Applying a Three-Antenna GPS and Suspension Displacement Sensors to a Road Vehicle

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Abstract—This paper presents a novel sensor fusion system for road vehicles, which mainly composed of a three-antenna global positioning system (GPS), and four suspension displacement sensors. This sensor system not only obtains accurate six degreeof-freedom (DOF) information for vehicle dynamics but also road angles in real time. The road angles and vehicle attitude are difficult to be detected using on-board sensors because they are often coupled in the sensor measurements. Most approaches ignore this coupling effect; only a few try to obtain them by state estimations using vehicle models. The proposed method solve this problem, without requiring any vehicle model, by incorporating a non-inertial sensor (suspension displacement sensors) with inertial sensors (three-antenna GPS). In a demonstrating case, an additional inertial measurement unit (IMU) is included in the senor system to improve the estimation accuracy. Simulation results indicate that the accuracy of the estimated vehicle attitude is less than 0.85 deg, and that of the estimated road angles is less than 0.3 deg. Additionally, the estimation accuracy of the vertical displacement is improved from 3 m to 0.248 m.

I. INTRODUCTION

Many research reports have shown that road angles have direct influences on vehicle dynamics and those effects need to be identified for a satisfactory performance. For example, lacking of road information, rollover prediction systems may produce false alarms [1]; vehicle stability controls may either initiate false activation or need large actuation power [2]. Road angles are difficult to be measured in real time using sensors attached to a vehicle body because they are often coupled with other vehicle dynamics in sensor measurements [3], [4]. Many researches ignore this road angle effect in their applications [5], [6]; only a few try to obtain them using vehicle dynamics models [3], [4]. The former approach is inapplicable in many cases, such as obtaining vehicle dynamics on a sloped road. The latter approach is feasible but greatly relies on the accuracy of the employed vehicle model.

Lots of sensor fusion systems employ a GPS and an IMU (a three-axis accelerometer and a three-axis gyroscope) to measure six-DOF motions of an object. In that case, the object attitude (rotation angles) is determined by integrating the gyroscope measurements (angular rates). This sensor system has been widely used many applications such as: aircrafts, ships, and road vehicles [7]–[9]. However, it has some drawbacks when applied to road vehicles. First, the rotation angles obtained by integrating the angular rates may suffer from the

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initial value ambiguity problem and the error accumulation problem [9]. Second, both GPS and IMU are inertial sensors. The vehicle attitude and road angles are mingled in those sensor measurements. Third, the position accuracy determined by GPS is inadequate for the vehicle displacement in the vertical direction.

In recent years, a GPS with two or three antennas has been proposed in various sensor applications because it not only provides absolute position measurements but also robust angle measurements [4], which can avoid the error accumulation problems and initial value ambiguity problem. However, its sensing accuracy may still be unsatisfactory to some applications. For this reasons, serval papers proposed using additional instruments to improve the sensing accuracy such as: laser distance sensor, ultrasonic sensors, and etc. [10].

In this paper, we represent a novel sensor fusion system to estimate the road angles and improve the sensing accuracy of the six-DOF vehicle motions without the knowledge of vehicle models. This is done by selecting a proper non-inertial sensor (suspension displacements sensors) in synergy with two inertial sensors (a three-antenna GPS and an IMU). Specifically, they are done by the following: (1) three coordinate systems and three sets of Euler angles (see, Fig. 1) are used to describe vehicle dynamics on a sloped road. Thus, the vehicle attitude relative to a road and road angles can be clearly represented; (2) Using a kinematic model and a multi-rate extended Kalman filter, a sensor fusion system is constructed to estimate those six-DOF dynamics and road angles. The procedures of constructing proposed sensor fusion system are discussed in details in this paper.

II. EULER ANGLES AND COORDINATE SYSTEMS

Three coordinate systems (see Fig. 1) are introduced to describe a vehicle moving on a sloped road. These three coordinate systems are: global frame $\{g\}$, road frame $\{r\}$, and vehicle frame $\{v\}$. Similar to conventional research, the global frame is fixed to a point on Earth, while the vehicle frame is fixed to the center of gravity (CG) of the vehicle and rotates with the vehicle. Additionally, a road frame is introduced to describe the vehicle dynamics on a sloped road. The road frame is fixed to a road and rotated with the road.

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Fig. 1. Three coordinate systems: global frame, road frame, and vehicle frame.

Three sets of Euler angles are used to describe the relationships between any two out of three coordinate systems. The first set of Euler angles (ψ_g , θ_g , ϕ_g), which are referred to in this paper as the "absolute yaw angle," "absolute pitch angle," and "absolute roll angle," are used to describe the absolute attitude of the vehicle (global frame vs. vehicle frame). The rotation order of this set of Euler angles is yaw-pitch-roll. Its direction cosine matrix (C_g^v) can be written as follows:

$$
\mathbf{C}_{g}^{V} = \mathbf{R}(x, \phi_{g}) \mathbf{R}(y, \theta_{g}) \mathbf{R}(z, \psi_{g}) \qquad (1)
$$

$$
\mathbf{R}(x, \phi_{g}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi_{g}) & \sin(\phi_{g}) \\ 0 & -\sin(\phi_{g}) & \cos(\phi_{g}) \end{bmatrix}
$$

$$
\mathbf{R}(y, \theta_{g}) = \begin{bmatrix} \cos(\theta_{g}) & 0 & \sin(\theta_{g}) \\ 0 & 1 & 0 \\ -\sin(\theta_{g}) & 0 & \cos(\theta_{g}) \end{bmatrix}
$$

$$
\mathbf{R}(z, \psi_{g}) = \begin{bmatrix} \cos(\psi_{g}) & \sin(\psi_{g}) & 0 \\ -\sin(\psi_{g}) & \cos(\phi_{g}) & 0 \\ 0 & 0 & 1 \end{bmatrix}.
$$

The second set if Euler angles (θ_r , ϕ_r , ψ_r), which are referred to in this paper as the "road grade angle," "road bank angle," and "road curve angle," are used to describe the road angles (global frame vs. road frame). The rotation order of this set of Euler angles is pitch-roll-yaw. Since a vehicle may move on a terrain irrelevant to the human-defined road path, it is impossible to determine the road curve angle from vehicle dynamics. Thus, it is assumed to be zero ($\psi_r = 0$) for simplicity. Its direction cosine matrix (C_g^r) can be written as:

$$
\mathbf{C}_{g}^{\mathrm{r}} = \mathbf{R}(z, \psi_{r}) \mathbf{R}(x, \phi_{r}) \mathbf{R}(y, \theta_{r}). \tag{2}
$$

The third set of Euler angles (ψ_v , θ_v , ϕ_v), which are referred to in this paper as the "vehicle yaw angle," "vehicle pitch angle," and "vehicle roll angle," are used to describe the vehicle attitude relative to a road plane (road frame vs. vehicle frame). The rotation order of this set of Euler angles is yawpitch-roll. Its direction cosine matrix (C_r^v) can be written as:

$$
\mathbf{C}_{\mathbf{r}}^{\mathbf{v}} = \mathbf{R}(x, \phi_{v}) \mathbf{R}(y, \theta_{v}) \mathbf{R}(z, \psi_{v}). \tag{3}
$$

Since two sets of Euler angles are enough to describe the relationships between three coordinate systems, complying with the above angle definitions, the following relationship can be established for these angles:

$$
\mathbf{C}_{g}^{\mathrm{v}} = \mathbf{C}_{\mathrm{r}}^{\mathrm{v}} \, \mathbf{C}_{g}^{\mathrm{r}}.\tag{4}
$$

Equation (4) provide three constrained equations for those nine angles.

III. A CONVENTIONAL KINEMATIC MODEL FOR THE SENSOR SYSTEM

Normally, an IMU sensor (a three-axis accelerometer and a three-axis gyroscope) is installed at the center of gravity of an object to measure the six-DOF movements. A kinematic model that can coordinate the outputs of those inertial sensors is:

$$
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_{acc} \begin{bmatrix} A_{acc}^{acc} \\ A_{g}^{acc} \\ A_{g}^{acc} \end{bmatrix} + \mathbf{B}_{gyro} \begin{bmatrix} \omega_{g}^{gyro} \\ \omega_{g}^{gyro} \\ \omega_{g}^{gyro} \end{bmatrix} (5)
$$
\n
$$
\mathbf{x} = [x_g, y_g, z_g, \dot{x}_g, \dot{y}_g, \dot{z}_g, \phi_g, \theta_g, \psi_g]^{\mathrm{T}}
$$
\n
$$
\mathbf{A} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{6\times9} & \mathbf{0}_{6\times9} \end{bmatrix}
$$
\n
$$
\mathbf{B}_{acc} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{C}_{g}^{\mathrm{v}-1} & \mathbf{0}_{3\times3} \end{bmatrix}^{\mathrm{T}}
$$
\n
$$
\mathbf{B}_{gyro} = \begin{bmatrix} \mathbf{0}_{3\times6} & \mathbf{C}_{\omega}^{-1} \end{bmatrix}^{\mathrm{T}}
$$
\n
$$
\mathbf{C}_{\omega} = \begin{bmatrix} 1 & 0 & -\sin\theta_g \\ 0 & \cos\phi_g & \cos\theta_g \sin\phi_g \\ 0 & -\sin\phi_g & \cos\theta_g \cos\phi_g \end{bmatrix}
$$

where $(A_x^{acc}, A_y^{acc}, A_z^{acc})$ represents the measurements from
a three axis accelerometer: (A_y^{gyro}, A_z^{gyro}) represents a three-axis accelerometer; $(\omega_x^{gyro}, \omega_y^{gyro}, \omega_z^{gyro})$ represents
the measurements from a three axis gyroscope; (x, y, z) the measurements from a three-axis gyroscope; (x_g, y_g, z_g) and $(\dot{x}_q, \dot{y}_q, \dot{z}_q)$ represent position and velocity observed in a global frame.

The above kinematic model has been widely used in various applications for the absolute position and attitude determinations. However, the above model does not contain any road angle information. Thus, it is not applicable to road vehicles on a sloped road.

IV. A SENSOR FUSION SYSTEM FOR ROAD VEHICLES

As proposed in this paper, nine angles are employed to parameterize the vehicle attitude determination system. The relationships stated in (4) provide three constrained equations. Therefore, six angles should be employed as system states to describe the vehicle attitude. If three angle states are chosen from the vehicle absolute attitude $(\phi_q, \theta_q, \psi_q)$, the same as the conventional approach does, another three angle states should come from the remaining six angles. And, which angles to be chosen should be determined by the senor elements incorporated in the sensor system. For this reason, the selected sensor elements are discussed first and then a new kinematic model is introduced for constructing a sensor fusion system.

Fig. 2. Three-antenna GPS, IMU, and suspension displacement sensors are installed on a vehicle body

A. Sensor Selections

1) Three-antenna GPS system: Different to a conventional GPS system, a three-antenna GPS is used because it not only provides absolute position measurements $(x_g^{qps}, y_g^{qps}, z_g^{qps})$ but also absolute angle measurements $(\phi_g^{gps}, \theta_g^{gps}, \psi_g^{gps})$.
Thus the angle information does not solely come from gyro Thus, the angle information does not solely come from gyroscope, which obtain rotation angles by integrating the angular rates.

2) Suspension Displacement Sensors: As shown in Fig. 2, four suspension displacement sensors are installed at four corners of a vehicle. The suspension deflection mainly comes from the relative vehicle attitude and the vertical displacement of the vehicle CG. Hence, if the vehicle geometric is known beforehand, the suspension displacement can be related to the following vehicle dynamics:

$$
z_r^{sus} = \frac{-1_r (H_1^{sus} + H_2^{sus}) - 1_f (H_3^{sus} + H_4^{sus})}{21_f + 21_r}
$$
(6)
\n
$$
\theta_v^{sus} = \sin^{-1} \left\{ \frac{H_1^{sus} + H_2^{sus} - H_3^{sus} - H_4^{sus}}{21_f + 21_r} \right\}
$$

\n
$$
\phi_v^{sus} = \sin^{-1} \left\{ \frac{-H_1^{sus} + H_2^{sus} + H_3^{sus} - H_4^{sus}}{(2t_f + 2t_r)\cos\theta_v} \right\}
$$

where the superscript (sus) denotes the physical quantities measured by the suspension displacement sensors; z_r is the vertical displacement of the vehicle CG observed in the road frame $\{r\}$; H_i represents the displacement of suspension at the corner i; the subscript (i) refers to four suspension corners in a way: $1 \rightarrow$ front-left, 2 to 4 in a clockwise direction; 1_f and l_r are the distances from CG to the front and rear axis, respectively; t_f and t_r are one half of the distances of the front and rear track, respectively.

B. A New Kinematic Model

As mentioned before, three out of the six angles (three vehicle angles relative to a road and three road angles) should be chosen to describe the vehicle dynamics. Since (1) the vehicle roll angle (ϕ_v^{sus}) and the vehicle pitch angle (θ_v^{sus})
can be obtained from the measurements of four suspension can be obtained from the measurements of four suspension

TABLE I SENSOR OUTPUT RATES AND NOISE CHARACTERISTICS

	Sampling	Noise		
	frequency	Mean	Standard deviation	
GPS	5 Hz	Ω	0.4	\deg
(attitude measurement)				
GPS	5 H _Z	Ω	horizontal: 1	m
(position measurement)			vertical: 3	m
Suspension	$100\,\mathrm{Hz}$	Ω	0.001	m
displacement sensor				
Accelerometer	$100\,\mathrm{Hz}$	Ω	0.2	$\rm m/s^2$
Gyroscope	$100\,\mathrm{Hz}$	0	0.08	\deg/s

displacement sensors; (2) the road curve angle is assumed to be zero ($\psi_r = 0$), the remaining three angles ϕ_r , θ_r , ψ_v are chosen as new states. And thus, equation (4) becomes three constrained equations for these three unknowns. Therefore, even without a kinematic model, those three unknown angles can be solved.

In order to apply existing state estimation techniques to this problem for additional benefits of robustness and noise reduction, the "dynamic equations" of three unknown angles should be obtained beforehand and added to the conventional kinematic model. Since it is neither practical to use additional sensors to measure the change rate of those three angles, nor to obtain this information for a specific case, the change rates of three angles are assumed to be zero.

$$
\dot{\phi}_r = 0
$$
\n
$$
\dot{\theta}_r = 0
$$
\n
$$
\dot{\psi}_v = 0.
$$
\n(7)

By combing the state vector (**x**) in (5) and three unknown angles, a new kinematic model with a state vector (**z**) is formulated as:

$$
\dot{\mathbf{z}} = \mathbf{Fz} + \mathbf{G}_{acc} \begin{bmatrix} A_{xc}^{acc} \\ A_{yc}^{acc} \\ A_{zc}^{acc} \end{bmatrix} + \mathbf{G}_{gyro} \begin{bmatrix} \omega_{xy}^{gyro} \\ \omega_{y}^{gyro} \\ \omega_{z}^{gyro} \end{bmatrix}
$$
(8)

where

$$
\mathbf{z} = \begin{bmatrix} \mathbf{x}, & \phi_r, & \theta_r, & \psi_v \end{bmatrix}^\mathrm{T}
$$

$$
\mathbf{F} = \begin{bmatrix} \mathbf{A} & \mathbf{0}_{9 \times 3} \\ \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 3} \end{bmatrix}
$$

$$
\mathbf{G}_{acc} = \begin{bmatrix} \mathbf{B}_{acc} & \mathbf{0}_{3 \times 3} \end{bmatrix}^\mathrm{T}
$$

$$
\mathbf{G}_{gyro} = \begin{bmatrix} \mathbf{B}_{gyro} & \mathbf{0}_{3 \times 3} \end{bmatrix}^\mathrm{T}
$$

C. Observer Design

For a dynamic model shown in (8), a state observer that can estimate each state value is constructed as follows:

$$
\dot{\hat{\mathbf{z}}} = \mathbf{F}\hat{\mathbf{z}} + \hat{\mathbf{G}}_{acc} \begin{bmatrix} A_{gc}^{acc} \\ A_{y}^{acc} \\ A_{z}^{acc} \end{bmatrix} + \hat{\mathbf{G}}_{gyro} \begin{bmatrix} \omega_{x}^{gyro} \\ \omega_{y}^{gyro} \\ \omega_{z}^{gyro} \end{bmatrix} + \mathbf{L}(\mathbf{y} - \hat{\mathbf{y}}) \tag{9}
$$

where the (ˆ) denotes the estimated state value; **y** is the system output equation; **L** is the matrix of observer gains.

The system output equation **y** in (9) is carefully chosen as (10) to ensure the system observability.

$$
\mathbf{y} = \begin{bmatrix} \mathbf{y}_{1}^{gps}, \ \mathbf{y}_{2}^{gps}, \ \mathbf{y}_{1}^{sus}, \ \mathbf{y}_{2}^{sus} \end{bmatrix}^{T} \qquad (10)
$$
\n
$$
\mathbf{y}_{1}^{gps} = \begin{bmatrix} x_{g}^{gps}, \ y_{g}^{gps}, \ z_{g}^{gps}, \ \phi_{g}^{gps}, \ \theta_{g}^{gps}, \ \psi_{g}^{gps} \end{bmatrix}
$$
\n
$$
\mathbf{y}_{2}^{gps} = \begin{bmatrix} \mathbf{C}_{g}^{v}(1,1), \ \mathbf{C}_{g}^{v}(1,2) \end{bmatrix}
$$
\n
$$
= \begin{bmatrix} \mathbf{C}_{g}^{v}\mathbf{C}_{g}^{r}(1,1), \ \mathbf{C}_{g}^{v}(\mathbf{C}_{g}^{r}(1,2)) \end{bmatrix}
$$
\n
$$
\mathbf{y}_{1}^{sus} = \begin{bmatrix} \mathbf{C}_{g}^{v}(\mathbf{C}_{g}^{r-1}(1,1), \ \mathbf{C}_{g}^{v}(\mathbf{C}_{g}^{r-1}(1,2), \ \mathbf{C}_{g}^{v}(\mathbf{C}_{g}^{r-1}(1,3)) \end{bmatrix}
$$
\n
$$
= \begin{bmatrix} \mathbf{C}_{g}^{v}\mathbf{C}_{g}^{r-1}(1,1), \ \mathbf{C}_{g}^{v}\mathbf{C}_{g}^{r-1}(1,2), \ \mathbf{C}_{g}^{v}\mathbf{C}_{g}^{r-1}(1,3) \end{bmatrix}
$$
\n
$$
\mathbf{y}_{2}^{sus} = z_{r}^{sus}
$$
\n
$$
= \mathbf{C}_{g}^{r}(3,1) x_{g} + \mathbf{C}_{g}^{r}(3,2) y_{g} + \mathbf{C}_{g}^{r}(3,3) z_{g}
$$

where $\mathbf{M}(m, n)$ denotes the element located at the $m - th$ row, $n-th$ column of the matrix **M**. The output equation y_1^{qps} provides the information of the state vector (**x**) and its values is obtained from the measurements of a three-antenna GPS. The output equation \mathbf{y}_2^{gps} is a function of $(\phi_r, \theta_r, \psi_v)$ and its values can be calculated from the measurements of a three-antenna GPS. The output equation y_1^{sus} is a function of (ϕ_r, θ_r) and its values can be calculated from the measurements of the suspension displacement senors. The output equation y_2^{sus} is a function of (ϕ_r, θ_r) and (x_g, y_g, z_g) and its values can be calculated from the measurements of the suspension displacement senors (6) and three-antenna GPS.

It should be emphasized that the output equations (y_2^{gps}, y_1^{sus}) are designed to consist of two or three elements from the corresponding direction cosine matrix. Each element consists of multiplication terms of two or more trigonometric functions. In most cases, only one element in each output equation is enough to ensure the success of state estimations. However, since it is multiplications of trigonometric functions, the estimation process would fail at certain angle values. Therefore, redundant equations are used to ensure the success at every angles.

Since the outputs of the GPS, IMU, and suspension displacement sensors are unsynchronized and with different noise characteristics (see Tab. I), a multi-rate Kalman filter [11] is chosen to construct a state observer to coordinate those sensor outputs. The numerical algorithms of a multi-rate Kalman filter is similar to that of a conventional extended Kalman filter with the only difference in updating the estimated values [12]. When the GPS measurement is available, the estimated state value is updated by the measurements of the GPS and suspension displacement sensors. When the GPS measurement is unavailable, the estimated state value is updated only by the measurements of the suspension displacement sensors. It is done by following:

If the GPS measurements is available,

$$
\mathbf{y} \;\; = \;\; \left[\begin{array}{cccc} \mathbf{y}_{1}^{gps} , & \mathbf{y}_{2}^{gps} , & \mathbf{y}_{1}^{sus} , & \mathbf{y}_{2}^{sus} \end{array} \right]^{\mathrm{T}}
$$

If the GPS measurements are unavailable,

$$
\mathbf{y} = [0_{1 \times 6}, 0_{1 \times 2}, y_1^{sus}, y_2^{sus}]^{T}
$$

Fig. 3. Using the measurements from suspension displacement sensors to calculate the values of vehicle roll angle, vehicle pitch angle, and vertical displacement relative to a road. The calculated values are almost identical to the vehicle dynamics.

Therefore, with the system equation described in (9) and output equation described in (10), one can follow the standard procedures of the extend Kalman filter to estimate each state values.

V. SIMULATION RESULTS

In a simulation case, a vehicle moves at the speed of 25 m/s on a sloped road and then makes a left-hand turn at the fourth second. This turn is initiated by a change in the steering wheel angle from the fourth to fifth second. The steering wheel is held at this angle from the fifth to eighth second and then switched back to the original from the eighth to ninth second. In this case, two road angles (road bank angle and road grade angle) are modeled as two time-dependent functions (sinusoidal, frequency: 0.25 Hz, magnitude: 5 deg). The fullstate vehicle model proposed in [1] is used to mimic the real vehicle dynamics on a sloped road. The simulation results are shown in Fig. 3∼5. In these plots, the vehicle dynamics (from the full-state vehicle model) are shown in dashed-blue line. The sensor outputs are shown in dashed-dotted-green line. The states of the proposed sensor fusion system are shown in solidred line.

Figure 3 shows the relative vehicle attitude and vertical displacement relative to the road, which are calculated by the measurements of four suspension displacement sensors. The standard deviations (1σ) between the calculated values and vehicle dynamics are 0.04 deg for the vehicle roll angle, 0.023 deg for the vehicle pitch angle, and 5.13×10^{-4} m for the vehicle vertical displacement. These measured vehicle dynamics obtained from the measurements of suspension displacement sensors are accuracy enough to lots of applications.

Figure 4 shows that the proposed sensor fusion system can accurately estimate three unknowns angles (road bank angle, road grade angle and vehicle yaw angle). The standard

Fig. 4. Comparison of the vehicle dynamics and the output of the proposed sensor fusion system. Three angles for the absolute vehicle attitude are shown in the left column; three unknown angles (road bank angle, road grade angle, and vehicle yaw angle) are shown in the right column.

deviations (1σ) between the estimated values and real values are 0.283 deg for road bank angle, 0.134 deg for road grade angle, and 0.824 deg for vehicle yaw angle. Furthermore, the standard deviation (1σ) in the vertical displacement estimation is 0.248 m, which is 3 m from the GPS measurements (see, Fig. 5).

VI. CONCLUSION

In this paper, a sensor fusion system for estimating the six-DOF motions of a vehicle and two road angles is presented and verified by simulation results. Three types of sensor are employed in this sensor fusion system, which are: three-antenna GPS, an IMU, and four suspension displacement sensors. The three-antenna GPS provides information of the absolute position and attitude; suspension displacement sensors provide information of the vehicle attitude relative to a road; an IMU improves the sensing accuracy of the sensor system. These sensor outputs are coordinated by a newly developed kinematic model and a multi-rate extended Kalman filter. According to simulation results, this sensor fusion system can accurately estimate two road angles and vehicle attitude. The estimation accuracy of two angles is less than 0.3 deg, and that of the relative vehicle attitude is less than 0.85 deg. Additionally, the estimation accuracy of the vertical displacement is improved from 3 m to 0.248 m.

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Fig. 5. Comparison of the vehicle position observed in the global frame and the output of GPS system.

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