Efficient and robust shooting algorithm for numerical design of bidirectionally pumped Raman fiber amplifiers

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An efficient and robust shooting algorithm for the design of bidirectionally pumped Raman fiber amplifiers is proposed. First, a parameter $\boldsymbol{S}$, new to our knowledge, called scaling vector, is introduced. This parameter is used in combination with the physical picture of stimulated Raman scattering to generate accurate initial guesses for the powers of backward pumps in the bidirectionally pumped Raman fiber amplifiers. Second, a modified Newton–Raphson method is developed. With an appropriate restriction for the adjustment of increments of initial guess attached, the modified Newton–Raphson method is used to correct the initial guesses to approach the true solution of the problem. By combining the method of initial value determination and the correction mechanism, 14 types of bidirectionally pumped Raman fiber amplifiers are designed. The simulation results show that the proposed shooting algorithm is more efficient and stable than most of the existing ones. Comparison with three other relevant methods reported in the literature is made to reveal the advantages of the new method.

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1. INTRODUCTION

Raman fiber amplifiers (RFAs) have been widely used in wavelength-division-multiplexing (WDM), long-haul (LH), and ultra-long-haul optical fiber communication systems [1–3]. The RFAs used in these systems can be forward pumped, counterpumped or bidirectionally pumped. Among them the bidirectionally pumped RFAs have more even gain spectrum and noise figure than the forward-pumped or counterpumped RFAs have [4,5]. In numerically dealing with the equations of these RFAs, the equations of forward-pumped RFAs are the easiest to solve. The power evolution of pumps and signals along the fiber can be directly obtained by any integration method. In the counterpumping and bidirectional pumping configuration, on the other hand, some of the optical channels are known for their powers at one end of the gain fiber, while the others are known at the other end of the gain fiber; an ordinary integration method cannot be applied directly. Finding the solution of the Raman amplifier equations in the counterdirectional or bidirectional pumping configuration is a two-point boundary value problem. This problem can be solved by the shooting algorithm in general. Several schemes using the shooting algorithm have been applied to the counterpumped or bidirectionally pumped Raman amplifier equations [6–11]. In the shooting algorithm, accuracy and the correction mechanism of initial guesses are very important issues for the efficiency and convergence criteria of the scheme. A linear correction mechanism for the correction of initial guesses was used in [6,7]. The linear correction mechanism has advantage of easy implementation. However, from the consideration of transmission capacity, a large number of optical channels is often desired. In order to reduce the system cost, the number of amplifiers needs to be minimized, so longer relay distance is preferred. To improve optical signal–noise ratio, strong input optical powers are frequently used. In all these practical cases, the adjustments involved in the linear correction mechanism to find a solution of an RFA are vast. Even so, divergence occurs occasionally. A better correction mechanism would be good in situations like these. Recently, several correction mechanisms based on the Newton–Raphson method have been proposed [8–11] for the design of counterpumped RFAs. However, these correction mechanisms are sensitive to the initial guesses used. In [8,9] a method for the determination of initial guess was provided for counterpumped RFAs. In this method, all counterpumps are viewed as forward pumps. In [10,11] a novel initial value determination method was proposed by making use of a contraction factor as well as the physical picture of stimulated Raman scattering (SRS). These methods are effective for the counterpumped Raman coupled equations. Unfortunately, they are not fully applicable to the bidirectionally pumped Raman coupled equations due to the different pump power evolution scenarios of the bidirectionally pumped RFAs and of the counterpumped RFAs. A better initial value determination method is required for the design of the bidirectionally pumped RFAs.

In this work, an efficient and robust shooting algorithm for the design of bidirectionally pumped RFAs is proposed. First, a scaling vector $\boldsymbol{S}$ is introduced for the determination of initial values of the Raman amplifier equations in the bidirectionally pumped configuration. It is found that the forward pumps in the bidirectionally pumped RFAs have larger influence on the power evolution of the backward pumps than the forward signals in counterpumped RFAs have. Furthermore, the influence varies with wavelengths of the forward pumps. For example, when forward pumps of shorter wavelengths are applied to the RFAs, the appropriate initial powers of the backward pumps are usually increased by SRS interaction of the backward and the added forward pumps. In contrast, when forward pumps of longer wavelengths are applied to the RFAs, the appropriate initial powers of the backward pumps generally decrease to cooperate with the forward pumps. Therefore, the previously introduced contraction factor [10,11], which weighs all backward pumps equally, is not suitable for the case where some pumps propagate forward, while some other pumps propagate backward. The replacement of the contraction factor is the scaling vector $\boldsymbol{S}$, where a different component deals with a different backward pump. The second point of the proposed shooting algorithm is to replace the linear correction method with a modified Newton–Raphson correction method so that the correction procedure for the initial guesses can be performed efficiently. By combining the scaling vector $\boldsymbol{S}$ with the modified Newton–Raphson correction mechanism, an efficient and robust shooting algorithm for bidirectionally pumped RFA equations is constructed.

2. COUPLED EQUATIONS OF RFAs

When pumps and signals propagate simultaneously along the fiber of an RFA, by considering the interactions between the pumps, between the signals, and between the pumps and signals, the power in each channel of the bidirectionally pumped RFAs is mathematically modeled by the coupled equations in the steady state [6,10,11],

$$\pm\frac{dP_i}{dz} = \sum_{j=1,j\neq i}^{n_1+n_2+m} g(v_j, v_i)P_jP_i - \alpha_iP_i,$$
$$(i = 1, 2, \cdots, n_1 + n_2 + m), \tag{1}$$

where n_1 is the number of backward pump channels, n_2 is the number of forward pump channels, m is the number of signal channels, P_i, v_i, and α_i are the power, frequency, and attenuation coefficient of the ith optical channel, respectively. The "+" is designated to the forward traveling pump and signal waves, and the "−" is designated to the backward traveling pump waves. The Raman gain coefficient $g(v_j, v_i)$ describes the power transfer between the jth and the ith optical channels. It is given by $g(v_j, v_i) = (1/K_{\mathrm{eff}}A_{\mathrm{eff}})g_j(v_j - v_i)$ for $v_j > v_i$ and $g(v_j, v_i) = -(1/K_{\mathrm{eff}}A_{\mathrm{eff}})(v_i/v_j)g_i(v_i - v_j)$ for $v_j < v_i$, where $g_k(\Delta v)$ is the Raman gain coefficient spectrum measured at the optical frequency v_k, A_{eff} is the effective overlap core area between waves of different optical channels, $K_{\mathrm{eff}} \approx 2$ is the polarization factor. Without loss of generality, the optical channels are so numbered that the frequency is descending from the first optical channel to the $(n_1 + n_2 + m)$th optical channel, i.e., $v_i > v_j$ for $i < j$. Equations (1) indicate that, when the light wave of frequency v_i propagates along the fiber with the light waves of other frequencies, due to the SRS, it receives power from the light waves of frequencies larger than v_i; in the meantime it loses power to the light waves of frequencies smaller than v_i. It should be noted that the attenuation and Raman gain coefficient spectra of a transmission fiber, as illustrated in Fig. 1, are so irregular that they cannot be expressed in a closed form. There are also highly nonlinear interactions existing between different optical

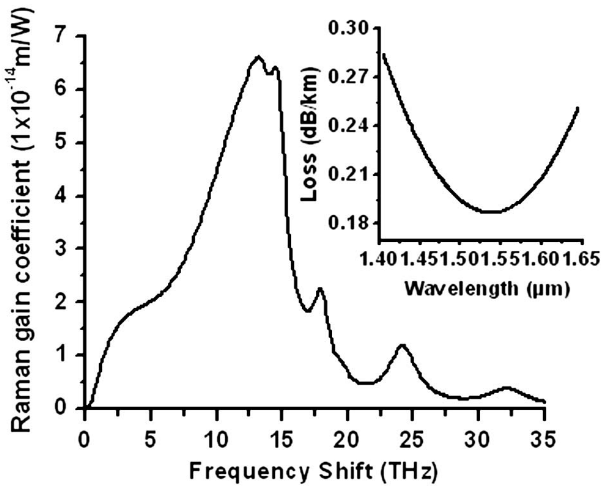

Fig. 1. Raman gain coefficient (measured at 1 μm pump wavelength) and (inset) loss spectrum of a single-mode fiber.

channels. Because of these two reasons, Eqs. (1) has no analytical solution in general. It has to be dealt with numerically.

3. NEW SHOOTING ALGORITHM

Generally speaking, a shooting algorithm applicable to the two-point boundary value problem of a set of differential equations consists of three parts, namely, the determination of accurate initial guesses, a correction mechanism for the improvement of the initial guesses, and a numerical integration procedure. For the numerical integration, any computational method can be used as long as it performs the integration accurately. The Runge–Kutta method with adaptive step size is used in this work. In general, the initial guesses and their correction mechanism are crucial for a shooting algorithm to converge quickly and stably. In the following sections, these two issues will be discussed in detail.

A. Scaling Vector and Determination of Accurate Initial Guesses

For a bidirectionally pumped RFA, the n_2 forward pumps and m signal channels propagate from the beginning of the fiber (hereinafter, Port I) to the end of the fiber (hereinafter, Port II), and the n_1 backward pumps propagate from Port II to Port I. The powers of the forward pumps and signals are therefore known at Port I as $\boldsymbol{P}_{\mathrm{I}} = (P_{\mathrm{I}1}, P_{\mathrm{I}2}, \ldots, P_{\mathrm{I}(n_2+1)}, \ldots P_{\mathrm{I}(n_2+m)})$, while the powers of the backward pumps are known at Port II as $\boldsymbol{P}_{\mathrm{IIp}} = (P_{\mathrm{IIp}1}, P_{\mathrm{IIp}2}, \ldots, P_{\mathrm{IIp}n_1})$. Because the powers of the backward pumps are unknown at Port I, an initial guess $\boldsymbol{P}'_{\mathrm{I}} = (P'_{\mathrm{I}1}, P'_{\mathrm{I}2}, \ldots, P'_{\mathrm{I}n_1})$ at Port I has to be made to start the integration of Eqs. (1). Starting from the given powers of the forward pumps and signals $\boldsymbol{P}_{\mathrm{I}} = (P_{\mathrm{I}1}, P_{\mathrm{I}2}, \ldots, P_{\mathrm{I}(n_2+1)}, \ldots P_{\mathrm{I}(n_2+m)})$ and the guessed powers of the backward pumps $\boldsymbol{P}'_{\mathrm{I}} = (P'_{\mathrm{I}1}, P'_{\mathrm{I}2}, \ldots, P'_{\mathrm{I}n_1})$ at port I, Eqs. (1) can be integrated step by step to obtain all powers of the optical channels along the fiber until Port II. This process is referred to as "shooting." The final results obtained at Port II are designated as $\boldsymbol{P}_{\mathrm{II}} = (P_{\mathrm{II}1}, P_{\mathrm{II}2}, \ldots, P_{\mathrm{II}(n_2+1)}, \ldots P_{\mathrm{II}(n_2+m)})$ for the forward pumps and signals and $\boldsymbol{P}'_{\mathrm{IIp}} = (P'_{\mathrm{IIp}1}, P'_{\mathrm{IIp}2}, \ldots, P'_{\mathrm{IIp}n_1})$ for the backward pumps. These values are called the "shot" values. The shot powers $\boldsymbol{P}'_{\mathrm{IIp}}$ are then compared with the actual backward pump powers $\boldsymbol{P}_{\mathrm{IIp}} = (P_{\mathrm{IIp}1}, P_{\mathrm{IIp}2}, \ldots, P_{\mathrm{IIp}n_1})$ at port II. Their difference is used by the correction mechanism, which will be discussed in Subsection 3.B, to generate an improved new initial guess for the next round of shooting. This procedure will be repeated until a prescribed criterion is met.

In the above shooting process, the accuracy of initial guess for the backward pump powers $\boldsymbol{P}'_{\mathrm{I}}$ at port I plays a crucial rule in the efficiency and convergence of the shooting. A good initial guess could not only avoid waste of computational time but also minimize the risk of computational breakdown for the shooting process. A primary initial guess could be derived from the following concerns [10,11]. First, let the first individual backward pump with power P_{IIp1} propagate on its own along the fiber from Port II to Port I. As it reaches Port I, its power is numerically obtained as $P'_{\mathrm{I}1t}$. Second, let both the first and the second backward pumps with powers P_{IIp1} and P_{IIp2}, respectively, propagate along the fiber from Port II to Port I, and the power of the second pump at Port I is obtained as $P'_{\mathrm{I}2t}$. This process is repeated until the last backward pump. The obtained vector $\boldsymbol{P}'_{\mathrm{I}t} = (P'_{\mathrm{I}1t}, P'_{\mathrm{I}2t}, \ldots, P'_{\mathrm{I}n_1 t})$ can then be used as a primary initial guess. When all the pumps and signals propagate together, due to SRS, every pump will transfer its power to pumps and signals of longer wavelengths. In the meantime, every pump will receive power from the other channels of shorter wavelengths. The actual backward pump power emerging form Port I, $P'_{\mathrm{I}j}$, could therefore be larger or smaller than the corresponding primary initial guess $P'_{\mathrm{I}tj}$, depending on the position of its wavelength among the wavelengths of the other pumps. In order to give a more appropriate initial guess, a scaling vector $\boldsymbol{S} = (1/S_1, 1/S_2, \ldots, 1/S_{n1})$ is introduced and is multiplied by $\boldsymbol{P}'_{\mathrm{I}t} = (P'_{\mathrm{I}1t}, P'_{\mathrm{I}2t}, \ldots, P'_{\mathrm{I}n_1 t})$ to produce a better initial guess for powers of the backward pumps at Port I,

$$\begin{aligned}\boldsymbol{P}'_{\mathrm{I}} &= (P'_{\mathrm{I}1}, P'_{\mathrm{I}2}, \ldots, P'_{\mathrm{I}n_1}) = \boldsymbol{S} \cdot \boldsymbol{P}'_{\mathrm{I}t} \\ &= (P'_{\mathrm{I}1t}/S_1, P'_{\mathrm{I}2t}/S_2, \ldots, P'_{\mathrm{I}n_1 t}/S_{n_1})\end{aligned} \quad (2)$$

where $S_j \in (0, \infty)$ is a parameter to be determined. The general guideline for selecting the values of S_j is as follows. For the jth pump, if it loses more power to channels of larger wavelength than it receives from channels of shorter wavelength, S_j should be set to a value larger than 1.0. Otherwise, S_j should be set to a value smaller than 1.0. The bigger the difference between loss and gain, the larger S_j should be set to, and vice versa. The use of appropriate $\boldsymbol{S}$ is tested in many numerical simulations of this work in the design of bidirectionally pumped RFAs, and efficient convergence of the shooting program is observed in all cases.

B. Correction Mechanism for Initial Guesses

In the shooting procedure, due to the use of guessed initial values for the backward pump powers at Port I, the resulting $P'_{\mathrm{IIp}j}$ is usually not equal to $P_{\mathrm{IIp}j}$, $j = 1, 2, \ldots, n_1$. Their discrepancy is designated as $\boldsymbol{D} = (P'_{\mathrm{IIp1}} - P_{\mathrm{IIp1}}, P'_{\mathrm{IIp2}} - P_{\mathrm{IIp2}}, \ldots, P'_{\mathrm{IIp}n_1} - P_{\mathrm{IIp}n_1})$. If all the components of vector $\boldsymbol{D}$ are zero, the guessed initial values $\boldsymbol{P}'_{\mathrm{I}} = (P'_{\mathrm{I}1}, P'_{\mathrm{I}2}, \ldots, P'_{\mathrm{I}n_1})$ correctly represent the actual backward pump powers that would emerge from Port I. To reveal the true solution, the component values of the vector $\boldsymbol{D}$ are minimized by the Newton–Raphson method with the following formulas to update the guessed values in $\boldsymbol{P}'_{\mathrm{I}}$ [10–13]:

$$\boldsymbol{P}'^{(k)}_{\mathrm{I}} = \boldsymbol{P}'^{(k-1)}_{\mathrm{I}} + \alpha \Delta \boldsymbol{P}'_{\mathrm{I}}, \quad (3)$$

$$\Delta \boldsymbol{P}'_{\mathrm{I}} = -\boldsymbol{J}^{-1} \cdot \boldsymbol{D}, \quad (4)$$

$$\boldsymbol{J} = \begin{bmatrix} J_{11} & J_{12} & \ldots & J_{1n_1} \\ J_{21} & J_{22} & \ldots & J_{2n_1} \\ \vdots & \vdots & \ldots & \vdots \\ J_{n_1 1} & J_{n_1 2} & \ldots & J_{n_1 n_1} \end{bmatrix}, \quad (5)$$

where $\Delta \boldsymbol{P}'_{\mathrm{I}}$ and $\boldsymbol{J}$ are the increment of $\boldsymbol{P}'_{\mathrm{I}}$ and the Jacobian matrix, respectively, for the next shooting process. The element of $\boldsymbol{J}$ is determined by $J_{kj} = \partial D_k / \partial P_{\mathrm{Ip}j}$ (k, $j = 1, 2, 3, \ldots, n_1$), and in the numerical calculation the partial differential operator is replaced by finite difference formula.

In Eq. (3), an appropriate value of α is crucial for the convergence of the shooting algorithm. Too large negative adjustment of $\boldsymbol{P}'_{\mathrm{I}}$ may result in $P'_{\mathrm{I}j}$ (the component of $\boldsymbol{P}'_{\mathrm{I}}$) being less than zero. Obviously, negative $P'_{\mathrm{I}j}$ corresponds to no physical circumstance. On the other hand, too small adjustment of $\boldsymbol{P}'_{\mathrm{I}}$ may result in extra iterations for the shooting. In [10] a sufficiently small value of α was chosen. Later it was found that a small value of α is not always necessary. In [11] α is initially set to 1.0. If the resulting adjustment $\alpha \Delta \boldsymbol{P}'_{\mathrm{I}}$ could not meet the criteria, α is decreased gradually until all the criteria are met. The value of α is chosen according to two criteria. One is to ensure that all the components of $\boldsymbol{P}'_{\mathrm{I}}$, $P'_{\mathrm{I}j}$, are between 0 and $P'_{\mathrm{I}jt}$. The other one is to ensure that the Euclidean norm of the error vector $\boldsymbol{D}$, $\|\boldsymbol{D}\|$, decreases in the succeeding shooting. When all the pumps are applied backward from Port II to Port I, the above criteria are proven effective [11]. In this work, however, since some of the pumps are launched from Port I, it is found that the limitation set by $P'_{\mathrm{I}jt}$ is no longer valid. According to the discussion just before Eq. (2), the actual backward pump power emerging from Port I, $P'_{\mathrm{I}j}$, could be smaller or larger than the corresponding $P'_{\mathrm{I}jt}$, depending on the relative position of the wavelength of the jth pump among the wavelengths of the other pumps. Therefore, the limitation of $P'_{\mathrm{I}j} < P'_{\mathrm{I}jt}$ is removed for the bidirectional pumping scheme.

In summary, the whole procedure of new shooting algorithm used in this work is as follows.

1. Determine the initial values $\boldsymbol{P}'_{\mathrm{I}}$ for the backward pumps according to Subsection 3.A.
2. Use the Runge–Kutta method to integrate Eqs. (1) with the initial guess $\boldsymbol{P}'_{\mathrm{I}}$ from Port I to Port II along the fiber.
3. Generate the error vector $\boldsymbol{D}$ and its Euclidean norm $\|\boldsymbol{D}\|$ for the kth iteration.
4. Compare $\|\boldsymbol{D}\|$ to a prespecified small quantity ε and compare k to the maximum allowed number of shootings N_{smax}. If $\|\boldsymbol{D}\| < \varepsilon$ or $k > N_{\mathrm{smax}}$, terminate the shooting procedure and output the numerical results; otherwise go to Step 5.
5. Generate the new $\boldsymbol{P}'_{\mathrm{I}}$ according to Subsection 3.B, and go to Step 2.

4. RESULTS AND DISCUSSION

Using the proposed method, 14 different bidirectionally pumped RFAs are simulated, and the results are compared with three other existing methods in terms of efficiency and stability. The Raman gain coefficient and attenuation

Table 1. Parameters Used in Simulations of Four-Pump RFAs

Parameter Name	Unit	Values
Fiber length	km	50
Effective core area of fiber	μm^2	80
Wavelength separation of signals	nm	1.0
Power of each signal channel	mW	0.1
Power of each pump channel	mW	300
Starting wavelength for signals	nm	1530
Ending wavelength for signals	nm	1625
Number of pumps		4
Wavelength of the first pump	nm	1430
Wavelength of the second pump	nm	1455
Wavelength of the third pump	nm	1480
Wavelength of the fourth pump	nm	1505

spectra of the fiber used in the simulations are illustrated in Fig. 1. The other parameters are listed in Table 1. Under these conditions the following three schemes of bidirectionally pumped RFAs are simulated, namely, (1) one forward pump and three backward pumps, (2) two forward and two backward pumps, and (3) three forward pumps and one backward pump. The wavelengths of pumps increase with their channel indices. There are 14 different arrangements in total derivable from these schemes, as summarized in Tables 2–4, where F represents the forward pumping and B denotes the backward pumping. Regarding the simulation results, Y is an indication of convergence of the corresponding case. To the contrary, N represents that the corresponding method fails to deliver a solution for the corresponding case. The numeric character following Y indicates the required iterations of adjustments from the initial guesses in the shooting process. The values of contraction factor f used in Method III and of the scaling vector component S_j used in Method IV are given in the parentheses. Note that here $1/f$ is equivalent to the parameter c of [10] or d of [11].

Method I uses the undepleted pump approximation [14] to guess the initial values of the backward pumps. In the guess of initial values, any SRS between pumps and between pumps and signals are not taken into account. The equivalent method [8,9] is used by Method II to guess the initial values of backward pumps. In the equivalent method all counterpumps are viewed as forward pumps. Method III uses a contraction factor [10,11] to obtain the initial values of backward pumps. The correction mechanism for these initial values used in Methods I–III is the modified Newton–Raphson method. The new shooting algorithm proposed in this work is represented by Method IV.

From Tables 2–4, it is found that Method I fails in every case except in Case I, while Methods II and III find the solution in several cases. On the other hand, Method IV is much more robust. It successfully identifies the true solution in all the 14 cases. Because the correction mechanism used in all the four methods is the same as the one illustrated in Subsection 3.B, the four methods are different mainly in the way their initial guess is decided. It is evident from Tables 2–4 that the initial guess method has a big impact on the convergence performance of a shooting algorithm. The evolutions of powers from initial guesses of Methods I—IV and from the true solution are plotted in Figs. 2 and 3 for the second and the fourth backward pumps of Case II of Table 3. It is seen that the initial guess of Method IV is close to the true solution. The accurately guessed initial values of Method IV play an important role on the stability and efficiency of this method.

Method I does not take the SRS into account in making the initial guess for powers of the backward pumps. It therefore

Table 2. Simulation Results of Four-Pump RFAs with One Forward Pump and Three Backward Pumps

	First Pump	Second Pump	Third Pump	Fourth Pump	Method I	Method II	Method III	Method IV
Case I	F	B	B	B	Y, 11	N	N	Y, 3 (3, 2, 2)
Case II	B	F	B	B	N	N	Y, 4 (4)	Y, 3 (5, 4, 3)
Case III	B	B	F	B	N	N	Y, 6 (20)	Y, 5 (30, 20, 10)
Case IV	B	B	B	F	N	N	Y, 4 (35)	Y, 3 (50, 40, 30)

Table 3. Simulation Results of Four-Pump RFAs with Two Forward and Two Backward Pumps

	First Pump	Second Pump	Third Pump	Fourth Pump	Method I	Method II	Method III	Method IV
Case I	F	F	B	B	Y, 4	N	N	Y, 3 (0.9, 0.8)
Case II	F	B	F	B	N	N	N	Y, 2 (20, 0.7)
Case III	F	B	B	F	N	N	Y, 2 (20)	Y, 2 (25, 20)
Case IV	B	F	F	B	N	N	Y, 3 (30)	Y, 3 (30, 25)
Case V	B	F	B	F	N	Y, 5	Y, 6 (70)	Y, 5 (75, 70)
Case VI	B	B	F	F	N	Y, 5	Y, 4 (70)	Y, 4 (80, 60)

Table 4. Simulation Results of Four-Pump RFAs with Three Forward Pumps and One Backward Pump

	First Pump	Second Pump	Third Pump	Fourth Pump	Method I	Method II	Method III	Method IV
Case I	F	F	F	B	Y, 3	Y, 4	Y, 3 (2)	Y, 3 (2)
Case II	F	F	B	F	N	Y, 4	Y, 2 (30)	Y, 2 (30)
Case III	F	B	F	F	N	Y, 2	Y, 1 (35)	Y, 1 (35)
Case IV	B	F	F	F	N	Y, 2	Y, 2 (200)	Y, 2 (200)

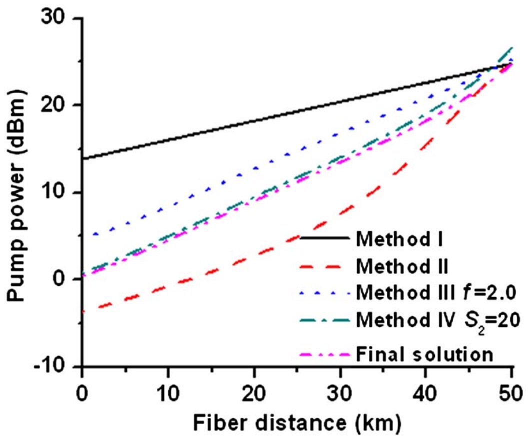

Fig. 2. (Color online) Evolutions of the second pump power for different initial guesses and final solution in simulation of the four-pump RFA in Case II of Table 3.

produces a poor outcome in complex circumstances such as high input powers, a large number of channels, and/or a longer relay distance. In these situations, discrepancy between the true solution and the initial guess for the backward pump powers at Port I is quite large due to the lack of consideration of the strong SRS between different optical channels. As shown in Figs. 2 and 3, the guessed initial values of the second and the fourth backward pumps derived by Method I are 23.0 mW higher and 46.6 mW lower, respectively, than the true values of the corresponding pumps at Port I.

Method II is not a good method either for the determination of initial values of the backward pumps when the RFAs are bidirectionally pumped, as will be explained below. In this method the backward pumps are treated as forward pumps when their initial values are decided. Although this scheme works for backward-pumped RFAs, its reliability reduces when forward pumps coexist with backward pumps. It can be imagined that, when all pumps are launched into one port of the transmission fiber with signals, as was done in Method II, all pumps transfer their powers to signals as propagating along the fiber. Because the pumps also receive power from other pumps of short wavelength due to SRS between pumps, the powers of some pumps may increase initially. After the initial transition stage, due to the attenuation by fiber and depletion by signals, the power will decrease monotonically along the fiber for any pump. This phenomenon is clearly observable from Figs. 2 and 3. The power of the second pump decreases monotonically from right to left according to Method II, while the power of the fourth pump derived by this method grows at the beginning then decreases monotonically from right to left. In a bidirectionally pumped RFA, however, the forward and backward pumps are applied simultaneously, and the real picture of the power evolution is more complicated. At a particular fiber position, the actual power level of a pump depends not only on its distance from its source but also on the wavelengths of its neighboring pumps. As illustrated by the curves marked with “Final solution” in Figs. 2 and 3, the power of the second backward pump monotonically decreases along the transmission fiber from right to left. On the other hand, the power of the fourth backward pump decreases initially then increases from right to left along the transmission fiber. Because the treatment of Method II does not correctly reflect the true situation, a big difference between its initial guess and the actual value results. The initial guesses derived for the second and the fourth backward pumps by Method II are 4.1 and 25.1 dB lower, respectively, than those of the true solution.

Method III also works well for backward-pumped RFAs [10,11]. In a backward-pumped RFA the actual output pump power $P'_{\mathrm{I}j}$ from Port I is always smaller than the corresponding $P'_{\mathrm{I}jt}$, as analyzed in [11]. $P'_{\mathrm{I}jt}$ can therefore be used as the bound of initial guess. After being weighed by an appropriate contraction factor $f > 1$, the initial guess $P'_{\mathrm{I}jt}/f$ approximates the actual pump powers $P'_{\mathrm{I}j}$ fairly accurately. However, such bound does not exist when forward and backward pumps are applied simultaneously. The actual output pump powers $P'_{\mathrm{I}j}$ from Port I could be smaller or larger than the corresponding $P'_{\mathrm{I}jt}$, depending on the actual wavelength of the pump. As shown in Figs. 2 and 3, the actual output power of the second backward pump is 4.3 dB lower than the initial guess of Method III, while the actual output power of the fourth backward pump is 10.1 dB higher than the corresponding initial guess. For the fourth backward pump, although it suffers from power loss due to pump depletion and material attenuation, it also receives more power from other pumps of short wavelengths, especially those forward pumps, so its overall balance increases when it reaches Port I. Based on a similar argument, the overall balance of the second backward pump decreases at Port I. Because different pumps follow different evolution tracks, it is not a good idea to use the equally weighed quantity $\boldsymbol{P}'_{\mathrm{I}t}/f$ as the initial guess. For example, a larger f results in a smaller difference between $P'_{\mathrm{I}2t}/f$ and $P'_{\mathrm{I}2}$ but in the meantime results in a larger difference between $P'_{\mathrm{I}4t}/f$ and $P'_{\mathrm{I}4}$, and vice versa. This explains why Method III does not work as well for bidirectionally pumped RFAs as it does for backward-pumped RFAs.

In Method IV, the different pump evolution tracks are taken care of by replacing the contraction factor f with the scaling vector $\boldsymbol{S}$. The improvement is clearly demonstrated by the examples listed in Tables 2–4, where all the cases dealt with by Method IV are shown convergent. The initial guesses of the second and fourth backward pumps derived by Method IV are 0.3 and 0.5 dB higher, respectively, than those of the true solution, very close to the points they respectively converge to. The reason for the better performance of Method IV can be explained as following. Because of the exponential growth of

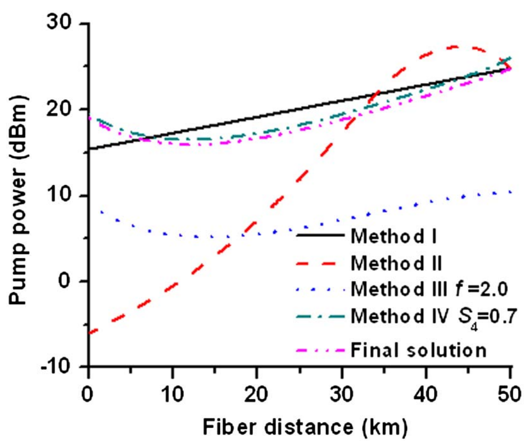

Fig. 3. (Color online) Evolutions of the fourth pump power for different initial guesses and final solution in simulation of the four-pump RFA in Case II of Table 3.

the backward pump powers, a too large initial guess of the backward pump could directly result in divergence of the shooting process, while a too small initial guess could incur an overly large increment of backward pump power at Port I for the next shooting. Similar to the effect of the overly large initial guess, the overly large increment of pump power could also result in numerical overflow. By the use of scaling vector $\boldsymbol{S}$, the different P'_{Ijt} are weighed differently, so the initial guess of pump power P'_{Ijt}/S_j is very close to the actual true solution P'_{Ij}. The curves of Method III and IV in Figs. 2 and 3 are power evolutions of the second and fourth backward pumps, calculated from their corresponding initial guesses using the contraction factor f and the scaling factor $\boldsymbol{S}$, respectively, with all pumps and signals applied. The improvement brought out by $\boldsymbol{S}$ is noticeable. Obviously, when the number of backward pumps reduces to one, the scaling vector $\boldsymbol{S}$ contains only one component and effectively reduces to the contraction factor f.

Similar to Case II of Table 3, it is found that, in all the other cases listed in Tables 2 and 4, the shooting procedure will converge if the initial guess is close enough to its corresponding true solution. For example, there is one backward pump, three forward pumps of shorter wavelengths, and 96 signal channels of longer wavelengths in the RFA of Case I of Table 4. Because of SRS, the fourth backward pump receives enough power from the three forward pumps to compensate for its power loss to the signals. In this case, all four methods provide a good-enough initial guess so all the shooting procedures converge. Because the initial guess derived by Method IV with the use of an appropriate scaling factor $\boldsymbol{S}$ is always close enough to the corresponding true solution, the new proposed shooting algorithm using the modified Newton–Raphson method as a correction mechanism for the initial values always succeeds in finding the true solution. It is very robust in the simulation of bidirectionally pumped RFAs, even in extreme cases such as extended fiber length, high level pump and/or signal power, vast number of pumps and signals, variable pump schemes, etc. It is worth pointing out that, besides Method IV, another numerical method called the average power analysis technique [15–17] can also be used to solve the Raman coupled equations under conditions of Case II of Table 3. However, since the average power analysis technique is based on the so-called small-signal travelling-wave approximation and since the conditions of Case II of Table 3 involve intense signal power and/or strong SRS, which violate the small-signal travelling-wave approximation, larger numerical error is expected.

The calculation times of different methods in simulations of the bidirectionally pumped RFA for Case I of Table 4 are summarized in Table 5. It is found that the calculation efficiency of Method IV is the highest one. In Case I of Table 4, only one backward pump is applied; Method IV degenerates to Method III. This is why the calculation time of Method III is almost the same as that of Method IV; the small discrepancy between them is due to computational error. It is interesting to note that, although the number of iterations of Method I and Method IV are both 3 in Case I of Table 4, the calculation time of Method I is about 30% bigger than that of Method IV. The reason lies in the different initial conditions. Because of a worse initial guess, Method I needs more steps for adjustment using the Runge–Kutta method along the fiber to satisfy the required accuracy in a iteration, so it takes longer than Method IV. The corresponding gain profiles generated by Method I to Method IV in Case I of Table 4 are illustrated in Fig. 4. It is seen that, after convergence, all these methods achieve almost the same gain profiles on a four-pump RFA. This suggests that the gain profile in Fig. 4 is a correct one and that these methods confirm their correctness on the design of RFAs by each other.

Table 5. Calculation Time of Different Methods in Case I of Table 4

	Method I	Method II	Method III	Method IV
Calculation time (s)	9.172	10.266	7.109	7.093

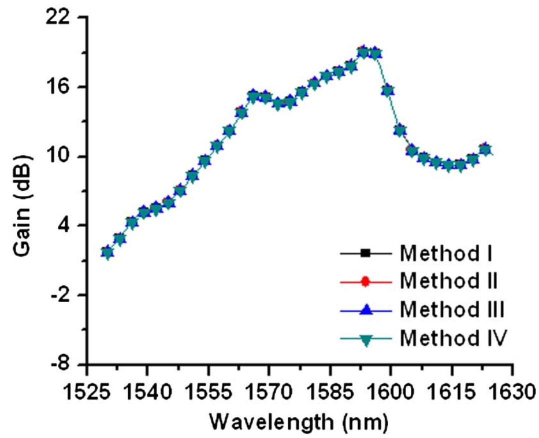

Fig. 4. (Color online) Comparison of gain profiles of a four-pump RFA generated by four different numerical methods for Case I of Table 4.

5. CONCLUSION

In summary, an efficient and robust shooting algorithm for the simulation of bidirectionally pumped RFAs is proposed in this work. With this method, the bidirectionally pumped Raman coupled equations can be solved under various conditions. This new shooting algorithm improves not only the efficiency of calculation but also the stability of the shooting algorithm. In this scheme, a scaling vector $\boldsymbol{S}$ is introduced to produce accurate initial guesses for the shooting algorithm. With the appropriate limitations for the increments of initial values applied, a better correction mechanism based on the Newton–Raphson method is introduced to evolve the initial guess to the final solution quickly. In total, 14 types of bidirectionally pumped RFAs with four pumps arranged differently in direction and wavelength are tested. The results show that the proposed shooting algorithm is convergent in many practical situations, including the cases when high optical powers, long transmission fiber, or a large number of pumps and signals are involved. In most of these extreme cases, the other three relevant methods discussed in this paper for comparison are shown to fail to deliver a solution. This shooting algorithm provides an efficient and robust methodology for the design of bidirectionally pumped RFAs in an LH WDM system.

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