JOURNAL OF COMBINATORIAL THEORY, Series A **73**, 190–192 (1996)
Article No. 0016

Corrigendum

Volume **61**, Number 1 (1992), in the article "A Comparison Theorem for Permanents and a Proof of a Conjecture on (t, m)-Families," by Joseph Y.-T. Leung and W.-D. Wei, pages 98–112: There is a flaw in the proof of a conjecture on (t, m)-families. Suppose $F = (F_1, F_2, ..., F_m)$ is a family of subsets of S. A *system of distinct representatives* (SDR) of the family F is a sequence $(f_1, f_2, ..., f_m)$ of m distinct elements of S such that $f_i \in F_i$ for $1 \leqslant i \leqslant m$. Let $N(F)$ denote the number of distinct SDRs of the family F. The problem of finding the value and the bounds for $N(F)$ has been investigated extensively in the literature. For a non-negative integer t, a family $F = (F_1, F_2, ..., F_m)$ is called a (t, m)-family if

$$\left| \bigcup_{i \in I} F_i \right| \geqslant |I| + t \qquad \text{for any nonempty subset } I \subseteq \{1, 2, ..., m\}.$$

Chang [1] proposed the problem of determining the value

$$M(t, m) = \min\{N(F) : F \text{ is a } (t, m)\text{-family}\}.$$

It is easy to see that for the family $F^*_{t,m} = (F^*_1, F^*_2, ..., F^*_m)$ with

$$F^*_i = \{i, m+1, m+2, ..., m+t\}, \qquad 1 \leqslant i \leqslant m,$$

the value of $N(F^*_{t,m})$ is

$$U(t, m) = \sum_{j=0}^{\min(t, m)} j! \binom{t}{j} \binom{m}{j}.$$

It was proved in [1] that

$$M(t, m) = U(t, m) \qquad \text{for} \quad 0 \leqslant t \leqslant 2,$$

and that $F^*_{2,m}$ is the only $(2, m)$-family F with $N(F) = M(2, m)$. All (t, m)-families F with $N(F) = M(t, m)$ for $t = 0$ and 1 were also determined. It was then conjectured in [1] that

$$N(F) = U(t, m) \qquad \text{for all} \quad t \geqslant 3,$$

and that $F^*_{t,m}$ is the only (t, m)-family with $N(F) = M(t, m)$ for all $t \geqslant 3$.

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Leung and Wei [2] gave a proof of the above conjecture by means of a comparison theorem for permanents. Let $B(b_{i,j})$ be an $m \times n$ matrix over a ring R. The permanent of B is defined as

$$\operatorname{per}(B) = \sum_{j_1 j_2 \cdots j_m} \prod_{i=1}^{m} b_{i, j_i}, \tag{1}$$

where $j_1 j_2 \cdots j_m$ is an m-permutation of $\{1, 2, ..., n\}$. When $m > n$, the sum on the right-hand side of (1) is 0. For a family $F = (F_1, F_2, ..., F_m)$, where each $F_i \subseteq S \equiv \{s_1, s_2, ..., s_m\}$, the *incidence matrix* of F is the $m \times n$ matrix $A = (a_{i,j})$ defined by

$$a_{i,j} = \begin{cases} 1, & \text{if } s_j \in F_i, \\ 0, & \text{otherwise.} \end{cases}$$

It is easy to see that $N(F) = \operatorname{per}(A)$. Let A_i be the ith row of A. Then the family F is a (t, m)-family if and only if for any nonempty subset I of $\{1, 2, ..., m\}$, $\sum_{i \in I} A_i$ has at least $|I| + t$ nonzero components. A (0, 1)-matrix with this property is called a (t, m)-matrix. Then we have

$$M(t, m) = \min\{\operatorname{per}(A) : A \text{ is a } (t, m)\text{-matrix}\}.$$

The key to Leung and Wei's proof of Chang's conjecture is the following comparison theorem for permanents.

THEOREM 1. *Let $B = (b_{i,j})$ be an $m \times n$ (0, 1)-matrix, $m \leqslant n$, and let p and q be given, $1 \leqslant p < q \leqslant n$. Suppose $\hat{B} = (\hat{b}_{i,j})$ is obtained from B by changing the pth and qth columns as:*

$$(\hat{b}_{i,p}, \hat{b}_{i,q}) = \begin{cases} (1, 0), & \textit{if } \ (b_{i,p}, b_{i,q}) = (0, 1), \\ (b_{i,p}, b_{i,q}), & \textit{otherwise.} \end{cases}$$

Then $\operatorname{per}(B) \geqslant \operatorname{per}(\hat{B})$, *and the strict inequality holds if and only if there are two indices i and j such that*

$$\begin{pmatrix} b_{i,p} & b_{i,q} \\ b_{j,p} & b_{j,q} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \textit{or} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and $\operatorname{per}(B(i, j \mid p, q)) \neq 0$, *where $B(i, j \mid p, q)$ is the submatrix of b formed by deleting row i, row j, column p, and column q.*

Note that there is no guarantee that the fact that B is a (t, m)-family implies $\hat{B}$ is a (t, m)-family. Theorem 1 was used to prove Chang's conjecture in Theorem 2 of [2]. The place that may cause a problem is at the bottom of page 109. For a (t, m)-family, the new matrix A' obtained from A by applying Theorem 1 is not necessarily a (t, m)-family. So Leung and Wei's argument breaks down here. The following is an example of a $(3, 3)$-matrix A for which A' is not a $(3, 3)$-matrix when we use $p=2$ and $q=3$ as in Theorem 1:

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

and

$$A' = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

References

1. G. J. Chang, On the number of SDR of a (t, m)-family, *European J. Combin.* **10** (1989), 231–234.
2. J. Y.-T. Leung and W.-D. Wei, A comparison theorem for permanents and a proof of a conjecture on (t, m)-families, *J. Combin. Theory Ser. A* **61** (1992), 98–112.

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* Supported partly by the National Science Council of the Republic of China under Grant NSC83-0208-M-009-050, E-mail: gchang@cc.nctu.edu.tw.

Printed in Belgium
Uitgever: Academic Press, Inc.
Verantwoordelijke uitgever voor België:
Hubert Van Maele
Altenastraat 20, B-8310 Sint-Kruis