

## A heterodyne interferometer using an electro-optic modulator for measuring small displacements

This content has been downloaded from IOPscience. Please scroll down to see the full text.

1996 J. Opt. 27 19

(<http://iopscience.iop.org/0150-536X/27/1/003>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 140.113.38.11

This content was downloaded on 28/04/2014 at 15:04

Please note that [terms and conditions apply](#).

INSTITUTE OF ELECTRO-OPTICAL ENGINEERING  
NATIONAL CHIAO TUNG UNIVERSITY

1001, Ta-Hsueh Road, Hsinchu, Taiwan, R.O.C.

## A HETERODYNE INTERFEROMETER USING AN ELECTRO-OPTIC MODULATOR FOR MEASURING SMALL DISPLACEMENTS

DER-CHIN SU, MING-HORNG CHIU and CHENG-DER CHEN

KEY WORDS :

Heterodyne interferometer  
Electro-optic modulator

MOTS CLÉS :

Interférométrie hétérodyne  
Modulateur électro-optique

**Un interféromètre hétérodyne, utilisant un modulateur électrooptique, pour la mesure de petits déplacements**

**RÉSUMÉ :** Un modulateur électro-optique est utilisé comme décodeur de fréquence dans cet interféromètre hétérodyne. Cet interféromètre présente quelques avantages : insensibilité aux vibrations, structure simple et compacte, traitement du signal classique et bon marché sans perte de résolution et avec une utilisation aisée en temps nul.

**SUMMARY :** In this heterodyne interferometer for measuring small displacements, an electro-optic modulator is used as a frequency shifter. It has some merits, such as, free from the mechanical vibration, simple and compact optical structure, and a general and inexpensive electrical signal processing circuit can be used without decreasing the measuring resolution and easily operated in real time.

### INTRODUCTION

In a heterodyne interferometer [1-5], two light beams with slightly different frequencies interfere and the interference intensity being modulated at the difference frequency between these two beams is converted to the electrical signals for easier and accurate measurements [5, 6]. Due to its high resolution, the heterodyne interferometer is suitable for measuring small displacements in the submicron manufacturing processes. The measurement resolution of a heterodyne interferometer depends on the angular resolution of the phase meter [7, 8], and the angular resolution of the phase meter is determined by the ratio of the reference clock frequency to the signal frequency. A reference clock with  $10^3 \sim 10^4$  times higher than the difference frequency is necessary for high resolution measurements. In a common heterodyne interferometer, a Zeeman laser [4, 9, 10], an acoustic-optic modulator [6, 11], a rotating polarization component [1, 3, 12], or a moving diffraction grating [13, 14] is used for generating two light beams. For a Zeeman laser or an acoustic-optic

modulator, the difference range is from several MHz to 100 MHz. Hence the frequency of the reference clock is about 10 GHz  $\sim$  100 GHz. For such high frequencies, the reference clock is not only expensive, but also very difficult to operate in a common electrical signal processing system. For easier operation and low cost, it is better to decrease the difference frequency between these two interfering beams without decreasing the measurement resolution. Although either a rotating polarization component or a moving diffraction grating can meet this requirement, the mechanical vibration will influence on the stability of the optical structure and decrease the measurement resolution.

To avoid the mechanical vibration and the difficulty of high frequency electrical signal processing, a heterodyne interferometer using an electro-optic modulator for measuring small displacements is presented. The necessary difference frequency between two interfering beams is generated by an electro-optic modulator. A polarization beam-splitter with a special reflection layer is introduced for reducing a part of the influence coming from air turbulence and

makes the optical setup to be compact. Besides, it has some other merits, such as, the resolution is as good as that of a common heterodyne interferometer, capability of operation in real-time, low cost, simple optical setup, etc.

**ANGULAR RESOLUTION OF THE PHASE METER**

Consider the case that the two sinusoidal signals with a period  $T_s$  shown in *figure 1(a)* are the input signals coming from the photodetectors. A phase meter for measuring the phase difference between these two signals performs as shown in *figure 1*. First, these two sinusoidal signals are converted into the square waves as shown in *figure 1(b)*. Next, the phase meter compares these two square waves and generates a square wave as shown in *figure 1(c)* with a time interval  $T_d$ , where  $T_d$  is the time difference between the zero-crossing points of these two waves. For convenience, *figure 1(c)* is redrawn as shown in the upper part of *figure 1(d)*. Finally, the phase meter measures the time interval  $T_d$  by a series of very short reference square waves generating from a reference clock, as shown in the lower part of *figure 1(d)*. Let the period of the very short reference square waves be  $T_c$ , and the counting number be  $N$ , then the phase difference between these two waves is

$$\phi = \frac{T_d}{T_s} \times 360^\circ = \frac{NT_c}{T_s} \times 360^\circ = N \frac{f_s}{f_c} \times 360^\circ. \quad (1)$$

Where  $f_s$  and  $f_c$  are the frequencies of input waves and the reference clock, respectively, i.e.,  $f_s = 1/T_s$  and  $f_c = 1/T_c$ . From Eq. (1) it is obvious that the angular resolution of the phase meter is

$$\Delta\alpha = \frac{f_s}{f_c} \times 360^\circ. \quad (2)$$

Hence, the angular resolution will be enhanced as the ratio of  $f_c/f_s$  increases. *Table 1* shows the desired frequencies of the reference clock in some cases for  $\Delta\alpha = 0.1^\circ$  and  $0.01^\circ$ , respectively.

**OPTICAL SETUP AND PRINCIPLE**

The schematic diagram of this heterodyne interferometer using an electro-optic modulator is shown in *figure 2*, for the measuring small displacements of a mirror  $M$ . The linearly polarized light passing through an electro-optic modulator  $EO$  incidents on a beam-splitter  $BS$ , and is divided into two parts; the reflected light and the transmitted light. The reflected light passes through an analyzer  $AN_r$ , and en-

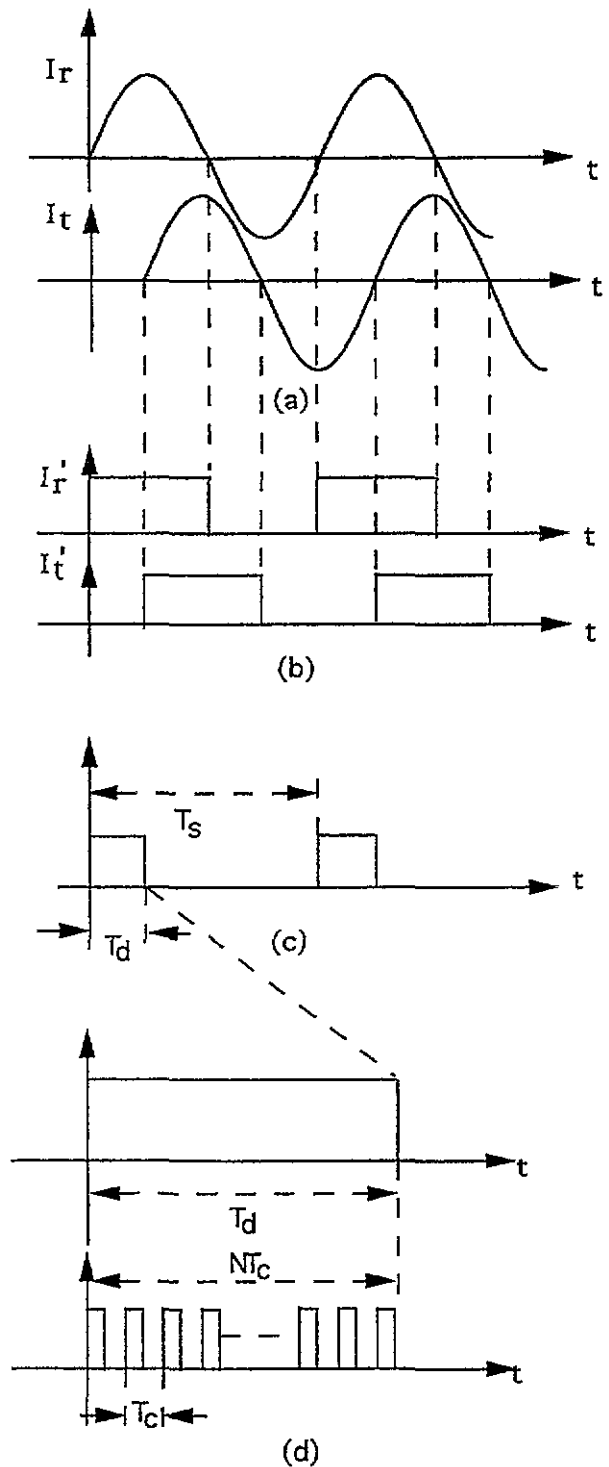


FIG 1. — Diagrams illustrating how a phase meter measures the phase difference between two signals: (a) two input sinusoidal signals with same frequency; (b) the square waves corresponding with the input signals; (c) the square wave with a time interval  $T_d$ , which is the time difference between the zero-crossing points of the above square waves; (d) the time interval  $T_d$  is measured by a series of very short reference square waves generated from a reference clock.

ters into a photodetector  $D_r$ . If the amplitude of the light detected by  $D_r$  is  $E_1$ , then the intensity mea-

TABLE I

The desired frequencies of the reference clock in some cases for  $\Delta\alpha = 0.1^\circ$  and  $0.01^\circ$ , respectively.

$\Delta\alpha = 0.1^\circ$		$\Delta\alpha = 0.01^\circ$	
$f_s$	$f_c$	$f_s$	$f_c$
100 Hz	360 kHz	100 Hz	3.6 MHz
1 kHz	3.6 MHz	1 kHz	36 MHz
10 kHz	36 MHz	10 kHz	360 MHz
100 kHz	360 MHz	100 kHz	3.6 GHz
1 MHz	3.6 GHz	1 MHz	36 GHz
10 MHz	36 GHz	10 MHz	360 GHz

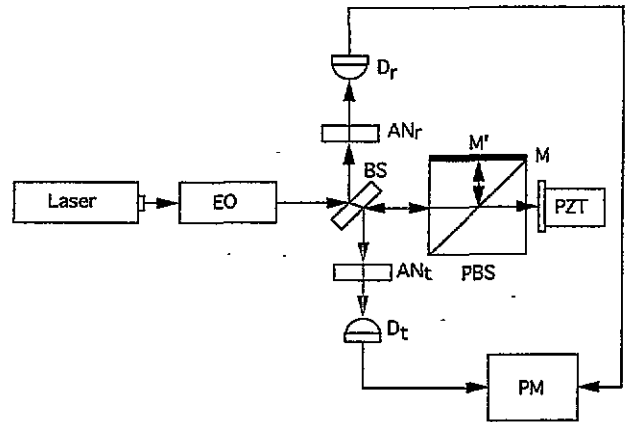


FIG. 2. — Schematic diagram of this new type of low frequency heterodyne interferometer for measuring small displacements. EO: electro-optic modulator; BS: beam-splitter; AN: analyzer; PBS: polarization beam-splitter; M': reflection layer; M: mirror; PZT: piezo-electric transducer; PM: phase meter.

sured by  $D_r$  is  $I_r = |E_1|^2$ . Here,  $I_r$  is the reference signal. On the other hand, the transmitted light enters a modified Michelson interferometer and is divided into two parts by the polarization beam-splitter PBS; the reflected s-polarization light and the transmitted p-polarization light. The former is reflected by the reflection layer  $M'$  coated intentionally on one side of PBS, then is reflected by PBS and BS again. And it passes through an analyzer  $AN_r$  and is detected by the photodetector  $D_r$ . It acts as the reference light in the modified Michelson interferometer. The latter is reflected by  $M$  and returns along the original path. After being reflected by BS, it passes through  $AN_t$  and also enters into  $D_t$ . It acts as the test light in the modified Michelson interferometer. If the amplitudes of the reference light and the test light are  $E_2$  and  $E_3$ , respectively, the  $D_t$  measures the interference intensity of  $E_2$  and  $E_3$ , i.e.,  $I_t = |E_2 + E_3|^2$ . Here,  $I_t$  is the test signal.

**The intensities of the reference signal and the test signal**

For convenience, the +z axis is chosen along the propagation direction and the y-axis is along the vertical direction. Let the incident light be linearly polarized at  $45^\circ$  with respect to the x-axis, then its Jones vector [15] can be written as

$$E_{in} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{3}$$

If the fast axis of EO is along the x-axis, and an external sawtooth voltage signal with angular frequency  $\omega$  and amplitude  $V_{\lambda/2}$ , the half-wave voltage of EO, is applied to EO, then the retardation pro-

duced by EO can be expressed as  $\omega t$ . And if the transmission axis of  $AN_r$  is  $45^\circ$  with respect to the x-axis, then we have

$$\begin{aligned} E_1 &= AN_r(45^\circ) EO(\omega t) E_{in} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{i(\omega t/2)} & 0 \\ 0 & e^{-i(\omega t/2)} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ &= \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} (e^{i(\omega t/2)} + e^{-i(\omega t/2)}). \end{aligned} \tag{4}$$

Hence, the intensity of the reference signal is

$$I_r = |E_1|^2 = \frac{1}{2} (1 + \cos \omega t). \tag{5}$$

On the other hand, let the angle between the transmission axis of  $AN_t$  and the x-axis be  $45^\circ$ , then we have

$$\begin{aligned} E_2 &= AN_t(45^\circ) R_{PBS} EO(\omega t) E_{in} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i(\omega t/2)} & 0 \\ 0 & e^{-i(\omega t/2)} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ &= \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-i(\omega t/2)} \end{aligned} \tag{6}$$

and

$$\begin{aligned} E_3 &= AN_t(45^\circ) T_{PBS} EO(\omega t) E_{in} e^{i\phi} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} e^{i(\omega t/2)} & 0 \\ 0 & e^{-i(\omega t/2)} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{i\phi} \\ &= \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{i[(\omega t/2) + \phi]}; \end{aligned} \tag{7}$$

where  $R_{PBS}$  and  $T_{PBS}$  are the transformation matrices of *PBS* for the reflected light and the transmitted light, respectively.  $\phi$  is the phase difference being corresponding to the optical path difference between the test light and the reference light in the modified Michelson interferometer. Therefore, the intensity of the test signal is given by

$$I_t = |E_2 + E_3|^2 = \frac{1}{2} [1 + \cos(\omega t + \phi)] \quad (8)$$

**Evaluation of small displacements**

From Eq. (5) and Eq. (8), it is obvious that both the reference signal and the test signal are sinusoidal with angular frequency  $\omega$ . These two signals are introduced into the phase meter, the phase difference  $\phi_1$  between them can be obtained. After *M* is displaced, another corresponding phase difference  $\phi_2$  can be obtained similarly. Then,

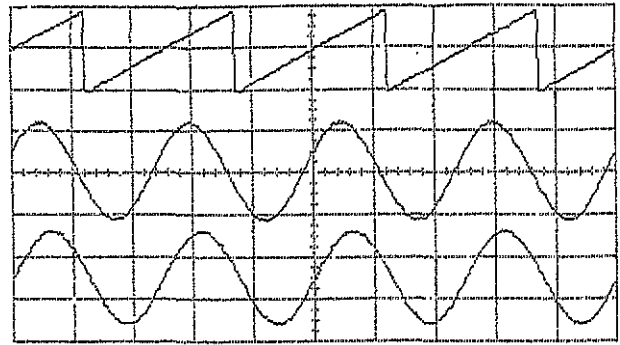
$$\text{displacement} = \frac{\lambda}{2} \times \frac{(\phi_2 - \phi_1)}{360} \quad (9)$$

where  $\lambda$  is the wavelength of light.

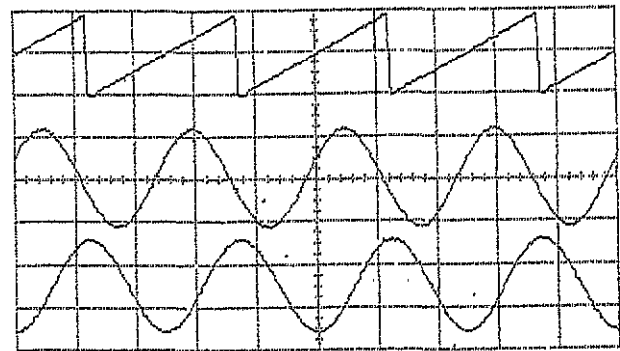
**Experiments and results**

In order to show the feasibility of this heterodyne interferometer, an He-Ne laser of wavelength 632.8 nm and an electro-optic modulator (product number: PC200/2) manufactured by Electro-Optics Developments Ltd., with half-wave voltage 170 V are used. A piezo-electric transducer PZT (product number: PZ-91) manufactured by Burleigh Instruments, Inc. with motion sensitivity 0.002  $\mu\text{m}/\text{V}$  is used to displace *M*. And the frequencies of the external modulated sawtooth signal applied to *EO* and the reference clock of the phase meter are chosen to be 2 kHz and 7.2 MHz, respectively, so the angular resolution is 0.1°. For easier observation, these signals are monitored by an oscilloscope and shown as figure 3(a) and figure 3(b), which are the recorded signals before and after the displacement of *M*, respectively. The upper parts of these two figures represent the external modulated sawtooth signal applied to *EO*, the middle parts and the lower parts represent the reference signal and the test signal, respectively. The phase difference can be easily read from the digital display connected with the phase meter. Here,  $\phi_1 = 28.8^\circ$ ,  $\phi_2 = 115.2^\circ$ , and the displacement is 76 nm.

The measured data can be transferred to a computer for real time measurements, and figure 4 is the results printed by a laser printer. The horizontal coordinate represents the electric voltage applied to *PZT*, and the vertical coordinate represents the corresponding displacement of *M*. The measured curve



(a)



(b)

FIG. 3. — The recorded signals (a) before ; (b) after *M* is displaced ; upper trace -- external modulated signal ; middle trace -- reference signal ; lower trace -- test signal.

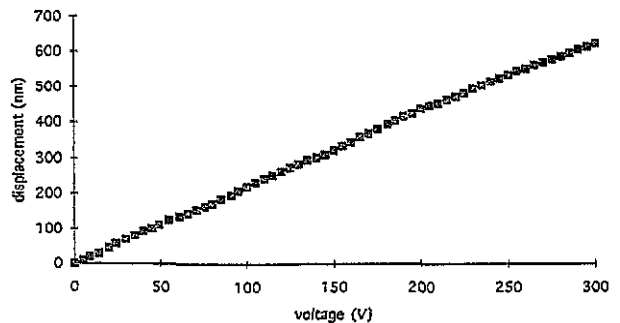


FIG. 4. — Displacement of *M* driven by a *PZT*.

shows that the displacement of *M* is linearly proportional to the voltage applied to *PZT*.

**DISCUSSION**

The frequency range of this interferometer depends on the characteristics of the electro-optic modulator and the phase meter. Theoretically,  $f_c / f_s$  may be enhanced by 2 orders in the above experi-

ments, but the air turbulence may wash out the fringe quality and decrease the measurement resolution. For general automatic manufacturing processes, the test signal with several kHz is enough for measuring small displacements in real-time. The polarization beam splitter with a reflective layer makes the optical setup to be compact, and reduce a part of the influence coming from the air turbulence.

### CONCLUSION

A heterodyne interferometer for measuring small displacements is presented. The necessary difference frequency between the interfering beams is generated and controlled by an electro-optic modulator. So, it is free from the mechanical vibration. Although it uses a general and inexpensive electrical signal processing circuits for the phase meter, it has the same high resolution as that of a common heterodyne interferometer and can be easily operated in real time. And its optical setup is simple and compact. Moreover, its feasibility is demonstrated.

### ACKNOWLEDGEMENT

This study was supported by the National Science Council, R.O.C., under the contract number: NSC81-0417-009-628.

### REFERENCES

[ 1 ] Sommargren GE. Up/down frequency shifter for optical heterodyne interferometry. *J Opt Soc Am* 1975 ; 65 : 960-1.

- [ 2 ] Crane R. Interference Phase measurement. *Appl Opt* 1969 ; 8 : 538-42.
- [ 3 ] Shagam RN and Wyant JC. Optical frequency shifter for heterodyne interferometers using multiple rotating polarization retarders. *Appl Opt* 1978 ; 17 : 3034-5.
- [ 4 ] Sommargren GE. Optical heterodyne profilometry. *Appl Opt* 1981 ; 20 : 610-8.
- [ 5 ] Massie NA, Nelson RD and Holly S. High performance real-time heterodyne interferometry. *Appl Opt* 1979 ; 18 : 1797-803.
- [ 6 ] Nakadate S. High-precise retardation measurement using a phase detection of Young's fringes. *Proc SPIE* 1989 ; 1163 : 30-8.
- [ 7 ] Bi DG, Kubo S, Nishizawa A, Takahashi C and Fujita J. A new method for measuring phase shift in heterodyne interferometric measurement. *Rev Sci Instrum* 1992 ; 61 : 1442-7.
- [ 8 ] Oldham NM, Kramar JA, Hetrick PS and Teague EC. Electronic limitations in phase meters for heterodyne interferometry. *Prec Eng* 1993 ; 15 : 173-9.
- [ 9 ] Takasaki H, Umeda N and Tsukiji M. Stabilized transverse Zeeman laser as a new light source for optical measurement. *Appl Opt* 1980 ; 19 : 435-41.
- [ 10 ] Umeda N, Tsukiji M and Takasaki H. Stabilized  $^3\text{He-}^{20}\text{Ne}$  transverse Zeeman laser. *Appl Opt* 1980 ; 19 : 442-50.
- [ 11 ] Ehrlich MJ, Phillips LC and Wagner JW. Voltage-controlled acousto-optic phase shifter. *Rev Sci Instrum* 1988 ; 59 : 2390-2.
- [ 12 ] Hu HZ. Polarization heterodyne interferometry using a simple rotating analyzer. 1: Theory and error analysis. *Appl Opt* 1983 ; 22 : 2052-6.
- [ 13 ] Suzuki T and Hioki R. Translation of light frequency by a moving grating. *J Opt Soc Am* 1967 ; 57 : 1551-1.
- [ 14 ] Stevenson WH. Optical frequency shifting by means of a rotating diffraction grating. *Appl Opt* 1970 ; 9 : 649-52.
- [ 15 ] Yariv A and Yeh P. *Optical waves in crystal*. 121-54. John Wiley and Sons, New York, 1984.

(Manuscript received in February 28<sup>th</sup> 1995.)