan n\theta \cot n\theta_c - Z_1^2 \sec n\theta \cot n\theta_c \qquad (3d)$$

One can validate that  $Z_{oe}(f_1)Z_{oo}(f_1) = Z_{oe}(f_2)Z_{oo}(f_2) = Z_1^2$  when  $Z_1 = Z_T$ . Here, the dispersion of microstrip characteristic impedance is neglected. Once n,  $\theta_c$  and  $\theta$  are given, by enforcing  $Z_{oe}(f_1) = Z_{oe}(f_2)$  or  $Z_{oo}(f_1) = Z_{oo}(f_2)$ , the structure parameters of the dual-band element can be solved as follows. Figure 2(a) plots the changes of  $Z_{oe}(f_1)$  and  $Z_{oe}(f_2)$  with respect to the variation of  $\theta$  for  $\theta_c = 50^\circ$ , 47.5° and 45° for n = 2.12. This specific n value is used when  $f_1 = 2.45$  GHz and  $f_2 = 5.2$  GHz. As shown in Fig. 2(a), when  $\theta_c$  is decreased from 50° to 45°, the  $Z_{oe}$  solution, i.e. the intersected points, is increased.

When  $\theta_c = 45^\circ$ , the solution reads  $Z_{oe} = 130 \ \Omega$  and  $Z_{oo} = 2Z_o^{2/}Z_{oe} = 38.45 \ \Omega$ . Implemented by coupled microstrips, the gap size over substrate thickness ratio S/h will be 0.08 for a substrate with  $\varepsilon_r = 10.2$ . When the substrate thickness h = 1 mm, such gap sizes will be critical for microstrip realization by the standard fabrication process since it is close to being beyond the resolution limit. One may increase  $\theta_c$  to release the tight gap size as indicated in Fig. 2(a). Figure 2(b) draws the  $Z_{oe}(f_1)$  and  $Z_{oe}(f_2)$  solutions against the variation of  $\theta$  for n = 2.1, 2.2, ..., 3 for  $\theta_c = 45^\circ$ . When n is increased, the  $Z_{oe}$  solution decreases. Note that when  $\theta = 0$  and n = 3, the solution  $Z_{oe} = 70.7 \ \Omega$ . It validates the fact that when the two-port is used to imitate a  $\lambda/4$  section at  $f_1$  and  $3\lambda/4$  at  $f_2 = 3f_1$ , the *C*-section becomes a folded section with  $Z_{oe} = Z_{oo}$ .



Fig. 2.  $Z_{oe}(f_1)$  and  $Z_{oe}(f_2)$  against the variation of  $\theta$ , where  $Z_1 = 70.7 \Omega$ . (a) n = 2.12. (b)  $\theta_c = 45^\circ$ .



Fig. 3. Simulated and measured responses of the fabricated rat-race coupler at 2.45/5.2 GHz. (a)  $|S_{11}|$  and  $|S_{21}|$ . (b)  $|S_{31}|$  and  $|S_{41}|$ . (c)  $\angle S_{31} - \angle S_{21}$ . (d)  $\angle S_{42} - \angle S_{12}$ . n = 2.12,  $\theta_c = 48^\circ$ ,  $\theta = 23.85^\circ$ ,  $Z_1 = 70.7 \Omega$ ,  $Z_{oe} = 120.65 \Omega$  and  $Z_{oo} = 41.45 \Omega$ .



Fig. 4. Photograph of the dual-band rat-race.

#### **III. SIMULATION AND MEASUREMENT**

Two rat-race couplers designed at 2.45/5.2 GHz and 2.45/5.8 GHz are fabricated and measured for demonstration Figures 3 and 4 show the simulated and measured results. The circuit has a substrate of  $\varepsilon_r = 10.2$  and h = 1.27 mm. Simulation data are obtained by the electromagnetic software package IE3D [16]. Figure 3(a) plots  $|S_{11}|$  and  $|S_{21}|$  responses. It can be observed that all measured  $|S_{11}|$  at both the designated frequencies are better than 18 dB. If a 15-dB return loss is referred, measured data indicate that the circuit has bandwidths of 30% and 9% at  $f_1$  and  $f_2$ , respectively. Figure 3(b) shows the  $|S_{31}|$  and  $|S_{41}|$  curves. The measured isolations  $|S_{41}|$  at  $f_1$  and  $f_2$  are better than 25 dB. For a 20-dB reference, the measured isolations possess bandwidths of 27% and 8% at  $f_1$  and  $f_2$ , respectively. Also, detailed data show that the total power loss of the circuit  $P_L = 1 - |S_{11}|^2 - |S_{21}|^2 - |S_{31}|^2 - |S_{41}|^2$  at  $f_1$  and  $f_2$  are 4% and 8.2%, respectively.

Figures 3(c) and 3(d) plot the responses of relative phase  $\angle S_{31} - \angle S_{21}$  and  $\angle S_{42} - \angle S_{12}$ , respectively. One can observe that these responses have relatively smooth variations over the first band as compared with those over the second. It reflects the fact that the circuit has a bandwidth at  $f_1$  larger than that at  $f_2$ . Good agreement between the simulation and measured responses for the experiment circuits can be observed. Figure 4 shows the photograph of the experimental circuit.

Figures 5(a) and 5(b) plot the magnitude responses of  $S_{11}$ ,  $S_{21}$ ,  $S_{31}$  and  $S_{41}$  of the second experimental dual-band rat-race. At both  $f_1$  and  $f_2$ , the measured  $|S_{11}|$  results are better than 20 dB. The measurement indicates that the circuit has bandwidths of 37% and 8% at  $f_1$  and  $f_2$ , respectively, for a 15-dB return loss. These bandwidths are closely related to the relative phases  $\angle S_{31} - \angle S_{21}$  and  $\angle S_{42} - \angle S_{12}$  shown in Fig. 5(c) and 5(d), respectively. The experimental  $|S_{41}|$  data show that the isolations are better than 35 dB, and the total power losses  $P_L$  are 3.2% and 7.3% at  $f_1$  and  $f_2$ , respectively.

Figure 6 shows the photograph of the experimental circuit. In this design, all the *C*-sections can be placed inside the ring circumference. This is because this circuit has a larger *n* value than the previous one so that the  $\theta_c$  solution shown in Fig. 2 can be shorter. It occupies only 31% of the area of a conventional rat-race coupler. It is believed that this circuit has the best size reduction comparing with the microstrip dual-band rat-race couplers in open literature.

#### IV. CONCLUSION

The function of a microwave *C*-section is exploited for dual-band and circuit miniaturization. For design of a dualband rat-race coupler, the *C*-section together with two short microstrip sections at its ends is used to replace each of the six  $\lambda/4$  sections of a traditional coupler at the two design frequencies. This approach can save more than 69% of the circuit area. The proposed two-port is viable for development of other dual-band microwave passive devices.

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Fig. 5. Simulated and measured responses of the fabricated dualband rat-race coupler at 2.45/5.8 GHz. (a)  $|S_{11}|$  and  $|S_{21}|$ . (b)  $|S_{31}|$  and  $|S_{41}|$ . (c)  $\angle S_{31} - \angle S_{21}$ . (d)  $\angle S_{42} - \angle S_{12}$ . n = 2.37,  $\theta_c = 36.5^\circ$ ,  $\theta = 43.22^\circ$ ,  $Z_1 = 70.7 \Omega$ ,  $Z_{oe} = 121 \Omega$  and  $Z_{oo} = 41.33 \Omega$ .



Fig. 6. Photograph of the fabricated 2.45/5.8 GHz rat-race coupler.

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