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Diagnosability of star graphs with missing edges ☆

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ABSTRACT

In this paper, we study the system diagnosis on an n-dimensional star under the comparison model. Following the concept of local diagnosability [3], the strong local diagnosability property [7] is discussed; this property describes the equivalence of the local diagnosability of a node and its degree. We prove that an n-dimensional star has this property, and it keeps this strong property even if there exist $n - 3$ missing edges in it.

1. Introduction

In recent years, with the continuing advancements in semiconductor technology, large multiprocessor systems such as very-large-scale integration (VLSI) systems have become increasingly popular. Such systems must be capable of uninterrupted processing, and therefore, the reliability of the processors in these systems should be considered. The diagnosis of such systems involves the identification of all the faulty processors in the system. The diagnosability of the system refers to the maximum number of faulty processors that can definitely be identified.

Several approaches to system diagnosis have been developed in previous researches. One major approach, called the comparison diagnosis model, was proposed by Maeng and Malek [13,14]. In this model, diagnosis is performed by simultaneously sending two identical signals from a processor to two other linked processors and then comparing the responses. The test results are collected and analyzed to identify all the faulty processors. Following the traditional concept of diagnosability, many variants of diagnosability measurements have been presented. A different measurement called conditional diagnosability was proposed [8], and a more precise concept called strong diagnosability has been widely applied to various networks [4–6].

In contrast to the traditional concept of diagnosability, Chiang and Tan [3] introduced a different concept for system diagnosis called local diagnosability; this method requires only the correct identification of the status of a single processor. Each processor has its own local diagnosability, and there exists a strong relationship between the local diagnosability and the traditional diagnosability. In the comparison diagnosis model for a given processor, a local structure called an extended star has also been presented for guaranteeing a processor's local diagnosability.

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Among all well-known topologies, the star graph is one of the most popular ones. Its features include node symmetry, edge symmetry, regular and low degree of node, and small diameter. Since its introduction, this topology has attracted considerable attention. Some studies have discussed the diameter and fault diameters [9,15,16]. When linearly many vertices are deleted in a star graph, the resulting graph has a large connected component containing almost all remaining vertices [2]. The problem of embedding a linear array of vertices (or a ring) into the star graph has also been solved, even when there exist some vertex faults in the target star graph [10]. With edge faults, the star graph has been proved to have fault-tolerant Hamiltonian laceability [12]. The robustness of star graphs with edge faults has been addressed [11], and the improvement of bounds on edge failure tolerance has also been investigated [18]. For system diagnosis, Zheng et al. [20] showed that the traditional diagnosability of an n-dimensional star is $n-1$ for $n \geqslant 4$.

In this paper, we study system diagnosis by following the concept of local diagnosability [3]. Based on this concept, we obtain a simple proof of the fact that the diagnosability of an n-dimensional star S_n is $n-1$ for $n \geqslant 4$; this is the same result as that obtained by Zheng et al. [20]. Moreover, we study the diagnosability of a star graph in the presence of arbitrary distributed missing edges under the comparison diagnosis model. A relative study was discussed for the case of a hypercube by Wang [19]. Furthermore, we have studied the strong local diagnosability property [7]. A given processor has the strong local diagnosability property if its local diagnosability equals its degree, where the degree is defined as the number of links incident to this processor. A system has the strong local diagnosability property if every processor in it has this property. We prove that each processor in an n-dimensional star S_n has this strong local diagnosability property, and this property is maintained even if S_n has up to $n-3$ missing edges. The number $n-3$ is tight in the sense that the strong local diagnosability property cannot be guaranteed if there are $n-2$ missing edges.

The remainder of this paper is organized as follows. In Section 2, we present some definitions, notations, and terminologies. The concept of local diagnosability for system diagnosis is also introduced in this section. Then, in Section 3, we prove that an n-dimensional star keeps the strong local diagnosability property even if there exist $n-3$ missing edges in it. Finally, some conclusions are presented in Section 4.

2. Preliminaries and local diagnosability

The topology of a multiprocessor system can be modeled as an undirected graph $G = (V,E)$, where the set of nodes V represents the set of all processors and the set of edges E represents the set of all connecting links between the processors. Let G' be a subgraph of G and v be a node in G'; then, $deg_{G'}(v)$ denotes the degree of v in subgraph G'. The neighborhood set of a node v, denoted by $N(v)$, is defined as the set of all nodes adjacent to v.

Let n be a positive integer and $\langle n\rangle$ be the set $\{1, 2 \ldots n\}$. An n-dimensional star [1], denoted by S_n, is a graph whose set of nodes consists of all permutations on $\langle n\rangle$. Each node is uniquely assigned a label $x_1x_2 \ldots x_n$, where $x_i \in \langle n\rangle$ for $1 \leqslant i \leqslant n$ and $x_i \neq x_j$ for $i \neq j$. Each node $x_1x_2 \ldots x_{i-1}x_ix_{i+1} \ldots x_n$ is adjacent to the nodes $x_ix_2 \ldots x_{i-1}x_1x_{i+1} \ldots x_n$ for $2 \leqslant i \leqslant n$, that is, nodes obtained by the transposition of the first coordinate with the ith coordinate of the node. Consequently, there exist $n!$ nodes in an n-dimensional star, and each node has degree $n-1$. Let $\mathbf{x} = x_1x_2 \ldots x_n$ be a node in an n-dimensional star S_n. We use $(\mathbf{x})_i$ to denote the ith coordinate x_i of $\mathbf{x}$ for $1 \leqslant i \leqslant n$. We say that two nodes $\mathbf{x}$ and $\mathbf{y}$ in S_n are adjacent to each other with an ith edge or an edge in dimension i if $\mathbf{x}$ can be obtained by the transposition of the first coordinate with the ith coordinate of $\mathbf{y}$. Then, $\mathbf{x}$ is said to be the ith neighbor of $\mathbf{y}$ and it is denoted as $\mathbf{x} = \mathbf{y}^i$, and vice versa. In addition, we use S_n^i to denote the subgraph of S_n that is induced by the nodes $\mathbf{x}$'s with $(\mathbf{x})_n = i$ for $1 \leqslant i \leqslant n$. Thus, S_n can be decomposed into n subgraphs S_n^i for $1 \leqslant i \leqslant n$ and each S_n^i is isomorphic to S_{n-1}.

Under the comparison model [13,14], a system performs diagnosis by the specific procedure described below. For each processor w linked to two distinct processors u and v, the diagnosis is performed by simultaneously sending two identical signals from w to u and from w to v, and then comparing their returning responses. The comparison result of w for the two responses from u and v is denoted by $r((u, v)_w)$. An agreement is denoted by $r((u, v)_w) = 0$, whereas a disagreement is denoted by $r((u, v)_w) = 1$. Because the comparator processor might be faulty, if $r((u, v)_w) = 1$, at least one member of $\{u, v, w\}$ is faulty; or, if $r((u, v)_w) = 0$ and w is known to be fault-free, both u and v are fault-free. Furthermore, a special case of the comparison model, called the MM* model [17], assumes that a comparison is performed by each processor for each pair of distinct connected neighbors.

A labeled multigraph $M = (V,C)$, called a comparison graph, is usually used to model this diagnosis strategy, where V represents the set of all processors in G and C represents the set of labeled edges. Each labeled edge $(u, v)_w \in C$ implies that processors u and v are being compared by processor w.

The collection of all test results of a test assignment is called a syndrome. Formally, a syndrome is a function σ:$C \rightarrow \{0,1\}$. For a given syndrome σ, a subset of processors $F \subset V(G)$ is said to be consistent with σ if the syndrome σ can be produced when all processors in F are faulty and all processors in $V-F$ are fault-free. Let σ_F denote the set of syndromes that are consistent with F. We say that a system is diagnosable if for every syndrome σ, a unique set of processors $F \subset V$ is consistent with it. A system is defined to be t-diagnosable if the system is diagnosable as long as the number of faulty processors does not exceed t. In other words, a system is t-diagnosable if given a test syndrome σ_F produced by the system under the presence of a set of faulty nodes F with $|F| \leqslant t$, any set of faulty nodes F' consistent with σ_F with $|F'| \leqslant t$ must be $F' = F$. The maximum number t for which a system is t-diagnosable is called the diagnosability of the system. Two distinct subsets of processors $F_1, F_2 \subset V$ are distinguishable if and only if every syndrome consistent with F_1 differs from that consistent with F_2.

The next lemma follows trivially from the definition of the t-diagnosability of a system.

Lemma 1 [17]. *A system $G(V,E)$ is t-diagnosable if and only if, for each pair of distinct set of nodes (F_1,F_2) with $|F_1|,|F_2| \leqslant t$, (F_1,F_2) is a distinguishable pair.*

The following lemma is a useful characterization for the distinguishability of two sets of nodes under the comparison model.

Lemma 2 [17]. *Let F_1 and F_2 be two distinct subsets of nodes. (F_1,F_2) is a distinguishable pair if and only if at least one of the following conditions is satisfied: (See Fig. 1 for an illustration.)*

(1) $\exists u, w \in V - F_1 - F_2$ and $\exists v \in (F_1 - F_2) \cup (F_2 - F_1)$ such that $(u,v)_w \in C$.
(2) $\exists u, v \in F_1 - F_2$ and $\exists w \in V - F_1 - F_2$ such that $(u,v)_w \in C$, or
(3) $\exists u, v \in F_2 - F_1$ and $\exists w \in V - F_1 - F_2$ such that $(u,v)_w \in C$.

In contrast to the global sense in system diagnosis, Chiang and Tan [3] present a local concept called the local diagnosability of a given node in a system. This method requires only the correct identification of the faulty or fault-free status of a single node. Below are two definitions that introduce the concept of local diagnosability.

Definition 1 (3). A system $G(V,E)$ is locally t-diagnosable at node x if, given a test syndrome σ_F produced by the system under the presence of a set of faulty nodes F containing node x with $|F| \leqslant t$, every set of faulty nodes F' consistent with σ_F and $|F'| \leqslant t$, must also contain node x.

Definition 2 (3). The local diagnosability $t_l(x)$ of a node x in a system $G(V,E)$ is defined to be the maximum number of t for G being locally t-diagnosable at x, that is, $t_l(x) = max\{t|G \text{ is locally } t-\text{diagnosable at } x\}$.

The close relationship between the local diagnosability and the traditional diagnosability is stated as follows.

Lemma 3 (3). *A system $G(V,E)$ is t-diagnosable if and only if G is locally t-diagnosable at every node.*

Lemma 4 (3). *The diagnosability $t(G)$ of a system $G(V,E)$ is equal to the minimum value among the local diagnosability of every node in G, that is, $t(G) = min\{t_l(x)|\text{for all } x \in V(G)\}$.*

Under the comparison diagnosis model, an extended star structure for guaranteeing the local diagnosability of a given node is stated as below.

Definition 3 (3). Let x be a node in a graph $G(V,E)$. For $n \leqslant deg_G(x)$, we define an extended star $ES(x;n)$ of order n at node x with the node set $V(ES(x;n)) = \{x\} \cup \{v_{ij}|1 \leqslant i \leqslant n, 1 \leqslant j \leqslant 4\}$ and the edge set $E(ES(x;n)) = \{(x,v_{k1}),(v_{k1},v_{k2}),(v_{k2},v_{k3}),(v_{k3},v_{k4})|1 \leqslant k \leqslant n\}$. (See Fig. 2 for an illustration.)

We say that there exists an extended star structure $ES(x;n) \subseteq G$ at node x if G contains an extended star $ES(x;n)$ of order n at node x as a subgraph. Node x is called the *root* of $ES(x;n)$. The extended star is a useful structure for computing the local diagnosability of a given node.

Lemma 5 (3). *Let x be a node in a system $G(V,E)$ with $deg_G(x) = n$. The local diagnosability of x is n if there exists an extended star $ES(x;n) \subseteq G$ at x.*

Consequently, for every processor in a regular recursively constructed system, its local diagnosability can be easily determined by finding an extended star structure at this processor. Moreover, the diagnosability of the entire system can also be obtained accordingly.

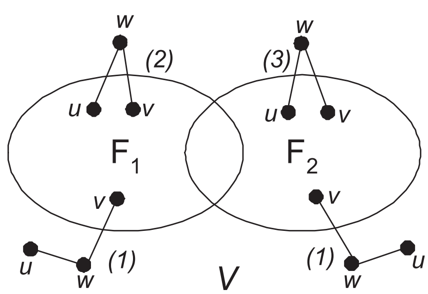

Fig. 1. Distinguishability of two sets of nodes for Lemma 2.

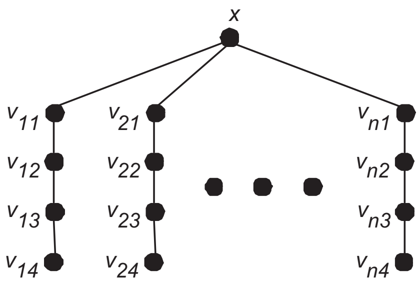

Fig. 2. Extended star structure $ES(x;n)$ of order n.

3. Strong local diagnosability property

In this section, we discuss the strong local diagnosability property [7]; this property describes the equivalence of the local diagnosability of a node and its degree. We prove that an n-dimensional star has this property, and it keeps this strong property even if there exists a bounded amount of missing edges.

Definition 4 (7). Let x be a node in a graph $G(V,E)$. Node x has the strong local diagnosability property if the local diagnosability of x equals to its degree in G. That is, $t_l(x) = deg_G(x)$.

Definition 5 (7). Let $G(V,E)$ be a graph. Graph G has the strong local diagnosability property if the local diagnosability of every node equals to its degree in G. That is, $t_l(x) = deg_G(x)$, for all $x \in V(G)$.

In the following, we show that an n-dimensional star with $n \geqslant 4$ has the strong local diagnosability property.

Lemma 6. *For each node x in an n-dimensional star S_n with $n \geqslant 4$, there exists an extended star $ES(x;n-1) \subseteq S_n$ of order $n-1$ at x.*

Proof. We use the notations mentioned in Definition 3 to find an extended star $ES(x;n-1)$ as a subgraph of an n-dimensional star S_n at a given node x. Because S_n is node symmetric, we arbitrarily choose $\mathbf{x} = x_1x_2\ldots x_n$ to be the root of an $ES(\mathbf{x};n-1)$.

For $n = 4$, we can find an extended star $ES(\mathbf{x};3)$ of order 3 at node $\mathbf{x} = x_1x_2x_3x_4$ (as shown in Fig. 3), where the set of nodes contains $\mathbf{x}$, $v_{11} = x_2x_1x_3x_4$, $v_{12} = x_3x_1x_2x_4$, $v_{13} = x_4x_1x_2x_3$, $v_{14} = x_2x_1x_4x_3$, $v_{21} = x_3x_2x_1x_4$, $v_{22} = x_4x_2x_1x_3$, $v_{23} = x_2x_4x_1x_3$, $v_{24} = x_3x_4x_1x_2$, $v_{31} = x_4x_2x_3x_1$, $v_{32} = x_2x_4x_3x_1$, $v_{33} = x_3x_4x_2x_1$, and $v_{34} = x_4x_3x_2x_1$, and the set of edges is $\{(\mathbf{x}, v_{k1}),(v_{k1}, v_{k2}),(v_{k2}, v_{k3}),(v_{k3}, v_{k4}) | 1 \leqslant k \leqslant 3\}$.

Suppose the result holds for all S_{n-1}, for some $n \geqslant 5$. Now we claim the result also holds for S_n, that is, there is an extended star $ES(\mathbf{x};n-1) \subseteq S_n$ of order $n-1$ at each node $\mathbf{x} \in V(S_n)$. Since the definition of star graphs, an S_n can be seen as a composition of n S_{n-1}'s. Let $S_{n-1}(\mathbf{x})$ be the subgraph of S_n induced by all nodes $\mathbf{z}$'s where their nth coordinates are the same as that of $\mathbf{x}$, that is, $(\mathbf{z})_n = (\mathbf{x})_n$. By the assumption, there exists an $ES(\mathbf{x};n-2) \subseteq S_{n-1}(\mathbf{x})$ at node $\mathbf{x}$. Pick the nth neighbor of $\mathbf{x}$ denoted $\mathbf{x}^n$. Let $S_{n-1}(\mathbf{x}^n)$ be the subgraph of S_n that $\mathbf{x}^n$ belongs to, in which all nodes has the same nth coordinate as $\mathbf{x}^n$. We can

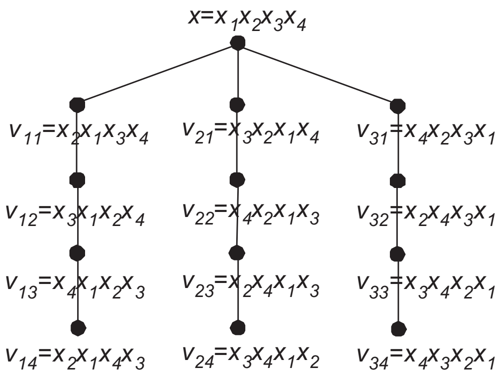

Fig. 3. The base case for the proof of Lemma 6: an $ES(\mathbf{x};3)$ of order 3 at $\mathbf{x} = x_1x_2x_3x_4$.

easily find another three different nodes **a**, **b**, and **c**, all in $S_{n-1}(\mathbf{x}^n)$, such that $\mathbf{x}^n$, **a**, **b**, and **c** form a path in $S_{n-1}(\mathbf{x}^n)$. The reason is that the girth of S_{n-1} is six and each node in it has degree $n-2$, which is larger than two when $n \geqslant 5$. Consequently, there exists an extended star $ES(\mathbf{x};n-1) \subseteq S_n$ of order $n-1$ at each node $\mathbf{x} \in V(S_n)$ for $n \geqslant 4$. □

Theorem 1. *Let S_n be an n-dimensional star and $n \geqslant 4$. Each node x in S_n has the strong local diagnosability property and graph S_n has the strong local diagnosability property.*

Proof. By Lemmas 5 and 6, the local diagnosability of each node $x \in V(S_n)$ is $n-1$, because the degree of x in S_n is $n-1$ and there exists an extended star $ES(x;n-1)$ of order $n-1$ at x for $n \geqslant 4$. Thus, every node in an n-dimensional star S_n with $n \geqslant 4$ has the strong local diagnosability property. Therefore, graph S_n has the strong local diagnosability property. □

By the theorem above, we conclude that the diagnosability of S_n is $n-1$ for $n \geqslant 4$; this is the same result as that obtained by Zheng et al. [20].

In some circumstances, some links in a multiprocessor system may be missing. A missing edge indicates a link between two processors that has broken or failed for some reason. The existence of missing edges in a system may reduce the diagnosability of the entire system and change the local diagnosability of each node in some manner. Meanwhile, the presence of missing edges changes the degrees of some of the nodes within the system. More precisely, in a regular graph, nodes adjacent to some missing edges have lower degrees than other nodes. Accordingly, with a small number of missing edges, the nodes connecting to these edges may not keep the strong local diagnosability property, and the graph may not keep the strong local diagnosability property as well. Therefore, these new degrees will be used to decide whether the incomplete graph keeps the strong local diagnosability property.

In the following, we show that an n-dimensional star S_n keeps the strong local diagnosability property even with up to $n-3$ missing edges for $n \geqslant 4$.

Before proving this claim, we present an example to show that an n-dimensional star S_n may not keep the strong local diagnosability property if there exist $n-2$ missing edges. For an arbitrary node **x** in S_n, **x** is labeled as a permutation on $\langle n\rangle$. Suppose there exist $n-2$ missing edges in S_n that are incident to node **x** (as shown in Fig. 4). Then, the remaining degree of **x** in this incomplete star with missing edges is 1. Let **y** be the only node adjacent to **x**. Let F_1 be the set of nodes $\{\mathbf{y}\} \cup N(\mathbf{y}) - \{\mathbf{x}\}$ with $|F_1| = n-1$, and F_2 be the set of nodes $N(\mathbf{y})$ with $|F_2| = n-1$. By Lemma 2, (F_1,F_2) is not a distinguishable pair under the comparison diagnosis model, and this incomplete star with missing edges is not $(n-1)$-local diagnosable at **y**. Because the local diagnosability of **y** (which is less than $n-1$) does not equal its degree (which is $n-1$) in this incomplete star graph S_n, node **y** has no strong local diagnosability property anymore. Therefore, an incomplete star S_n with $n-2$ missing edges cannot be guaranteed to have the strong local diagnosability property.

We now prove that an n-dimensional star S_n still keeps the strong local diagnosability property, provided that the number of missing edges is at most $n-3$ for $n \geqslant 4$. Note that for a given set of edges $L \subseteq E(G)$ in a system G, we use $G-L$ to denote the subgraph with node set $V(G)$ and edge set $E(G)-L$.

Lemma 7. *Let S_n be an n-dimensional star with $n \geqslant 4$, and let F be an arbitrary set of missing edges with $|F| \leqslant n-3$. For each node x in S_n, there exists an extended star $ES(x;deg_{S_n-F}(x)) \subseteq S_n - F$ at x, where $deg_{S_n-F}(x)$ denotes the remaining degree of node x in $S_n - F$.*

Proof. We prove this lemma by induction on n.

For the base case $n=4$, each node in S_4 is labeled as a permutation on $\langle 4\rangle$. Let $\mathbf{x} = x_1x_2x_3x_4$ be any node in S_4. We first construct two extended star structures of order 3 around node **x**; one of which is the same as that described in the proof of Lemma 6 (as shown in Fig. 5(a)), and the other one contains the node set $\{\mathbf{x}, v_{11} = x_2x_1x_3x_4, v_{12} = x_4x_1x_3x_2, v_{13} = x_1x_4x_3x_2, v_{14} = x_3x_4x_1x_2, v_{21} = x_3x_2x_1x_4, v_{22} = x_2x_3x_1x_4, v_{23} = x_1x_3x_2x_4, v_{24} = x_4x_3x_2x_1, v_{31} = x_4x_2x_3x_1, v_{32} = x_3x_2x_4x_1, v_{33} = x_1x_2x_4x_3, v_{34} = x_2x_1x_4x_3\}$ and the edge set $\{(\mathbf{x}, v_{k1}),(v_{k1}, v_{k2}),(v_{k2}, v_{k3}),(v_{k3}, v_{k4}) | 1 \leqslant k \leqslant 3\}$ (as shown in Fig. 5(b)). It is easy to check that in both structures, except for the 2nd, 3rd, or 4th edge of **x**, all edges are different.

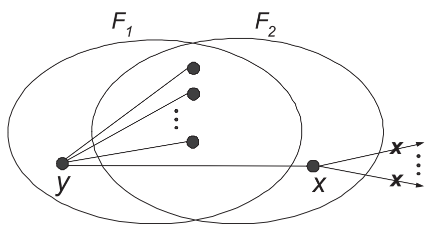

Fig. 4. Example showing that an n-dimensional star S_n has no strong local diagnosability property with $n-2$ missing edges.

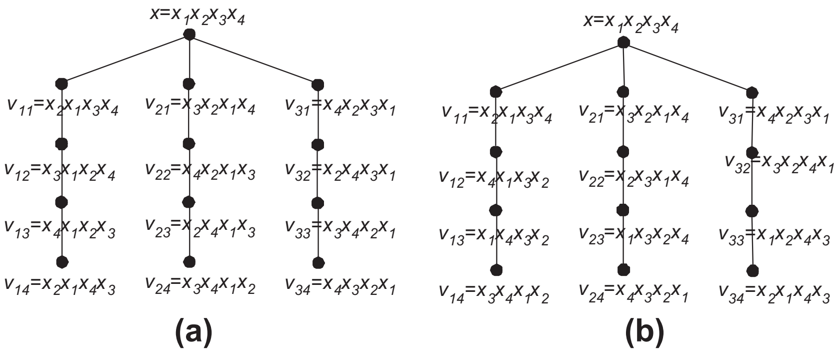

Fig. 5. Two possible extended stars $ES(\mathbf{x};3)$ at any node $\mathbf{x} = x_1x_2x_3x_4$ described in the proof of Lemma 7.

Since $n - 3 = 1$, we only need to consider two situations in which the number of missing edges is 0 or 1. If there exist no missing edges in S_4, an $ES(\mathbf{x};3)$ at $\mathbf{x}$ indeed exists. If there exists one missing edge in S_4, one of two following cases may occur: (1) if the missing edge is the 2nd, 3rd, or 4th edge of $\mathbf{x}$, the degree of $\mathbf{x}$ is 2 and there exists an $ES(\mathbf{x};2)$ at $\mathbf{x}$; (2) otherwise, we can pick any one of the two above structures to avoid the missing edge, in order to form an extended star $ES(\mathbf{x};3)$ at $\mathbf{x}$. As a consequence, there exists an extended star $ES(x;deg_{S_n-F}(x)) \subseteq S_n - F$ at each node x for $n = 4$ and $|F| = 0$ or 1.

For induction hypothesis, we suppose that the result is true for S_{n-1}, for some $n \geqslant 5$. That is, for any set of missing edges F with $|F| \leqslant (n-3) - 1$, there exists an $ES(x;deg_{S_{n-1}-F}(x)) \subseteq S_{n-1} - F$ at each node $x \in V(S_{n-1}) - F$.

Now, we claim that the result also holds for S_n, for all $|F| \leqslant n - 3$. We shall prove that for a set of missing edges F with $|F| \leqslant n - 3$, there exists an $ES(x;deg_{S_n-F}(x)) \subseteq S_n - F$ at each node $x \in V(S_n) - F$. Assume that the number of missing edges is at most $n - 3$ in an n-dimensional star S_n for $n \geqslant 5$. Let $f = (u, v)$ be an arbitrary missing edge. Because the star graph is edge symmetric, without lost of generality, we let $v = u^n$. The n-dimensional star S_n can be seen as the composition of n subgraphs S_n^k for $1 \leqslant k \leqslant n$, where S_n^k is a subgraph of S_n induced by the nodes $\mathbf{z}$'s with $(\mathbf{z})_n = k$. Thus, the number of all missing edges F except f in S_n is at most $n - 4$. Consider a node $\mathbf{x}$ in S_n; $\mathbf{x}$ is in one of the n induced subgraphs S_n^k, $1 \leqslant k \leqslant n$, and each S_n^k is isomorphic to an $(n-1)$-dimensional star S_{n-1}. Let $S_{n-1}(\mathbf{x})$ be the substar that $\mathbf{x}$ belongs to. By the induction hypothesis, there exists an extended star $ES(\mathbf{x};deg_{S_{n-1}(\mathbf{x})-F'}(\mathbf{x})) \subseteq S_{n-1}(\mathbf{x}) - F'$ at $\mathbf{x}$, where F' is the set of all missing edges in $S_{n-1}(\mathbf{x})$ and $|F'| \leqslant n - 4$.

If the nth edge of $\mathbf{x}$ is missing (Fig. 6(a)), the degree of $\mathbf{x}$ in $S_{n-1}(\mathbf{x}) - F'$ is equal to the degree of $\mathbf{x}$ in this incomplete star $S_n - F$ with at most $n - 3$ missing edges. If the nth edge of $\mathbf{x}$ is not missing (Fig. 6(b)), $\mathbf{x}$ is adjacent to its nth neighbor, denoted by $\mathbf{x}^n$, through the nth edge. Let $S_{n-1}(\mathbf{x}^n)$ be the subgraph that $\mathbf{x}^n$ belongs to. Since $|F - f| \leqslant n - 4$, each node in $S_{n-1}(\mathbf{x}^n) - F$ is adjacent to at least two other nodes in $S_{n-1}(\mathbf{x}^n) - F$. We note again that in a star graph S_{n-1}, each node has degree $n - 2$. Then, $\mathbf{x}^n$ is adjacent to a node $\mathbf{a}$ in $S_{n-1}(\mathbf{x}^n) - F$, $\mathbf{a}$ is adjacent to another node $\mathbf{b}$ in $S_{n-1}(\mathbf{x}^n) - F$, and $\mathbf{b}$ is adjacent to another node $\mathbf{c}$ other than $\mathbf{x}^n$, $\mathbf{a}$ in $S_{n-1}(\mathbf{x}^n) - F$ since the girth of S_{n-1} is six and the degree of each vertex in $S_{n-1}(\mathbf{x}^n) - F$ is not less than two. As a result, for $n \geqslant 5$, there exists an $ES(\mathbf{x};deg_{S_n-F}(\mathbf{x})) \subseteq S_n - F$ at $\mathbf{x}$ for all $|F| \leqslant n - 3$. The proof is complete. □

Theorem 2. *Let S_n be an n-dimensional star and $n \geqslant 4$, and let F be an arbitrary set of missing edges with $|F| \leqslant n - 3$. For each node x in S_n with missing edges F, node x has the strong local diagnosability property and graph $S_n - F$ has the strong local diagnosability property.*

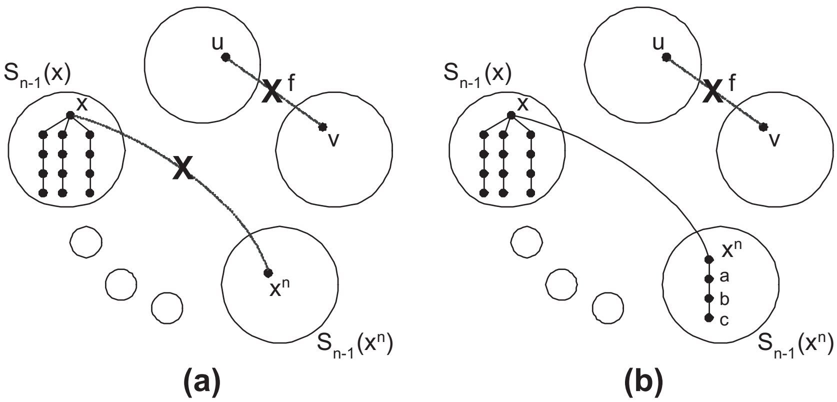

Fig. 6. Illustration of the inductive step in the proof of Lemma 7.

Proof. By Lemmas 5 and 7, the local diagnosability of each node x in an incomplete n-dimensional star $S_n - F$ is equal to its remaining degree for $n \geqslant 4$ and $|F| \leqslant n - 3$. Thus, every node in $S_n - F$ has the strong local diagnosability property. Consequently, graph $S_n - F$ has the strong local diagnosability property. □

4. Conclusions

In this paper, we studied the system diagnosis of an n-dimensional star under the comparison model. Following the concept of local diagnosability and the extended star structure proposed by Chiang and Tan [3], the diagnosability of a system can be determined in a straightforward manner. By the definition of the strong local diagnosability property [7], we proved that an n-dimensional star has this property, and it keeps this strong property even if there exist up to $n - 3$ missing edges in it. As a result, the diagnosability of an incomplete n-dimensional star system with arbitrary missing links can be obtained as the minimum value among the remaining degree of every processor, provided that the cardinality of the set of missing links does not exceed $n - 3$.

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